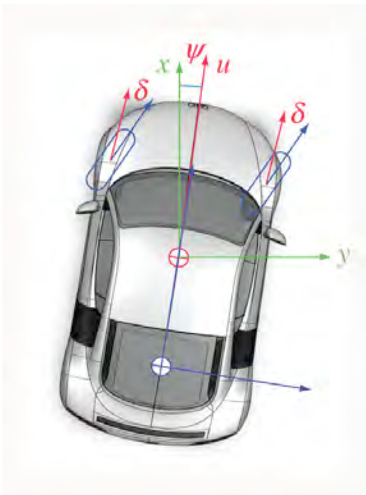
MAE 546
Optimal Control and Estimation
Instructor: Robert Stengel
Assignment #4
due: April 5, 2012



This assignment deals with the design of a sampled-data (digital) control system for adaptive cruise control and lane changing. The BMW 750i's horizontal dynamics are described by,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} \simeq \begin{bmatrix} \left[F - C_D\left(\frac{1}{2}\right)\rho x_1^2 S\right]/m + w_1 \\ x_1 \cos x_4 \\ x_1 \sin x_4 \\ \dfrac{x_1}{l_{wb}} \tan\delta_{FW} + w_2 \end{bmatrix} \quad (1$$

where the state, control, and disturbance variables are

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} u \\ x \\ y \\ \psi \end{bmatrix} = \begin{bmatrix} \text{Axial velocity, m/s} \\ \text{Northerly position, m} \\ \text{Easterly position, m} \\ \text{Yaw angle, rad} \end{bmatrix} \quad (2$$

$$\begin{bmatrix} F = F_{\max}\delta_F \\ \delta_{FW} \end{bmatrix} = \begin{bmatrix} F_{\max}u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \text{Engine force – braking force, N, } -0.25 < \delta_F < 1 \\ \text{Front wheel angle, rad} \end{bmatrix} \quad (3$$

$$F_{\max} = \frac{C_D \frac{1}{2}\rho V_{\max}^2 S\left(V_c + V_{\max}\right)}{V_c + x_1} \quad (4$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \text{Longitudinal disturbance acceleration, m/s}^2 \\ \text{Yaw rate disturbance, rad/s} \end{bmatrix} \quad (5$$

and the automobile parameters are defined as before.

Assignment

1) <u>Steady-state throttle settings for straight driving</u> – Using eq. 1, determine the steady-state throttle setting for speeds from 0 to 35 m/s in 1 m/s increments, assuming no disturbances.

2) <u>Linear, time-invariant models of automobile dynamics and linear-quadratic feedback control</u>
a) Calculate the continuous-time LTI model matrices, **F**, **G**, and **L** at the steady-state operating points, x_1*, x_2*, x_3*, and x_4*, determined in (1). Note that x_2 and x_3 are integrals of x_1 and x_4; therefore, in this case, "steady state" means steady velocity, and quasi-steady positions are actually integrals of x_1* and x_4* (see pp. 122 and 512-514, *Optimal Control and Estimation*).
b) Calculate and plot the continuous-time, constant-gain, linear-quadratic regulator (LQR) feedback gain matrix, **C**, as a function of the operating points using MATLAB's *lqr* function. Choose the cost function weighting matrices, as

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$$

c) Calculate the discrete-time LTI model matrices, $\mathbf{\Phi}$, $\mathbf{\Gamma}$, and $\mathbf{\Lambda}$ corresponding to **F**, **G**, and **L** at the steady-state operating points, assuming that the sampling interval, Δt, is 0.1 sec.
d) Calculate and plot the sampled-data linear-quadratic regulator (LQR) feedback gain matrix, **C**, as a function of the operating points, assuming that the sampling interval, Δt, is 0.1 sec. Choose the cost function weighting matrices, $\hat{\mathbf{Q}},\hat{\mathbf{M}},\hat{\mathbf{R}}$, to be the sampled-data equivalents of the continuous-time matrices, **Q** and **R**. Compare the sampled-data and continuous-time gain matrices.

3) <u>Adaptive cruise control</u> – Simulate your nonlinear system using eq. 1 with the sampled-data feedback controller derived in Part 2.d). Every tenth of a second the controller sends a new command to the throttle and front wheels. The command is held constant in the 1/10-sec interval.
a) Demonstrate your controller's response for a nominal straight-ahead trajectory that ramps up the velocity from 0 to 24 m/s in 10 sec, holds that velocity for 5 sec, and ramps down to 0 m/s after another 10 sec. Schedule the control gain matrix, $\mathbf{C}(x_1^*)$, and nominal throttle setting, $u_1^*(x_1^*)$, as functions of desired velocity by interpolating between the control gain matrices computed in Part 2.d).
b) Repeat (a) with $u_1^* = 0$. Is there a significant difference in the result?
c) The automobile is cruising at 20 m/s and is 100 m behind a school bus traveling at the same velocity. Instruct the control system to close the distance to 50 m in 10 sec using feedback control, maintaining your own speed at 20 m/s at the end of the period.

d) The school bus speeds up from 20 m/s to 24 m/s in 10 sec. You wish to maintain your distance behind the bus; your bumper-mounted radar detects the changing distance but not the velocity. With no additional guidance, how does your feedback control system respond? How would you change the control system to improve its performance?

4) Lane-changing control - For all cases, simulate your nonlinear system using eq. 1 with the sampled-data feedback controller derived in Part 2.d).

a) Instruct the control system to perform a smooth lane change (3.6 m) to the left in 10 sec while cruising at 20 m/s, beginning and ending with zero yaw angle.

b) Having performed the lane change, pass the 12-m-long school bus in 10 sec and pull back into the lane within 10 sec, 50 m ahead of the bus, and resuming a velocity of 20 m/s. Your BMW 750i is 5.2 m long.

c) Describe how your lane-changing controller would execute a passing maneuver fully autonomously if the school bus slows to 18 m/s and the radar detects decreasing range between the vehicles.

d) You are cruising at 24 m/s when a sudden side wind produces a constant yaw rate disturbance of 2 deg/s. How does your system respond? How would you change the control system to improve its performance?