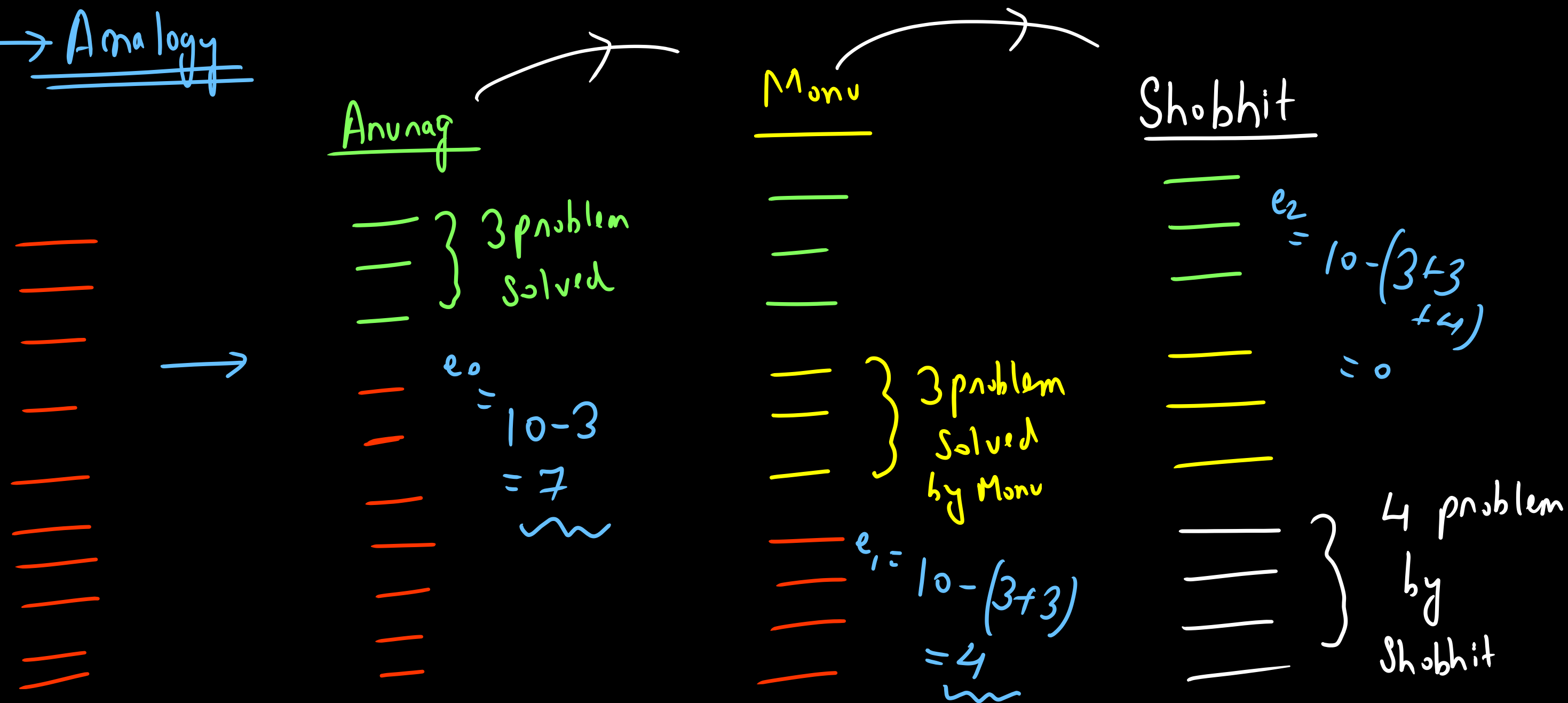


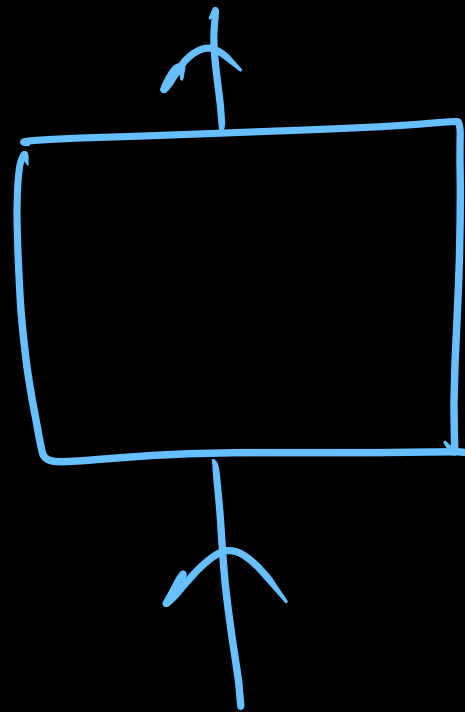
→ Boosting

→ Analogy



→ Each friend is solving the unsolved question left by his/her previous friend

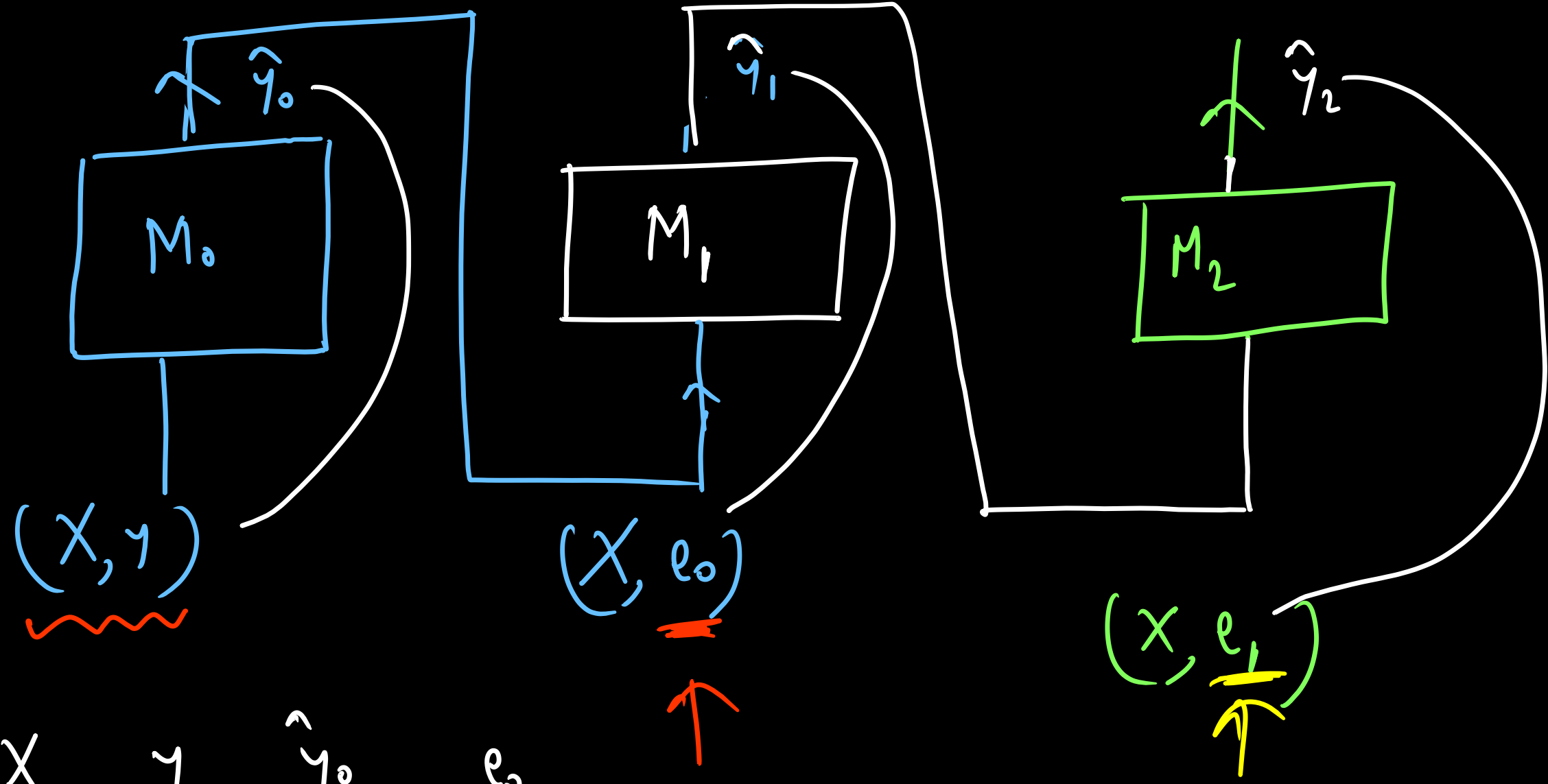
$$e = (y - \hat{y})$$

 \hat{y}  (x, y)

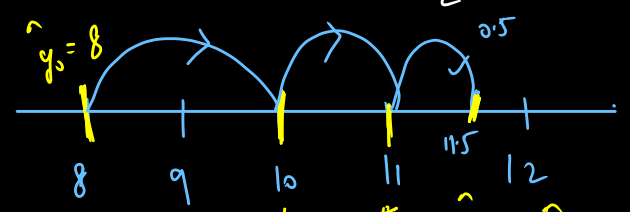
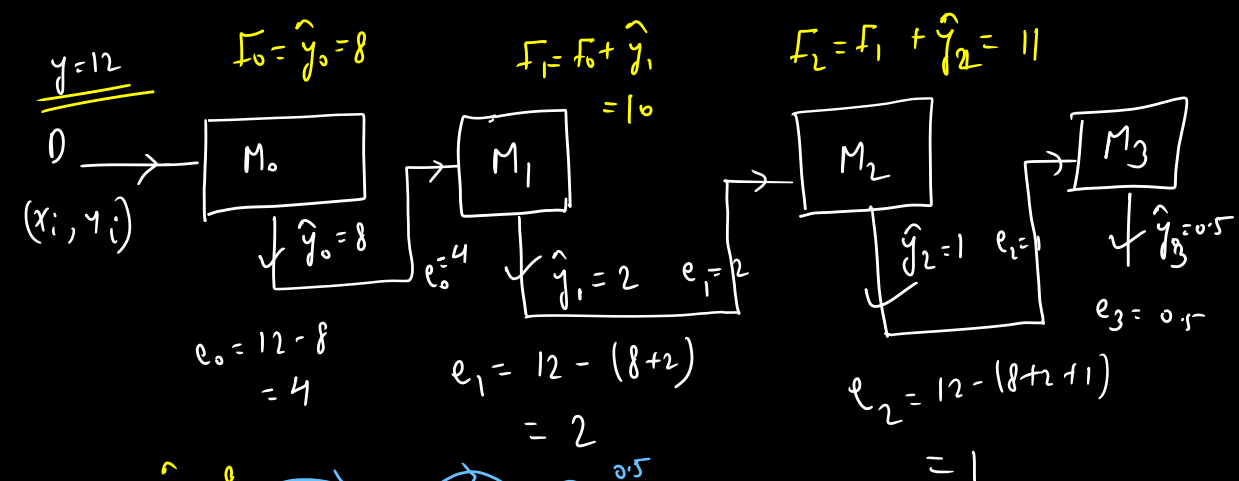
$$e_0 = y - \hat{y}_0$$

$$e_1 = y - (\hat{y}_0 + \hat{y}_1)$$

$$e_2 = y - (\hat{y}_0 + \hat{y}_1 + \hat{y}_2)$$



<u>X</u>	<u>y</u>	<u>\hat{y}_0</u>	<u>e_0</u>
3.5	2	1.5	
3.8	2	1.8	
9	2.5	6.5	



$F_0 = \hat{y}_0 = 8$
 $F_1 = \hat{y}_0 + \hat{y}_1 = 10$
 $F_2 = \hat{y}_0 + \hat{y}_1 + \hat{y}_2 = 11$

$e_0 = 4$
 $e_1 = 2$
 $e_2 = 1$

F_2 low bias
 4 low variance

→ High bias + Low Variance

→ Low bias + Low Variance

✓
 $\gamma = 0.1$
 ↳ Contribution of each of Base learner reduces

$F_0 = \hat{y}_0$
 $F_1 = F_0 + \gamma \hat{y}_1$
 $F_2 = F_1 + \gamma \hat{y}_2$
 $= F_0 + \gamma (\hat{y}_1 + \hat{y}_2)$

$F_m = F_0 + \gamma \sum_{i=1}^m \hat{y}_i$

$F_0 + \gamma_1 \hat{y}_1 + \gamma_2 \hat{y}_2$
 $F_0 + \gamma (\hat{y}_1 + \hat{y}_2)$

•> Each model is called base learner or weak learner

•> Initial base learner will have higher error

•> Later base learner try to reduce the error made by previous base learner.

M_0 \rightarrow Average Model

x	y
-	-
-	-
-	-
-	-
-	-

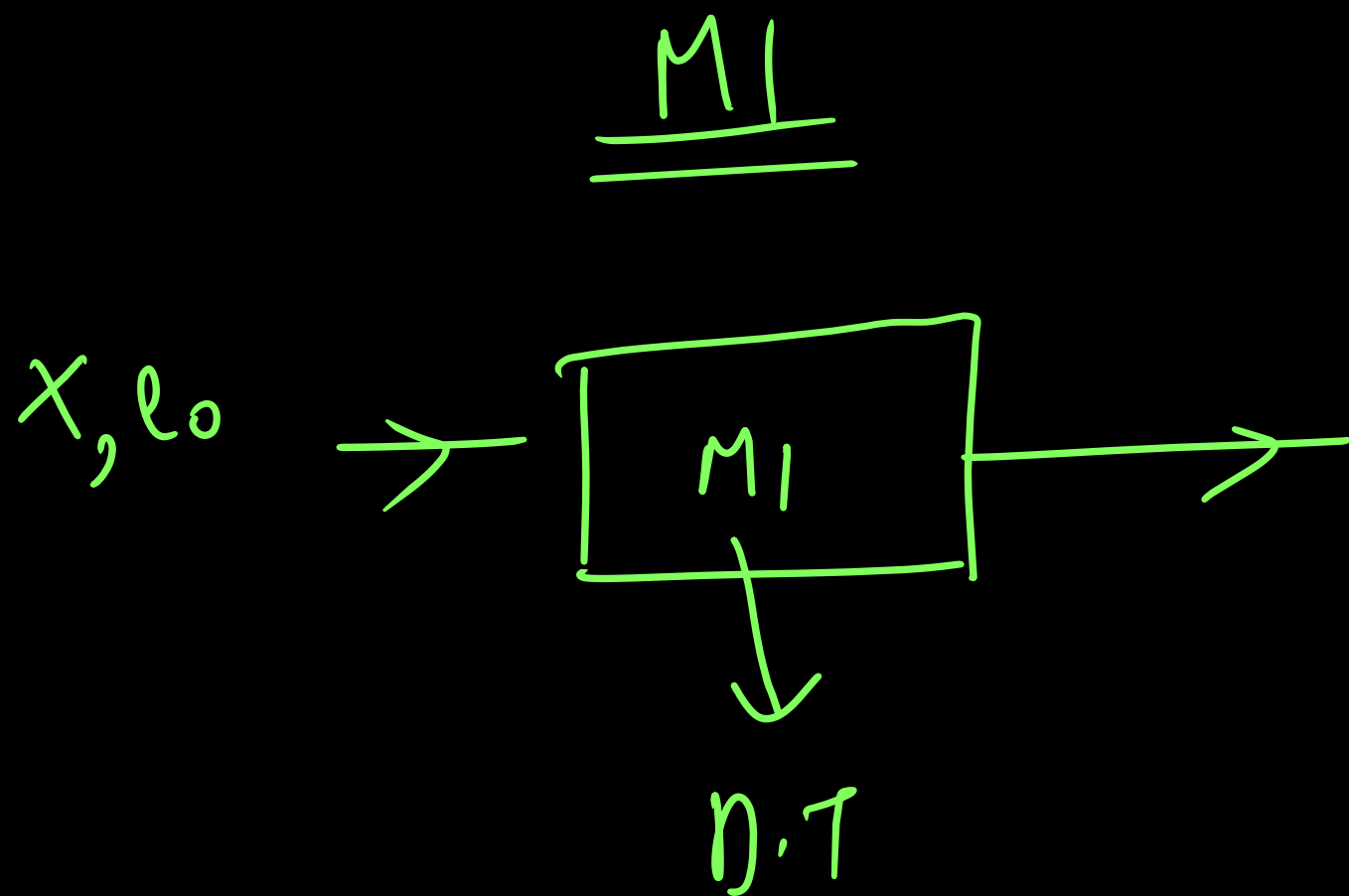
$\rightarrow \bar{y}$



\rightarrow High bias
(Underfit)

& low variance

$$e_0 = y - \hat{y}_0 = y - \bar{y}$$



Underfit (high bias & variance)

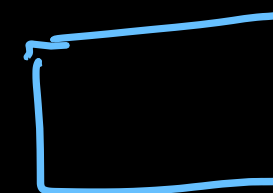
→ Break until 22:18

X		y	$f_0(x)$	$e_0 = y - f_0(x)$	
Height	Gender	Weight(y)	\hat{y}_0		
1.6	M	82	65.75	16.25	$\rightarrow M_0$
1.5	F	55	65.75	-10.75	
1.4	F	61	65.75	-4.75	
1.4	■ M	65	65.75	-0.75	
					$\hat{y}_0 = \frac{82 + 55 + 61 + 65}{4}$ $= 65.75$

(x, e_0)

Height	Gender	Weight(y)	err _o
1.6	M	82	16.25
1.5	F	55	-10.75
1.4	F	61	-4.75
1.4	M	65	-0.75

f_0



65.75

Height	Gender	Weight(y)	err ₀
1.6	M	82	16.25
1.5	F	55	-10.75
1.4	F	61	-4.75
1.4	M	65	-0.75

e_0

\hat{y}_1
7.75
-7.75
-7.75
7.75

F_0
65.75
65.75
65.75
65.75

$0.1 \leftarrow F_1 = F_0 + \gamma \hat{y}_1$

y | e_1
82 | 15.48
55 | -9.97
61 | -3.97
65 | -1.52

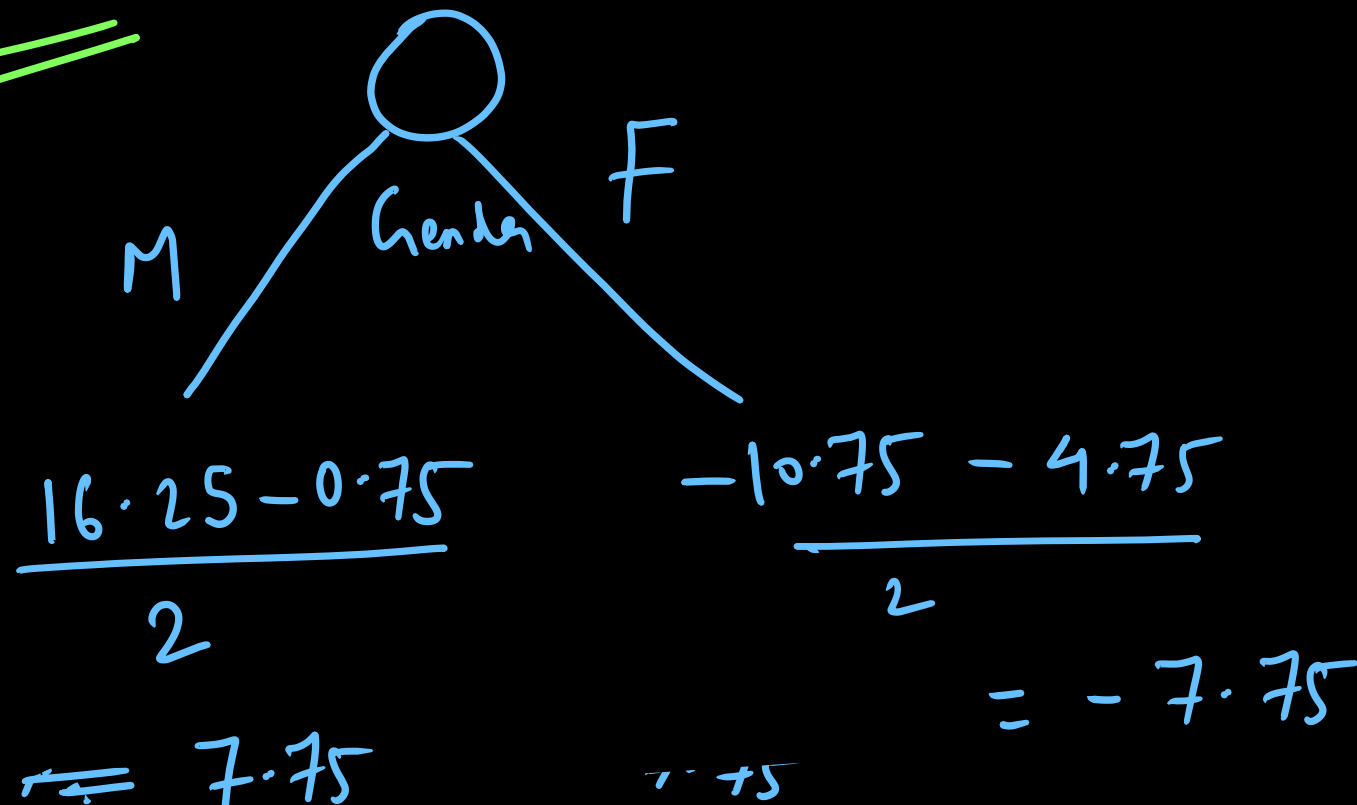
$F_0 = 0$

M_0

65.75

+

M_1



Height	Gender	Weight(y)	err ₀
1.6	M	82	16.25
1.5	F	55	-10.75
1.4	F	61	-4.75
1.4	M	65	-0.75

e₀

$$F_1 = F_0 + \gamma \hat{y}_1$$

66.52
64.97
64.97
66.52

y

82

55

61

65

e₁

15.48

-9.97

-3.97

-1.52

\hat{y}_2

15.48

-5.15

-5.15

-5.15

$$F_2 = F_1 + \gamma \hat{y}_2$$

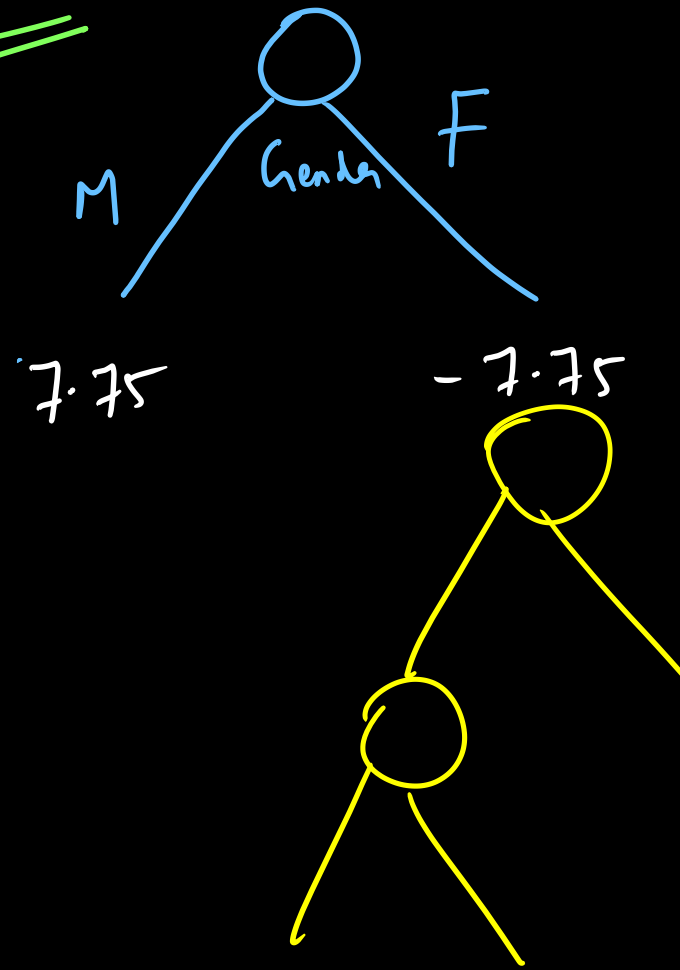
F₀ = 0

M₀

65.75

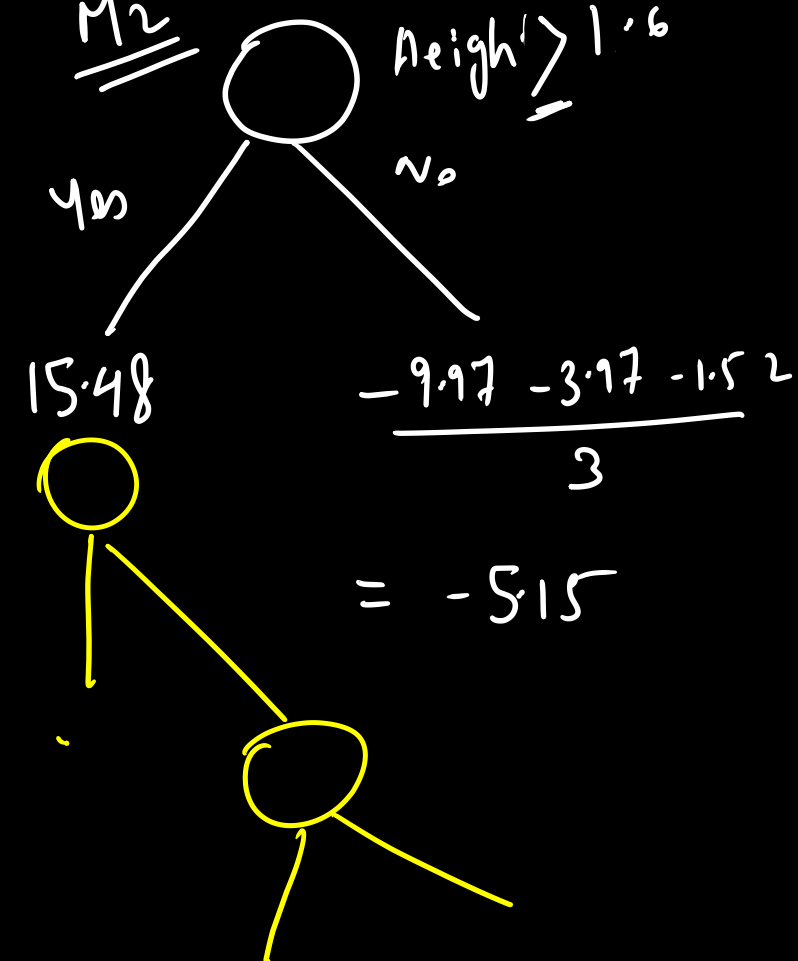
+

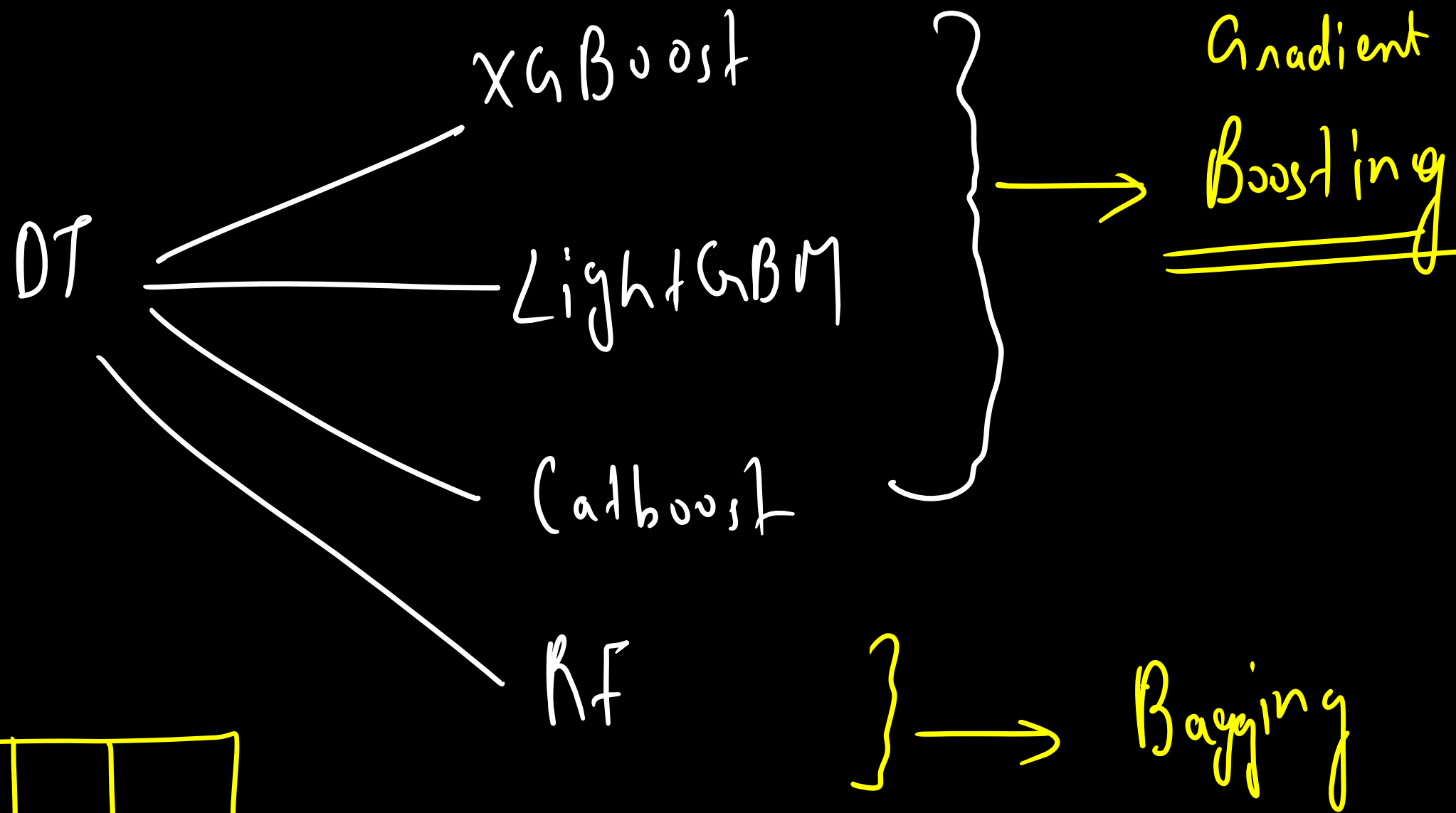
M₁



+

M₂





Tabular

Gradient Boosting Algorithm

1. Initialize model with a constant value:

$$F_0(x) = \underset{\gamma}{\operatorname{argmin}} \sum_{i=1}^n L(y_i, \gamma)$$

2. for $m = 1$ to M :

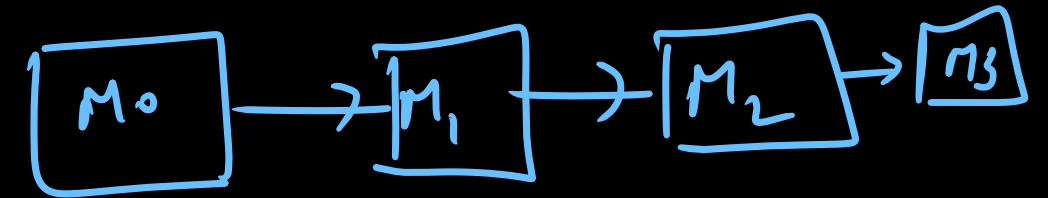
2-1. Compute residuals $r_{im} = - \left[\frac{\partial L(y_i, F(x_i))}{\partial F(x_i)} \right]_{F(x)=F_{m-1}(x)}$ for $i = 1, \dots, n$

2-2. Train regression tree with features x against r and create terminal node reasions R_{jm} for $j = 1, \dots, J_m$

2-3. Compute $\gamma_{jm} = \underset{\gamma}{\operatorname{argmin}} \sum_{x_i \in R_{jm}} L(y_i, F_{m-1}(x_i) + \gamma)$ for $j = 1, \dots, J_m$

2-4. Update the model:

$$F_m(x) = F_{m-1}(x) + v \sum_{j=1}^{J_m} \gamma_{jm} \mathbf{1}(x \in R_{jm})$$



$y - \hat{y}_0 \rightarrow M_1$
 $y - \hat{F}_1 \rightarrow M_2$

T.A \uparrow T.E \downarrow
Test A \uparrow Test E \downarrow
] \rightarrow Best fit \leftarrow

T.A \uparrow T.E \downarrow
Test A \downarrow Test E \uparrow
] \rightarrow Overfit \leftarrow

T.A \downarrow T.E \uparrow
Test A \downarrow Test E \uparrow
] \rightarrow Underfit \leftarrow

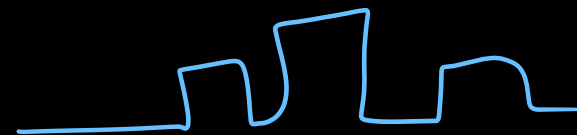
Data drift

Test 1 month



X	Y
x_1, x_2	

Training



Test 1 year

Test



$$y^{(i)} = \sum w_j x_j^{(i)} + w_0$$

$w_j, w_0 \rightarrow \text{Normal}$

$$y^{(i)} = \sigma(\sum w_j x_j^{(i)} + w_0) \rightarrow \text{Bernoulli}$$

Height	Weight	Y