

Logistic Regression-2

L MLE - Maximum Likelihood Estimation

loss \rightarrow $-\log \text{loss} + \lambda \sum_{i=1}^d |w_i|$ - lasso

$$\hat{y} = \sigma(\underline{w}^T \cdot x + w_0)$$

$-\log \text{loss} + \lambda \sum_{i=1}^d w_i^2$ - Ridge.

(1) ✓

Regularization + logistic Regression

Validation \rightarrow Hyperparameter tuning

$$SD = \frac{(x_i - \mu)}{\sigma}$$

(μ) - ? ✓

(σ) - ✓

Accuracy y \hat{y} if $y == \hat{y}$ # correct prediction

y	\hat{y}	
$\rightarrow 1$	$1 \checkmark$	Corrected
1	0	Wrong
0	$0 \checkmark$	
1	$0 \times$	

$y \neq \hat{y}$ Wrong prediction

$$\text{Accuracy} = \frac{\# \text{ of Correct Predictions}}{\text{Total \# of Predictions}} = \frac{80}{100} = 0.8$$

(80%)

Regularization, Hyperparameter tuning

Feature importance

$$\hat{y} = \frac{1}{1 + e^{-(w_0 + w_1 x_1 + w_2 x_2 + \dots + w_d x_d)}} \quad w x?$$

odds

Prob of success
Prob of failure

$$\frac{0.5}{0.5} \quad ? \quad 1:1$$

or Prob of success $\frac{4/5}{1/5}$ $\underline{4} : \underline{1}$
Prob of failure

feature imp.

Linear Reg.

$$\hat{y} = w_0 + w_1 x_1$$

$\hat{y} \uparrow$ (w_1) $x_1 \uparrow$ unit

$$\therefore \frac{P}{P(y=1|x)}$$

$$P = \frac{1}{1 + e^{-z}}$$

$$z = w_0 + w_1 x_1 + \dots + w_d x_d$$

$$P(1 + e^{-z}) = 1$$

$$P + P \cdot e^{-z} = 1$$

$$P e^{-z} = 1 - P$$

$$e^{-z} = \frac{1-P}{P}$$

$$-z = \log\left(\frac{1-P}{P}\right)$$

$$z = \log\left(\frac{P}{1-P}\right)$$

$$\log\left(\frac{P}{1-P}\right) = w_0 + w_1 x_1 + \dots + w_d x_d$$

$$\frac{P}{1-P} = \frac{\text{Prob of } y=1 \text{ (succm)}}{\text{prob of } y=0 \text{ (failure)}} = \text{odds?}$$

$$y=1 \uparrow \text{ } P \uparrow \text{ } w_{+ve} \quad \log(\text{odds}) \uparrow = w_0 + \overset{2.6}{w_1} \overset{\uparrow}{x_1} + \overset{1.2}{w_2} x_2 - \dots - \overset{-0.9}{w_d} x_d$$

or $y=1 \downarrow \text{ } w_{-ve}$

$w_1 \quad x_1 \uparrow$

Predict heart failure

$$P(y=1 \text{ heart failure}) \uparrow = w_0 + w_1 \text{ age} \uparrow + w_2 \text{ BMI} \uparrow - w_3 \text{ good con (HDL)} \uparrow$$

$$P(y=0 \text{ no heart})$$

$$\boxed{\log(\text{odds})}$$

w ?

w high value - very imp

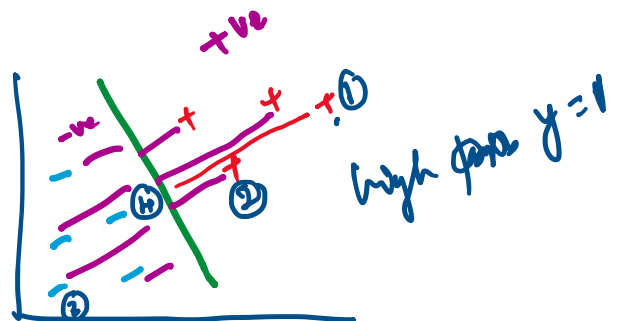
w - small - less imp

w +ve - increase odds, increase heart risk

w -ve - decreases heart risk

10:11

Geometrical intuition.



$$\textcircled{Z} = \frac{w^T \cdot x + w_0}{\|w\|}$$

$$\boxed{\|w\|=1}$$

$$\log\left(\frac{P}{1-P}\right) = Z \quad \textcircled{\uparrow}$$

$$\boxed{\frac{\text{prob of } y=1}{\text{prob of } y=0}}$$

$$\log\left(\frac{P}{1-P}\right) = Z$$

$$P = \boxed{\frac{1}{1 + e^{-Z}}}$$

-? Sigmoid

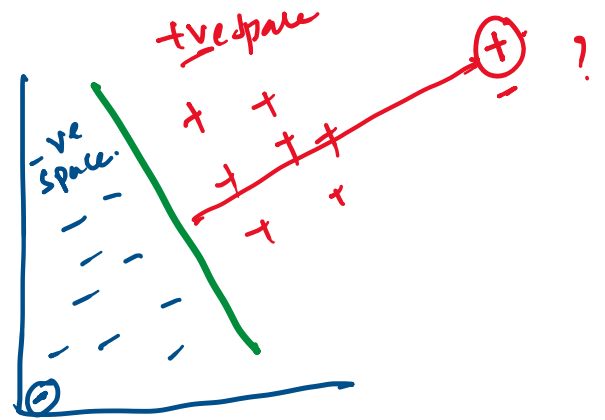
Assumptions ?

① NO multicollinearity]

EDA ✓

② outlier effect.

outlier is on the same half space.

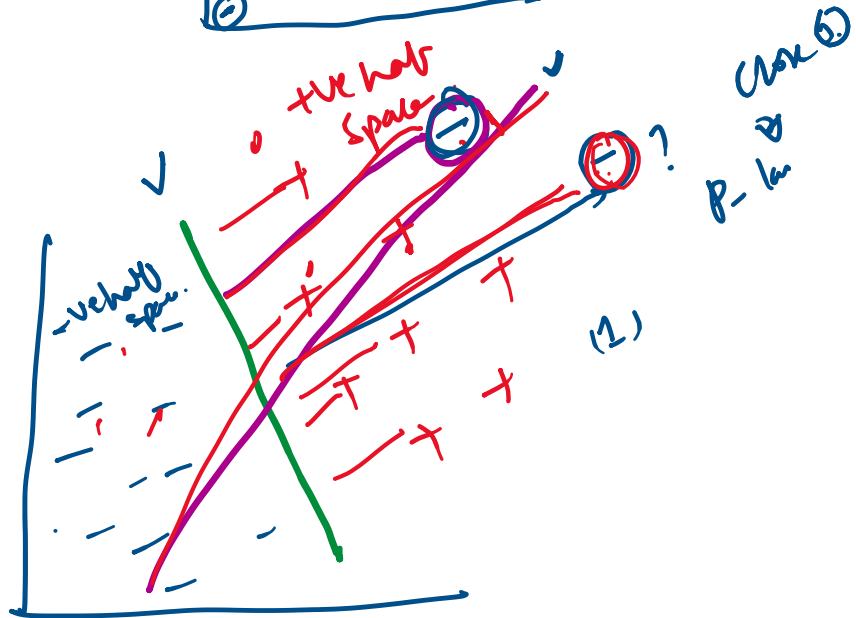


tve
-ve
tve half
-ve half

Metric?

log(Pi)

(0)



log loss

$$- [y_i \log P_i + (1 - y_i) \log (1 - P_i)]$$

0 log(Pi)

1 $\log(1 - 0.99)$

$$- [\textcircled{4.6} ?]$$

$\log(0.01)$

① if outliers are on same half space
No impact

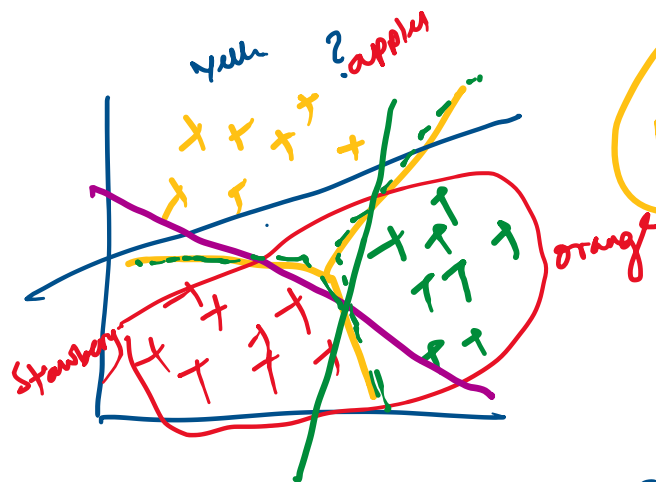
② if outliers are on opp half spaces
hyperplane will be twisted to reduce
error of outlier



Binary Classification

multiclass Classification

logistic Regression?



Hack :

one vs rest (OVR)

(X)

y	y_Orange ✓	y_Apple	y_Strawberry
Apple	0	1	0
Orange	1	0	0
Strawberry	0	0	1
Apple orange	1	0	0

$$x \rightarrow \hat{y}$$

$$x_1, y\text{-orange} \rightarrow M_1 \rightarrow P(y=\text{orange})$$

$$\hat{y}_{\text{orange}} \quad \textcircled{0.9} \quad \text{Avg Max}$$

$$x_2, y\text{-Apple} \rightarrow M_2 \rightarrow P(\text{Apple})$$

$$\hat{y}_{\text{apple}} \quad 0.1$$

$$x_3, y\text{-straw}$$

$$M_3 \rightarrow P(y=\text{straw}) \quad \hat{y}_{\text{straw}} \quad 0.15$$

one

$$\left. \begin{array}{l} P(\text{orange}) = 0.7 \\ P(\text{Apple}) = 0.1 \\ P(\text{straw}) = 0.25 \end{array} \right\}$$

$$\begin{array}{r} 0.1 \\ 0.15 \\ 0.12 \\ 0.17 \\ \boxed{0.5} ? \\ 0.18 \end{array}$$

$$\begin{array}{r} 0.22 \\ 0.25 \\ 0.24 \\ 0.26 \\ \vdots \end{array}$$

Soft max :

$$\frac{P(\text{orange})}{0.7 + 0.3 + 0.25}$$

$$\frac{0.3}{0.7 + 0.3 + 0.25} \quad \frac{0.25}{0.7 + 0.3 + 0.25} \quad 0.11$$

$$x \quad y \quad \hat{y} \quad \text{straw} \quad 0.11$$