Simulation of a Two-Link Robotic Manipulator

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Abstract

In this assignment, a Type 2 two-link robotic manipulator was simulated. A schematic of the manipulator and its workspace is shown in Figure [FIG]. The dynamics of the manipulator were provided, and are given in Equations [EQN'S]. The dynamic system was represented in a MatLab SIMULINK model which was designed in accordance with the assignment specifications.

This assignment was motivated by the need to establish an understanding of physical dynamic systems in the context of robotics. It also served to establish familiarity with SIMULINK, a powerful software system for modeling and designing dynamic systems.

Part I

SIMULINK Model

The system was translated into the SIMULINK block model shown in Figure [FIGURE]. As can be seen in the figure, System parameters were represented as constant blocks, and the equations of motion were represented using function blocks, and integrated using integrator blocks in feedback loops.

Part II

Link 2 in Equilibrium

We desire the required torque as a function of joint angle to keep link 2 suspended in equilibrium at angles ranging from -90º to 90º. Since we are concerned with equilibrium conditions, and only the second link, $x_1$is unimportant, and $x_2$must be zero. We derive an formula for $u_2$as a function of $x_3$by setting the equation for $x_4$equal to zero:

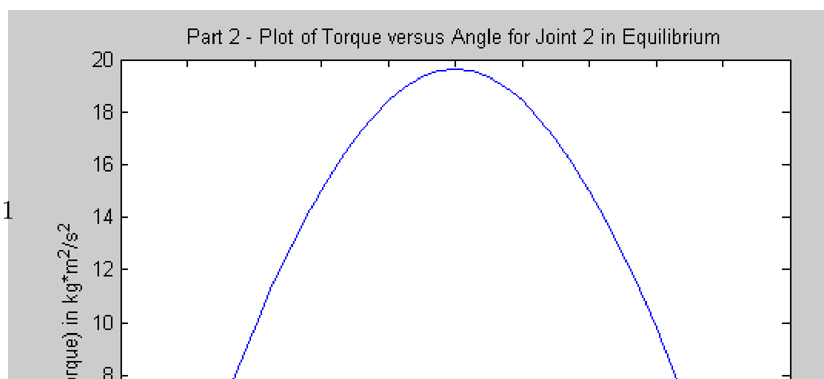
$$x_4 = \dot{x_3} = -\frac{g}{l}\cos(x_3) - \frac{x_2}{2}\sin(2x_3) + \frac{u_2}{ml^2}$$

cancelling the second term and solving for u_2:

$$u_2(x_3) = mlg\cos(x_3)$$

A plot of this function is show in Figure 1:

The shape of this plot agrees with our natural intuitions. As we would expect, the greatest torque is required at $\theta_2 = 0^{\text{o}}$, when the arm is horizontal to the ground. At $\theta_2 = \pm 90^{\text{o}}$, the arm is vertical, and no torque is required. A sinusoidal shape is



also consistent with a simple pendulum, which is essentially what this dynamic system represents.

Part III

θ_1Oscillation

We drive $u_1$with a sine wave of frequency 3.14 rad/sec.

1 Five Degree Oscillation at 3.14 rad/sec

With $\theta_2 = x_3 = 0 - 45^{\text{o}}$, we search for an amplitude of $u_1$that will produce an oscillation amplitude $\triangle\theta_1 = 5^{\text{o}}$.

We find the required amplitude of$u_1\left(t\right)$by integrating the equation of motion for $\theta_1$to come up with an analytic solution for $\theta_1$as a function of u_1:

We begin with the dynamics for θ_1:

$$\dot{x}_2 = \ddot{\theta}_1 = \frac{1}{\cos^2(x_3)}\left(x_2 x_4 \sin\left(2x_3\right) + \frac{u_1}{ml_2}\right)$$

As $x_3 = -45^o$, $x_4 = \dot{x}_3 = 0$, and we may cancel the first term in the parentheses. Plugging in $x_3 = -45^o$:

$$\ddot{\theta}_1 = 2\frac{u_1}{ml^2}$$

$u_1$is specified as a sinusoidal function, so we assume it will take the form $u_1 = A\sin\left(t\right) + B\sin\left(t\right)$. We may now integrate to find $\theta_1\left(t\right)$.

$$\frac{d^2\theta}{dt^2} = \frac{2}{ml^2}\left(A\sin\left(\omega t\right) + B\cos\left(\omega t\right)\right)$$

$$\frac{d\theta}{dt} = \frac{2}{ml^2\omega}\left(-A\left(\cos\left(\omega t\right) + 1\right) + B\left(\sin\left(\omega t\right)\right)\right)$$

$$\theta\left(t\right) = \frac{-2}{ml^2\omega^2}\left(A\left(\sin\left(\omega t\right) + t\right) + B\left(\cos\left(t\right) + 1\right)\right)$$

We see that A must equal zero, or $\theta\left(t\right)$will grow to infinity. We solve for B:

$$\triangle\theta\left(t\right) = \frac{-2B}{ml^2\omega^2} = 5^o = 0.0873rad$$

$$B = 0.0436ml^2\omega^2$$

For $\omega = 3.14$, $B = 0.8598$. As can be seen in Figure 2, this yeilds an oscillation of 5^o.

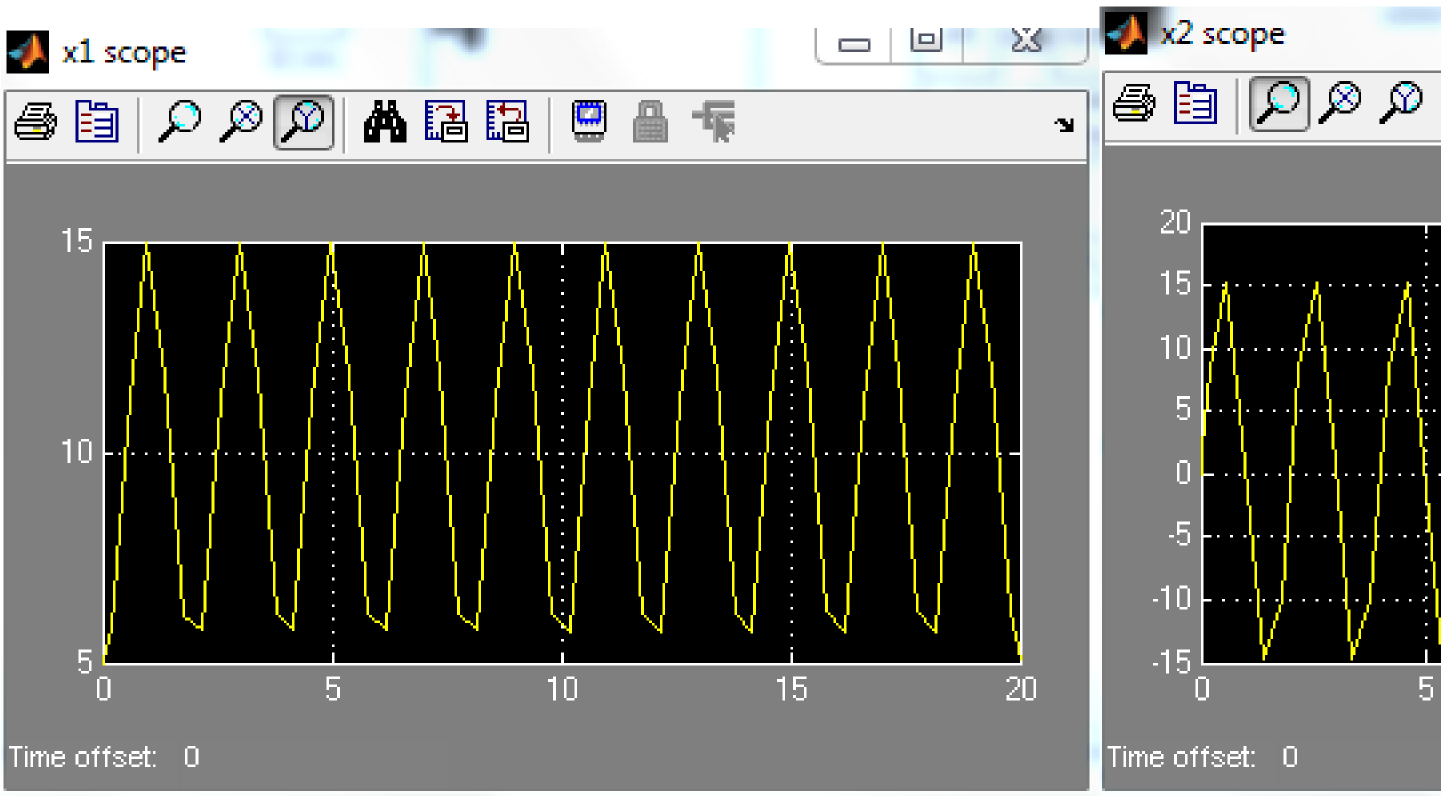


Figure 2: Plots of $u_1\,(t)$(right) and $x_1,(t)$(left) at 3.14rad/s

Figure 3: Plots of $u_1(t)$(right) and $x_1,(t)$(left) at 6.28rad/s

2 Five Degree Oscillation at 6.28 rad/sec

We now seek the required amplitude for a five degree oscillation at a frequency of 6.28 rad/sec. This can be easily computed using the formula for B derived in the first part of this equation. Pluggin in $\omega = 6.28$, we find $B = 3.439$. Again, in Figure 3we see that this yeilds an oscillation of 5^o.

Part IV

θ_2Oscillation

Here we drive $u_2$with a sinusoidal function at 3.14 rad/sec plus a fixed torque u_{2_0}which holds Link 2 at an equilibrium position.

3 Oscillation about -45º

We seek an amplitude for $u_2$which will cause an oscillation of 5º about an equilibrium value of -45º.

The value of u_{2_0}can be easily found using the formula from part 1:

$$u_{2_0}(x_3) = mlg\cos(x_3)$$

For $x_3 = -45^o$, $u_{2_0} = 13.86$

The amplitude of $u_2$was found by trial-and-error. For a five degree oscillation about -45^o, this amplitude was found to be 0.45. This oscillation is shown in Figure 4

4 Oscillation about -1º and 45º

Applying the same methods as above:

$u_{2_0}(-1^o) = 19.60$

$u_{2_0}(45^o) = 13.86$

The required amplitude for oscillation about -1º was found to be ,and about 45º to be

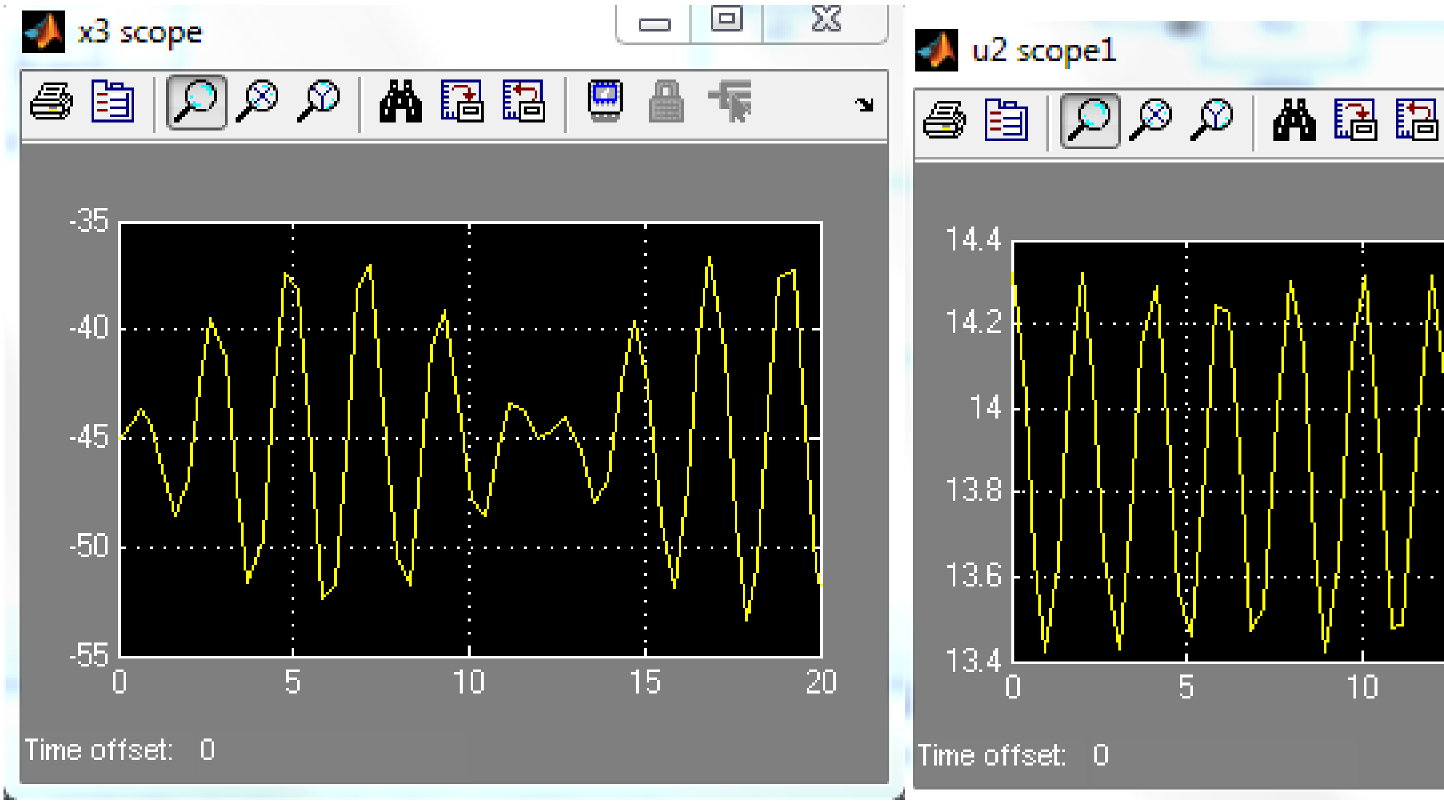


Figure 4: Plots of Plots of $u_2\,(t)$(right) and $x_3,(t)$(left) oscillating about -45^o.