Simulation of a Two-Link Robotic Manipulator

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Abstract

In this assignment, a Type 2 two-link robotic manipulator was simulated. The dynamics of the system were represented in a MatLab SIMULINK model. This assignment was motivated by the need to establish an understanding of physical dynamic systems in the context of robotics. It also served to establish familiarity with SIMULINK, a powerful software system for modeling and designing dynamic systems.

1 SIMULINK Model

The system was translated into the SIMULINK block model shown in Figure 1. As can be seen in the figure, system parameters were represented as constant blocks, and the equations of motion were represented using function blocks, and integrated using integrator blocks in feedback loops. Controllers for each part of the assignment were designed as subsystem blocks that could be switched in and out easily.

2 Link 2 in Equilibrium

We desire the required torque as a function of joint angle to keep link 2 suspended in equilibrium at angles ranging from -90º to 90º. Since we are only concerned with the second link under equilibrium conditions, x_1 is unimportant and x_2 must be zero. We derive a function for u_2 in terms of x_3 by setting the equation for x_4 equal to zero. A plot of this function is shown in Figure 2.

$$x_4 = \dot{x_3} = -\frac{g}{l}\cos(x_3) - \frac{x_2}{2}\sin(2x_3) + \frac{u_2}{ml^2}$$

cancelling the second term and solving for u_2:

$$u_2(x_3) = mlg\cos(x_3)$$

The shape of this plot agrees with our natural intuitions. As we would expect, the greatest torque is required at $\theta_2 = 0^{\text{o}}$, when the arm is horizontal and gravity's moment arm is largest. At $\theta_2 = \pm 90^{\text{o}}$, the arm is vertical, and no torque is required. A sinusoidal shape is also consistent with a simple pendulum, which is essentially what this dynamic system represents.

3 θ_1 Oscillation

3.1 Five Degree Oscillation at 3.14 rad/sec

With $\theta_2 = x_3 = 0 - 45^{\text{o}}$, we search for an amplitude of $u_1$that will produce an oscillation amplitude $\triangle\theta_1 = 5^{\text{o}}$. We find the required amplitude of $u_1\,(t)$by integrating the equation of motion for $\theta_1$to come up with an analytic solution for $\theta_1$as a function of u_1:

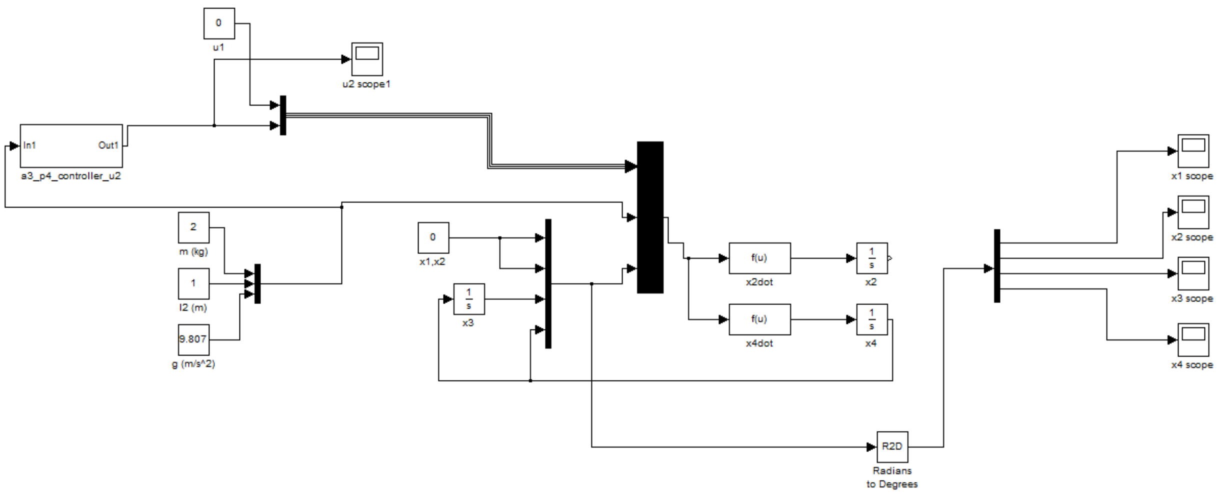

Figure 1: Screenshot of the Simulink model used in this assignment

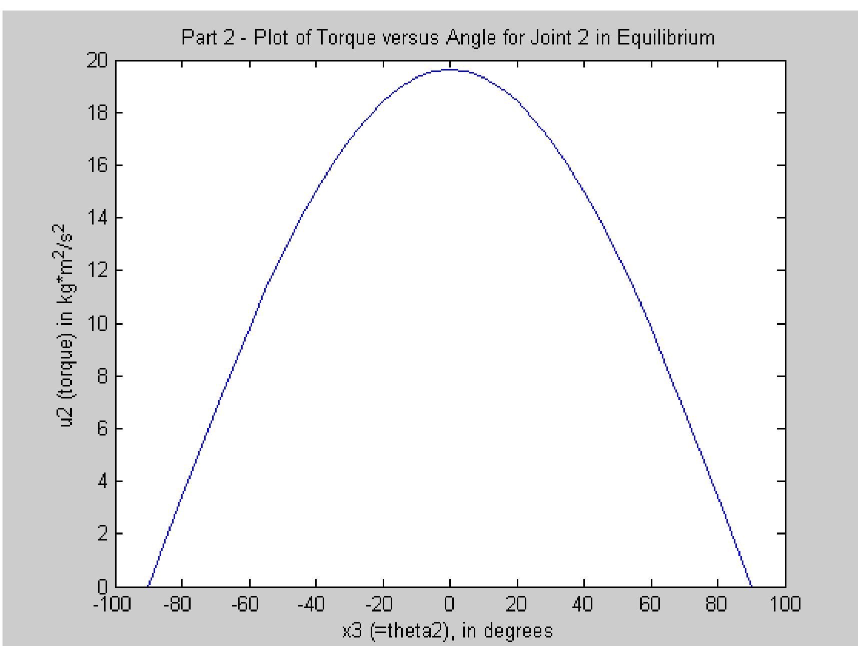

Figure 2: Plot of $u_2(x_3)$

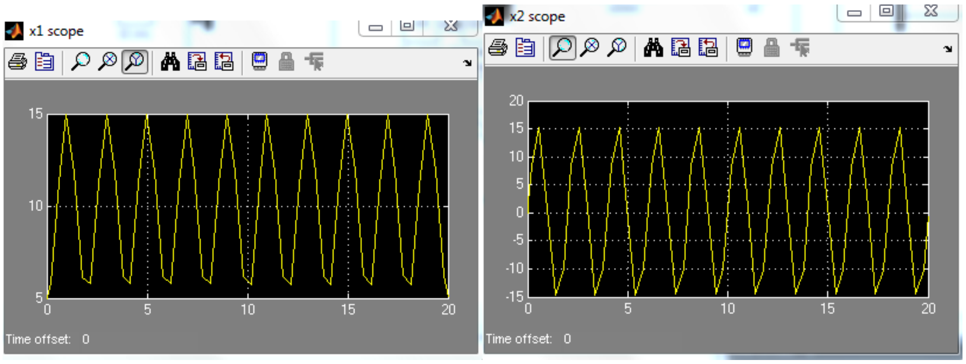

Figure 3: Plots of $u_1(t)$(right) and $x_1,(t)$(left) at 3.14rad/s

We begin with the dynamics for θ_1:

$$\dot{x}_2 = \ddot{\theta}_1 = \frac{1}{\cos^2(x_3)}\left(x_2 x_4 \sin(2x_3) + \frac{u_1}{ml_2}\right)$$

As $x_3 = -45^o$, $x_4 = \dot{x}_3 = 0$, and we may cancel the first term in the parentheses. Plugging in $x_3 = -45^o$:

$$\ddot{\theta}_1 = 2\frac{u_1}{ml^2}$$

$u_1$is specified as a sinusoidal function, so we assume it will take the form $u_1 = A\sin(t) + B\sin(t)$. We may now integrate to find $\theta_1(t)$.

$$\frac{d^2\theta}{dt^2} = \frac{2}{ml^2}(A\sin(\omega t) + B\cos(\omega t))$$

$$\frac{d\theta}{dt} = \frac{2}{ml^2\omega}(-A(\cos(\omega t) + 1) + B(\sin(\omega t)))$$

$$\theta(t) = \frac{-2}{ml^2\omega^2}(A(\sin(\omega t) + t) + B(\cos(t) + 1))$$

We see that A must equal zero, or $\theta(t)$ will grow to infinity. We solve for B:

$$\triangle\theta(t) = \frac{-2B}{ml^2\omega^2} = 5^o = 0.0873rad$$

$$B = 0.0436ml^2\omega^2$$

For $\omega = 3.14$, $B = 0.8598$. As can be seen in Figure 3, this yeilds an oscillation of 5^o.

3.2 Five Degree Oscillation at 6.28 rad/sec

We now seek the required amplitude for a five degree oscillation at a frequency of 6.28 rad/sec. This can be easily computed using the formula for B derived in the first part of this equation. Pluggin in $\omega = 6.28$, we find $B = 3.439$. Again, in Figure 4 we see that this yeilds an oscillation of 5^o.

Comparing the solutions to the first and second parts of this question, we see that the only real difference lies in the frequency response of the system. The system is more resonant at $\omega = 3.14$ that it is at $\omega = 6.28$, evidenced by the fact that a much smaller input amplitude is required to drive the system to the desired response amplitude at $\omega = 3.14$. Beyond the amplitude of the response, however, the dynamics of the system do not change with frequency.

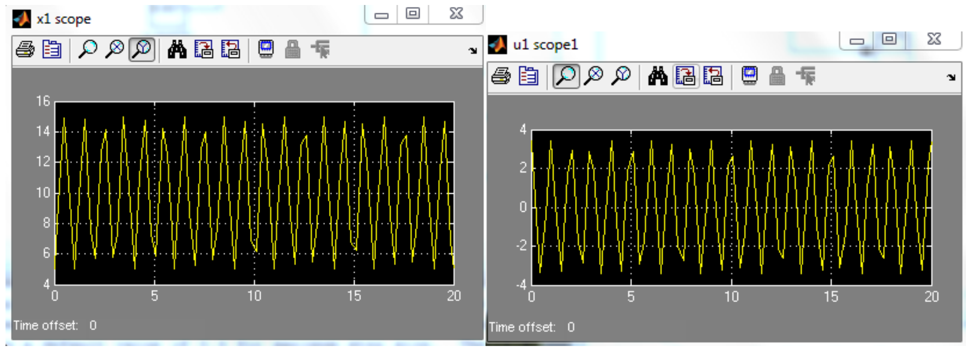

Figure 4: Plots of $u_1\,(t)$(right) and $x_1, (t)$(left) at 6.28rad/s

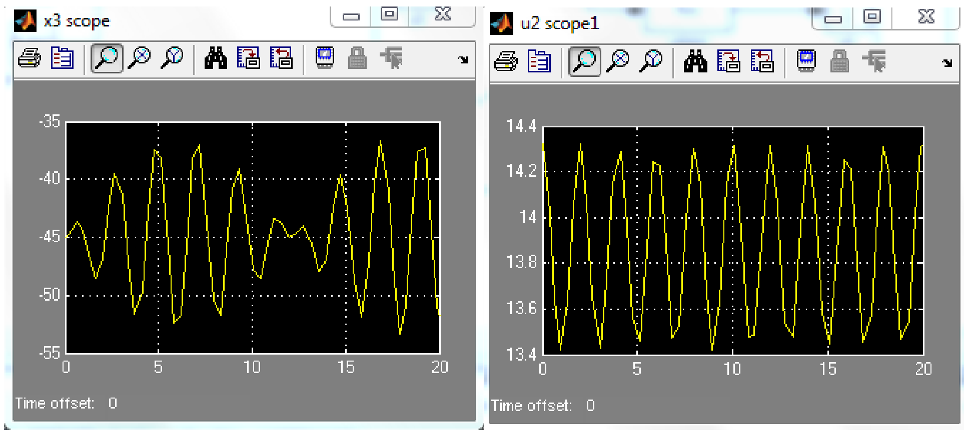

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Figure 5: Plots of Plots of $u_2\,(t)$(right) and $x_3, (t)$(left) oscillating about -45^o.

4 θ_2 Oscillation

Here we drive u_2 with a sinusoidal function at 3.14 rad/sec plus a fixed torque u_{2_0}which holds Link 2 at an equilibrium position.

4.1 Oscillation about -45º

We seek an amplitude for u_2 which will cause an oscillation of 5º about an equilibrium value of -45º at 3.14 rad/sec.

The value of u_{2_0} can be easily found using the formula from part 1:

$$u_{2_0}(x_3) = mlg\cos(x_3)$$

For $x_3 = -45^o$, $u_{2_0} = 13.86$

The amplitude of $u_2$was found by trial-and-error. For a five degree oscillation about -45^o, this amplitude was found to be 0.35. This oscillation is shown in Figure 5

It can be clearly seen in Figure 5 that there is a second oscillation in amplitude present which is slower than the intended osciallation at 3.14 rad/sec. The explanation for this oscillation can be found in Figure 2. If we look at the region in the vicinity of -45^o, we see that the torque required to keep the arm at static equilibrium decreases as the angle decreases and increases as the angle increases. When the arm moves above the equilibrium position of -45^o,the force of gravity is greater than u_{2_0}, and acts to move the arm back

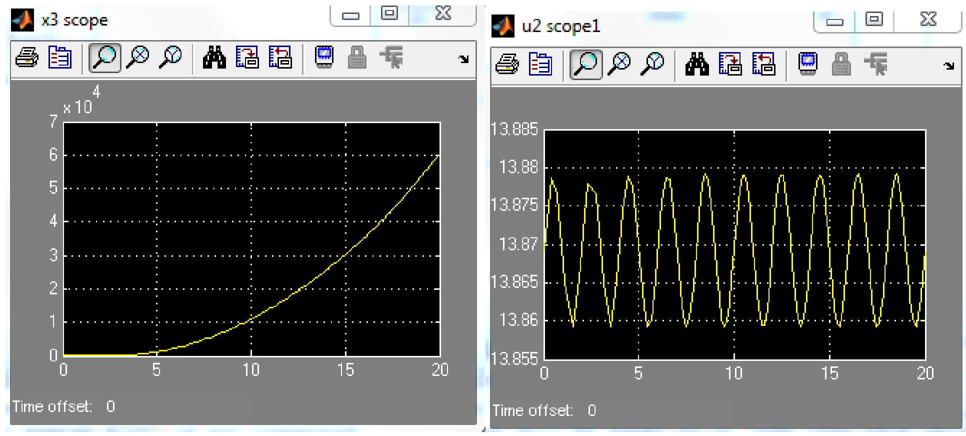

Figure 6: Plots of Plots of $u_2\,(t)$(right) and $x_3,(t)$(left) about 45^o.

towards equilibrium. When the arm moves below the equilibrium position, $u_{2_0}$is greater than the force of gravity, and it also acts to move the arm back towards equilibrium. The result is a net restoring force that is related to the deviation from equilibrium, which introduces a second, slower oscillatory mode.

4.2 Oscillation about -1º and 45º

Applying the same methods as above, $u_{2_0}(-1^o) = 19.60$ and $u_{2_0}(45^o) = 13.86$. With these values and a sinusoidal oscillation as input, however, it is impossible to generate a stable oscillation about either of these points. We will consider the 45^o case first. Figure 6 shows the input and response. As can be seen in the figure, x_3 quickly grows to infinity with even the smallest amplitude of oscillation in u_2 (in the case shown, the amplitude is only 0.01). This is because 45^o is not a stable equilibrium point. Consulting Figure 2, we see that the required equilibrium torque decreases as the angle increases and increases as the angle decreases about 45^o. As a result, the combination of u_{2_0} and gravity acts always acts away from the desired equilibrium point, driving the system unstable.

When we attempt to oscillate about -1^o, we are successful only at very small amplitudes. Figure 7 shows an oscillation about -1^o with an amplitude of 0.01. The oscillation is stable, but the low-frequency amplitude oscillation is very pronounced because the system is so close to the unstable point of 0^o. However, we can see in Figure 8 that an amplitude of 0.1 is enough to push the system into the unstable range, causing x_3 to blow up after about ten seconds.

Conclusions

This assignment demonstrated that the dynamics of even a simple physical system can exhibit complex and sometimes surprising behavior. The erratic behavior of the robot arm at and near the system's region of natural instability is proof of the value of feedback controllers, which can shape an unstable system's dynamics into something much more manageable. In practice, a robot such as this one should have a well designed PID or LQR controller in the inner loop of its control system in order to stabilize the system dynamics and make it easy for an operator to move the robot without worrying about driving the system unstable or damaging the actuators.

A Final Note:

You may have noticed that my SIMULINK scope output figures are strangely angular, rather than smooth. This is due to some sort of problem in SIMULINK which I have not been able to figure out. Several times

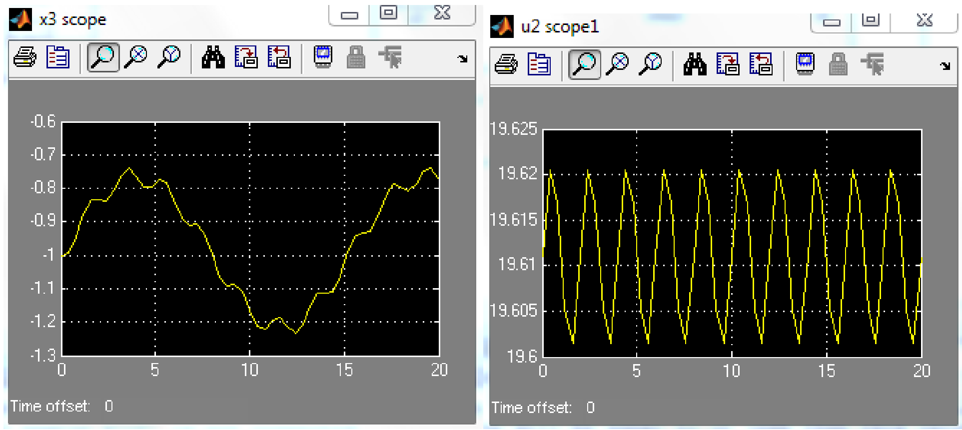

Figure 7: Plots of $u_2\,(t)$(right) and $x_3,(t)$(left) about $-1^o$with amplitude of 0.01

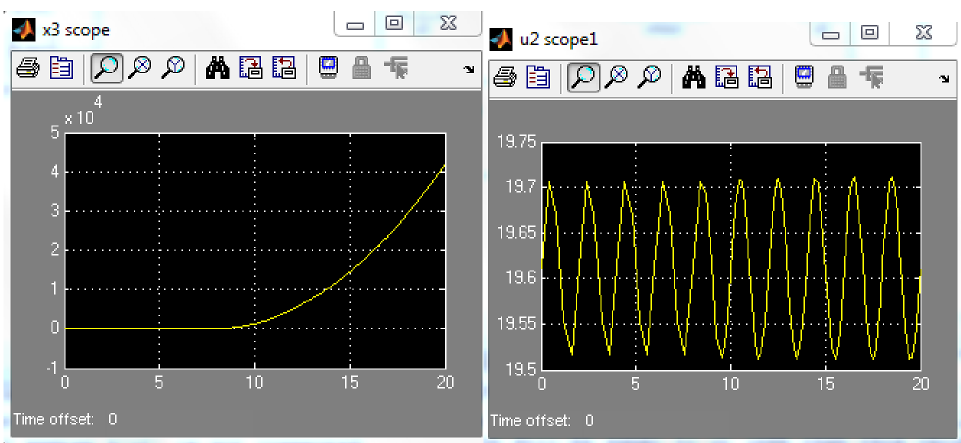

Figure 8: Plots of $u_2\,(t)$(right) and $x_3,(t)$(left) about $-1^o$with amplitude of 0.1

when I output scope data I did have smooth lines, but after a MatLab crash and restart, all my lines appeared angular. I am sorry I was not able to fix the problem, but the graphs should be otherwise correct, so I hope you can forgive their unusual appearance.