Two-Stage Healthcare Optimization

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Stage-1: Healthcare Facility Placement

Sets and Parameters:

- $i, j \in \{1, \ldots, n\}$: Indices for communities and potential facility locations.
- p_i: Population of community i.
- d_{ij}: Distance between community i and location j.
- C: Capacity of each healthcare unit.
- m: Number of healthcare units to be deployed.

Decision Variables:

- $b_{ij} \in \{0, 1\}$: 1 if community i is assigned to a unit at location j.
- $z_j \in \{0, 1\}$: 1 if a unit is placed at location j.
- $Z \in \mathbb{R}_{\geq 0}$: Maximum weighted distance.

Objective: $\min Z$
Subject to:

$$Z \geq \sum_{j=1}^{n} p_i d_{ij} b_{ij}, \qquad \forall i \tag{1}$$

$$\sum_{i=1}^{n} p_i b_{ij} \leq C z_j, \qquad \forall j \tag{2}$$

$$\sum_{j=1}^{n} b_{ij} = 1, \qquad \forall i \tag{3}$$

$$\sum_{j=1}^{n} z_j = m, \tag{4}$$

$$b_{ij}, z_j \in \{0, 1\}, \; Z \geq 0 \tag{5}$$

Stage-2: Equipment Distribution (CVRP)

Sets and Parameters:

- $i, j \in \{0, 1, \ldots, m\}$: Nodes (0 is depot, $1 \ldots m$ are healthcare units).
- $k \in \{1, \ldots, M\}$: Ambulance (vehicle) indices.
- q_j: Equipment need at node j (based on population).
- Q: Capacity of each ambulance (e.g., 10,000).
- d_{ij}: Distance between node i and j.

Decision Variables:

- $x_{ijk} \in \{0, 1\}$: 1 if ambulance k travels from node i to j.
- $y_k \in \{0, 1\}$: 1 if ambulance k is deployed.
- $u_i \in \mathbb{R}_{\geq 0}$: Load variable for MTZ subtour elimination.

Objective: $\min \sum_{i,j,k} d_{ij} \cdot x_{ijk}$
Subject to:

$$\sum_{j \neq i} x_{ijk} = 1, \qquad \forall i,\ \forall k \tag{6}$$

$$\sum_{j \neq 0} x_{0jk} \leq 1, \qquad \forall k \tag{7}$$

$$\sum_{i \neq j} q_j x_{ijk} \leq Q y_k, \qquad \forall k \tag{8}$$

$$\sum_{j \neq 0} x_{0jk} = y_k, \quad \sum_{i \neq 0} x_{i0k} = y_k, \qquad \forall k \tag{9}$$

$$x_{iik} = 0, \qquad \forall i, k \tag{10}$$

$$\sum_{j \neq i} x_{ijk} = \sum_{j \neq i} x_{jik}, \qquad \forall i,\ \forall k \tag{11}$$

$$u_i - u_j + Q x_{ijk} \leq Q - q_j, \qquad \forall i \neq j,\ \forall k \tag{12}$$

Solution and Implementation

Solution Approach

The core problem addressed in this project revolves around the optimal placement of healthcare units and the efficient routing of supply distributions from a central depot. Our methodology involved decomposing the overall problem into two distinct stages.

In **Stage 1**, we tackled the facility location problem, which aims to determine the best positions for healthcare units such that all demand points (communities) are covered optimally. Due to computational advantages and solution reliability, we formulated this as a linearized version of a min-max problem. This transformation enabled us to leverage powerful MILP solvers efficiently.

In **Stage 2**, we addressed the capacitated vehicle routing problem (CVRP), where ambulances transport equipment from a central depot to each healthcare unit. Each route begins and ends at the depot, and each healthcare unit must be visited exactly once. We modeled this using binary decision variables and incorporated subtour elimination, flow continuation constraints to guarantee valid route construction.

Implementation Details

Our entire implementation was conducted in Python, chosen for its flexibility and extensive library support. The optimization solver used was **Gurobi**, selected after comparison with CPLEX. Gurobi stood out for its industry-standard reputation, state-of-the-art algorithm performance, ease of integration, and highly effective solver parameters. It is important to note that use of these parameters were specific to each dataset, meaning every dataset had a different combination of parameters that improved the solution.

All experiments were executed on a local laptop using Visual Studio Code (VSCode) as the development environment. The solver was configured to terminate after a time limit of about 600-800 seconds, ensuring practical runtime limits while still returning the best feasible solution found.

To enhance solver performance, we applied several techniques:

- **Agglomerative clustering** was used to preprocess community locations and generate meaningful clusters. This step enabled the placement of healthcare units at cluster centers.
- With the units pre-positioned, we **reshaped the problem as an assignment model**, drastically simplifying the formulation and yielding rapid solutions.
- The resulting assignment solution served as a **high-quality initial feasible solution**. We leveraged Gurobi's `Start` parameter to input this solution into more complex models, significantly improving solver performance and reducing the branch-and-bound tree.
- A custom-built `plot_function()` was used extensively to visualize community nodes, healthcare unit locations, and assignments. This also provided insight into the quality of the clustering-based initialization and helped validate routing decisions.