

# Determination of Z Matrices

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EEE 306 – FINAL PROJECT

GROUP B1-1B

1006071-74

# Objectives

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1. Creating a system that can take as input at least a six bus system.
2. Find the Z matrices, that is positive sequence matrix ( $Z_1$ ), negative sequence matrix ( $Z_2$ ) and zero sequence matrix ( $Z_0$ ).

# Approach to the Solution

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Take the system as input in database (with a specific schema).



Load the system into memory and create positive and zero sequence graph of the network.



Use the Zbus building algorithm on the graph to obtain the  $Z_1$ ,  $Z_2$  and  $Z_0$  matrices.

# Assumptions

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- All the elements are balanced.
- All impedance data are given in per phase equivalent.
- There are no regulating transformers in the network (for which  $Z_{bus}$  is asymmetric).

# Database Schema

```
◦ {  
◦ "bus1":{  
◦   "N":1,  
◦   "V":13.8  
◦ },  
◦ "bus2":{  
◦   "N":2,  
◦   "V":69  
◦ }  
◦ }
```

Name of the bus

Bus ID

Bus Voltage in KV

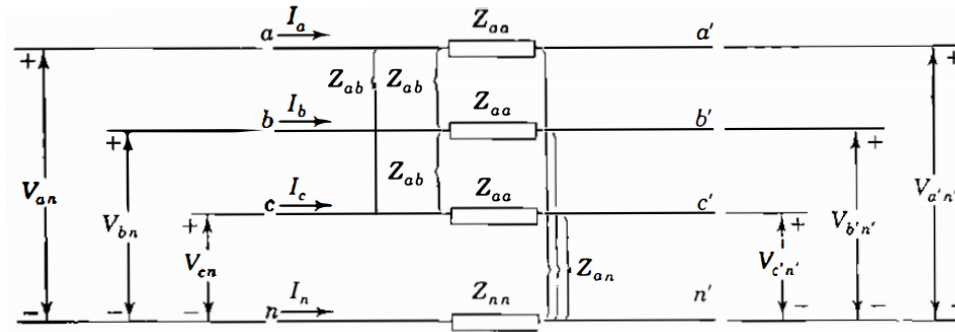
The database used is an open standard known as **JavaScript Object Notation**. Some of the properties of such schema is:

- Each curly braces { } there is a structure inside it.
- Each elements in the structure has a (key, value) pair.
  - For example, “N” is a key, and “1” is it’s value.
  - For the base structure “bus1” is a key and the structure that defines the bus is a value.
- Base structure does not have any key.
- A key can be any string that can be name of a variable in the programming environment. In our case MATLAB.
- Value can be string or numeric.

Each network is described by the following files:

- busdata.json
- transformerdata.json
- generatordata.json
- linedata.json
- loaddata.json

# Transmission Line



$Z_{aa}$  = Balanced transmission line impedance  
 $Z_{ab}$  = Mutual Impedance between a, b, c phase  
 $Z_{nn}$  = Impedance of neutral line  
 $Z_{an}$  = Mutual Impedance between a, b, c line and neutral

$$Z_0 = Z_{aa} + 2 Z_{ab} + 3 Z_{nn} - 6 Z_{an}$$

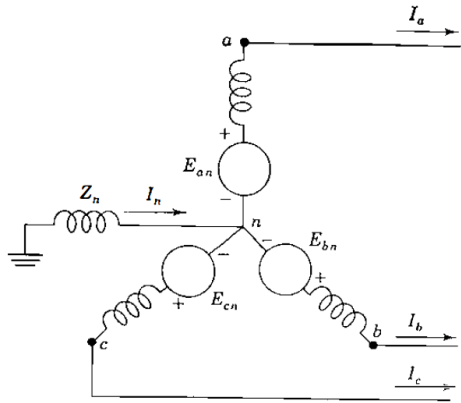
$$Z_1 = Z_2 = Z_{aa} - Z_{ab}$$

In database:

```

"L34":{
  "bus1": 2,
  "bus2": 3,
  "Raa": 0,
  "Xaa": 0.2,
  "Xab": 0.05,
  "Xan": 0.08333,
  "Rnn": 0,
  "Xnn": 0.1
}
    
```

# Synchronous Machine



$$Z_0 = Z - 2 Z_m + 3 Z_n$$

$$Z_1 = Z_2 = Z + 2 Z_m$$

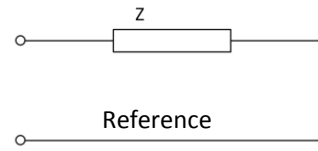
$Z$  = Machine Impedance  
 $Z_m$  = Mutual Impedance  
 $Z_n$  = Neutral to reference impedance

In Database (one machine is shown):

```
"g2":{  
  "bus": 4,  
  "rated_kv": 13.8,  
  "rated_mva": 200,  
  "sub_tr_R": 0,  
  "sub_tr_X": 0.14667,  
  "sub_tr_M": 0.05333,  
  "tr_R": 0,  
  "tr_X": 0,  
  "tr_M": 0,  
  "R": 0.01,  
  "X": 0.12,  
  "M": 0,  
  "winding": "Yg",  
  "Rn": 0,  
  "Xn": 0.05,  
  "activeGeneration_MVA": 100  
}
```

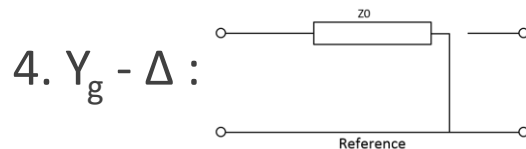
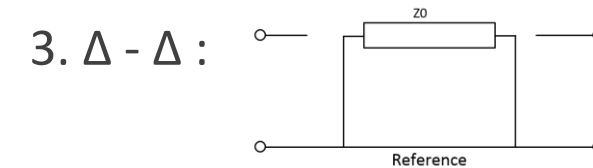
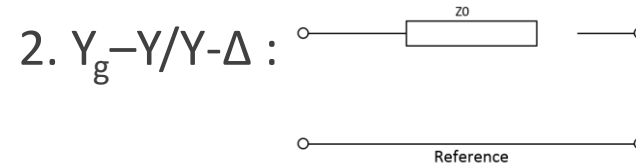
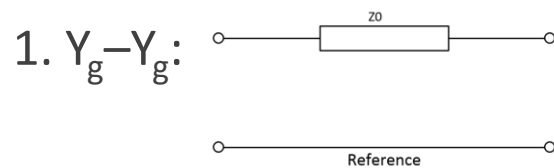
# Fixed Tap Transformers

Positive/Negative Sequence Network:



$$Z_1 = Z_2 = Z$$

Zero Sequence Networks:



1.  $Z_0 = Z + 3 Z_N + 3 Z_n$
2.  $Z_0 = Z + 3 Z_N$  or  $Z_0 = Z + 3 Z_n$  (A new bus at floating impedance is created in the zero sequence network)
3.  $Z_0 = Z$  (Can be neglected in the network)
4.  $Z_0 = Z + 3 Z_N$  or  $Z_0 = Z + 3 Z_n$



# Creating the Network Graph

The diagram shows a bus impedance matrix with rows and columns indexed by bus numbers. Annotations with arrows point to specific elements:

- Reference Bus:** Points to the column header '0'.
- Other Bus ID:** Points to the row header '1'.
- Impedance between Bus 1 and reference:** Points to the element  $Z$  at row 1, column 0.
- Impedance between Bus 1 and Bus 2:** Points to the element  $Z2$  at row 1, column 2.

	0	1	2	....
0	0	$Z$	...	
1	$Z$	0	$Z2$	
2	...	$Z2$	0	
.				
.				
.				

- The algorithm for building network graph goes through all the information given, one by one.
- For each element  $Z0$ ,  $Z1$  (or  $Z2$ ) impedance is added in their respective graph.
- For the floating impedance (Usually from transformers or synchronous machines) in  $Z0$  network, a new node is created in the corresponding network graph.

# Building $Z_1$ , $Z_2$ And $Z_0$ Matrices

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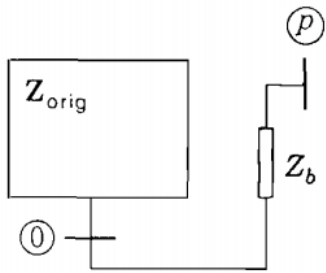
The following approaches are taken for all sequence networks ( $Z_1$  and  $Z_2$  matrices are equal, so calculating one should suffice):

1. First, the impedances connected to the reference are dealt with (Case 1).
2. Then all other buses which has a impedance between two of them are processed (Case 2 & Case 3). Kron reduction is used where necessary.

# Z Matrices: Case 1

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Adding an impedance to new bus from reference:

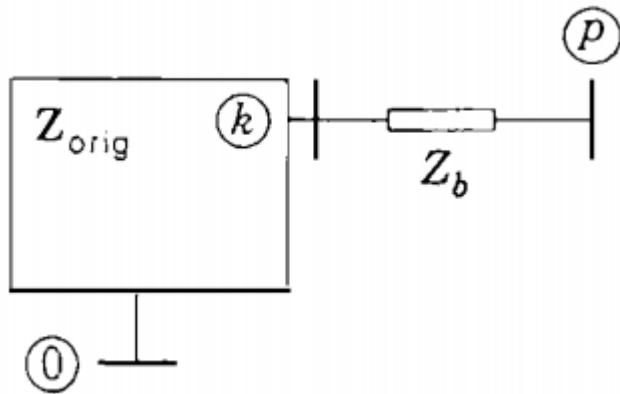


1. Add a new column and row (assuming  $k$ ) in  $Z_{bus}$  matrix.
2. All elements are zero initialized.
3.  $Z_{kk} = Z_b$

# Z Matrices: Case 2

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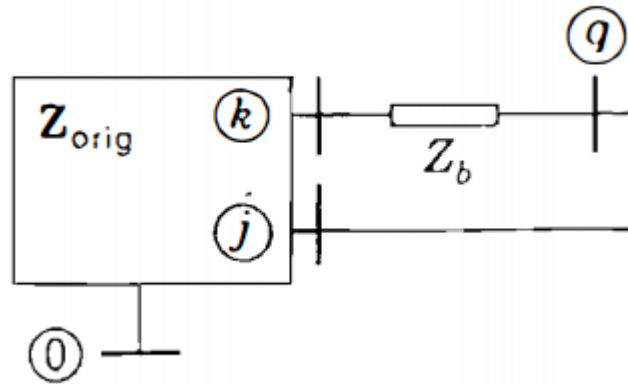
Adding branch  $Z_b$  from existing bus  $k$  to a new bus  $p$



1. Create a new column and row in  $Z_{bus}$  matrix
2. Copy column  $k$  and row  $k$  into new column  $p$  and row  $p$  respectively
3.  $Z_{pp} = Z_{kk} + Z_b$

# Z Matrices: Case 3

Adding branch  $Z_b$  from existing bus  $k$  to a existing bus  $j$



1. Add temporary node  $q$ .
2. Copy (column  $k$ -column  $j$ ) to column  $q$  and copy (row  $k$  - row  $j$ ) to row  $q$ .
3.  $Z_{qq} = Z_{jj} + Z_{kk} - 2 Z_{jk} + Z_b$
4. Use Kron reduction.

For all index other than  $q$ :

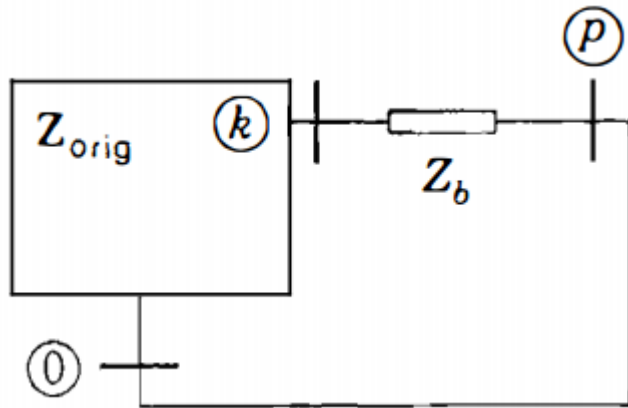
$$Z_{hi(new)} = Z_{hi} - Z_{hq} Z_{qi} / Z_{qq}$$

5. Delete node  $q$ .

# Z Matrices: Case X

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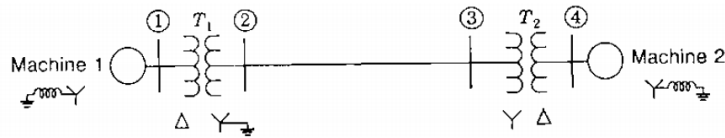
Adding a impedance to an existing bus from reference:



This case is not considered because we have already considered all the impedances from reference to bus in first step. For which, all such impedances are considered in case 1.

# Tests: 4 Bus System

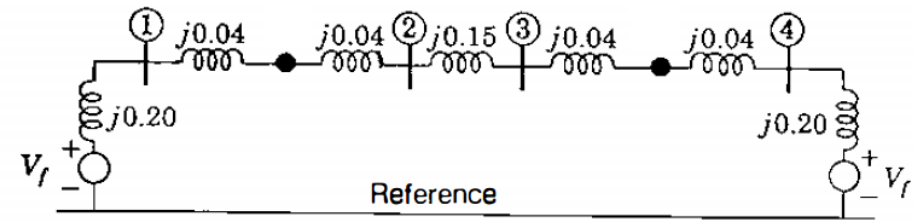
This system is taken from the book 'Power System Analysis' (Indian Edition) by John J. Grainger and William D. Stevenson, Jr. Chapter 12, Example 12.1.



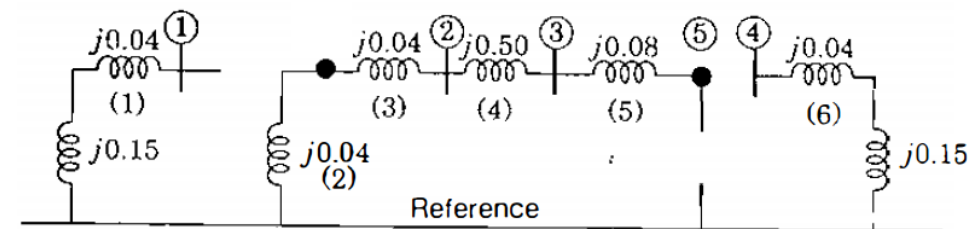
One Line Diagram (Above)

Positive Sequence Network (Top Right)

Zero Sequence Network (Bottom Right)



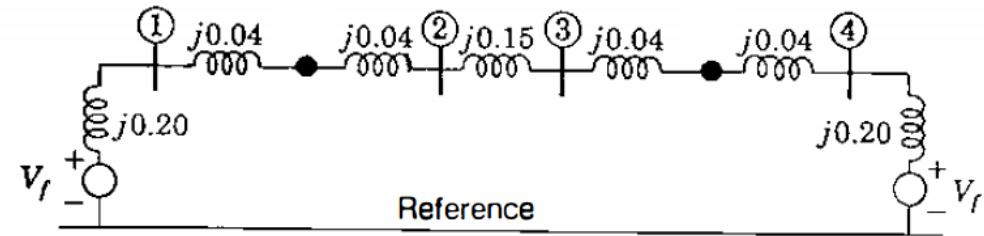
(a)



(b)

# Tests: 4 Bus System (Network Graph)

Positive Sequence Graph:



	0	1	2	3	5
0	0	$0 + 0.2000i$	0	0	$0 + 0.2000i$
1	$0 + 0.2000i$	0	$0 + 0.0800i$	0	0
2	0	$0 + 0.0800i$	0	$0 + 0.1500i$	0
3	0	0	$0 + 0.1500i$	0	$0 + 0.0800i$
4	$0 + 0.2000i$	0	0	$0 + 0.0800i$	0



# Tests: 4 Bus System ( $Z_1$ )

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Calculated  $Z_1 = Z_2$ :

	1	2	3	4
1	$0 + 0.1437i$	$0 + 0.1211i$	$0 + 0.0789i$	$0 + 0.0563i$
2	$0 + 0.1211i$	$0 + 0.1696i$	$0 + 0.1104i$	$0 + 0.0789i$
3	$0 + 0.0789i$	$0 + 0.1104i$	$0 + 0.1696i$	$0 + 0.1211i$
4	$0 + 0.0563i$	$0 + 0.0789i$	$0 + 0.1211i$	$0 + 0.1437i$

This matches the  $Z_{\text{bus}}^{(1)}$  of the network:

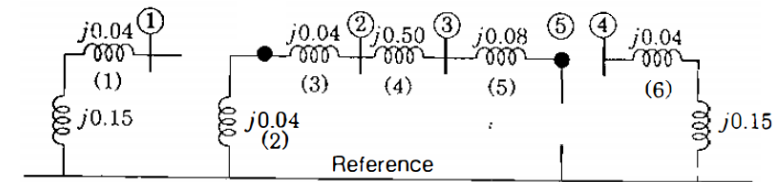
$$Z_{\text{bus}}^{(1)} = Z_{\text{bus}}^{(2)} = \begin{matrix} & \textcircled{1} & \bullet & \textcircled{3} & \textcircled{4} \\ \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} & \begin{bmatrix} j0.1437 & j0.1211 & j0.0789 & j0.0563 \\ j0.1211 & j0.1696 & j0.1104 & j0.0789 \\ j0.0789 & j0.1104 & j0.1696 & j0.1211 \\ j0.0563 & j0.0789 & j0.1211 & j0.1437 \end{bmatrix} \end{matrix}$$

# Tests: 4 Bus System ( $Z_0$ )

Similarly  $Z_0$  is calculated.

Network Graph:

	0	1	2	3	4	5
0	0	$0 + 0.1900i$	$0 + 0.0800i$	0	$0 + 0.1900i$	0
1	$0 + 0.1900i$	0	0	0	0	0
2	$0 + 0.0800i$	0	0	$0 + 0.5000i$	0	0
3	0	0	$0 + 0.5000i$	0	0	$0 + 0.0800i$
4	$0 + 0.1900i$	0	0	0	0	0
5	0	0	0	$0 + 0.0800i$	0	0



$Z_0$  Matrix:

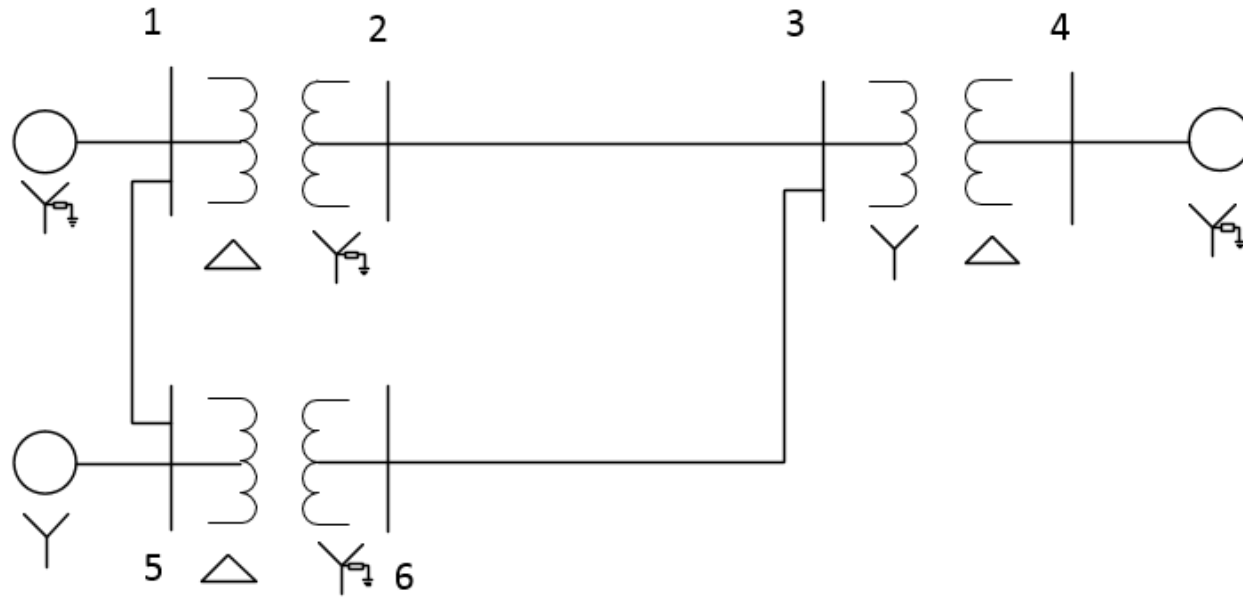
	1	2	3	4
1	$0 + 0.1900i$	0	0	0
2	0	$0 + 0.0800i$	$0 + 0.0800i$	0
3	0	$0 + 0.0800i$	$0 + 0.5800i$	0
4	0	0	0	$0 + 0.1900i$

Which matches the result from the book:

$$\mathbf{Z}_{\text{bus}}^{(0)} = \begin{matrix} & \textcircled{1} & \bullet & \textcircled{3} & \textcircled{4} \\ \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} & \begin{bmatrix} j0.19 & 0 & 0 & 0 \\ 0 & j0.08 & j0.08 & 0 \\ 0 & j0.08 & j0.58 & 0 \\ 0 & 0 & 0 & j0.19 \end{bmatrix} \end{matrix}$$

# Tests: 6 Bus System

One Line Diagram (System Described in the Database folder: Sample 3):



# Tests: 6 Bus System (Results)

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$Z_1$  or  $Z_2$  :

0 + 0.1055i	0 + 0.0885i	0 + 0.0566i	0 + 0.0404i	0 + 0.0541i	0 + 0.0550i
0 + 0.0885i	0 + 0.1374i	0 + 0.0792i	0 + 0.0566i	0 + 0.0550i	0 + 0.0634i
0 + 0.0566i	0 + 0.0792i	0 + 0.1216i	0 + 0.0869i	0 + 0.0566i	0 + 0.0792i
0 + 0.0404i	0 + 0.0566i	0 + 0.0869i	0 + 0.1192i	0 + 0.0404i	0 + 0.0566i
0 + 0.0541i	0 + 0.0550i	0 + 0.0566i	0 + 0.0404i	0 + 0.1055i	0 + 0.0885i
0 + 0.0550i	0 + 0.0634i	0 + 0.0792i	0 + 0.0566i	0 + 0.0885i	0 + 0.1374i

$Z_0$ :

0 + 0.1900i	0	0	0	0 + 0.1900i	0
0	0 + 0.0622i	0 + 0.0400i	0	0	0 + 0.0178i
0	0 + 0.0400i	0 + 0.0900i	0	0	0 + 0.0400i
0	0	0	0 + 0.1900i	0	0
0 + 0.1900i	0	0	0	0 + 0.4180i	0
0	0 + 0.0178i	0 + 0.0400i	0	0	0 + 0.0622i

# Thank You

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Sincerely,

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# Questions?

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