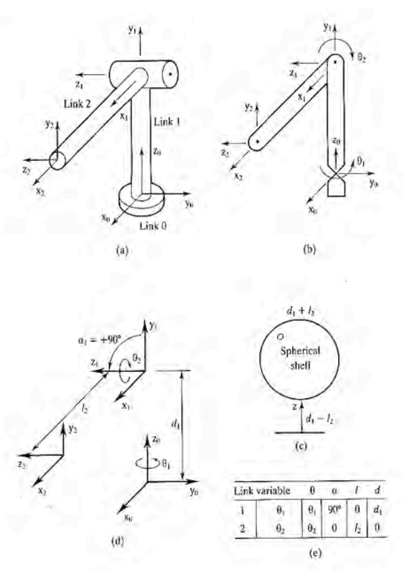
MAE 345
Robotics and Intelligent Systems

Assignment #4
due: October 20, 2011

In Assignment #3, you produced a nonlinear simulation of the robot arm shown below. The goal of this assignment is to examine linear, time-invariant models of small-perturbation motions around various operating points.

Figure 4.7 Type 2 two-link manipulator. (a) Manipulator in zero position; (b) Line diagram; (c) Workspace; (d) Assignment of coordinate frames; (e) Link parameters.



The fourth-order, nonlinear, time-invariant state-space model is

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{\cos^2 x_3}\left(x_2 x_4 \sin 2x_3 + \frac{u_1}{ml_2^2} \right) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -\frac{g}{l_2}\cos x_3 - \frac{x_2}{2}\sin 2x_3 + \frac{u_2}{ml_2^2}
\end{aligned}
\qquad (1\text{-}4
$$

or

$$
\dot{\mathbf{x}}(t) = \mathbf{f}\left[\mathbf{x}(t), \mathbf{u}(t)\right] \qquad (5
$$

The state is defined as,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} Angle\,of\;1^{st}\;link, \theta_1, rad \\ Angular\,rate\,of\;1^{st}\;link, \dot{\theta}_1, rad\,/\,\sec \\ Angle\,of\;2^{nd}\;link, \theta_2, rad \\ Angular\,rate\,of\;2^{nd}\;link, \dot{\theta}_2, rad\,/\,\sec \end{bmatrix}$$

and the control vector consists of the torques at each joint:

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

The angles, θ_1 and θ_2 (positive up), are zero when they are aligned with their respective x axes. Inertial properties are modeled simply by a point mass, m, at the distal end of the manipulator, and the length of the second link is l_2. Thus the mass and inertia of the links are neglected. Link 2 is mounted at distance d_1 from the base. The nominal model parameters are

d_1 = 1.5 m; m = 2 kg; l_2 = 1 m; g = 9.807 m/s2

1) Equation 5 can be expanded to first degree, identifying the nominal nonlinear part and a linear perturbation part:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \dot{\mathbf{x}}_N(t) + \Delta\dot{\mathbf{x}}(t) \\ &\approx \mathbf{f}[\mathbf{x}_N(t), \mathbf{u}_N(t)] + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(t)\Delta\mathbf{x}(t) + \frac{\partial \mathbf{f}}{\partial \mathbf{u}}(t)\Delta\mathbf{u}(t) \\ &= \mathbf{f}[\mathbf{x}_N(t), \mathbf{u}_N(t)] + \mathbf{F}(t)\Delta\mathbf{x}(t) + \mathbf{G}(t)\Delta\mathbf{u}(t) \end{aligned} \quad (6$$

We assume that the nominal and perturbation problems can be solved separately:

$$\begin{aligned} \dot{\mathbf{x}}_N(t) &= \mathbf{f}[\mathbf{x}_N(t), \mathbf{u}_N(t)], \quad \mathbf{x}_N(0) \;\text{given} \\ \Delta\dot{\mathbf{x}}(t) &= \mathbf{F}(t)\Delta\mathbf{x}(t) + \mathbf{G}(t)\Delta\mathbf{u}(t), \quad \Delta\mathbf{x}(0) \;\text{given} \end{aligned} \quad (7,8$$

a) Express the linear model matrices, $\mathbf{F}(t)$ (4 x 4) and $\mathbf{G}(t)$ (4 x 2), as functions of the nominal state.
b) Under what circumstances are **F** and **G** constant?

2) Assume that **F** and **G** are constant; hence,

$$\Delta\dot{\mathbf{x}}(t) = \mathbf{F}\Delta\mathbf{x}(t) + \mathbf{G}\Delta\mathbf{u}(t), \quad \Delta\mathbf{x}(0) \text{ given} \qquad (9$$

Assume that $\theta_{1_N} = x_{1_N}$ is arbitrary and constant and that $\theta_{2_N} = x_{3_N}$ takes three values: –45°, 0°, and 45°. The remaining nominal values, x_{2_N} and x_{4_N}, are zero.

a) Express the numerical values of the matrices, **F** (4 x 4) and **G** (4 x 2) for these three conditions.
b) What are the eigenvalues of **F** for these three conditions?
c) What values of Δu_1 and Δu_2 produce initial angular rate perturbations, $\Delta x_2(0)$ and $\Delta x_4(0)$, of 0.1°/sec for these three conditions (with remaining initial conditions = 0)?
d) Compute and plot the response to step inputs for Δu_1 equal to the three values computed in (c).
e) Compute and plot the response to step inputs for Δu_2 equal to the three values computed in (c).
f) Show the Bode plots for Δu_2 inputs:

$$\frac{\Delta x_1(j\omega)}{\Delta u_2(j\omega)}, \quad \frac{\Delta x_2(j\omega)}{\Delta u_2(j\omega)}, \quad \frac{\Delta x_3(j\omega)}{\Delta u_2(j\omega)}, \quad \frac{\Delta x_4(j\omega)}{\Delta u_2(j\omega)}$$

3) Assume that the angular rate about the vertical axis, $\dot{\theta}_{1_N} = x_{2_N}$, is constant, taking values between 0 and 720 degrees/sec. What effect does this nominal angular rate have on the eigenvalues for the three conditions?

. Provide some discussion of methods and results using a format similar to the one that we discussed in class.
. Be sure to make your reasoning clear in presenting your results.
. Grading is based not only on numbers and graphs but also on how well you explain the significance of the results.