

Logistic Regression - 2

L MLE - Maximum Likelihood Estimation

Loss → - log loss + $\lambda \sum_{j=1}^d |w_j|$ - Lasso

$$\hat{y} = \sigma(\underline{w^T} \cdot \underline{x} + w_0)$$

- log loss + $\lambda \sum_{j=1}^d w_j^2$ - Ridge

① ✓

Regularization + logistic regression

Validation → Hyperparameter tuning

$$SD = \frac{\overline{(x_i - \mu)^2}}{\sigma}$$

\overline{w} - ? ✓

σ - ✓

Accuracy y \hat{y} if $y = \hat{y}$ # correct prediction

$\rightarrow | \quad \hat{y}$
| \hat{y}
| ✓ - Corrected

$y \neq \hat{y}$ Wrong prediction

| 0 - Wrong
0 0 ✓
| 0 ✗

$$\text{Accuracy} = \frac{\# \text{ of Correct Predictions}}{\text{Total # of Predictions}} = \frac{80}{100} = 0.8$$

(80%)

Regularization, Hyper Parameter tuning

Feature importance

$$\hat{y} = \frac{1}{1 + e^{-(w_0 + w_1 x_1 + w_2 x_2 + \dots + w_d x_d)}}$$

Odds

Prob of success
Prob of failure

$$\frac{0.5}{0.5} \quad ? \quad 1:1$$

odds of success $\frac{4/5}{1/5}$ $4:1$

Prob of failure

feature imp.

Linear Reg.

$$\hat{y} = w_0 + w_1 x_1$$

$$\hat{y}_1 \quad (w_1)$$

$x_1 \uparrow$ unit

$$\therefore P(y=1|x) = \frac{1}{1+e^{-(z)}}$$

$$z = w_0 + w_1 x_1 + \dots + w_d x_d$$

$$P(1+e^{-z}) = 1$$

$$P + P \cdot e^{-z} = 1$$

$$P e^{-z} = 1 - P$$

$$e^{-z} = \frac{1-P}{P}$$

$$-z = \log\left(\frac{1-P}{P}\right)$$

$$z = \log\left(\frac{P}{1-P}\right)$$

$$\boxed{\log\left(\frac{P}{1-P}\right) = w_0 + w_1 x_1 + \dots + w_d x_d}$$

$$\frac{P}{1-P} = \frac{\text{Prob of } y=1 \text{ (succ)}}{\text{Prob of } y=0 \text{ (failure)}} = \text{odds?}$$

$$\begin{aligned} & \text{y=1} \uparrow \quad w_0 + w_1 x_1 \\ & \text{or y=1} \downarrow \quad w_0 + w_1 x_1 \end{aligned} \log(\text{odds}) = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_d x_d$$

$w_1 = x_1 \uparrow$

Predict heart failure

$$P(y=1 \text{ heart fail}) = w_0 + w_1 \text{ age} + w_2 \text{ BM/1} - w_3 \text{ good con (HDL)}$$

$P(y=0)$
no fail

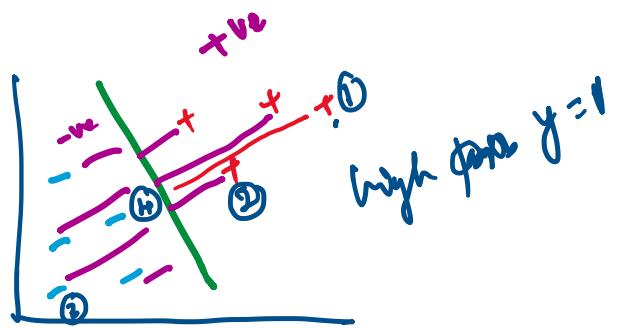
$$\boxed{\log(\text{odds})} - w ?$$

w - high value - very imp
w - small - less imp

- w +ve - increases odd,
increases chem risk
w -ve - decreases chem risk

10:11

Geometrical intuition.



$$z = \frac{w^T x + w_0}{\|w\|}$$

$$\|w\|=1$$

$$\log\left(\frac{P}{1-P}\right) = z \uparrow$$

prob of $y=1$
prob of $y=0$

$$\log\left(\frac{P}{1-P}\right) = z$$

$$P = \frac{1}{1+e^{-z}}$$

-? Sigmoid

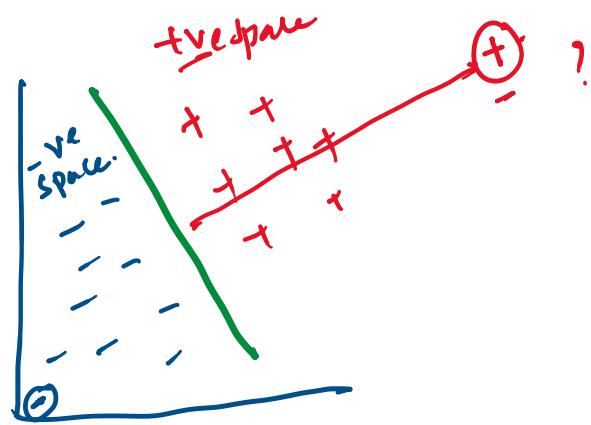
Assumptions ?

① NO multi collinearity

EDA ✓

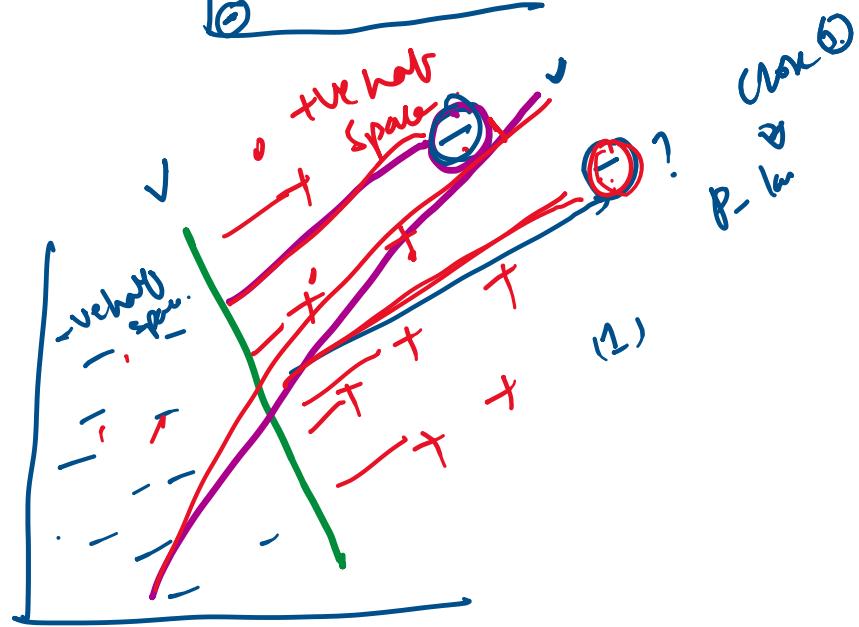
② outlier effect.

outlier is on the
Same half space.



tve
-ve tve half
-ve half
 $\log(p_i)$

Metric?



log loss

$$-\left[y_i \log p_i + (1-y_i) \log(1-p_i) \right]$$

$$0 \log(p_i)$$

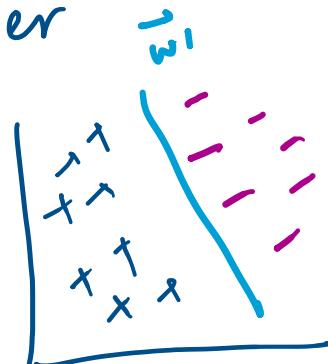
$$1 \log(1-0.99)$$

$$-\left[4.6 ? \right]$$

$$\log(0.01)$$

① if outliers are on same half space
No impact

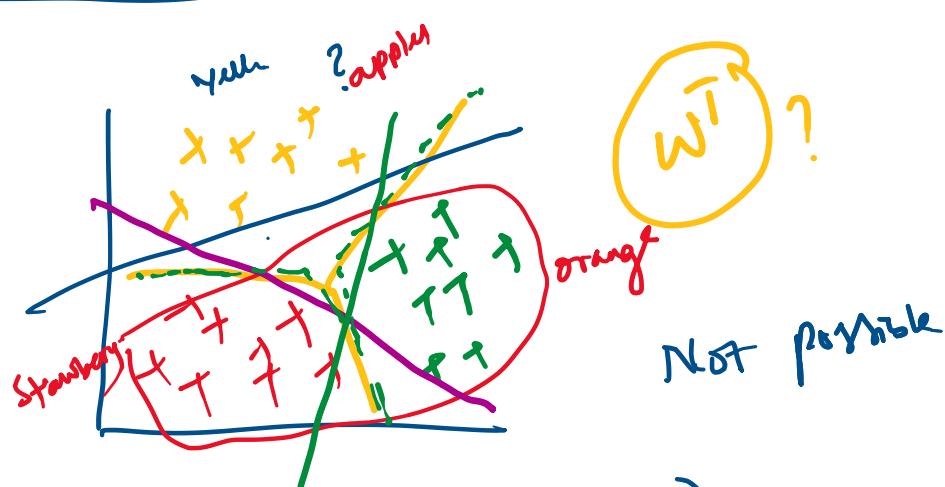
② if outliers are on opp half spaces
hyperplane will twisted to reduce
error of outlier \vec{w}_1



Binary Classification

Multiclass Classification

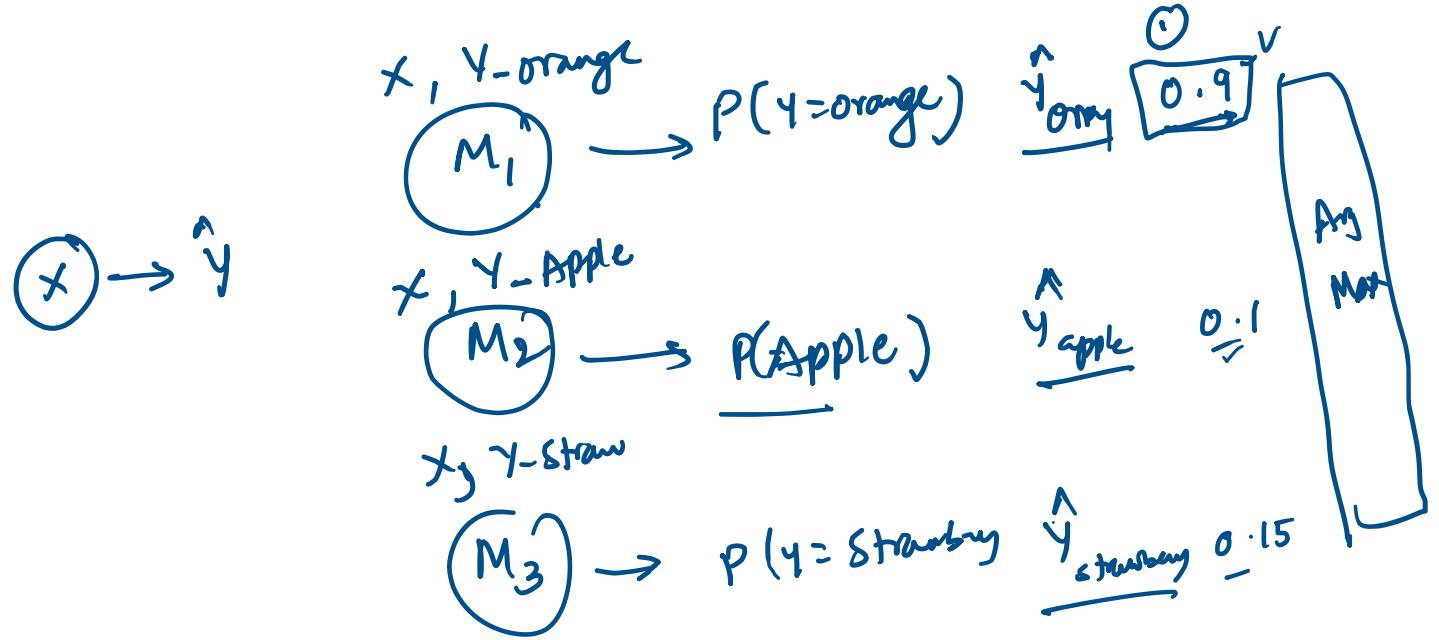
logistic Regression?



Hack : one vs rest (OVR)



y	y_{Orange}	y_{Apple}	$y_{\text{Strawberry}}$
Apple	0	1	0
Orange	1	0	0
Strawberry	0	0	1
Apple	1	0	0
Orange	0	0	0



one

$$P(\text{orange}) = 0.7$$

$$P(\text{apple}) = 0.1$$

$$P(\text{straw}) = 0.25$$

Soft max :

$$\frac{0.7}{0.7 + 0.3 + 0.25} = 0.5$$

$$\frac{0.3}{0.7 + 0.3 + 0.25} = 0.24$$

$$\frac{0.25}{0.7 + 0.3 + 0.25} = 0.22$$

