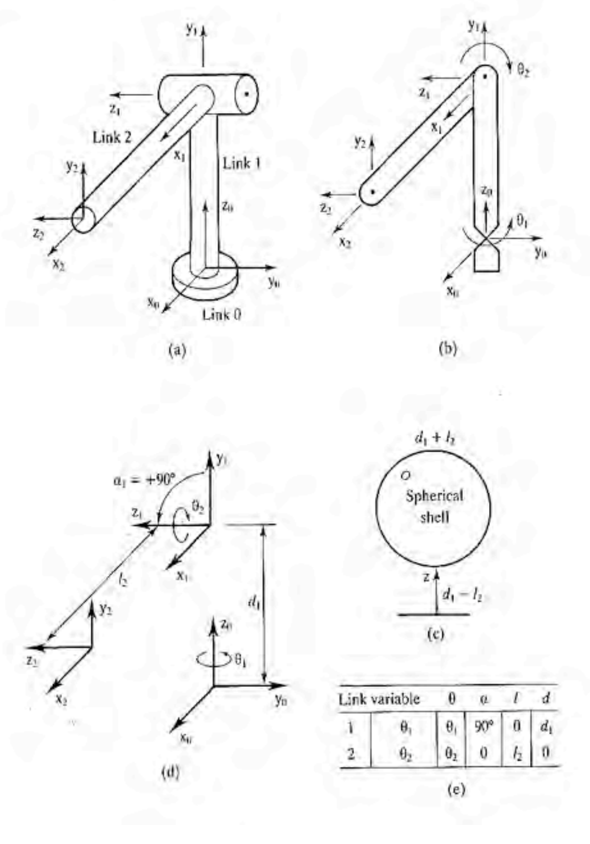
MAE 345
Robotics and Intelligent Systems

Assignment #3
due: October 11, 2011

The goal of this assignment is to simulate functions of a Type-2, two-link robotic manipulator using SIMULINK. The device is described in the figure below, which is Fig. 4.7 in *Introduction to Robotics*, P. J. McKerrow, Addison-Wesley, 1991, p. 188.

**Figure 4.7**
Type 2 two-link manipulator. (a) Manipulator in zero position; (b) Line diagram; (c) Workspace; (d) Assignment of coordinate frames; (e) Link parameters.




Equations for the manipulator's dynamics are derived in Example 7.1 (pp. 388-390, in Blackboard Course Materials). These equations lead to the following fourth-order, nonlinear state-space model:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{\cos^2 x_3}\left( x_2 x_4 \sin 2x_3 + \frac{u_1}{m l_2^2} \right) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -\frac{g}{l_2}\cos x_3 - \frac{x_2}{2}\sin 2x_3 + \frac{u_2}{m l_2^2}
\end{aligned}
$$

The state is defined as,

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \mathit{Angle\ of\ } 1^{st}\ \mathit{link}, \theta_1, \mathit{rad} \\ \mathit{Angular\ rate\ of\ } 1^{st}\ \mathit{link}, \dot{\theta}_1, \mathit{rad} / \sec \\ \mathit{Angle\ of\ } 2^{nd}\ \mathit{link}, \theta_2, \mathit{rad} \\ \mathit{Angular\ rate\ of\ } 2^{nd}\ \mathit{link}, \dot{\theta}_2, \mathit{rad} / \sec \end{bmatrix}
$$

and the control vector consists of the torques at each joint:

$$
\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}
$$

The angles, $\theta_1$ and $\theta_2$ (positive up), are zero when they are aligned with their respective $x$ axes. Inertial properties are modeled simply by a point mass, $m$, at the distal end of the manipulator, and the length of the second link is $l_2$. Thus the mass and inertia of the links are neglected. Link 2 is mounted at distance $d_1$ from the base.

1) Prepare a SIMULINK simulation of the system. Assign the following values to the model parameters:

| | | | |
|---|---|---|---|
| $d_1$ | = 1.5 m; | $m$ | = 2 kg |
| $l_2$ | = 1 m; | $g$ | = 9.807 m/s2 |

2) Neglecting any implied mechanical blockage, what equilibrium torque is required to hold Link 2 at equilibrium values of $\theta_{2_o}$ (= $x_3$) from –90 deg to 90 deg (plot your answer)? The Link 1 position does not matter, as it merely rotates Link 2 about a vertical axis and has no effect on the equilibrium torque at Joint 2.

3) Assume that the first control torque, $u_1$, is driven by a sine wave, with a frequency of 3.14 rad/sec.

a) With $\theta_2 = x_3$ fixed at –45 deg, find the amplitude of $u_1$ that produces an oscillation amplitude of $\Delta\theta_1$ (= $x_1$) of 5 deg. Your answer may be approximate, and you may use the simulation to derive and demonstrate the result. Plot the output for a 20-sec interval.

b) For the same torque amplitude, show the result when the driving frequency is 6.28 rad/sec. Comment on the differences between (a) and (b).

4) Assume that the second control torque, $u_2$, is driven by a sine wave, with a frequency of 3.14 rad/sec, plus the fixed torque, $u_{2_o}$, required to hold Link 2 at its equilibrium value.

a) With $\theta_1 = x_1$ fixed at 0 deg, find the amplitude of $u_2$ that produces an oscillation amplitude of $\Delta\theta_2$ (= $x_3$) of 5 deg, measured about an equilibrium value of $\theta_{2_o} = -45$ deg. Your answer may be approximate, and you may use the simulation to derive and demonstrate the result. Plot the output for a 20-sec interval.

b) Repeat the process with $\theta_{2_o} = -1$ deg, then $\theta_{2_o} = 45$ deg. Comment on the differences between (a) and (b).

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Provide some discussion of methods and results. Be sure to make your reasoning clear in presenting your results.