

→ Boosting

→ Analogy

Anunag

$$e_0 = 10^{-3} \approx 7$$

2

Mon

A hand-drawn diagram illustrating a menu structure. The word "Menu" is written in yellow at the top left. A white curved arrow originates from the letter "u" and points downwards and to the right. Below this, there are four horizontal lines: one yellow line and three green lines, representing levels of menu depth.

Shobhit

$$e_2 = 10 - (3 + 3 + 4) \geq 0$$

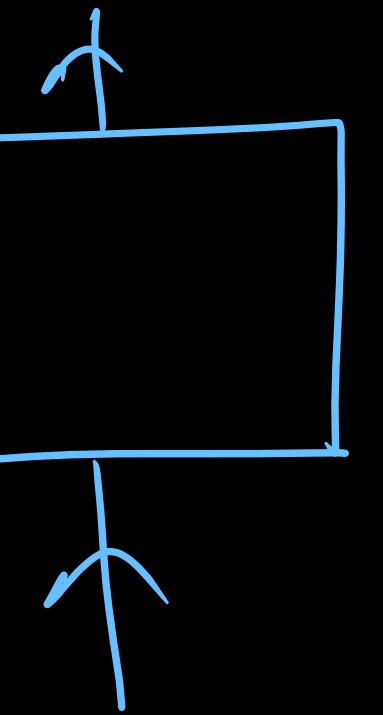
3 problem
Solved
by Monu

$$e_1 = 10 - (3+3) \\ = 4$$

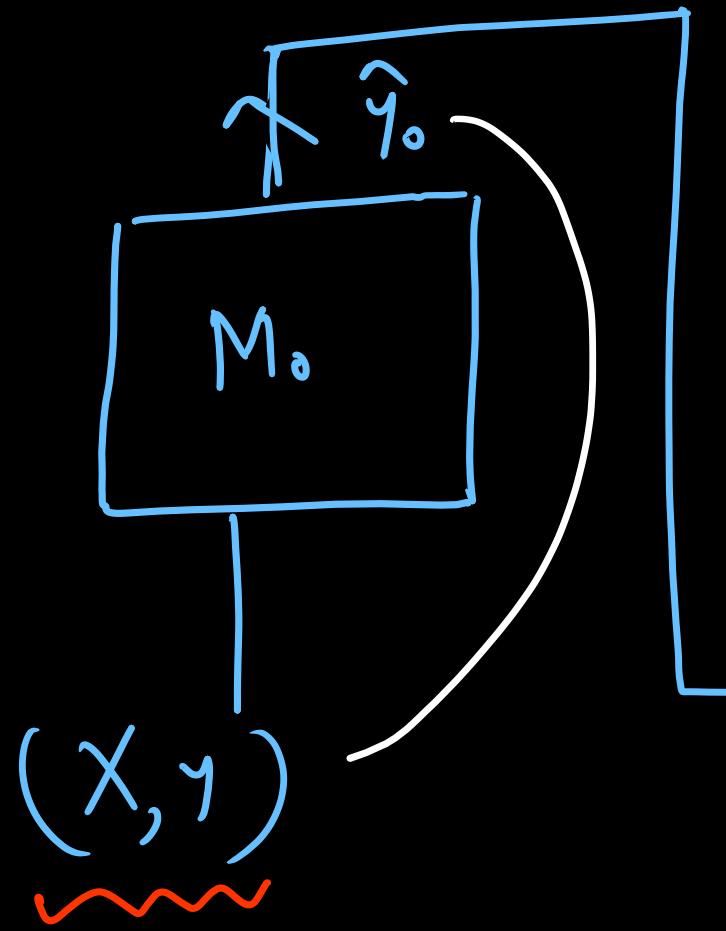
4 problem
by
Shobhit

→ Each friend is solving the unsolved question left by his/her previous friend

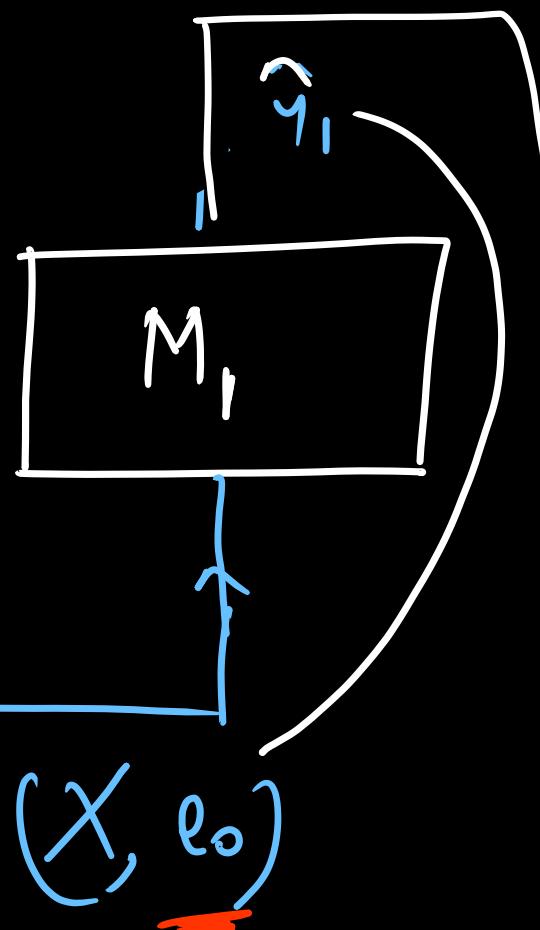
$$e = (\gamma - \hat{\gamma})$$


$$(x, \gamma)$$

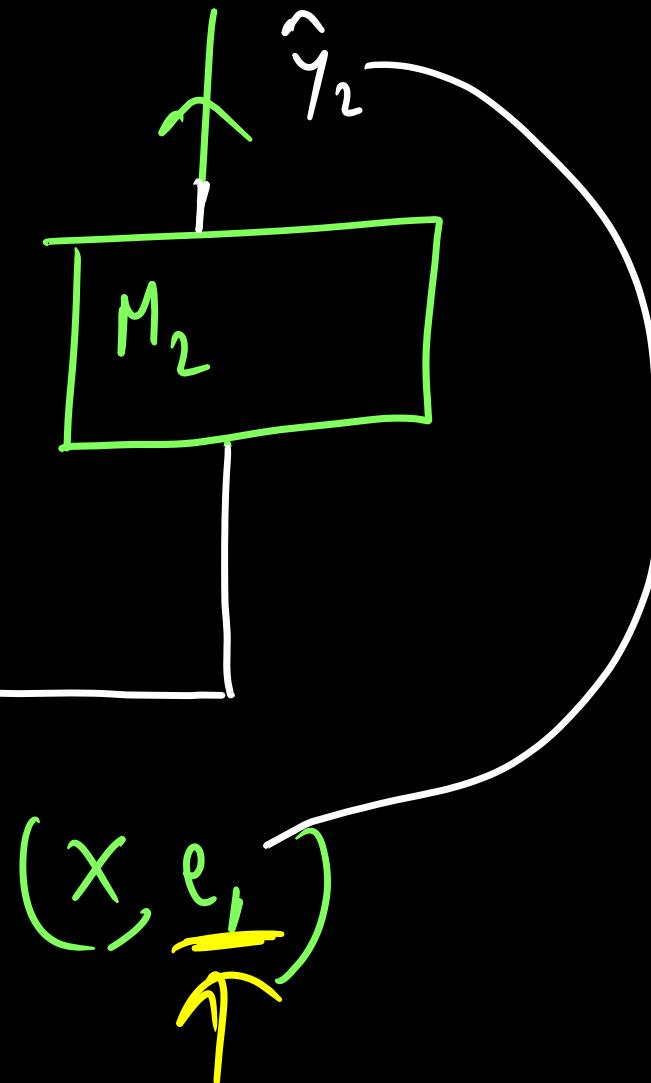
$$e_0 = \gamma - \hat{\gamma}_0$$



$$e_1 = \gamma - (\hat{\gamma}_0 + \hat{\gamma}_1)$$

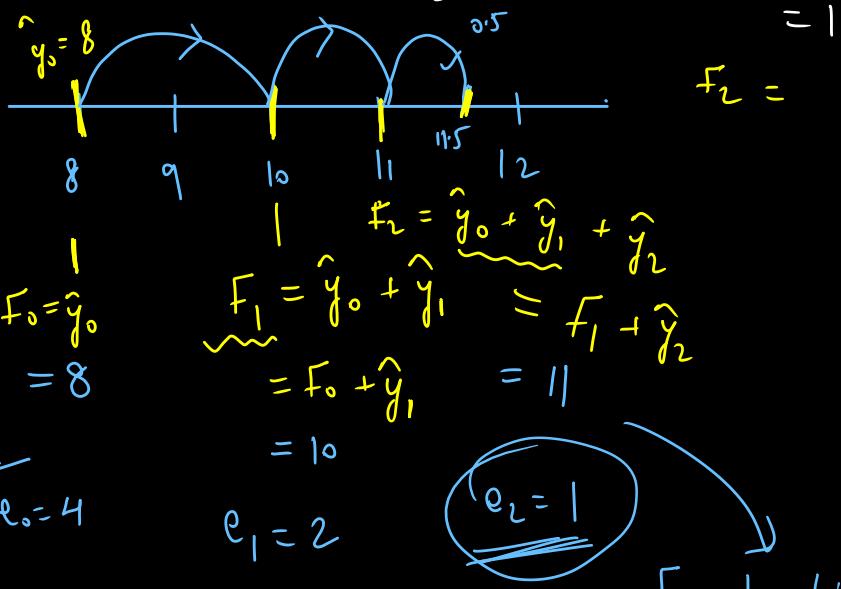


$$e_2 = \gamma - (\hat{\gamma}_0 + \hat{\gamma}_1 + \hat{\gamma}_2)$$



<u>X</u>	<u>γ</u>	$\hat{\gamma}_0$	e_0
3.5	2	1.5	
3.8	2	1.8	
9	2.5	6.5	

$$\begin{array}{l}
 \text{y=12} \\
 0 \rightarrow M_0 \quad f_0 = \hat{y}_0 = 8 \\
 (x_i, y_i) \quad \downarrow \hat{y}_0 = 8 \quad e_0 = 12 - 8 = 4 \\
 M_1 \quad f_1 = f_0 + \hat{y}_1 = 10 \\
 \downarrow \hat{y}_1 = 2 \quad e_1 = 12 - (8+2) = 2 \\
 M_2 \quad f_2 = f_1 + \hat{y}_2 = 11 \\
 \downarrow \hat{y}_2 = 1 \quad e_2 = 12 - (8+2+1) = 1 \\
 M_3 \quad \hat{y}_3 = 0.5 \quad e_3 = 0.5
 \end{array}$$



y=12
 $e_0 = 4$
 $e_1 = 2$
 $e_2 = 1$

\rightarrow High bias + Low Variance
 \rightarrow Low bias + Low Variance

\checkmark
 $\gamma = 0.1$
 \hookrightarrow Contribution of each of base learners reduces

$$f_m = f_0 + \gamma \sum_{i=1}^m \hat{y}_i$$

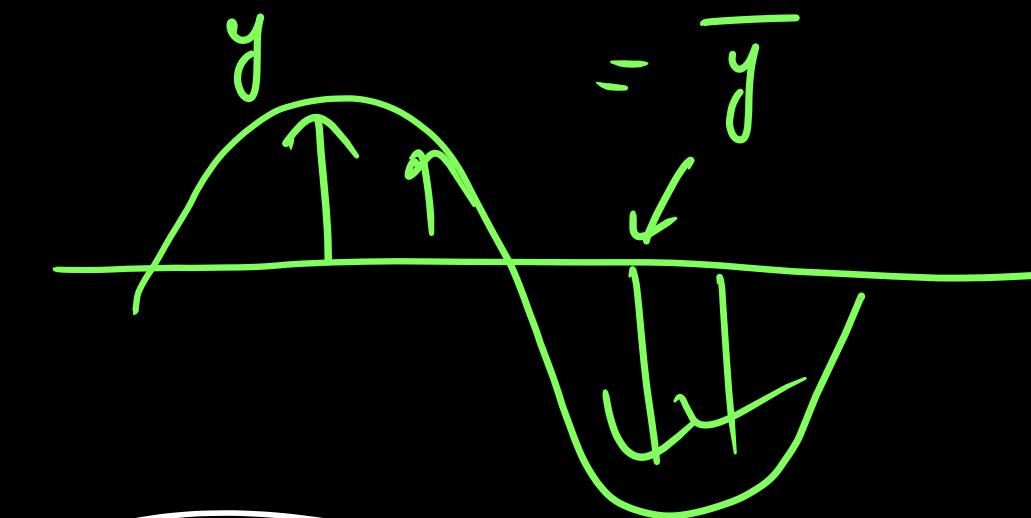
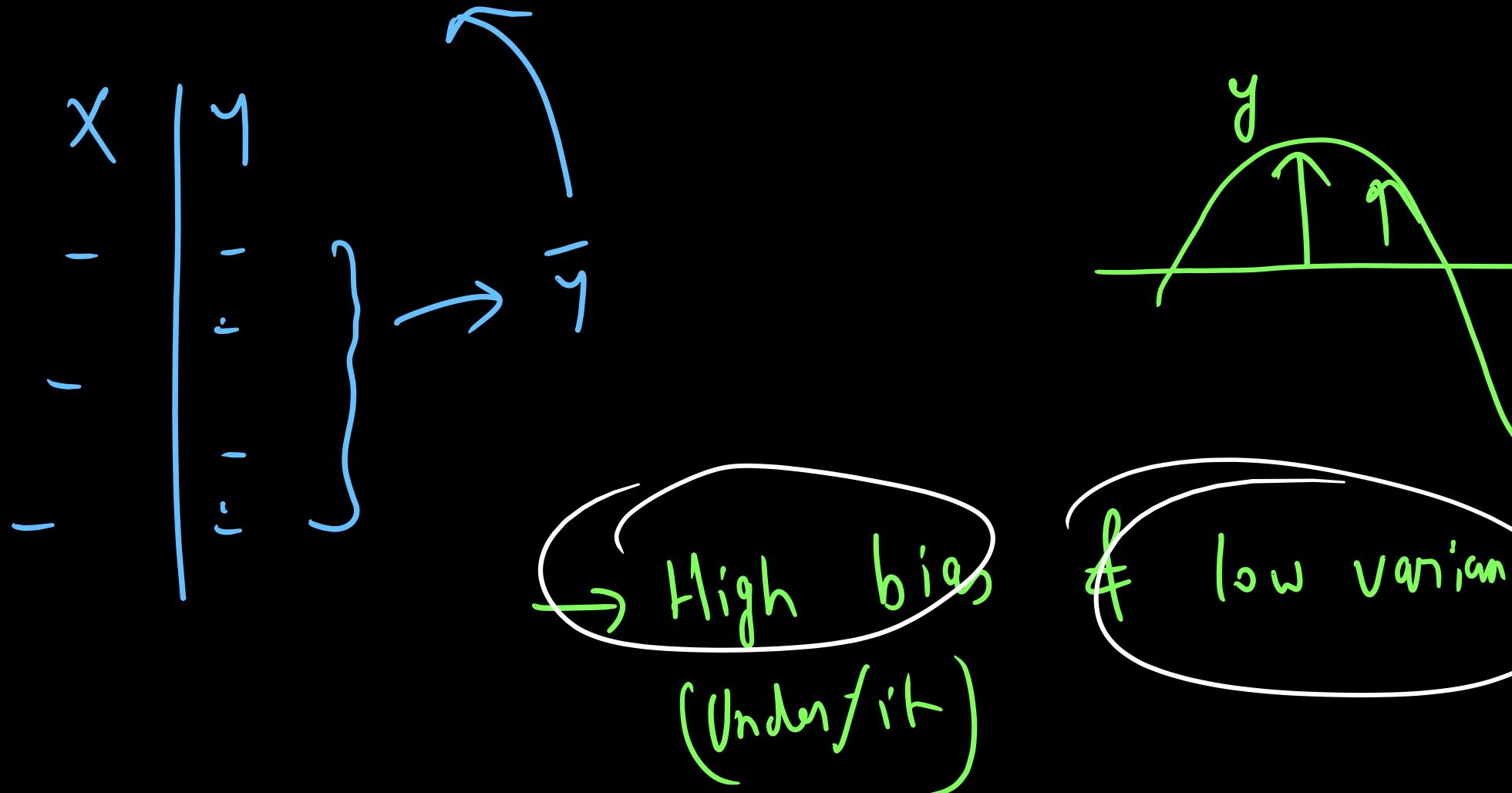
$$\frac{f_0 + \gamma_1 \hat{y}_1 + \gamma_2 \hat{y}_2}{\dots}$$

$$f_0 + \gamma (\hat{y}_1 + \hat{y}_2)$$

$$\begin{aligned}
 f_0 &= \hat{y}_0 && \checkmark \\
 f_1 &= f_0 + \gamma \hat{y}_1 \\
 f_2 &= f_1 + \gamma \hat{y}_2 && \checkmark \\
 &= f_0 + \gamma (\hat{y}_1 + \hat{y}_2)
 \end{aligned}$$

- > Each model is called base learner or weak learner
- > Initial base learner will have higher error
- > Later base learners try to reduce the error made by previous base learner.

M₀ → Average Model



$$e_0 = y - \hat{y}_0 = y - \bar{y}$$



→ break until 22:18

\tilde{x}	\tilde{y}	$f_0(x)$	\hat{y}_0	$e_0 = y - f_0(\tilde{x})$	M_0
Height	Gender	Weight(y)			
1.6	M	82	65.75	16.25	
1.5	F	55	65.75	-10.75	
1.4	F	61	65.75	-4.75	
1.4	M	65	65.75	-0.75	
					$\hat{y}_0 = \frac{82 + 55 + 61 + 65}{4} = 65.75$

Height	Gender	Weight(y)	err ₀
1.6	M	82	16.25
1.5	F	55	-10.75
1.4	F	61	-4.75
1.4	M	65	-0.75

f_0

 65.75

Height	Gender	Weight(y)	err_0
1.6	M	82	16.25
1.5	F	55	-10.75
1.4	F	61	-4.75
1.4	M	65	-0.75

e_0

\hat{y}_1	f_0	$f_1 = f_0 + \gamma \hat{y}_1$	y	e_1
7.75	65.75	66.52	82	15.48
-7.75	65.75	64.97	55	-9.97
-7.75	65.75	64.97	61	-3.97
7.75	65.75	66.52	65	-1.52

$F_0 = 0$ M_1 M_0 +

$16.25 - 0.75$
 $\frac{2}{2} = 7.75$

$-10.75 - 4.75$
 $\frac{2}{2} = -7.75$

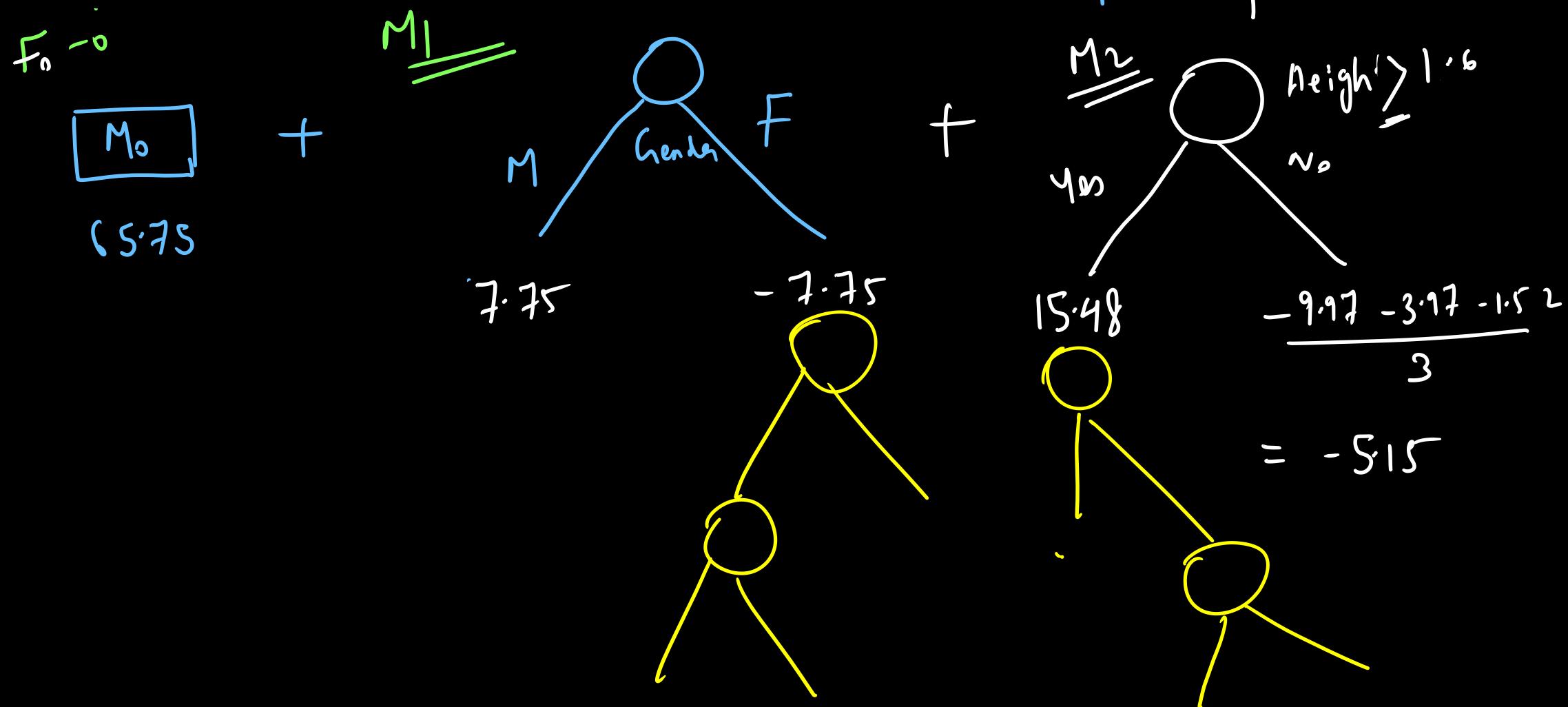
Height	Gender	Weight(y)	err_0
1.6	M	82	16.25
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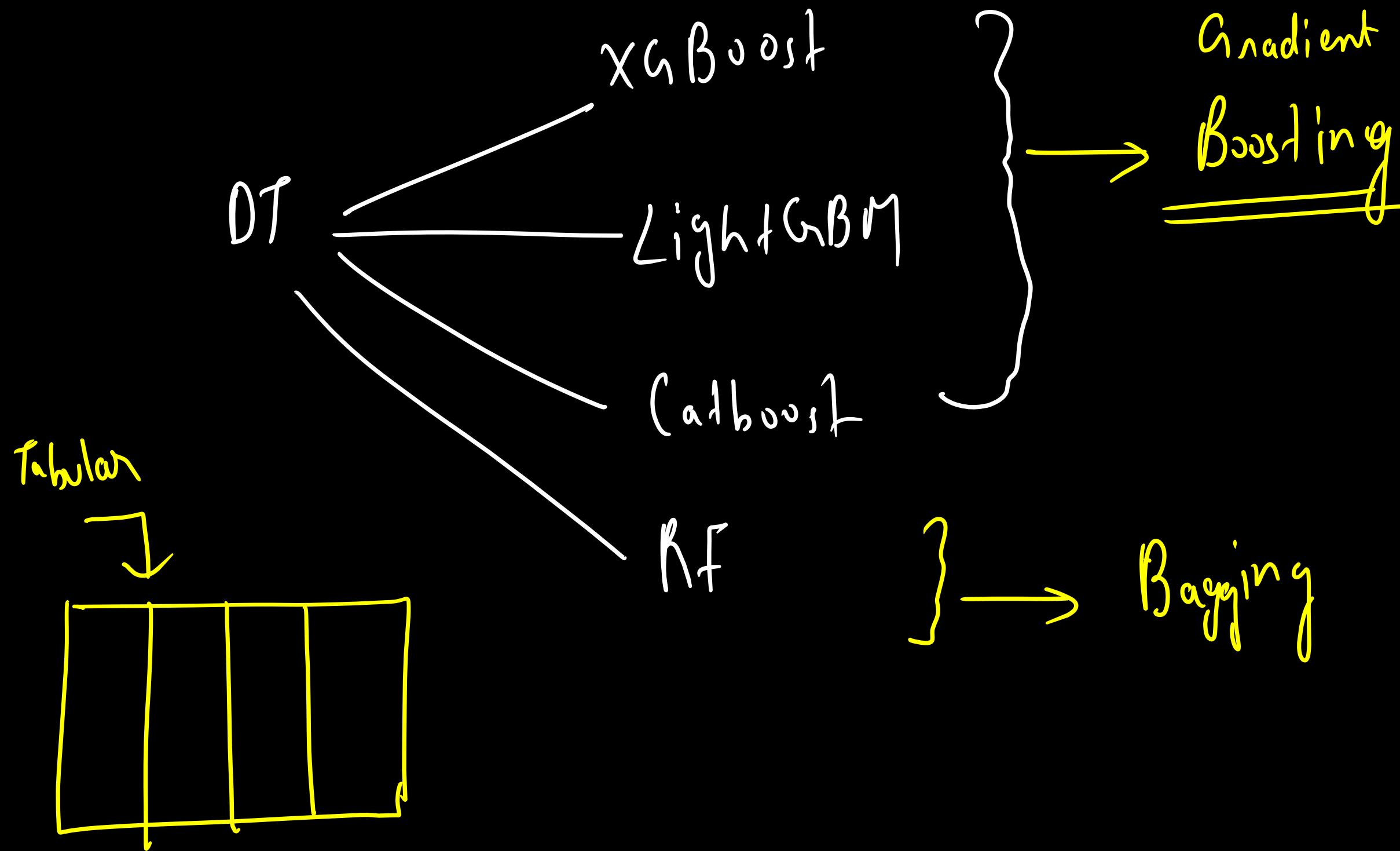
$$e_0$$

$$f_1 = f_0 + \gamma \hat{y}_1$$

$$f_2 = f_1 + \gamma \hat{y}_2$$

$$\begin{array}{c|c|c|c} \hat{y} & e_1 & \hat{y}_2 & \\ \hline 82 & 15.48 & 15.48 & \\ 55 & -9.97 & -5.15 & \\ 61 & -3.97 & -5.15 & \\ 65 & -1.52 & -5.15 & \end{array}$$





Gradient Boosting Algorithm

1. Initialize model with a constant value:

$$F_0(x) = \underset{\gamma}{\operatorname{argmin}} \sum_{i=1}^n L(y_i, \gamma)$$

2. for $m = 1$ to M :

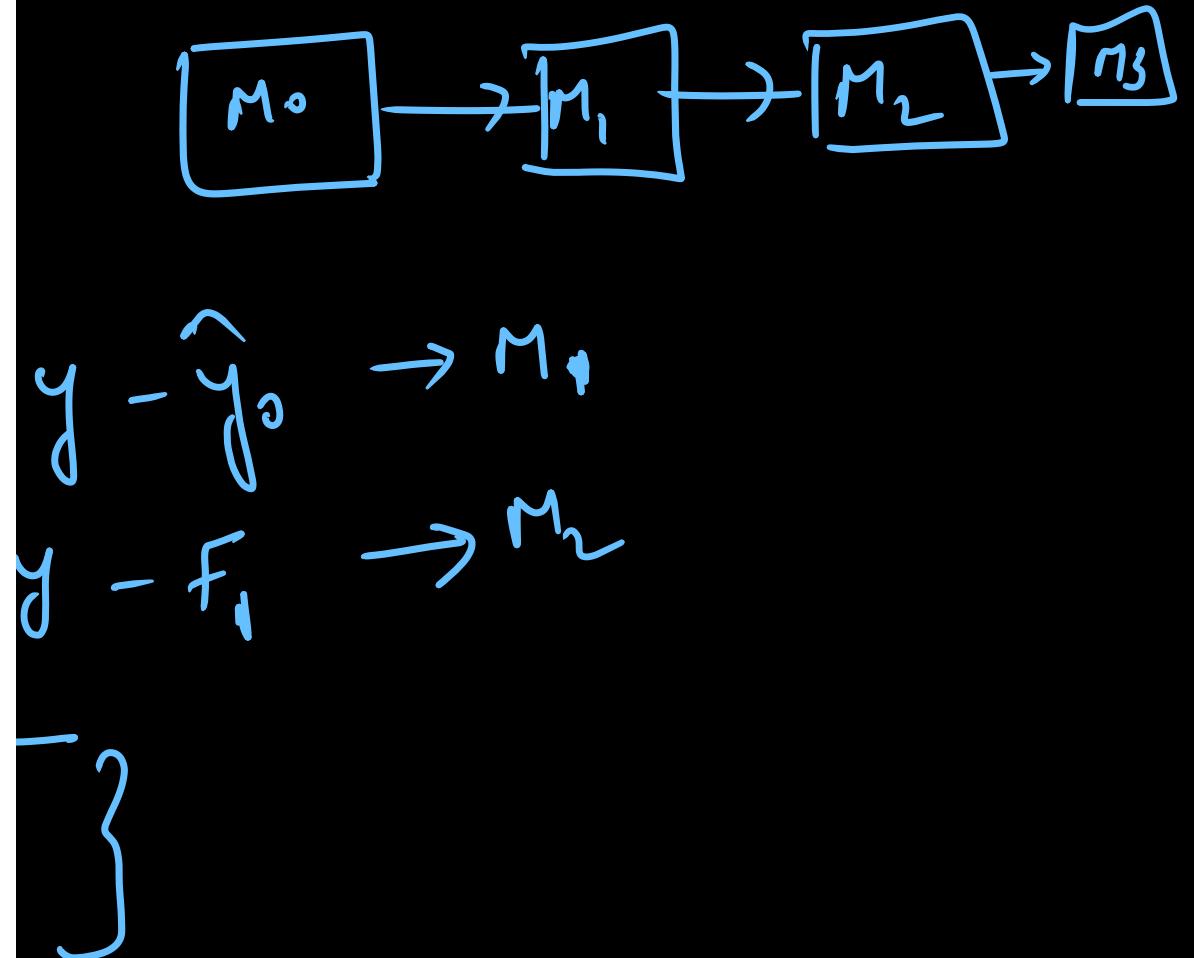
2-1. Compute residuals $r_{im} = - \left[\frac{\partial L(y_i, F(x_i))}{\partial F(x_i)} \right]_{F(x)=F_{m-1}(x)}$ for $i = 1, \dots, n$

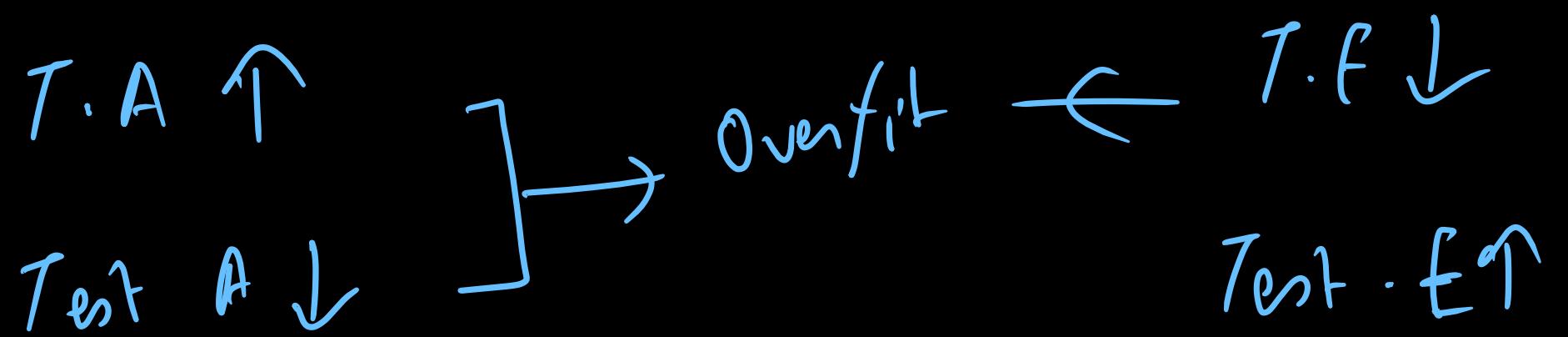
2-2. Train regression tree with features x against r and create terminal node regions R_{jm} for $j = 1, \dots, J_m$

2-3. Compute $\gamma_{jm} = \underset{\gamma}{\operatorname{argmin}} \sum_{x_i \in R_{jm}} L(y_i, F_{m-1}(x_i) + \gamma)$ for $j = 1, \dots, J_m$

2-4. Update the model:

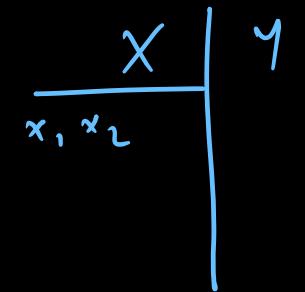
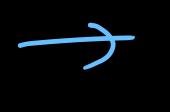
$$F_m(x) = F_{m-1}(x) + \nu \sum_{j=1}^{J_m} \gamma_{jm} \mathbf{1}(x \in R_{jm})$$



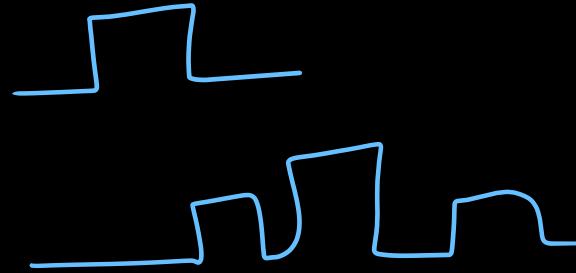


Data drift

Int 1 month

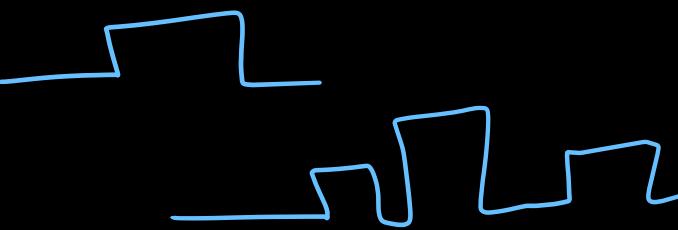


Training



Test corner

Test



$$y^{(i)} = \{ w_j x_j^{(i)} + w_0 \}$$

$w_j, w_0 \rightarrow \text{Normal}$

$$y^{(i)} = \sigma(\{ w_j x_j^{(i)} + w_0 \}) \rightarrow \text{Bernoulli}$$

