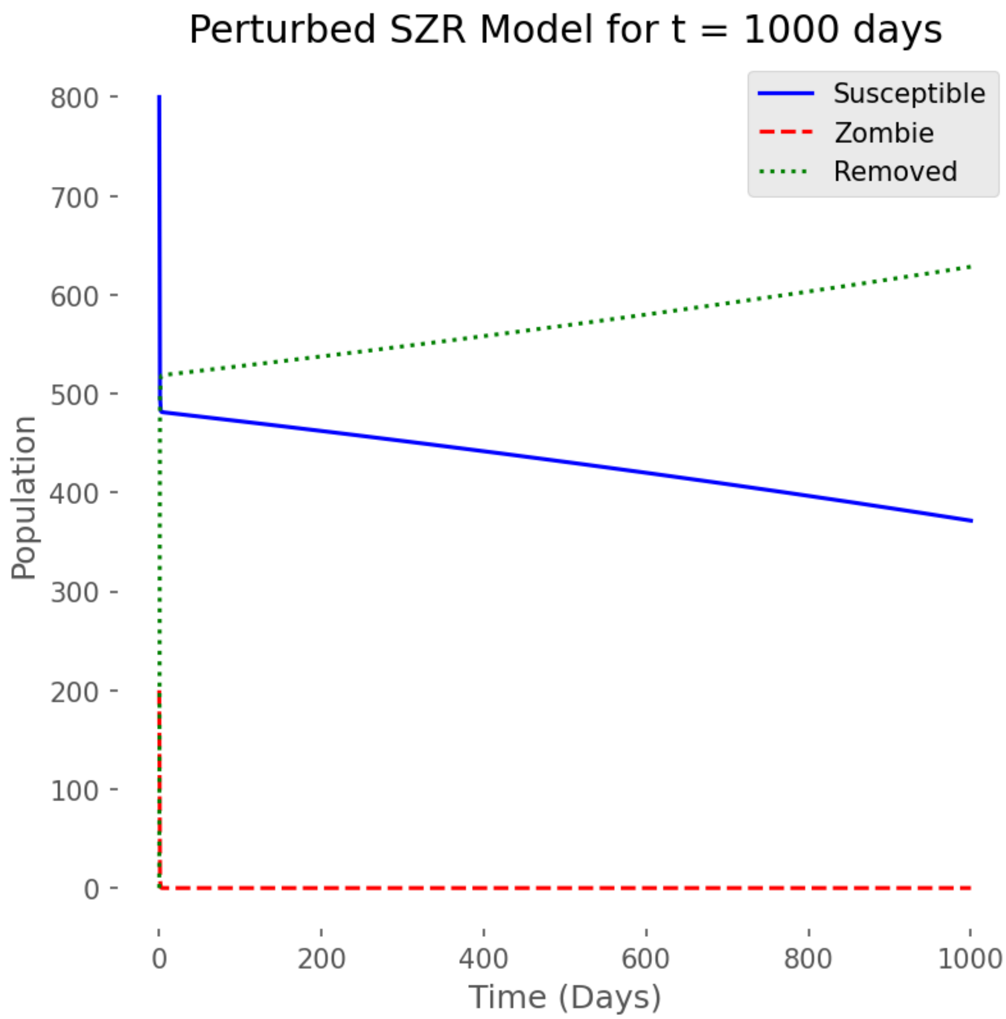
PerturbedSZR_model

September 28, 2021

[4]:
```python
#!/usr/bin/env python
# -*- coding: utf-8 -*-

"""This simple program tries to solve the ODE of the Perturbed SZR model and␣
↪create a numerical solution.
   After that, this program draws the solution graph for the Susceptible,␣
↪Zombie and Recovered
   Population. This simple graph helps us to understand the actual scenario of␣
↪the population
   from the SZR model after a certain number of days. We can change the value␣
↪of the Perturbation
   Parameter Mu to investigate the change of behaviour and stabality of the␣
↪SZR model.

   Reference: Allen, Robert F., Cassandra Jens, and Theodore J. Wendt. 2014.
  "Perturbations in Epidemiological Models". Letters in Biomathematics 1 (2),␣
↪173-80.
  Link- https://doi.org/10.1080/23737867.2014.11414478.

  Part of MS Thesis at Universität Koblenz-Landau
  Note: this program uses the following libraries-  Numpy, Matplotlib, Scipy
  Python Version 3.7
"""

# importing libraries
import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt
from matplotlib.pyplot import figure
figure(figsize=(6, 6), dpi=150)

# For a nice looking plot
plt.style.use('ggplot')

# Parameter values
beta = 0.0095
alpha = 0.005
```

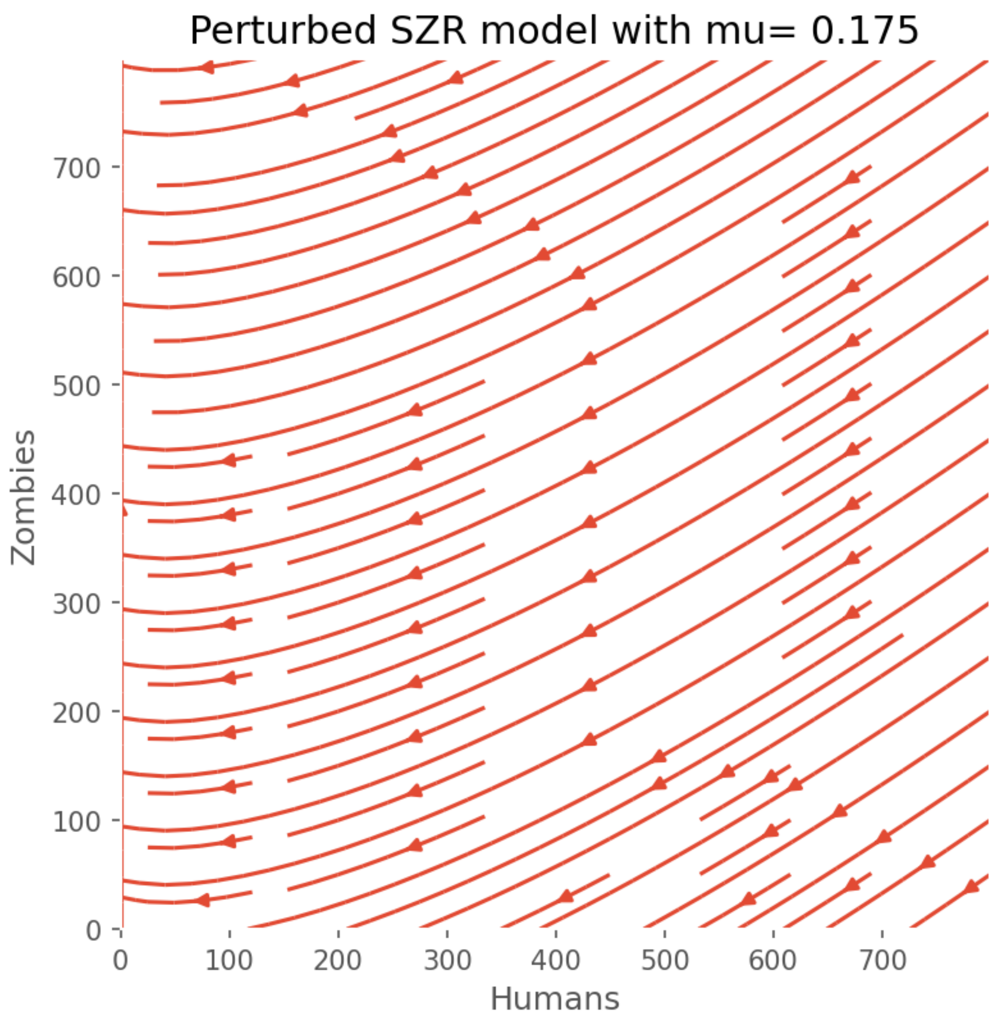
```python
zeta = 0.0001
N = 1000  # initial population

# Perturbation Parameter Mu, change this value to see different results
u = 0.175

# Defining Function of the SZR ODE
def model(t, x):
    S = x[0] # Intital Susceptible Population
    Z = x[1] # Zombies
    dsdt = - beta*S*Z # Susceptible
    dzdt = zeta*(N - S - Z) + beta*S*Z - alpha*(S**(1 + u))*Z # Zombies
    dxdt = [dsdt, dzdt]
    return dxdt

# Initial Condition. Susceptible= 800, Zombie= 200
x0 = [800, 200]

# Time, as in number of days. Total days= 1000
t = np.array([0, 1000])
tspan = np.linspace(t[0], t[1], 1001)

# Calculating numerical solution of the given ODEs
x = solve_ivp(model, t, x0, t_eval=tspan)

time = x.t
susceptible = x.y[0];
zombie = x.y[1];
recovered = (N - susceptible - zombie);

# Plot
plt.plot(time, susceptible, 'b-', label="Susceptible") # Susceptible Population␣
 ↪Graph
plt.plot(time, zombie, 'r--', label="Zombie") # Zombie Population Graph
plt.plot(time, recovered, 'g:', label="Removed") # Recovered Population Graph
plt.ylabel('Population')
plt.xlabel('Time (Days)')
plt.title('Perturbed SZR Model for t = 1000 days')
plt.legend(loc='best')
ax = plt.gca()
ax.set_facecolor('w')
plt.show()

__author__ = "Md Tariqul Islam"
__version__ = "1.0"
__maintainer__ = "Tariqul"
__email__ = "tariquldipu@uni-koblenz.de"
```

```
__status__ = "Final"
```



[10]:
```
#!/usr/bin/env python
# -*- coding: utf-8 -*-

"""This program plots the Phase Portrait for the Perturbed SZR Model

    Reference: Allen, Robert F., Cassandra Jens, and Theodore J. Wendt. 2014.
   "Perturbations in Epidemiological Models". Letters in Biomathematics 1 (2),␣
 ↪173-80.
   Link- https://doi.org/10.1080/23737867.2014.11414478.

   Part of MS Thesis at Universität Koblenz-Landau
   Note: this program uses the following libraries-  Numpy, Matplotlib
   Python Version 3.7
```

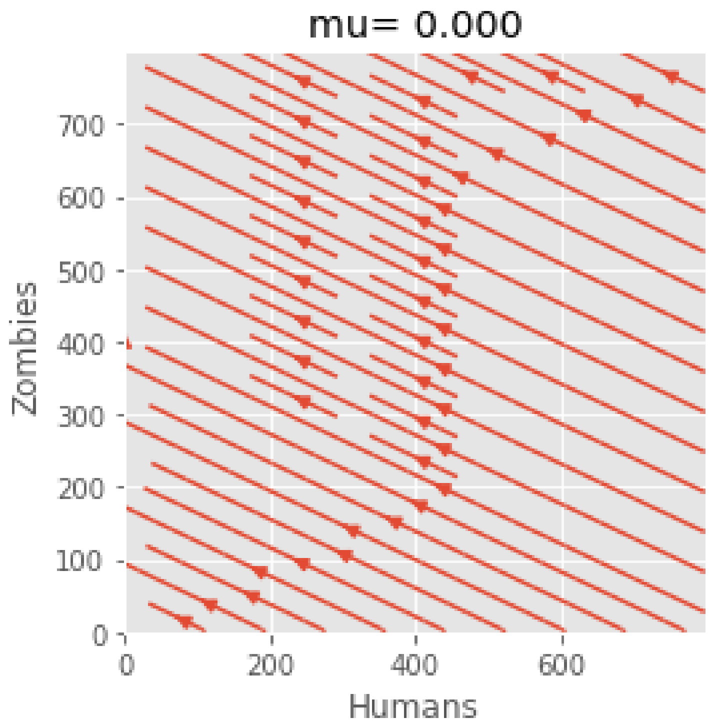
```python
"""
# Phase Portrait For Perturbed SZR Model

# importing libraries
import numpy as np
from matplotlib import pyplot as plt
from matplotlib.pyplot import figure
figure(figsize=(6, 6), dpi=150)


# Parameter values
beta = 0.0095
alpha = 0.005
zeta = 0.0001
N = 1000  #initial population

x = np.arange(0, 800, 1)
y = np.arange(0, 800, 1)
S, Z = np.meshgrid(x, y)

# range of values for Mu, with an increment of 0.005
mu = 0.175

S_dash = - beta*S*Z #SZR model differential equation
Z_dash = zeta*(N - S - Z) + beta*S*Z - alpha*(S**(1 + u))*Z #SZR model␣
 ↪differential equation
plt.streamplot(S, Z, S_dash, Z_dash, density=1.1) #phase portrait
plt.title("Perturbed SZR model with mu= %1.3f" %u)
plt.axis("scaled")
plt.draw()
plt.xlabel("Humans")
plt.ylabel("Zombies")
ax = plt.gca()
ax.set_facecolor('w')

__author__ = "Md Tariqul Islam"
__version__ = "1.0"
__maintainer__ = "Tariqul"
__email__ = "tariquldipu@uni-koblenz.de"
__status__ = "Final"
```

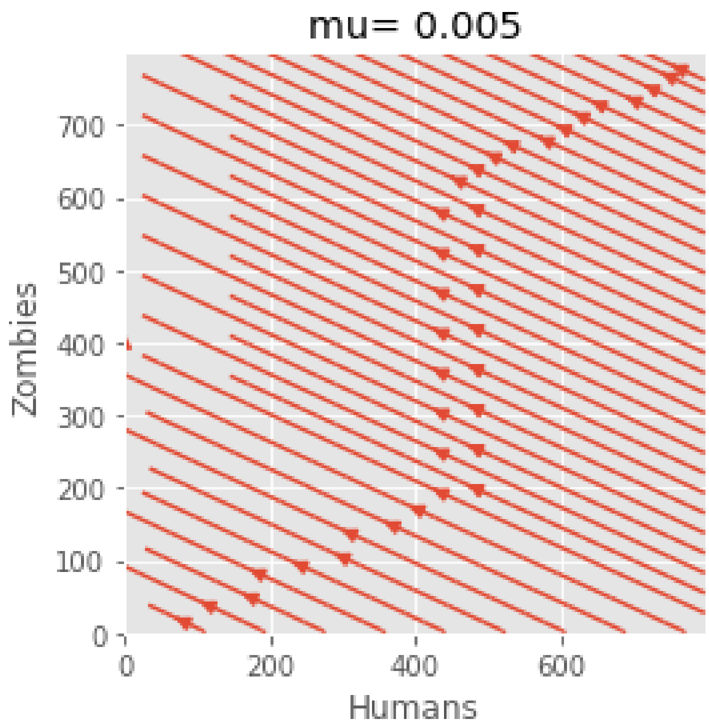


[10]:
```
#!/usr/bin/env python
# -*- coding: utf-8 -*-

"""This simple program tries to check the stability of the SZR model,
    for the different values of Mu (perturbation parameter), starting from 0,␣
↪with an increment of 0.005.
    The program will continue to run until it finds a optimal value of Mu,
    for which the SZR model reaches stability.

    Reference: Allen, Robert F., Cassandra Jens, and Theodore J. Wendt. 2014.
   "Perturbations in Epidemiological Models". Letters in Biomathematics 1 (2),␣
↪173-80.
   Link- https://doi.org/10.1080/23737867.2014.11414478.

   Part of MS Thesis at Universität Koblenz-Landau
```

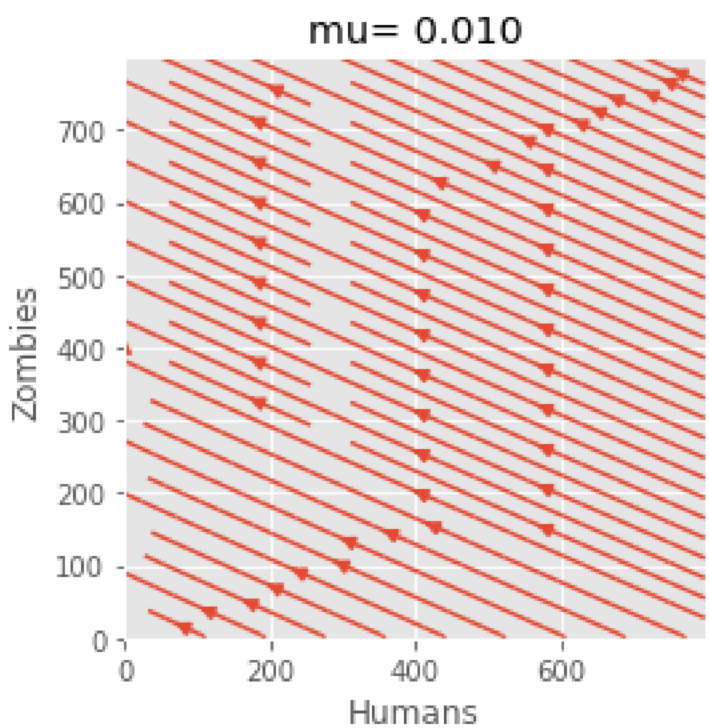
```python
    Note: this program uses the following libraries-  Numpy, Matplotlib
    Python Version 3.7
"""

# importing libraries
import numpy as np
from matplotlib import pyplot as plt


# Parameter values
beta = 0.0095
alpha = 0.005
zeta = 0.0001
N = 1000  #initial population

x = np.arange(0, 800, 1)
y = np.arange(0, 800, 1)
S, Z = np.meshgrid(x, y)

# range of values for Mu, with an increment of 0.005
mu_list = np.arange(0.0, 1.0, 0.005)


for u in mu_list:
    S_dash = - beta*S*Z #SZR model differential equation
    Z_dash = zeta*(N - S - Z) + beta*S*Z - alpha*(S**(1 + u))*Z #SZR model␣
 ↪differential equation
    plt.streamplot(S, Z, S_dash, Z_dash, density=1.0) #phase portrait
    plt.title("mu= %1.3f" %u)
    plt.axis("scaled")
    plt.draw()
    plt.xlabel("Humans")
    plt.ylabel("Zombies")
    plt.pause(0.0001)
    plt.clf()

    if (u > (((np.log((beta*N - zeta)/(alpha)))/(np.log(N))) - 1)):  #checking␣
 ↪for stable value point
        break
plt.show()
print("############################################")
print("So the model reaches stability at Mu= ", str(u))
print("############################################")


__author__ = "Md Tariqul Islam"
__version__ = "1.0"
```

```
__maintainer__ = "Tariqul"
__email__ = "tariquldipu@uni-koblenz.de"
__status__ = "Final"
```

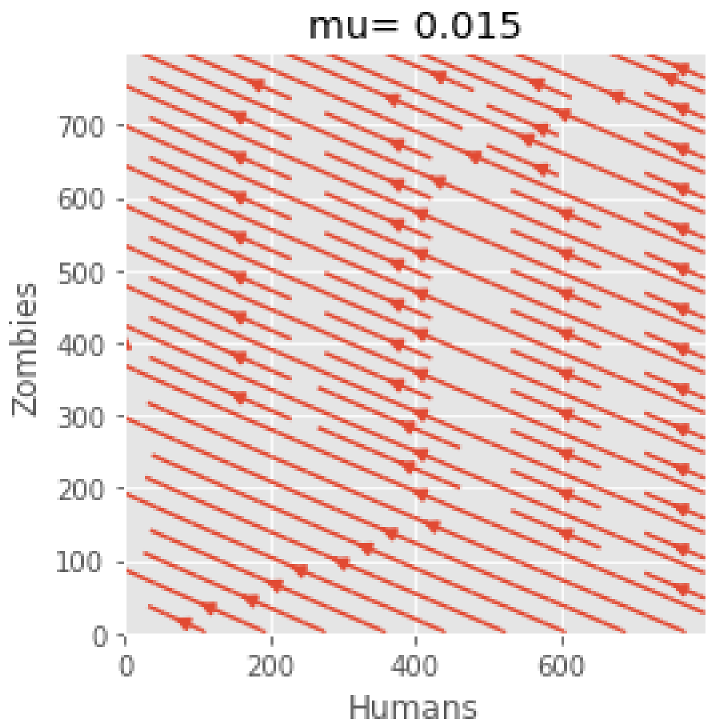


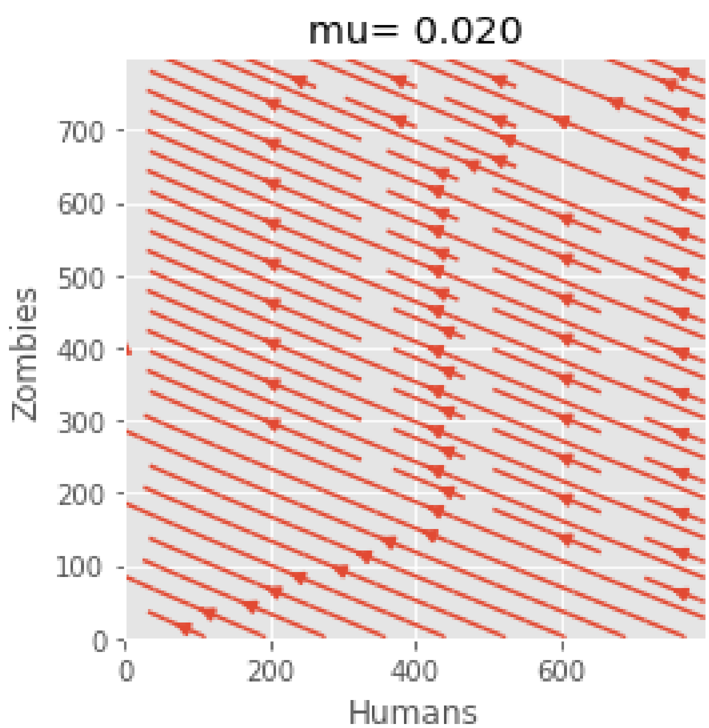


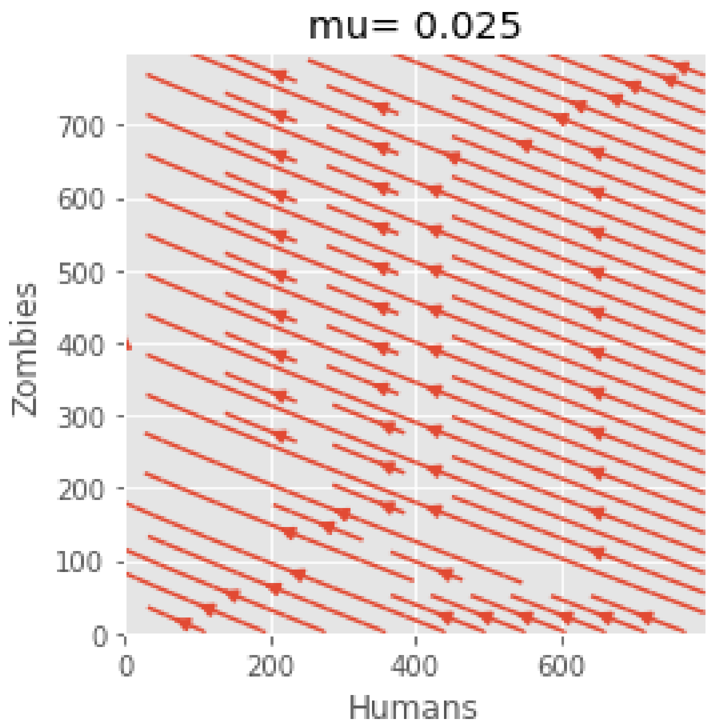
mu= 0.005
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600



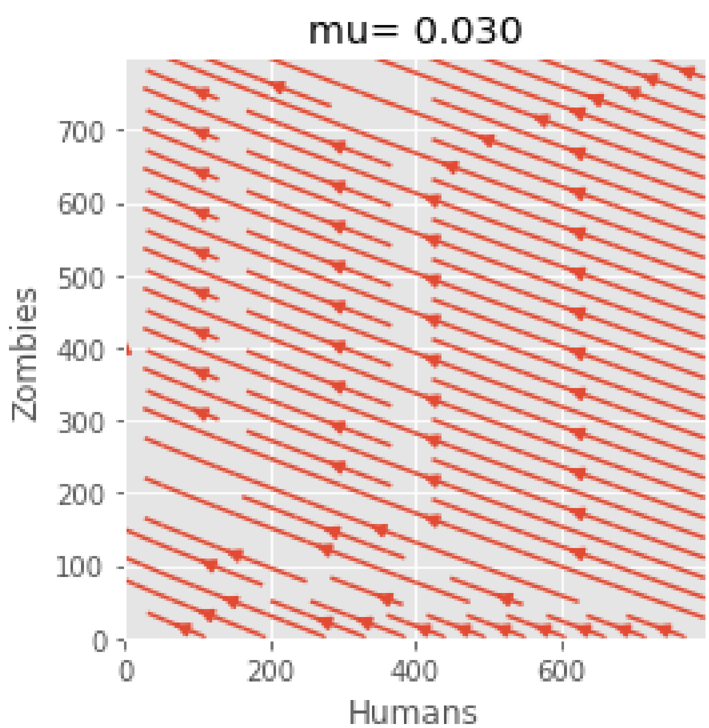
mu= 0.010
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600


mu= 0.015
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600

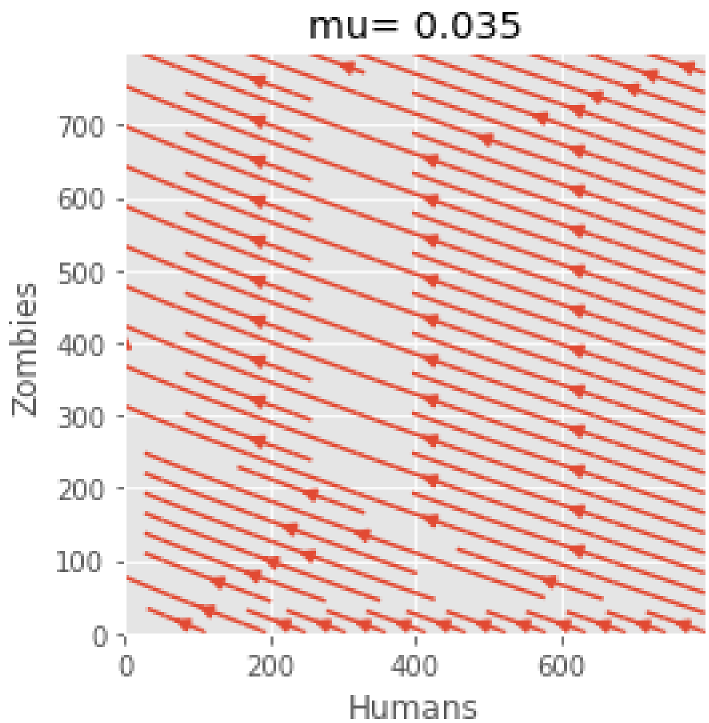

mu= 0.020
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600

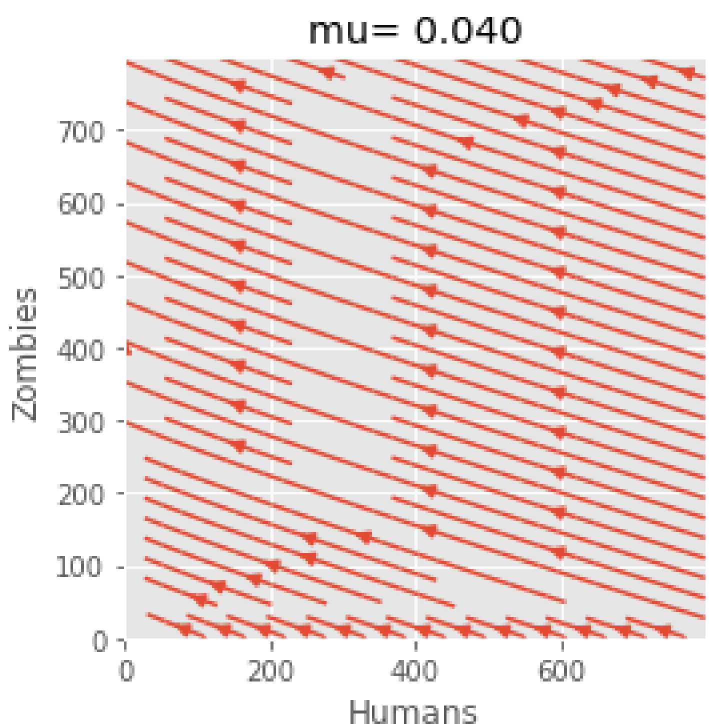


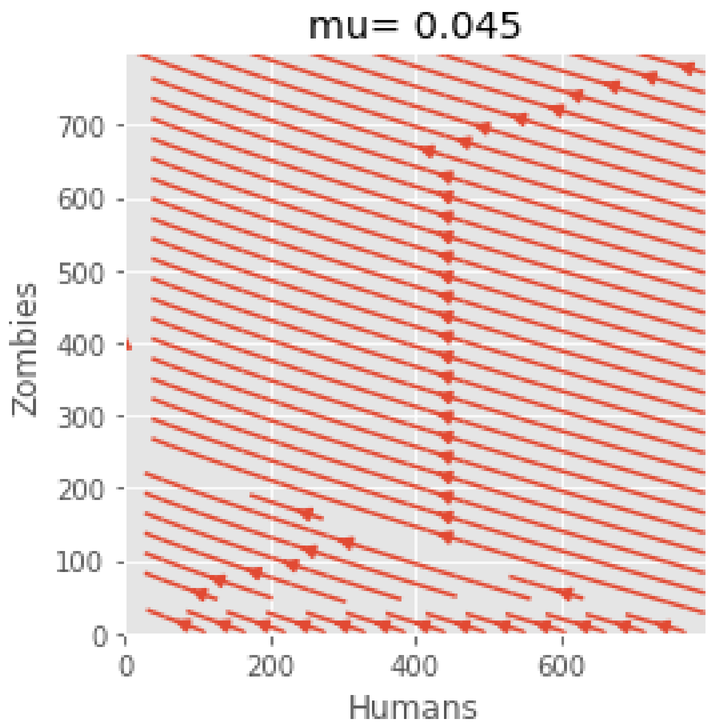
mu= 0.025
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600

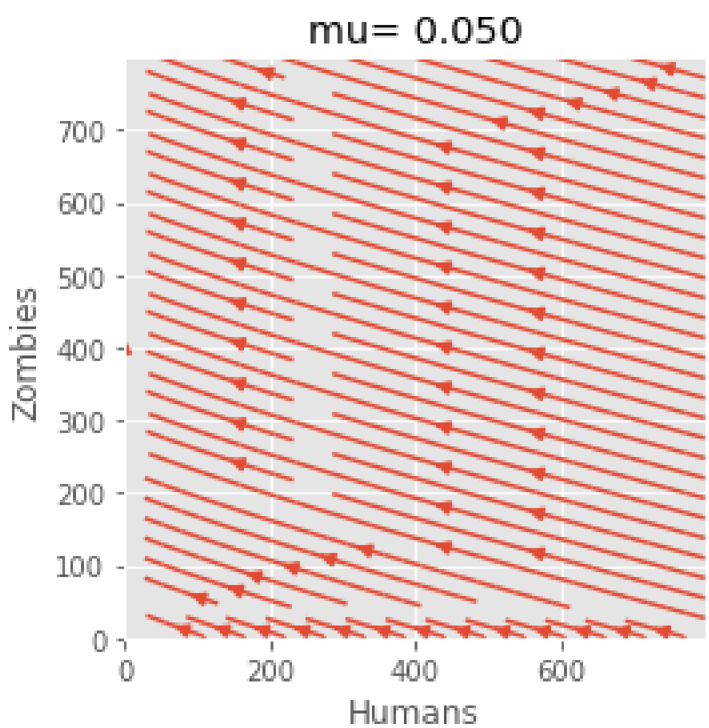


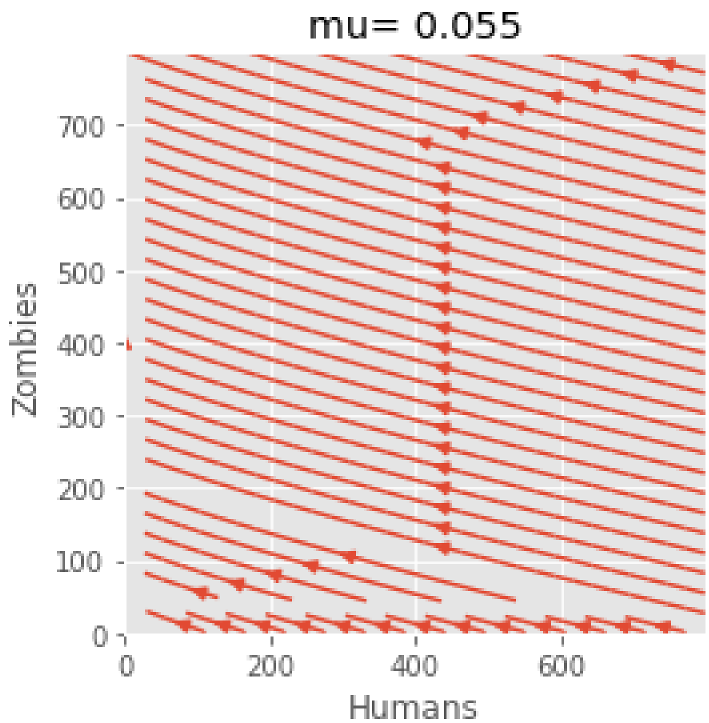
mu= 0.030
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600

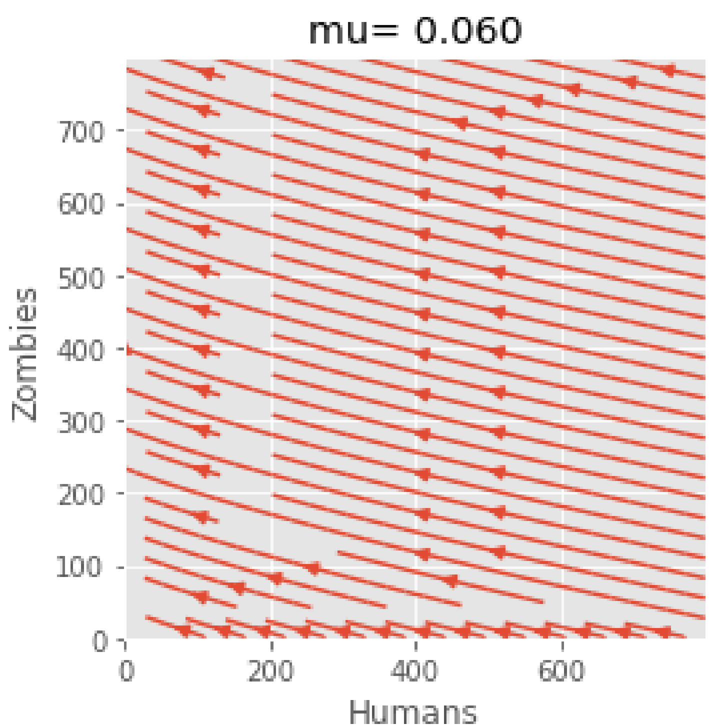

mu= 0.035
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600


mu= 0.040
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600

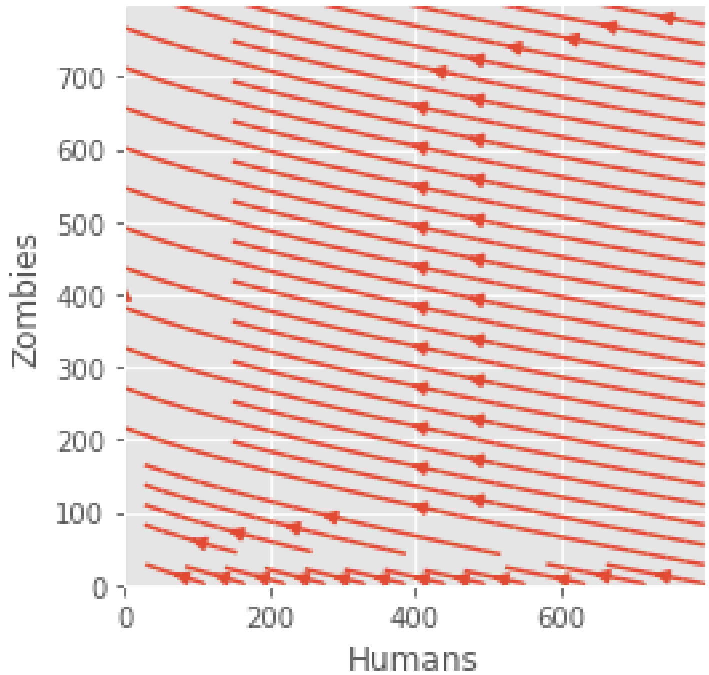

mu= 0.045
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600


mu= 0.050
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600

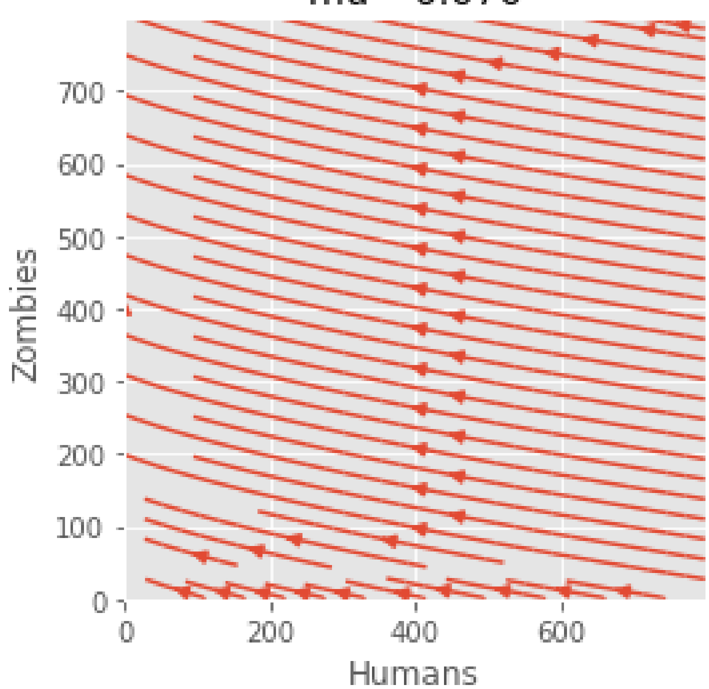

mu= 0.055
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600

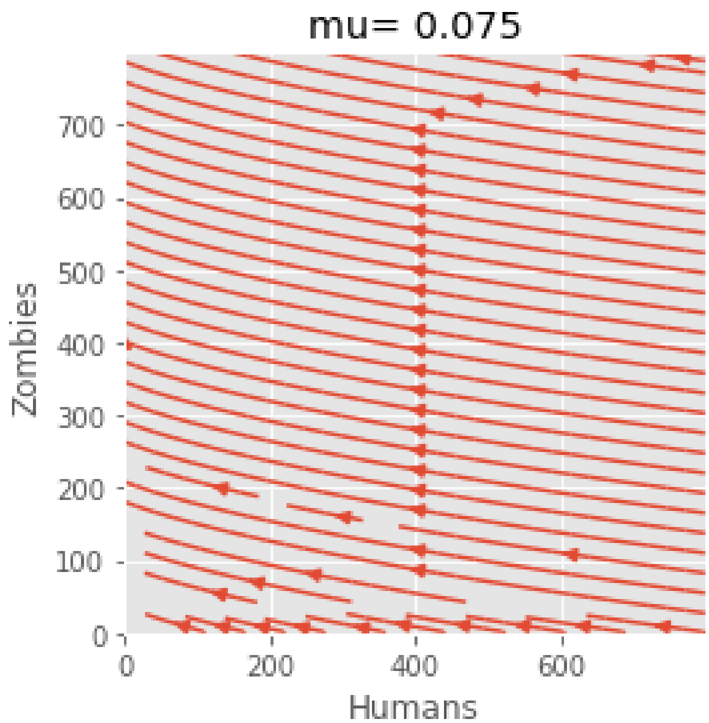

mu= 0.060
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600

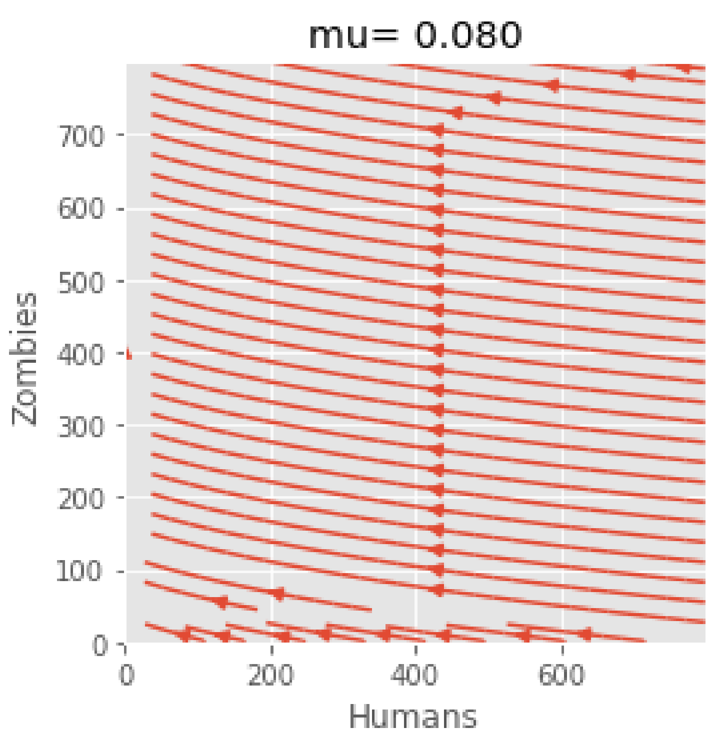
mu= 0.065

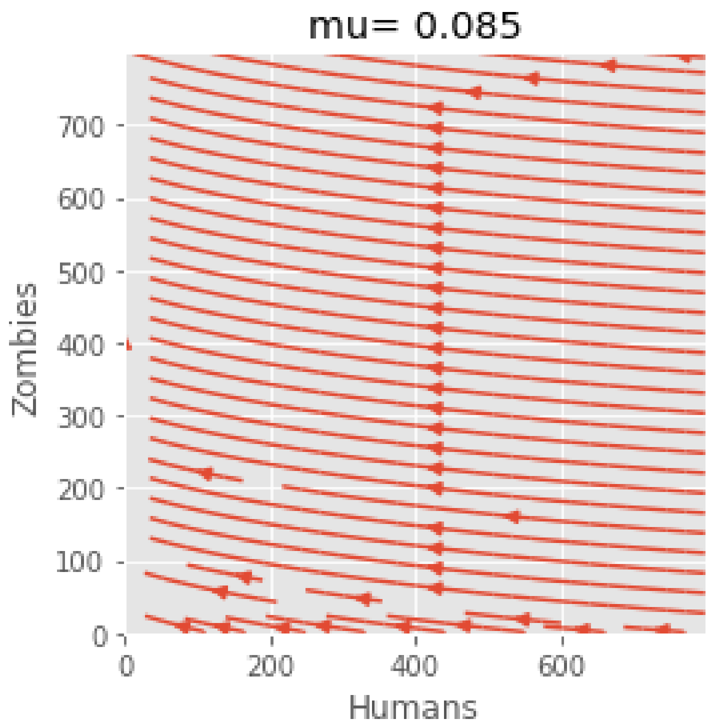


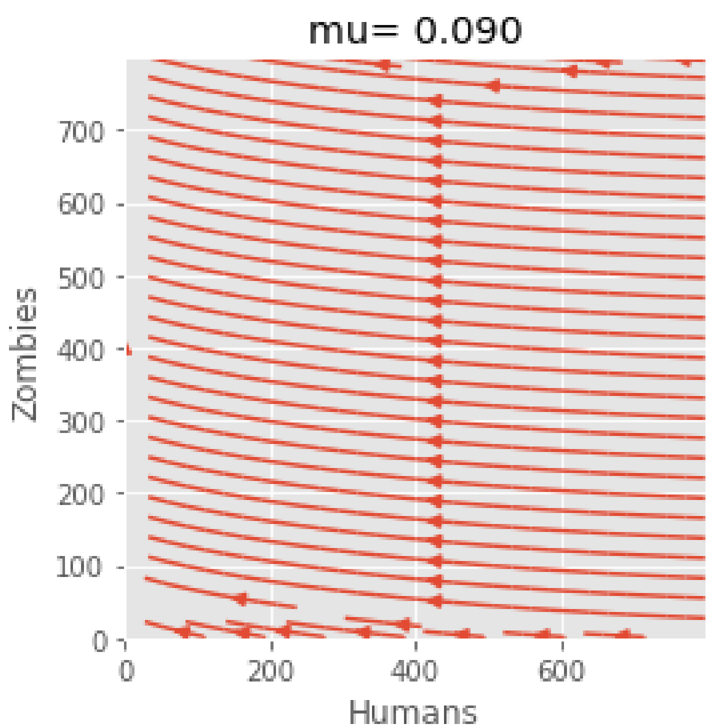
mu= 0.070

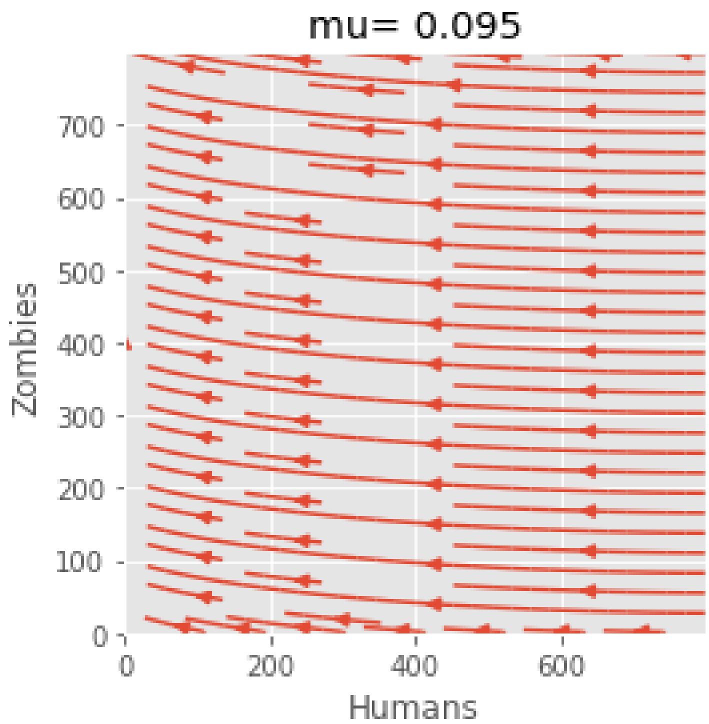



mu= 0.075
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600


mu= 0.080
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600


mu= 0.085
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600


mu= 0.090
Zombies
Humans
0
100
200
300
400
500
600
700
0
200
400
600




```
<Figure size 432x288 with 0 Axes>
```

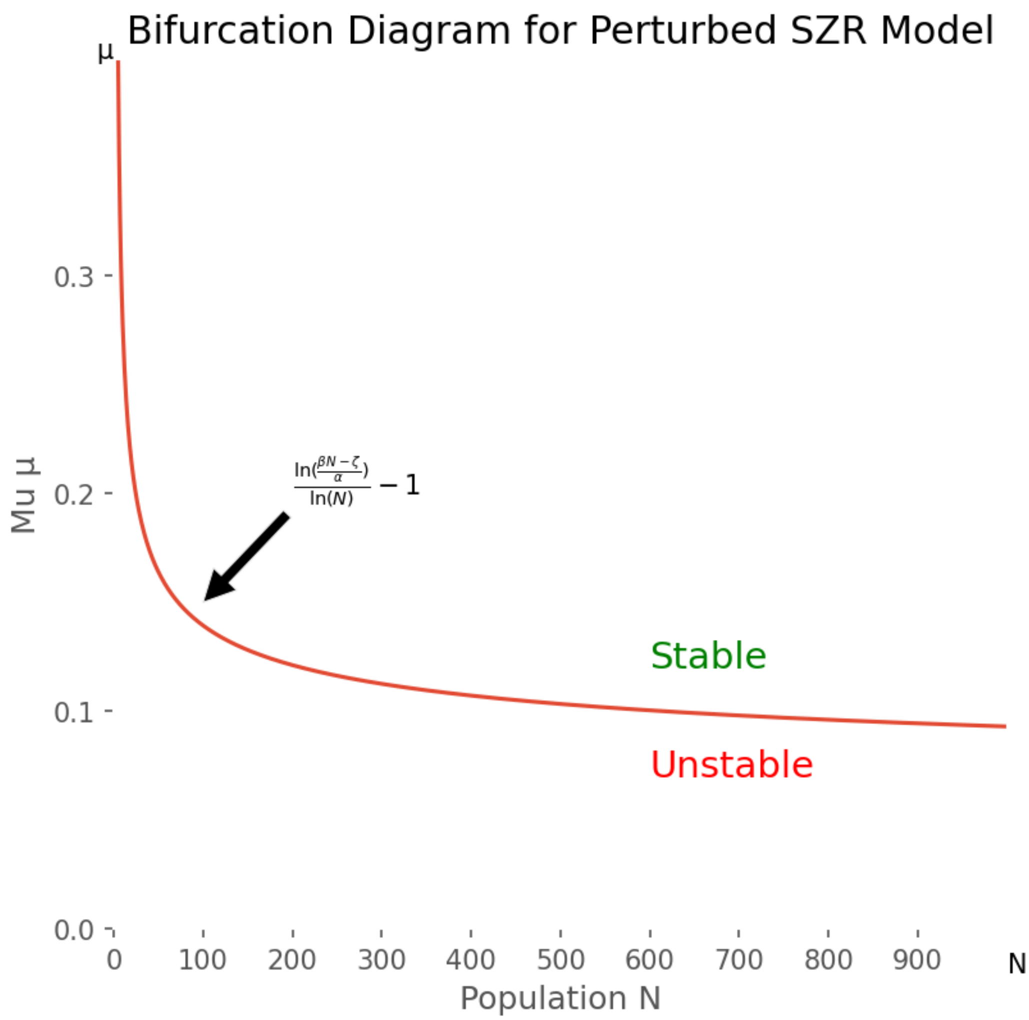
```
############################################
So the model reaches stability at Mu=  0.095
############################################
```

[15]:

```
# Bifurcation Diagram for The Perturbed SZR Model

# importing libraries
import numpy as np
from matplotlib import pyplot as plt
from matplotlib.pyplot import figure
figure(figsize=(6, 6), dpi=150)

print("The bifurcation diagram for the SZR model >>>>>")



# Parameter values
beta = 0.0095
alpha = 0.005
zeta = 0.0001
N = 1000  #initial population
```

```python
x = np.array(range(2,1000))
y = ((np.log((beta*x - zeta)/(alpha)))/(np.log(x))) - 1
# draw bifurcation diagram
plt.plot(x,y)
plt.ylim([0, 0.4])
plt.xlim([0, 1000])
plt.yticks(np.arange(0, 0.4, 0.1))
plt.xticks(np.arange(0, 1000, 100))
plt.xlabel("Population N")
plt.ylabel("Mu " + u"\u03bc")
plt.text(600, 0.12, "Stable",
        fontsize= 14,
        color= 'green')
plt.text(600, 0.07, "Unstable",
        fontsize= 14,
        color= 'red')
plt.text(1000, -0.02, "N")
plt.text(-20, 0.4, u"\u03bc")
plt.annotate(r"$ \frac{\ln(\frac{\beta N - \zeta}{\alpha})}{\ln (N)} - 1$",␣
 ↪xy=(100, 0.15), xytext=(200, 0.2),
            arrowprops=dict(facecolor='black', shrink=1))# theorem 1 condition
plt.title("Bifurcation Diagram for Perturbed SZR Model")
ax = plt.gca()
ax.set_facecolor('w')
ax.grid(True, which='both')
plt.show()

__author__ = "Md Tariqul Islam"
__version__ = "1.0"
__maintainer__ = "Tariqul"
__email__ = "tariquldipu@uni-koblenz.de"
__status__ = "Final"
```

The bifurcation diagram for the SZR model >>>>>

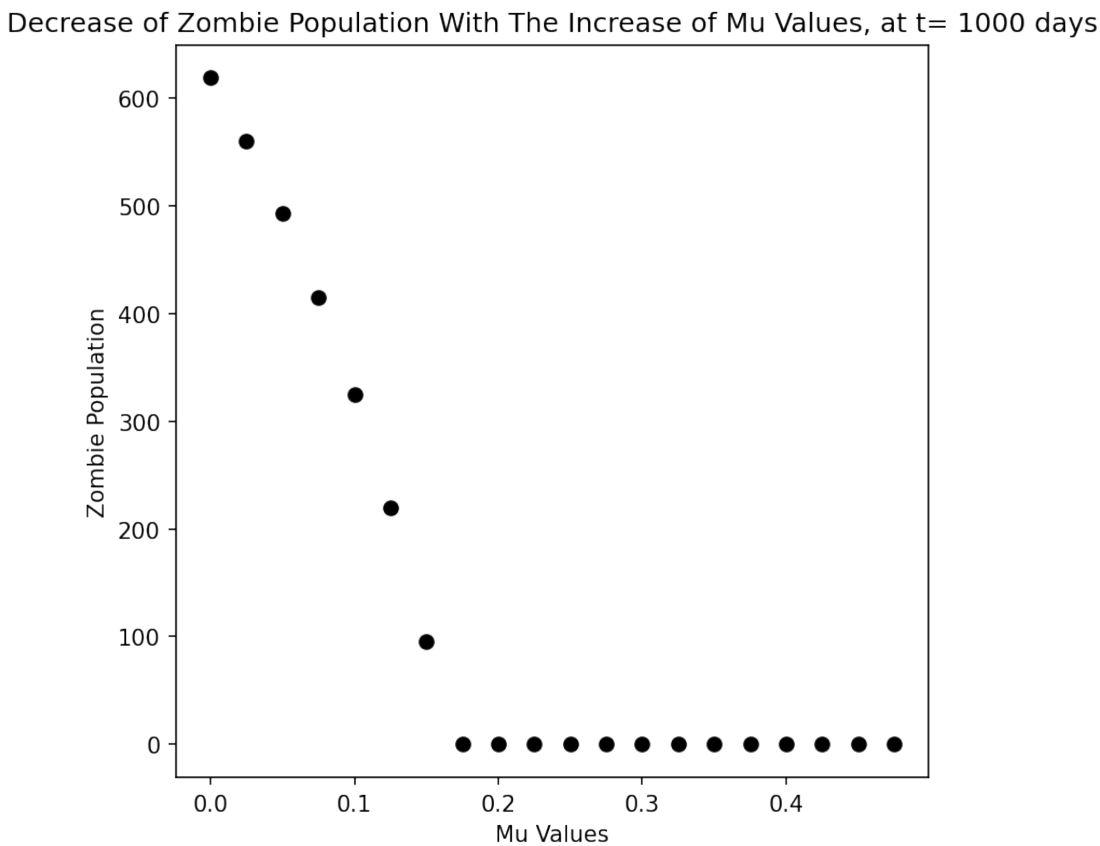


[20]:
```python
#!/usr/bin/env python
# -*- coding: utf-8 -*-


"""This simple program tries to show a relation between the number of zombies␣
↪and the perturbation value Mu.
    This program is calculated at the point t = 1000 days.
    It shows that, at t = 1000 days, as the value of the pertubation parameter␣
↪Mu increases, the number of total
    zombies decreases. Thus, establishing the fact that the perturbation␣
↪parameter has a direct impact on the
    zombie population.

    Reference: Allen, Robert F., Cassandra Jens, and Theodore J. Wendt. 2014.
```

```
    "Perturbations in Epidemiological Models". Letters in Biomathematics 1 (2),␣
 ↪173-80.
    Link- https://doi.org/10.1080/23737867.2014.11414478.

    Part of MS Thesis at Universität Koblenz-Landau
    Note: this program uses the following libraries-  Numpy, Matplotlib, Scipy
    Python Version 3.7
"""

# importing libraries
import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt
from matplotlib.pyplot import figure
figure(figsize=(6, 6), dpi=150)
import warnings
warnings.filterwarnings("ignore")

# Generating range of values for Mu, from 0.0 upto 0.5, with an increment of 0.
 ↪025
mu = np.arange(0.0, 0.5, 0.025)

# Parameter values
beta = 0.0095
alpha = 0.005
zeta = 0.0001
N = 1000  # initial population

# Defining Function of the SZR ODE
def model(t, x):
    S = x[0] # Intital Susceptible Population
    Z = x[1] # Zombies
    dsdt = - beta*S*Z # Susceptible
    dzdt = (zeta*(N - S - Z)) + (beta*S*Z) - (alpha*(S**(1 + value))*Z) #␣
 ↪Zombies
    dxdt = [dsdt, dzdt]
    return dxdt

# Initial Condition. Susceptible= 800, Zombie= 200
x0 = [800, 200]

# Time, as in number of days. Total days= 1000
t = np.array([0, 1000])
tspan = np.linspace(t[0], t[1], 1000)

i = 0;
```

```
# empty lists
zombie_new = []
recovered_new = []

for value in mu:
    x_new = solve_ivp(model, t, x0, t_eval=tspan)
    zombie = x_new.y[1];
    zombie_new.append(zombie[999])
    #susceptible = x_new.y[0];
    #recovered_new.append((N - susceptible[999] - zombie[999]))

# Drawing Scatter Plot at t = 1000 days position
# Change of zombie numbers with the increase of Mu Values
# When the number of zombies reaches 0 at t= 1000 days, the model reaches␣
 ↪stability at that Mu value
plt.plot(mu, zombie_new, 'o', color='black');
#plt.plot(mu, recovered_new, 'o', color='blue');
plt.ylabel('Zombie Population')
plt.xlabel('Mu Values')
plt.title("Decrease of Zombie Population With The Increase of Mu Values, at t=␣
 ↪1000 days")
ax = plt.gca()
ax.set_facecolor('w')
plt.show

__author__ = "Md Tariqul Islam"
__version__ = "1.0"
__maintainer__ = "Tariqul"
__email__ = "tariquldipu@uni-koblenz.de"
__status__ = "Final"
```

[20]:
```
<function matplotlib.pyplot.show(close=None, block=None)>
```



[10]:
```
#!/usr/bin/env python
# -*- coding: utf-8 -*-

"""This simple program tries to solve the ODE of the Perturbed SZR model
    and create a numerical solution.
    After that, this program draws the solution graph for only the Zombie␣
↪Population with the gradual
    increase of perturbation parameter.
    This simple graph helps us to understand the actual scenario of the zombie␣
↪population
    from the Perturbed SZR model with respect to the perturbation parameter.

   Part of MS Thesis at Universität Koblenz-Landau
   Note: this program uses the following libraries-  Numpy, Matplotlib, Scipy
   Python Version 3.7
"""

# Perturbed SZR Model

# importing libraries
```

```python
import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt
from matplotlib.pyplot import figure
figure(figsize=(6, 6), dpi=150)

# For a nice looking plot
plt.style.use('ggplot')

# Parameter values
beta = 0.0095
alpha = 0.005
zeta = 0.0001
N = 1000  # initial population

# Defining Function of the SZR ODE
def model(t, x):
    S = x[0] # Intital Susceptible Population
    Z = x[1] # Zombies
    dsdt = - beta*S*Z # Susceptible
    dzdt = zeta*(N - S - Z) + beta*S*Z - alpha*(S**(1 + mu))*Z # Zombies
    dxdt = [dsdt, dzdt]
    return dxdt

# Initial Condition. Susceptible= 800, Zombie= 200
x0 = [800, 200]

# Time, as in number of days. Total days= 1000
t = np.array([0, 10])
tspan = np.linspace(t[0], t[1], 1000)


# Calcuclate zombie population for different values of Mu
mu = 0.0
x = solve_ivp(model, t, x0, t_eval=tspan)
zombie01 = x.y[1]

mu = 0.05
x = solve_ivp(model, t, x0, t_eval=tspan)
zombie02 = x.y[1]

mu = 0.1
x = solve_ivp(model, t, x0, t_eval=tspan)
zombie03 = x.y[1]

mu = 0.15
x = solve_ivp(model, t, x0, t_eval=tspan)
```

```
zombie04 = x.y[1]

mu = 0.2
x = solve_ivp(model, t, x0, t_eval=tspan)
zombie05 = x.y[1]

mu = 0.25
x = solve_ivp(model, t, x0, t_eval=tspan)
zombie06 = x.y[1]

time = x.t
# Plot
plt.plot(time, zombie01, 'r--', label="Mu=0.0")
plt.plot(time, zombie02, 'b:', label="Mu=0.05")
plt.plot(time, zombie03, 'c-.', label="Mu=0.1")
plt.plot(time, zombie04, 'g-', label="Mu=0.15")
plt.plot(time, zombie05, 'y-', label="Mu=0.2")
#plt.plot(time, zombie06, 'k', label="Mu=0.25")

plt.title('Zombie Population For Different Mu')
plt.ylabel('Zombie Population')
plt.xlabel('Time (Days)')
plt.legend(loc='best')
ax = plt.gca()
ax.set_facecolor('w')
plt.show()

__author__ = "Md Tariqul Islam"
__version__ = "1.0"
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__status__ = "Final"
```
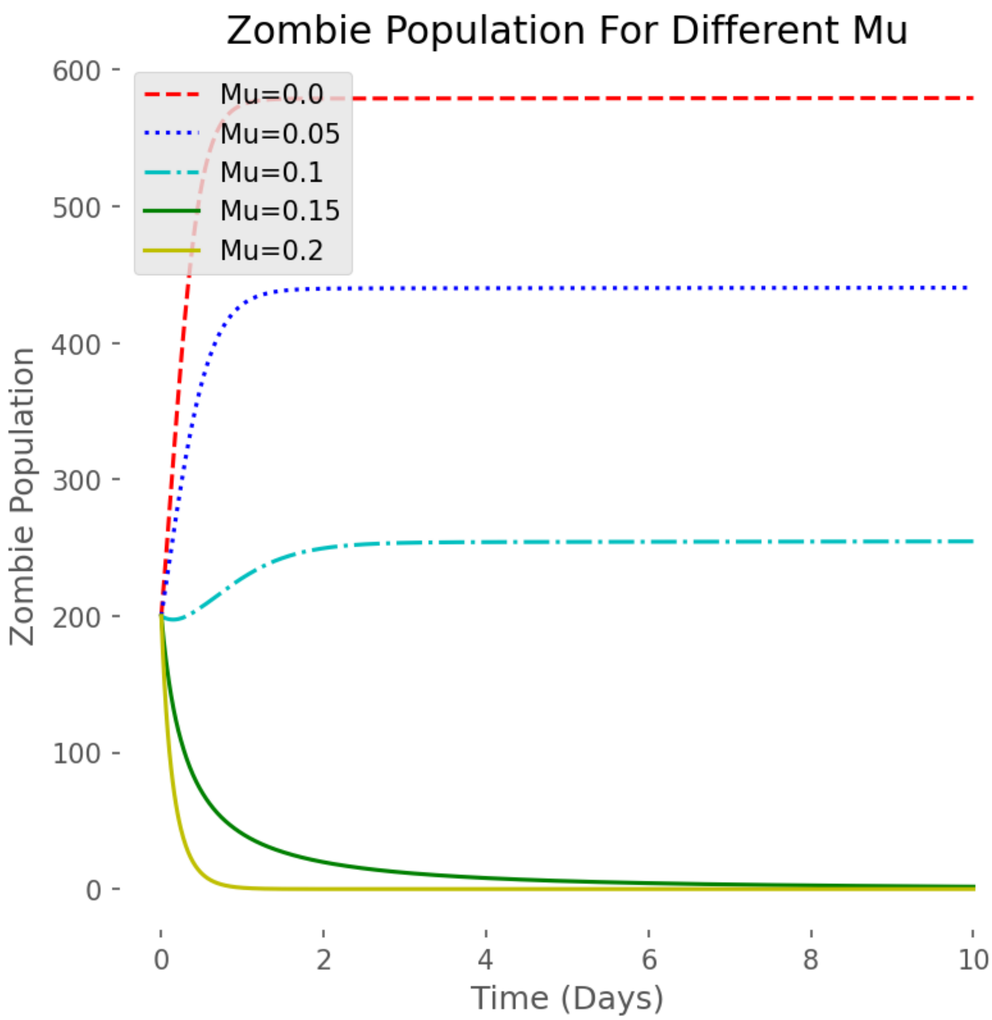

```
<ipython-input-10-71223a7d7ee9>:39: RuntimeWarning: invalid value encountered in
double_scalars
  dzdt = zeta*(N - S - Z) + beta*S*Z - alpha*(S**(1 + mu))*Z # Zombies
```


Zombie Population For Different Mu
Mu=0.0
Mu=0.05
Mu=0.1
Mu=0.15
Mu=0.2
Zombie Population
600
500
400
300
200
100
0
0
2
4
6
8
10
Time (Days)

```
[ ]:
```