Sample Report

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1 1.2.3 Exact solution

u(t,x,y) = e$^{i(k_x x+k_y y-\omega t)}$

(1)

$$\frac{\partial^2 u}{\partial t^2} = -\omega^2 e^{i(k_x x+k_y y-\omega t)}$$
$$\frac{\partial^2 u}{\partial x^2} = -k_x^2 e^{i(k_x x+k_y y-\omega t)}$$
$$\frac{\partial^2 u}{\partial y^2} = -k_y^2 e^{i(k_x x+k_y y-\omega t)}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$$
$$-\omega^2 e^{i(k_x x+k_y y-\omega t)} = -c^2(k_x^2 + k_y^2)e^{i(k_x x+k_y y-\omega t)}$$
$$c = \frac{\omega}{\sqrt{k_x^2 + k_y^2}}$$

2 1.2.4 Dispersion coefficient

$$e^{i(kh(i+j)-\tilde{\omega}(n+1)\Delta t)} - 2e^{i(kh(i+j)-\tilde{\omega}n\Delta t)} + e^{i(kh(i+j)-\tilde{\omega}(n-1)\Delta t)}$$
$$= 2c^2\frac{\Delta t^2}{h^2}(e^{i(kh(i+j+1)-\tilde{\omega}n\Delta t)} - 2e^{i(kh(i+j)-\tilde{\omega}n\Delta t)} + e^{i(kh(i+j-1)-\tilde{\omega}n\Delta t)})$$
$$e^{-i\tilde{\omega}\Delta t} - 2 + e^{i\tilde{\omega}\Delta t} = 2C^2(e^{ikh} - 2 + e^{-ikh})$$
$$cos(\tilde{\omega}\Delta t) = cos(kh)$$
$$\tilde{\omega}\Delta t = kh$$
$$\tilde{\omega} = k\frac{c}{C} = \sqrt{2}ck = \omega$$