



Antiamuny Code Library

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第零章

- 安装 typst:

- Linux, macOS, WSL

```
curl -fsSL https://typst.community/typst-install/install.sh | sh
```

- Windows

```
irm https://typst.community/typst-install/install.ps1 | iex
```

- 安装 VSCode 插件 tinymist:

- 打开 VSCode
- 搜索 tinymist 安装插件

赛前准备

测样例脚本(python 版)

```
1 import os
2 import sys
3 import zipfile
4 # linux 下去掉.exe 和 加./
5 c = sys.argv[1]
6 code = c.capitalize()
7 os.system(f'g++ {c}.cpp -O2 -Wall -o {c}.exe')
8 try:
9     x = zipfile.Zipfile(f'samples-{code}.zip')
10    x.extractall(f'samples-{code}')
11 except:
12     pass
13
14 for i in range(1,10000):
15     if not os.path.isfile(f'samples-{code}/{i}.ans'):
16         break
17     print(f'Test {i}')
18     os.system(f'{c}.exe < samples-{code}/{i}.in > samples-{code}/{i}.out')
19     out = open(f'samples-{code}/{i}.out').read().replace('\r\n', '\n')
20     ans = open(f'samples-{code}/{i}.ans').read().replace('\r\n', '\n')
21     if(out != ans):
22         print(f'WA {i}')
23     else:
24         print(f'AC {i}')
```

对拍(cpp 版本)

```
1  int t=10000,j=0;
2  while(t)
3  {
4      cout<<"test:"<<++j<<"\n";
5      t--;
6      system("testin.exe > data.txt");
7      system("abiao Cheng.exe < data.txt > biaoda.txt");
8      system("nedtest.exe < data.txt > aatest.txt");
9      if(system("fc aatest.txt biaoda.txt")){
10         cout<<"error"<<"\n";
11         break;
12     }
13 }
14 if(t==0) cout<<"no error"<<endl;
15 //system("pause");
16 return 0;
```

对拍(bat 版本)

```
1 @echo off
2 setlocal enabledelayedexpansion
3
4 set T=0
5 :loop
6 if %T% gtr 100000 (
7     echo "Finished"
8     exit /b
9 )
10 set /a T+=1
11 echo T=!T!
12 testin.exe > data.txt
13 abiao Cheng.exe < data.txt > biaoda.txt
14 nedtest.exe < data.txt > aatest.txt
15
16 fc aatest.txt biaoda.txt
17 if errorlevel 1 (
18     echo "WA"
19     exit /b
20 )
21
22 goto loop
```


杂项

Rand

```
1 mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
2 int myRand(int B) {
3     return (unsigned long long)rng() % B;
4 }
```

Time

```
1 struct gettime{
2     clock_t star,ends;
3     gettime(){
4         star = clock();
5     }
6     ~gettime(){
7         ends = clock();
8         cout <<"Running Time : "<<(double)(ends - star)/ CLOCKS_PER_SEC << endl;
9     }
10 } tim;
11 int main()
12 {
13     tim.begin();
14     tim.end();
15     return 0;
16 }
17 }
```

子集枚举

```
1 for(int i=0;i<(1<<n);i++){
2     for(int j=i;j;j=(j-1)&i){
3
4     }
5 }
```

高维前缀和

```
1 for(int j = 0; j < n; j++)
2     for(int i = 0; i < 1 << n; i++)
3         if(i >> j & 1) f[i] += f[i ^ (1 << j)];
```

三分

```
1 double f(double x){
2     //something
3 }
4 const double eps=1e-8;
5 double sanfen(double l, double r){
6     double mid,midr,ans;
7     while (fabs(r-l)>eps) {
8         mid=(l+r)/2;
9         midr=(mid+r)/2;
10        if(f(mid) < f(midr)) l=mid; else r=midr;    //求最大值
11    }
12    ans=f(l);
13    return ans;
14 }
```

Bitset

```
1 template<int LEN>void solve(){
2     sz=a.size();
3     if (LEN<=b.size()){
4         solve<min(N,LEN+10)>();
5         return;
6     }
7     using Bitset=bitset<LEN>;
8     Bitset is[sz+5];
9     for(int i=0;i<sz;i++)for(int j=0;j<b.size();j++){
10         auto&[x,l,r]=a[i];
11         auto&[y,L,R]=b[j];
12         if(l<=y&&y<=r&&L<=x&&x<=R)is[i][j]=1;
13     }
14     ll rs=0;
15     for(int i=0,x;i<sz;i++)for(int j=i+1;j<sz;j++){
16         x=(is[i]&is[j]).count();
17         rs+=x*(x-1)/2;
18     }
19     cout<<rs<<'\n';
20 }
```

Bitset 手写

```
1 const int N=3000;
2 typedef unsigned long long ull;
3
4 int lim=N/64+3;
5 struct Bitset{
6     ull v[N/64+5];
7     void init(){
8         memset(v,0,sizeof(v));
9         return;
10    }
11    void add(int x){
12        v[x>>6]|=(1ull<<(x&63));
13        return;
14    }
15    void shiftl(){
16        int lst=0;
17        for(int i=0;i<=lim;i++){
18            int cur=v[i]>>63;
19            v[i]<=1;v[i]|=lst;
20            lst=cur;
21        }
22        return;
23    }
24    int count(){
25        int res=0;
26        for(int i=0;i<=lim;i++) res+=__builtin_popcountll(v[i]);
27        return res;
28    }
29    Bitset operator|(const Bitset &x)const{
30        Bitset res;
31        for(int i=0;i<=lim;i++) res.v[i]=v[i]|x.v[i];
32        return res;
33    }
```

```

34 Bitset operator&(const Bitset &x) const{
35     Bitset res;
36     for(int i=0;i<=lim;i++) res.v[i]=v[i]&x.v[i];
37     return res;
38 }
39 Bitset operator^(const Bitset &x) const{
40     Bitset res;
41     for(int i=0;i<=lim;i++) res.v[i]=v[i]^x.v[i];
42     return res;
43 }
44 Bitset operator-(const Bitset &x) const{
45     Bitset res; ull lst=0;
46     for(int i=0;i<=lim;i++){
47         ull cur=(v[i]<x.v[i]+lst);
48         res.v[i]=v[i]-x.v[i]-lst;
49         lst=cur;
50     }
51     return res;
52 }
53 }

```

Lcslen(n2/w)

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int N=3010;
4  typedef unsigned long long ull;
5  const int lim=N/64+3;
6  struct Bitset{
7      ull v[N/64+5];
8      void init(){
9          memset(v,0,sizeof(v));
10         return;
11     }
12     void add(int x){
13         v[x>>6]|=(1ull<<(x&63));
14         return;
15     }
16     void shiftl(){
17         int lst=0;
18         for(int i=0;i<=lim;i++){
19             int cur=v[i]>>63;
20             v[i]<<=1;v[i]|=lst;
21             lst=cur;
22         }
23         return;
24     }
25     int count(){
26         int res=0;
27         for(int i=0;i<=lim;i++) res+=__builtin_popcountll(v[i]);
28         return res;
29     }
30     Bitset operator|(const Bitset &x) const{
31         Bitset res;
32         for(int i=0;i<=lim;i++) res.v[i]=v[i]|x.v[i];
33         return res;
34     }

```

```

35 Bitset operator&(const Bitset &x)const{
36     Bitset res;
37     for(int i=0;i<=lim;i++) res.v[i]=v[i]&x.v[i];
38     return res;
39 }
40 Bitset operator^(const Bitset &x)const{
41     Bitset res;
42     for(int i=0;i<=lim;i++) res.v[i]=v[i]^x.v[i];
43     return res;
44 }
45 Bitset operator-(const Bitset &x)const{
46     Bitset res;ull lst=0;
47     for(int i=0;i<=lim;i++){
48         ull cur=(v[i]<x.v[i]+lst);
49         res.v[i]=v[i]-x.v[i]-lst;
50         lst=cur;
51     }
52     return res;
53 }
54 }ch[26],f,g;
55 auto getans = [&](int mid){
56     int ll=mid,l2=n-mid;
57     f.init();g.init();
58     for(int i=0;i<26;i++){
59         ch[i].init();
60     }
61     for(int i=mid+1;i<=n;i++)ch[s[i]-'a'].add(i);
62     for(int i=1;i<=mid;i++){
63         g=f|ch[s[i]-'a'];
64         f.shiftl();
65         f.add(1);
66         f=g-f;
67         f=f^g;f=f&g;
68     }
69     return f.count();
70 };

```

wqs 二分

```

1 // 这里是 上凸 取 min
2 // 上凸取 max 二分的时候改变一下 mid 的变化方向
3 // 下凸取 min 改变 mid 算贡献的符号
4 //min max 指的是求的是最大值 还是最小值
5 int solve(int mid){
6     k=mid;
7     function<void(int,int)> dfs = [&](int x,int h){
8         info dp2[3];memset(dp2,0,sizeof(dp2));
9         for(auto [it,w]:ve[x])if(it!=h){
10             dfs(it,x);
11             gmax(dp2[0],dp[x][0]+dp[it][0]);
12             gmax(dp2[2],dp[x][0]+dp[it][1]);
13             gmax(dp2[2],dp[x][2]+dp[it][0]);
14             dp[x][0]=dp2[0];
15             dp[x][1]=dp2[1];
16             dp[x][2]=dp2[2];
17         }
18         dp[x][0]=dp2[2];

```

```

19     dp[x][1]=dp2[0]+info{a[x]+k,1};
20 };
21 dfs(1,0);
22 return dp[1][0].second;
23 }
24 signed main()
25 {
26     int l=-sum-1000000000000ll,r=sum+1000000000000ll;
27     int ans=1e18;
28     while(l<=r){
29         int mid=(l+r)/2ll;
30         if(solve(mid)>=m){
31             ans=dp[1][0].first-m*mid;
32             r=mid-1;
33         }
34         else l=mid+1;
35     }
36     if(ans!=(int)1e18)cout<<ans<<"\n";
37     else cout<<"Impossible\n";
38     return 0;
39 }

```

判断异或方程组是否有解

```

1  const int maxn=1e2+5;
2  //每个方程组一定是  $x_1^{i_1}x_2^{i_2}x_3^{i_3}\dots=1/0$  的形式
3  bitset<maxn>b[maxn]; //1 表示这个方程组存在  $x_i$  比如  $x_3^3x_5^5=1$  就应该是 3 和 5 的地方上是 1
4  //b[n+1]存方程右边等于 0/1
5  int sum[maxn];
6  bool check(int x)
7  {
8      for(int i=1;i<=n;i++)if(b[x][i])return true;
9      return !b[x][n+1];
10 }
11 bool solve()
12 {
13     for(int i=1,p=1;i<=n;i++,p++)
14     {
15         if(!b[p][i])
16         {
17             int pos=0;
18             for(int j=p+1; j<=n; j++)
19                 if(b[j][i])
20                 {
21                     pos=j;
22                     break;
23                 }
24             if(pos)swap(b[p],b[pos]);
25         }
26         int flag=b[p][i];
27         for(int j=p+1; j<=n; j++) if(b[j][i]) b[j]^=b[p],flag=1;
28         if(!flag) p--;
29     }
30     for(int i=1; i<=n; i++) if(!check(i)) return false;
31     return true;
32 }

```

可以判断不同或相同的并查集

```
1 int f[maxn];
2 int getf(int x){
3     if(x<0)return -getf(-x);
4     if(x==f[x])return x;
5     else return f[x]=getf(f[x]);
6 }
7 bool merge(int x,int y){//如果是 x!=y 将 y 取反 (x>0 y>0)
8     x=getf(x),y=getf(y);
9     if(x==y)return false;
10    if(x==y)return true;
11    if(x<0)f[-x]=-y;
12    else f[x]=y;
13    return true;
14 }
```

Int128

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 inline __int128 read()
4 {
5     __int128 x=0,f=1;
6     char ch=getchar();
7     while(ch<'0' || ch>'9')
8     {
9         if(ch=='-')
10             f=-1;
11         ch=getchar();
12     }
13     while(ch>='0'&&ch<='9')
14     {
15         x=x*10+ch-'0';
16         ch=getchar();
17     }
18     return x*f;
19 }
20 inline void write(__int128 x)
21 {
22     if(x<0)
23     {
24         putchar('-');
25         x=-x;
26     }
27     if(x>9)
28         write(x/10);
29     putchar(x%10+'0');
30 }
31
32 istream& operator >> (istream& in, __int128& num) {
33     string s;in>>s;
34     num=0;
35     for(auto it:s)num=num*10+it-'0';
36     return in;
37 }
38
39 ostream& operator << (ostream& out, __int128 num) {
```

```

40     string s;
41     do{
42         s.push_back(char(num%10+'0'));
43         num/=10;
44     }while(num>0);
45     reverse(s.begin(),s.end());
46     out<<s;
47     return out;
48 }

```

树哈希

```

1  #include <cctype>
2  #include <chrono>
3  #include <cstdio>
4  #include <random>
5  #include <set>
6  #include <vector>
7
8  typedef unsigned long long ull;
9
10 const ull mask = std::chrono::steady_clock::now().time_since_epoch().count();
11
12 ull h(ull x) {
13     return x * x * x * 1237123 + 19260817;
14 }
15 ull f(ull x) {
16     ull cur = h(x & ((1 << 31) - 1)) + h(x >> 31);
17     return cur;
18 }
19 ull shift(ull x) {
20     x ^= mask;
21     x ^= x << 13;
22     x ^= x >> 7;
23     x ^= x << 17;
24     x ^= mask;
25     return x;
26 }
27
28 const int N = 1e6 + 10;
29
30 int n;
31 ull hash[N];
32 std::vector<int> edge[N];
33 std::set<ull> trees;
34
35 void getHash(int x, int p) {
36     hash[x] = 1;
37     for (int i : edge[x]) {
38         if (i == p) {
39             continue;
40         }
41         getHash(i, x);
42         hash[x] += shift(hash[i]);
43     }
44     trees.insert(hash[x]);
45 }

```

```

46
47 int main() {
48     scanf("%d", &n);
49     for (int i = 1; i < n; i++) {
50         int u, v;
51         scanf("%d%d", &u, &v);
52         edge[u].push_back(v);
53         edge[v].push_back(u);
54     }
55     getHash(1, 0);
56     printf("%lu", trees.size());
57 }
58

```

求 s 所有前缀对于 t 的所有子串的最长公共子序列长度

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  struct PairLCS {
4      vector<vector<int>> ih, iv;
5      int n = 0, m = 0;
6      PairLCS(string s, string t) : n(s.size()), m(t.size()) {
7          ih = iv = vector<vector<int>>(n + 1, vector<int>(m + 1));
8          iota(ih[0].begin(), ih[0].end(), 0);
9          for (int i = 1; i <= n; i++) {
10             for (int j = 1; j <= m; j++) {
11                 if (s[i - 1] == t[j - 1]) {
12                     ih[i][j] = iv[i][j - 1];
13                     iv[i][j] = ih[i - 1][j];
14                 } else {
15                     ih[i][j] = std::max(ih[i - 1][j], iv[i][j - 1]);
16                     iv[i][j] = std::min(ih[i - 1][j], iv[i][j - 1]);
17                 }
18             }
19         }
20     }
21     int query(int a, int b, int c) const {
22         int res = 0;
23         for (int i = b + 1; i <= c; i++) res += ih[a][i] <= b;
24         return res;
25     } // s[0,a) t[b,c)
26 };
27
28
29
30 int cas;
31
32 void solution() {
33     int q;
34     std::string s, t;
35     std::cin >> q >> s >> t;
36
37     // int n = s.size(), m = t.size();
38     PairLCS solver(s, t);
39
40     for (int _ = 0; _ < q; _++) {
41         int a, b, c;

```



```

42     std::cin >> a >> b >> c;
43     std::cout << solver.query(a, b, c) << '\n';
44 }
45 }
46
47
48 int main() {
49     ios::sync_with_stdio(false);
50     cin.tie(0);
51     int T = 1;
52     // std::cin >> T;
53     for (cas = 1; cas <= T; cas++) solution();
54
55     return 0;
56 }

```

线性基

```

1  struct LinearBasis
2  {
3      static const int maxbase = 35;
4      bool flag = false;
5      ll a[maxbase + 1];
6      int tot;
7      LinearBasis()
8      {
9          memset(a, 0, sizeof a);
10         tot=0;
11     }
12     LinearBasis(ll *x, int n)
13     {
14         LinearBasis();
15         build(x, n);
16     }
17     void build(ll *x, int n)
18     {
19         for(int i = 1; i <= n; ++i)
20             insert(x[i]);
21     }
22     void clear()
23     {
24         memset(a, 0, sizeof a);
25     }
26     bool insert(ll t)
27     {
28         //暴力插入一个数，维护的是一个上三角型的线性基矩阵，时间复杂度低，当待插入元素能插入时，返
回 true
29         for(int i = maxbase; i >= 0; --i)
30         {
31             if(t & (1ll << i))
32             {
33                 if(!a[i])
34                 {
35                     a[i] = t; //这里表示插入成功
36                     break;
37                 }
38                 t ^= a[i];

```

```

39     }
40 }
41 if(t == 0)flag = true;
42 return t;
43 }
44 bool query(ll t)
45 {
46     // 询问 t 是否可以被当前线性基表示, 不插入
47     if(t > queryMax())return false;
48     if(t == 0)return true;
49     for(int i = maxbase; i >= 0; --i)
50     {
51         if(t & (1ll << i))
52         {
53             if(!a[i])
54             {
55                 return false;
56             }
57             t ^= a[i];
58         }
59     }
60     return true;
61 }
62 void Insert(ll t)
63 {
64     // 插入一个线性基, 利用高斯消元法维护一个对角矩阵
65     for(int i = maxbase; i >= 0; --i)
66     {
67         if(t >> i & 1)
68         {
69             if(a[i])t ^= a[i];
70             else
71             {
72                 a[i] = t;
73                 for(int j = i - 1; j >= 0; --j)if(a[j] && (a[i] >> j & 1))a[i]
74                 ^= a[j];
75                 for(int j = i + 1; j <= maxbase; ++j)if(a[j] >> i & 1)a[j] ^= a[i];
76                 break;
77             }
78         }
79     }
80 LinearBasis merge(const LinearBasis &l1, const LinearBasis &l2)
81 {
82     // 得到两个线性基的并
83     LinearBasis ret = l1;
84     for(int i = maxbase; i >= 0; --i)
85         if(l2.a[i])
86             ret.insert(l2.a[i]);
87     return ret;
88 }
89 LinearBasis intersection(const LinearBasis &l1, const LinearBasis &l2)
90 {
91     // 得到两个线性基的交
92     LinearBasis all, ret, full;
93     ret.clear();

```

```

94     for(int i = maxbase; i >= 0; --i)
95     {
96         all.a[i] = l1.a[i];
97         full.a[i] = l1 << i;
98     }
99     for(int i = maxbase; i >= 0; --i)
100    {
101        if(l2.a[i])
102        {
103            ll v = l2.a[i], k = 0;
104            bool flag = true;
105            for(int j = maxbase; j >= 0; --j)
106            {
107                if(v & (1ll << j))
108                {
109                    if(all.a[j])
110                    {
111                        v ^= all.a[j];
112                        k ^= full.a[j];
113                    }
114                    else
115                    {
116                        // l2's basis is not in l1's;
117                        flag = false;
118                        all.a[j] = v;
119                        full.a[j] = k;
120                        break;
121                    }
122                }
123            }
124            if(flag)
125            {
126                ll v = 0; // get intersection by k;
127                for(int j = maxbase; j >= 0; --j)
128                {
129                    if(k & (1ll << j))
130                    {
131                        v ^= l1.a[j];
132                    }
133                }
134                ret.insert(v);
135            }
136        }
137    }
138    return ret;
139 }
140 // 询问最大值
141 ll queryMax()
142 {
143     ll ret = 0;
144     for(int i = maxbase; i >= 0; --i)
145         if((ret ^ a[i]) > ret)
146             ret ^= a[i];
147     return ret;
148 }
149 ll queryMin()

```

```

150     {
151         for(int i = 0; i <= maxbase; ++i)
152             if(a[i])
153                 return a[i];
154         return 0;
155     }
156 ll Kth_Max(ll k)
157 {
158     ll res=0;
159     for(int i=62;i>=0;i--)
160         if (k&(1LL<<i)) res^=a[i];
161     return res;
162 }
163 };
164

```

维护多个二维向量能够表示的范围

```

1  int gcd(int x,int y){
2      if(y==0)return x;
3      else return gcd(y,x%y);
4  }
5  struct vec{
6      int a00,a01,a11;
7      void clear(){
8          a00=a01=a11=0;
9      }
10 void insert(int x,int y){
11     while(x!=0){
12         int t=a00/x;
13         a00-=x*t;
14         a01-=y*t;
15         swap(a00,x);
16         swap(a01,y);
17     }
18     a11=gcd(a11,abs(y));
19     if(a11!=0)a01%=a11;
20 }
21 bool query(int x,int y){
22     if(x!=0){
23         if(a00==0||x%a00!=0)return false;
24         int t=x/a00;
25         y-=a01*t;
26     }
27     if(y==0){
28         return true;
29     }
30     else return a11!=0&& y%a11==0;
31 }
32 };

```

字符串

AC 自动机

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  const int maxn=1e6+10;
4  int n;
5  char c[maxn];
6  struct AC{
7      int trie[maxn][26],tot;
8      int e[maxn],fail[maxn],old[maxn];
9      void init(){
10         memset(trie,0,sizeof(trie));
11         memset(e,0,sizeof(e));
12         memset(fail,0,sizeof(fail));
13         memset(old,0,sizeof(old));
14         tot=0;
15     }
16     void insert(char *t){
17         int x=0;
18         for(int i=1;t[i];i++){
19             if(!trie[x][t[i]-'a']){
20                 trie[x][t[i]-'a']=++tot;
21             }
22             x=trie[x][t[i]-'a'];
23         }
24         e[x]++;
25     }
26     queue<int> qu;
27     void build(){
28         for(int i=0;i<26;i++){
29             if(trie[0][i])qu.push(trie[0][i]);
30         }
31         while(!qu.empty()){
32             int x=qu.front();
33             qu.pop();
34             for(int i=0;i<26;i++){
35                 if(trie[x][i]){
36                     fail[trie[x][i]]=trie[fail[x]][i];
37                     qu.push(trie[x][i]);
38                 }
39                 else trie[x][i]=trie[fail[x]][i];
40             }
41             old[trie[x][i]]=e[fail[trie[x][i]]] ? fail[trie[x][i]] : old[fail[trie[x]
42             [i]]];
43         }
44     }
45     int query(char *t){//这里是统计有多少模板串出现在了文本串之中，所以统计到了就要变成-1
46         int x=0,res=0;
47         for(int i=1;t[i];i++){
48             x=trie[x][t[i]-'a'];
49             for(int j=x;j&&e[j]!=-1;j=old[j]){
50                 res+=e[j];
51                 e[j]=-1;
52             }
53         }
54     }
```

```

53         return res;
54     }
55 };
56 AC ac;
57 int main()
58 {
59     scanf("%d",&n);
60     for(int i=1;i<=n;i++){
61         scanf("%s",c+1);
62         ac.insert(c);
63     }
64     ac.build();
65     scanf("%s",c+1);
66
67     cout<<ac.query(c)<<endl;
68     return 0;
69 }

```

Dequehash

```

1  /*
2   严格 0base, 不用管任何函数里面的东西, 用就可以了, 不要越界
3   pair<int,int> first 表示哈希 sum, second 表示当前位置的值
4   */
5  #define int long long
6  #define sz(a) ((int)((a).size()))
7  const int maxn=3e5+10;
8  const int mod=1e9+7,base=1331;
9  int fpow(int n, int k, int p = mod) {int r = 1; for (; k; k >>= 1) {if (k & 1) r = r
    * n % p; n = n * n % p;} return r;}
10 void add(int& a, int val, int p = mod) {if ((a = (a + val)) >= p) a -= p;}
11 void sub(int& a, int val, int p = mod) {if ((a = (a - val)) < 0) a += p;}
12 int mul(int a, int b, int p = mod) {return (int) a * b % p;}
13 int inv(int a, int p = mod) {return fpow(a, p - 2, p);}
14 int p[maxn],ip[maxn];
15 void init()
16 {
17     p[0] = 1; for(int i=1;i<maxn;i++) p[i] = mul(p[i - 1], base, mod);
18     for(int i=0;i<maxn;i++) ip[i] = inv(p[i], mod);
19 }
20 struct extendable_sequence {
21     deque<pair<int,int>> dq;
22     int pow_offset;
23
24     extendable_sequence() {
25         pow_offset = 0;
26         dq.push_back(make_pair(0, 0));
27     }
28
29     int size() {
30         return sz(dq) - 1;
31     }
32
33     pair<int,int>& operator [] (int i) {
34         return dq[i+1];
35     }
36

```

```

37 void add_back(vector<int> vals) {
38     int t = dq.back().first;
39     for(int i=0;i<sz(vals);i++) {
40         add(t, mul(vals[i], mul(p[sz(dq) - 1], ip[pow_offset], mod), mod), mod);
41         dq.push_back(make_pair(t, vals[i]));
42     }
43 }
44
45 void add_front(vector<int> vals) {
46     pow_offset += sz(vals);
47     int t = dq.front().first;
48     for(int i=sz(vals)-1;i>=0;i--) {
49         dq.front().second = vals[i];
50         sub(t, mul(vals[i], mul(p[i], ip[pow_offset], mod), mod), mod);
51         dq.push_front(make_pair(t, 0));
52     }
53 }
54
55 int calc(int l, int r) {
56     l++, r++;
57     if (l > r) return 0;
58     int res = dq[r].first;
59     sub(res, dq[l - 1].first, mod);
60     res = mul(res, ip[l - 1], mod);
61     res = mul(res, p[pow_offset], mod);
62     return res;
63 }
64 };
65 //返回(x+y)[l 到 r]的哈希值
66 int calc(extendable_sequence& x, extendable_sequence& y, int l, int r) {
67     int res = x.calc(l, min(r, sz(x) - 1));
68     add(res, mul(y.calc(max(0ll, l - sz(x)), r - sz(x)), p[sz(x)], mod), mod);
69     return res;
70 }
71 //返回(x+y)[i]单个元素的值
72 int calc(extendable_sequence& x, extendable_sequence& y, int i) {
73     if (i < sz(x)) {
74         return x[i].second;
75     }
76     if (i - sz(x) < sz(y)) {
77         return y[i - sz(x)].second;
78     }
79     return -1;
80 }

```

Exkmp

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int maxn = 1e6 + 1e3;
4 struct EXKMP
5 { // S里找T
6
7     char S[maxn], T[maxn];
8     int next[maxn], extend[maxn];
9     void Get_Next()
10    {

```

```

11     int lenT = strlen(T + 1), p = 1, pos;
12     next[1] = lenT; // 对于 next[1] 要特殊考虑
13     while (p + 1 <= lenT && T[p] == T[p + 1])
14         ++p;
15     next[pos = 2] = p - 1; // next[2] 是为了初始化
16
17     for (int i = 3; i <= lenT; i++)
18     { // 注意此时 k + 1 = i
19         int len = next[i - pos + 1];
20         if (len + i < p + 1)
21             next[i] = len; // 对应上面第一种情况
22         else
23         {
24             int j = max(p - i + 1, 0); // 找到前面对于 子串 最靠后已经匹配的位置
25             while (i + j <= lenT && T[j + 1] == T[i + j])
26                 ++j; // 第二种需要暴力匹配
27             p = i + (next[pos = i] = j) - 1; // 记得更新 p, pos
28         }
29     }
30 }
31 void ExKMP()
32 {
33     int lenS = strlen(S + 1), lenT = strlen(T + 1), p = 1, pos;
34     Get_Next();
35     while (p <= lenT && S[p] == T[p])
36         ++p;
37     p = extend[pos = 1] = p - 1; // 初始化 extend[1]
38
39     for (int i = 2; i <= lenS; i++)
40     {
41         int len = next[i - pos + 1];
42         if (len + i < p + 1)
43             extend[i] = len;
44         else
45         {
46             int j = max(p - i + 1, 0);
47             while (i + j <= lenS && j <= lenT && T[j + 1] == S[i + j])
48                 ++j;
49             p = i + (extend[pos = i] = j) - 1;
50         }
51     } // 和上面基本一模一样啦
52 }
53 } sol;
54 int main()
55 {
56     scanf("%s", sol.S + 1);
57     scanf("%s", sol.T + 1);
58
59     sol.ExKMP();
60     int len = strlen(sol.S + 1);
61     for (int i = 1; i <= len; i++)
62         printf("%d%c", sol.extend[i], i == len ? '\n' : ' ');
63
64     return 0;
65 }

```


Hash

```
1  const int N=1e6+10;
2  typedef long long ll;
3  const ll p1=31,p2=131;
4  const ll mod1=1e9+7,mod2=1e9+9;
5  typedef pair<ll,ll> hs;
6  const hs p = make_pair(p1,p2);
7  hs &operator+=(hs &a, hs b) {
8      a.first=(a.first+b.first)%mod1;
9      a.second=(a.second+b.second)%mod2;
10     return a;
11 }
12 hs operator+(hs a, hs b) { return a += b; }
13 hs &operator-=(hs &a, hs b) {
14     a.first=(a.first-b.first+mod1)%mod1;
15     a.second=(a.second-b.second+mod2)%mod2;
16     return a;
17 }
18 hs operator-(hs a, hs b) { return a -= b; }
19 hs &operator*=(hs &a, hs b) {
20     a.first=(a.first*b.first)%mod1;
21     a.second=(a.second*b.second)%mod2;
22     return a;
23 }
24 hs operator*(hs a, hs b) { return a *= b; }
25 struct Hash{
26     int n;
27     vector<hs>has1,has2,Pow;
28     void Hash_init(string &s){
29         n=(int)s.size();
30         Pow.resize(n+2);
31         has1.resize(n+2);
32         has2.resize(n+2);
33         Pow[0]=make_pair(1ll,1ll);
34         for(int i=1;i<=n;i++)Pow[i]=Pow[i-1]*p;
35         for(int i=1;i<=n;i++)has1[i]=has1[i-1]*p+hs{s[i-1]-'a'+1,s[i-1]-'a'+1};
36         for(int i=n;i>=1;i--)has2[i]=has2[i+1]*p+hs{s[i-1]-'a'+1,s[i-1]-'a'+1};
37     }
38     hs get1(int l,int r){
39         return has1[r]-has1[l-1]*Pow[r-l+1];
40     }
41     hs get2(int l,int r){
42         return has2[l]-has2[r+1]*Pow[r-l+1];
43     }
44 };
```

Kmp

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  const int maxn=1e6+10;
4  struct KMP{//lbase
5      int len1,n,nxt[maxn]; //nxt 表示以 i 为结尾的前缀与后缀相同的长度
6      char s1[maxn],s2[maxn];
7      void build()
8      {
9          n=strlen(s+1);
```

```

10     nxt[1]=0;
11     int x=2,now=1;//x 是 s2 当前搜索到的位置, now 是前缀位置
12     while(x<=n){
13         if(s[x]==s[now]){
14             nxt[x]=now;
15             now++;
16             x++;
17         }
18         else{
19             if(now>1){
20                 now=nxt[now-1]+1;
21             }
22             else{
23                 nxt[x]=0;
24                 now=1;
25                 x++;
26             }
27         }
28     }
29 }
30 void find(){//s1 lbase
31     int now=1,tar=1;
32     len1=strlen(s1+1);
33     while(tar<=len1){
34         if(s1[tar]==s[now]){
35             tar++;
36             now++;
37         }
38         else{
39             if(now>1){
40                 now=nxt[now-1]+1;
41             }
42             else tar++;
43         }
44         if(now==n+1){
45             printf("%d\n",tar-now+1);
46         }
47     }
48 }
49 };
50 KMP sol;
51 int main()
52 {
53     scanf("%s%s",sol.s1+1,sol.s+1);
54     sol.build();
55     sol.find();
56     for(int i=1;i<=sol.n;i++)cout<<sol.nxt[i]<<" ";
57     return 0;
58 }

```

Manacher

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int maxn = 1.1e7 + 5;
4 struct ST{
5     char s[maxn * 2], str[maxn * 2];

```

```

6  int Len[maxn * 2], len;
7  void getstr() { // 重定义字符串
8      int k = 0;
9      len = strlen(s);
10     str[k++] = '@'; // 开头加个特殊字符防止越界
11     for (int i = 0; i < len; i++) {
12         str[k++] = '#';
13         str[k++] = s[i];
14     }
15     str[k++] = '#';
16     len = k;
17     str[k] = 0; // 字符串尾设置为 0, 防止越界
18 }
19 int manacher() {
20     int mx = 0, id; // mx 为最右边, id 为中心点
21     int maxx = 0;
22     for (int i = 1; i < len; i++) {
23         if (mx > i) Len[i] = min(mx - i, Len[2 * id - i]); // 判断当前点有没有超过 mx
24         else Len[i] = 1; // 超过了就让他等于 1, 之后再进行检查
25         while (str[i + Len[i]] == str[i - Len[i]]) Len[i]++; // 判断当前点是不是最长回文子串,
不断的向右扩展
26         if (Len[i] + i > mx) { // 更新 mx
27             mx = Len[i] + i;
28             id = i; // 更新中间点
29             maxx = max(maxx, Len[i]); // 最长回文子串长度
30         }
31     }
32     return (maxx - 1);
33 }
34 void writ(){
35     printf("%s\n", str);
36     for(int i=0; i<len; i++){
37         cout<<Len[i]<<" ";
38     }
39     cout<<"\n";
40 }
41 };
42 ST s1, s2;
43 int main() {
44     scanf("%s", s1.s);
45     s1.getstr();
46     printf("%d\n", s1.manacher());
47     return 0;
48 }

```

倍增 SA

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  const int N = 1e6 + 10; // 2*strlen
4  struct Suffix{
5      int ht[N], rk[N], sa[N], y[N], c[N];
6      int n, m;
7      char s[N];
8      int st[20][N];
9      void init(){

```

```

10     n=strlen(s+1);
11     m=300;
12     for(int i=0;i<=m;i++) c[i]=0;
13     for(int i=0;i<=2*n;i++) y[i]=0;
14     for(int i=1;i<=n;i++) c[rk[i]]=s[i]++;
15     for(int i=1;i<=m;i++) c[i]+=c[i-1];
16     for(int i=n;i>=1;i--) sa[c[rk[i]]--]=i;
17     for(int k=1;k<=n;k<=1){
18         int p=0;
19         for(int i=n-k+1;i<=n;i++) y[++p]=i;
20         for(int i=1;i<=n;i++){
21             if(sa[i]>k){
22                 y[++p]=sa[i]-k;
23             }
24         }
25         for(int i=0;i<=m;i++) c[i]=0;
26         for(int i=1;i<=n;i++) c[rk[i]]++;
27         for(int i=1;i<=m;i++) c[i]+=c[i-1];
28         for(int i=n;i>=1;i--) sa[c[rk[y[i]]]--]=y[i];
29         for(int i=0;i<=n;i++) swap(rk[i],y[i]);
30         rk[sa[1]]=p=1;
31         for(int i=2;i<=n;i++){
32             rk[sa[i]]=(y[sa[i]] == y[sa[i-1]] && y[sa[i]+k] == y[sa[i-1]+k] ? p : ++p);
33         }
34         if(p>=n) break;
35         m=p;
36     }
37     for(int i=1,k=0;i<=n;i++){
38         if(k)k--;
39         int j=sa[rk[i]-1];
40         while(s[i+k] == s[j+k])k++;
41         ht[rk[i]] = k;
42     }
43     for(int i=1;i<=n;i++)st[0][i]=ht[i];
44     for(int j=1;j<20;j++){
45         for(int i=1;i+(1<<j)-1<=n;i++)st[j][i]=min(st[j-1][i],st[j-1]
[i+(1<<(j-1))]);
46     }
47 }
48 int get(int l,int r){
49     int g=__lg(r-l+1);
50     return min(st[g][l],st[g][r-(1<<g)+1]);
51 }
52 int lcp(int x,int y){
53     x=rk[x],y=rk[y];
54     if(x==y)return n-x+1;
55     if(x>y)swap(x,y);
56     return get(x+1,y);
57 }
58 int query(int l1,int r1,int l2,int r2){
59     int len=lcp(l1,l2);
60     len=min({len,r1-l1+1,r2-l2+1});
61     if(len==min(r1-l1+1,r2-l2+1)){
62         if(r1-l1>r2-l2)return 1;
63         if(r1-l1<r2-l2)return -1;
64         if(r1-l1==r2-l2)return 0;

```

```

65     }
66     char p=s[l1+len],q=s[l2+len];
67     if(p>q)return 1;
68     if(p==q)return 0;
69     return -1;
70 }
71 void writ()
72 {
73     printf("%s\n",s+1);
74     for(int i=1;i<=n;i++)cout<<sa[i]<<" ";cout<<"\n";
75     for(int i=1;i<=n;i++)cout<<ht[i]<<" ";cout<<"\n";
76     for(int i=1;i<=n;i++)cout<<rk[i]<<" ";cout<<"\n";
77 }
78
79 };
80 Suffix suf;

```

后缀自动机 SAM

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int N=2e6;
4  struct SAM {
5      struct Node {
6          int tr[26];
7          int len, fa;
8          Node() { memset(tr, 0, sizeof(tr)); len = fa = 0; }
9      }ep[N << 1];
10     int last, tot, n;
11     char base;
12     vector<int> edg[N << 1];
13     int siz[N << 1];
14     void init(int _n) {
15         last = tot = 1;
16         base = 'a';
17         for (int i = 0; i <= 2 * _n; i++) {
18             ep[i] = Node();
19             edg[i].clear();
20             siz[i] = 0;
21         }
22     }
23     void insert(char x) {
24         int c = x - base;
25         int p = last;
26         int np = last = ++tot;
27         siz[np] = 1;
28         ep[np].len = ep[p].len + 1;
29         for (; p && !ep[p].tr[c]; p = ep[p].fa)
30             ep[p].tr[c] = np;
31         if (!p) ep[np].fa = 1;
32         else {
33             int q = ep[p].tr[c];
34             if (ep[q].len == ep[p].len + 1) ep[np].fa = q;
35             else {
36                 int nq = ++tot;
37                 ep[nq] = ep[q];
38                 ep[nq].len = ep[p].len + 1;

```

```

39         ep[q].fa = ep[np].fa = nq;
40         for (; p && ep[p].tr[c] == q; p = ep[p].fa)
41             ep[p].tr[c] = nq;
42     }
43 }
44 }
45 void construct() {
46     for (int i = 2; i <= tot; i++) {
47         edg[ep[i].fa].push_back(i);
48     }
49 }
50 void dfs(int u) {
51     for (auto v : edg[u]) {
52         dfs(v);
53         siz[u] += siz[v];
54     }
55 }
56 void build(string& s) {
57     n = s.size();
58     init(n);
59     for (int i = 0; i < n; i++) {
60         insert(s[i]);
61     }
62     construct();
63     dfs(1);
64 }
65 } sam;
66 }
67

```

回文自动机 PAM

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  const int N=2e6+10;
4  struct PAM_Trie
5  {
6      int ch[26];
7      int fail,len,num;
8  };
9  struct PAM
10 {
11     PAM_Trie b[N];
12     int n,length,last,cnt;
13     char s[N];
14     PAM()
15     {
16         b[0].len = 0; b[1].len = -1;
17         b[0].fail = 1; b[1].fail = 0;
18         last = 0;
19         cnt = 1;
20     }
21     int get_fail(int x)
22     {
23         while(s[n-b[x].len-1]!=s[n])
24         {
25             x=b[x].fail;

```

```

26     }
27     return x;
28 }
29 void insert()
30 {
31     int p=get_fail(last);
32     if(!b[p].ch[s[n]-'a'])
33     {
34         b[++cnt].len=b[p].len+2;
35         int tmp=get_fail(b[p].fail);
36         b[cnt].fail=b[tmp].ch[s[n]-'a'];
37         b[cnt].num=b[b[cnt].fail].num+1;
38         b[p].ch[s[n]-'a']=cnt;
39     }
40     last=b[p].ch[s[n]-'a'];
41     cout<<last<<"\n";
42     //如果要统计出现次数 f[last]++;
43 }
44 void init()
45 {
46     length=strlen(s+1);
47     for(n=1;n<=length;n++)
48     {
49         insert();
50     }
51 }
52 }pa;
53 int main()
54 {
55     scanf("%s",pa.s+1);
56     pa.init();
57     return 0;
58 }

```

最小表示

```

1 int getMin(string s) {
2     int i = 0, j = 1, k = 0;
3     int len = s.length();
4     while(i<len && j<len && k<len) {
5         int tmp = s[(i + k) % len] - s[(j + k) % len];
6         if(tmp==0) k++;
7         else {
8             if(tmp>0) i += k + 1;
9             else j += k + 1;
10            if(i==j) j++;
11            k = 0;
12        }
13    }
14    return min(i, j);
15 }

```

Runs

```

1 struct LongestCommonPrefix {
2     int n;
3     vector<int> p, rank;
4     vector<vector<int>> st;

```

```

5   LongestCommonPrefix(const string &s) : n(s.size()), p(n), rank(n) {
6       int k = 0;
7       vector<int> q, count;
8       for (int i = 0; i < n; i += 1) p[i] = i;
9       sort(p.begin(), p.end(), [&](int i, int j) { return s[i] < s[j]; });
10      for (int i = 0; i < n; i += 1)
11          rank[p[i]] = i and s[p[i]] == s[p[i - 1]] ? rank[p[i - 1]] : k++;
12      for (int m = 1; m < n; m *= 2) {
13          q.resize(m);
14          for (int i = 0; i < m; i += 1) q[i] = n - m + i;
15          for (int i : p)
16              if (i >= m) q.push_back(i - m);
17          count.assign(k, 0);
18          for (int i : rank) count[i] += 1;
19          for (int i = 1; i < k; i += 1) count[i] += count[i - 1];
20          for (int i = n - 1; i >= 0; i -= 1) p[count[rank[q[i]]] - 1] = q[i];
21          auto cur = rank;
22          cur.resize(2 * n, -1);
23          k = 0;
24          for (int i = 0; i < n; i += 1)
25              rank[p[i]] = i and cur[p[i]] == cur[p[i - 1]] and
26                  cur[p[i] + m] == cur[p[i - 1] + m]
27                  ? rank[p[i - 1]]
28                  : k++;
29      }
30      st.emplace_back(n);
31      for (int i = 0, k = 0; i < n; i += 1) {
32          if (not rank[i]) continue;
33          k = max(k - 1, 0);
34          int j = p[rank[i] - 1];
35          while (i + k < n and j + k < n and s[i + k] == s[j + k]) k += 1;
36          st[0][rank[i]] = k;
37      }
38      for (int i = 1; (1 << i) < n; i += 1) {
39          st.emplace_back(n - (1 << i) + 1);
40          for (int j = 0; j <= n - (1 << i); j += 1)
41              st[i][j] = min(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
42      }
43  }
44  int get(int i, int j) {
45      if (i == j) return n - i;
46      if (i == n or j == n) return 0;
47      i = rank[i];
48      j = rank[j];
49      if (i > j) swap(i, j);
50      int k = 64 - __builtin_clzll(u64(j - i)) - 1;
51      return min(st[k][i + 1], st[k][j - (1 << k) + 1]);
52  }
53 };
54 /*
55     input 0 base
56     output [l,r,p] lbase
57 */
58 vector<tuple<int, int, int>> run(const string &s) {
59     int n = s.size();
60     auto r = s;

```



```

61 reverse(r.begin(), r.end());
62 LongestCommonPrefix lcp(s), lcs(r);
63 vector<tuple<int, int, int>> runs;
64 for (bool inv : {false, true}) {
65     vector<int> lyn(n, n), stack;
66     for (int i = 0; i < n; i += 1) {
67         while (not stack.empty()) {
68             int j = stack.back(), k = lcp.get(i, j);
69             if (i + k < n and ((s[i + k] > s[j + k]) ^ inv)) break;
70             lyn[j] = i;
71             stack.pop_back();
72         }
73         stack.push_back(i);
74     }
75     for (int i = 0; i < n; i += 1) {
76         int j = lyn[i], t = j - i, l = i - lcs.get(n - i, n - j),
77         r = j + lcp.get(i, j);
78         if (r - l >= 2 * t) runs.emplace_back(l+1, r, t);
79     }
80 }
81 sort(runs.begin(), runs.end());
82 runs.erase(unique(runs.begin(), runs.end()), runs.end());
83 return runs;
84 }

```

图论

Johnson 全源最短路

```
1 struct graph {
2     vector<vector<pair<int, ll>>> e;
3     graph(int n) : e(n + 1) {}
4     void adde(int u, int v, ll w) { e[u].push_back({v, w}); }
5     vector<ll> h;
6     // initialize h(u), return false if there exists a negative cycle
7     bool init() {
8         int n = e.size();
9         h.assign(n, 0);
10        queue<int> que;
11        for (int u = 1; u < n; u++) que.push(u);
12        vector<int> vis(n, 0), cnt(n, n + 1);
13        while (que.size()) {
14            auto u = que.front();
15            que.pop();
16            vis[u] = false;
17            if (!cnt[u]--) return false; // exists a negative cycle
18            for (auto &[v, w] : e[u])
19                if (h[v] > h[u] + w) {
20                    h[v] = h[u] + w;
21                    if (!vis[v]) que.push(v), vis[v] = 1;
22                }
23        }
24        return true;
25    }
26    // single source shortest path from given sink based on h(u)
27    vector<ll> query(int s) {
28        int n = e.size();
29        vector<ll> dis(n, inf);
30        priority_queue<pair<ll, int>, vector<pair<ll, int>>,
31            greater<pair<ll, int>>>
32            que;
33        que.push({dis[s] = 0, s});
34        while (que.size()) {
35            auto [du, u] = que.top();
36            que.pop();
37            if (dis[u] < du) continue;
38            for (auto [v, w] : e[u]) {
39                auto dv = du + w + h[u] - h[v];
40                if (dis[v] > dv) que.push({dis[v] = dv, v});
41            }
42        }
43        for (int i = 0; i < n; i++) dis[i] += h[i] - h[s];
44        return dis;
45    }
46 };
```

Kosaraju

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 const int maxn = 100010;
4 vector<int> ve[maxn], ve2[maxn];
5 vector<int> sta; // 存第一次 dfs1() 的结果，即标记点的先后顺序，优先级小的点先进
```

```

6  int vis[maxn];    // vis[i]标记第一次 dfs1()点 i 是否访问过
7  int col[maxn];    // col[i]标记点 i 属于第几个强连通分量, 同时记录 dfs2()过程中点 i 是否访问过
8  int cnt;          // cnt 表示强连通分量的个数
9  void dfs1(int x){
10     vis[x] = 1;
11     for(auto it:ve[x])if(!vis[it])
12         dfs1(it);
13     sta.push_back(x); //记录点的先后顺序, 按照拓扑排序, 优先级大的放在 S 的后面
14 }
15
16 void dfs2(int x){
17     if(col[x]) return;
18     col[x] = cnt;
19     for(auto it:ve[x])if(!col[it])
20         dfs2(it);
21 }
22 void Kosaraju(int n) {
23     cnt = 0;
24     sta.clear();
25     memset(vis,0,sizeof(vis));
26     memset(col,0,sizeof(col));
27     for(int i=1; i<=n; i++) //搜索所有点
28         dfs1(i);
29     for(int i=n-1; i>=0; i--){
30         if(!col[sta[i]]){
31             cnt++;
32             dfs2(sta[i]);
33         }
34     }
35 }
36

```

K 短路

```

1  //复杂度 nlogn +mlogm+klogk
2  #include <bits/stdc++.h>
3  #include <queue>
4
5  template <class T, class U>
6  inline bool smin(T &x, const U &y) {
7      return y < x ? x = y, 1 : 0;
8  }
9  template <class T, class U>
10 inline bool smax(T &x, const U &y) {
11     return x < y ? x = y, 1 : 0;
12 }
13
14 using LL = long long;
15 using PII = std::pair<int, int>;
16
17 constexpr int N(2.5e5 + 5);
18
19 using T = LL;
20 struct Edge {
21     int x, y; T z;
22 };
23 struct Heap {

```

```

24     struct Node {
25         int ls, rs, h, v;
26         T w;
27     } t[N * 40];
28     int cnt;
29     int newNode(int v, T w) {
30         t[++cnt] = {0, 0, 1, v, w};
31         return cnt;
32     }
33     int merge(int x, int y) {
34         if (!x) return y;
35         if (!y) return x;
36         if (t[x].w > t[y].w) std::swap(x, y);
37         t[++cnt] = t[x], x = cnt;
38         t[x].rs = merge(t[x].rs, y);
39         if (t[t[x].ls].h < t[t[x].rs].h) std::swap(t[x].ls, t[x].rs);
40         t[x].h = t[t[x].rs].h + 1;
41         return x;
42     }
43 } h;
44
45
46 std::vector<T> kShortestPath(int n, int k, int s, int t, const std::vector<Edge> &e) {
47     int m = e.size();
48     std::vector<int> deg(n + 1), g(m);
49     for (auto &[x, y, z] : e) deg[y]++;
50     for (int i = 1; i <= n; i++) deg[i] += deg[i - 1];
51     for (int i = 0; i < m; i++) g[--deg[e[i].y]] = i;
52
53     std::vector<T> d(n, -1);
54     std::vector<int> fa(n, -1), p;
55
56     using Q = std::pair<T, int>;
57     std::priority_queue<Q, std::vector<Q>, std::greater<Q>> q;
58
59     {
60         p.reserve(n);
61         d[t] = 0, q.push({0, t});
62
63         std::vector<bool> vis(n);
64         while (!q.empty()) {
65             int x = q.top().second;
66             q.pop();
67             if (vis[x]) continue;
68             vis[x] = true;
69             p.push_back(x);
70             for (int i = deg[x]; i < deg[x + 1]; i++) {
71                 auto &[y, _, z] = e[g[i]];
72                 if (d[y] == -1 || d[y] > d[x] + z) {
73                     d[y] = d[x] + z, fa[y] = g[i];
74                     q.push({d[y], y});
75                 }
76             }
77         }
78     }
79 }

```

```

80     if (d[s] == -1) std::vector<T>(k, -1);
81     std::vector<int> heap(n);
82     h.cnt = 0;
83     for (int i = 0; i < m; i++) {
84         auto &[x, y, z] = e[i];
85         if (d[x] != -1 && d[y] != -1 && fa[x] != i) {
86             heap[x] = h.merge(heap[x], h.newNode(y, d[y] + z - d[x]));
87         }
88     }
89
90     for (int x : p) {
91         if (x != t) heap[x] = h.merge(heap[x], heap[e[fa[x]].y]);
92     }
93
94     if (heap[s]) q.push({d[s] + h.t[heap[s]].w, heap[s]});
95     std::vector<T> res = {d[s]};
96
97     for (int i = 1; i < k && !q.empty(); i++) {
98         auto [w, o] = q.top();
99         q.pop();
100
101         res.push_back(w);
102
103         int j = h.t[o].v;
104         if (heap[j]) q.push({w + h.t[heap[j]].w, heap[j]});
105         for (auto s : {h.t[o].ls, h.t[o].rs}) {
106             if (s) q.push({w + h.t[s].w - h.t[o].w, s});
107         }
108     }
109     res.resize(k, -1);
110     return res;
111 }
112
113 int a[N];
114 void solve() {
115     int n, k;
116     std::cin >> n >> k;
117
118     std::vector<Edge> e;
119     for (int i = 1; i <= n; i++) {
120         std::cin >> a[i];
121     }
122     e.push_back({0, 1, a[1]});
123     for (int i = 2; i <= n; i++) {
124         if (i - 3 > 0) e.push_back({i - 3, i, a[i]});
125         e.push_back({i - 2, i, a[i]});
126         e.push_back({i - 1, i, a[i]});
127     }
128     if (n - 1 >= 1) e.push_back({n - 1, n + 1, 0});
129     e.push_back({n, n + 1, 0});
130
131     auto res = kShortestPath(n + 2, k, 0, n + 1, e);
132
133     for (auto x : res) std::cout << x << "\n";
134 }
135

```

```

136 int main() {
137     // freopen("t.in", "r", stdin);
138
139     std::ios::sync_with_stdio(false);
140     std::cin.tie(nullptr);
141
142     int t = 1;
143
144     // std::cin >> t;
145
146     while (t--) {
147         solve();
148     }
149     return 0;
150 }

```

Maxflow 只算值版本

```

1 struct dinic{
2     struct E{
3         int to, cap, inv;
4     };
5     vector<E> g[N];
6     int dis[N], now[N];
7     void addedge(int u, int v, int w){
8         g[u].push_back({v, w, (int)g[v].size()});
9         g[v].push_back({u, 0, (int)g[u].size()-1});
10    }
11    void bfs(int st){
12        queue<int> q;
13        memset(dis, 0, sizeof dis);
14        q.push(st); dis[st]=1;
15        while(q.size()){
16            int u=q.front(); q.pop();
17            for(auto &[v, w, inv]:g[u]){
18                if(w&&!dis[v]){
19                    dis[v]=dis[u]+1;
20                    q.push(v);
21                }
22            }
23        }
24    }
25    int dfs(int u, int t, int flow){
26        if(u==t) return flow;
27        for(int &i=now[u], sz=g[u].size(), d; i<sz; i++){
28            auto &[v, w, inv]=g[u][i];
29            if(w&&dis[v]>dis[u]){
30                d=dfs(v, t, min(flow, w));
31                if(d>0){
32                    w-=d;
33                    g[v][inv].cap+=d;
34                    return d;
35                }
36            }
37        }
38        return 0;
39    }

```

```

40     int maxflow(int st,int ed){
41         for(int flow=0,res;;){
42             bfs(st);
43             if(!dis[ed])return flow;
44             memset(now,0,sizeof now);
45             while((res=dfs(st,ed,inf))>0){
46                 flow+=res;
47             }
48         }
49     }
50 };

```

Maxflow 网络流最大流

```

1 // 用 givest 定源点汇点
2 // addedge 一次加了正反两条边
3 // init 慎用
4 // S 必须是 0
5 // 输出方案注意是 head 开头
6 #include <bits/stdc++.h>
7 using namespace std;
8 const int N=2510,M=2510*10;
9 class Maxflow{
10 private:
11     int nedge=1,p[2*M],nex[2*M],head[N],c[2*M],cur[2*M];
12     int dist[2*N];
13     int S,T;
14     void Addedge(int a,int b,int v){
15         p[++nedge]=b;nex[nedge]=head[a];head[a]=nedge;
16         c[nedge]=v;
17     }
18     bool bfs(){
19         queue<int>q;
20         for(int i=S;i<=T;i++)dist[i]=-1;
21         dist[S]=0;q.push(S);
22         while(!q.empty()){
23             int now=q.front();q.pop();
24             for(int k=head[now];k;k=nex[k])if(dist[p[k]]==-1&&c[k]>0){
25                 dist[p[k]]=dist[now]+1;
26                 q.push(p[k]);
27             }
28         }
29         return dist[T]>-1;
30     }
31     int dfs(int x,int low){
32         if(x==T)return low;
33         if(low==0)return 0;
34         int used=0;
35         for(int &k=cur[x];k;k=nex[k])if(dist[p[k]]==dist[x]+1&&c[k]>0){
36             int a=dfs(p[k],min(c[k],low-used));
37             c[k]-=a;c[k^1]+=a;used+=a;
38             if(low==used)break;
39         }
40         if(used==0)dist[x]=-1;
41         return used;
42     }
43 public:

```

```

44 void init(int s,int t){
45     for(int i=S;i<=T;i++)head[i]=0;
46     S=s,T=t;
47     nedge=1;
48 }
49 void addedge(int a,int b,int v){
50     Addedge(a,b,v);
51     Addedge(b,a,0);
52 }
53 int dinic(){
54     int flow=0;
55     while(bfs()){
56         for(int i=S;i<=T;i++)cur[i]=head[i];
57         flow+=dfs(S,1e9);
58     }
59     return flow;
60 }
61 };

```

Tarjan 缩点

```

1 stack<int>s;
2 vector<int>ve[maxn];
3 int col[maxn],num,dfn[maxn],low[maxn],dfstime;
4 void tarjan(int u)
5 {
6     s.push(u);
7     dfn[u]=low[u]=++dfstime;
8     for(auto v:ve[u])
9     {
10         if(!dfn[v])
11         {
12             tarjan(v);
13             low[u]=min(low[u],low[v]);
14         }
15         else if(!col[v]) low[u]=min(low[u],dfn[v]);
16     }
17     if(dfn[u]==low[u])
18     {
19         col[u]=++num;
20         while(s.top()!=u)
21         {
22             col[s.top()]=num;
23             s.pop();
24         }
25         s.pop();
26     }
27 }

```

二分图匹配

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 int const N = 1510 * 4, M = 75010;
4 int e[M], ne[M], h[N], idx;
5 int n, m, k, match[N], backup[N], st[N];
6 void add(int a, int b) {
7     e[idx] = b, ne[idx] = h[a], h[a] = idx++;

```



```

8 }
9 int find(int x) {
10     for (int i = h[x]; ~i; i = ne[i]) {
11         int j = e[i];
12         if (!st[j]) {
13             st[j] = 1;
14             if (!match[j] || find(match[j])) {
15                 match[j] = x;
16                 return 1;
17             }
18         }
19     }
20     return 0;
21 }
22 int main() {
23     cin >> n >> m >> k;
24     memset(h, -1, sizeof h);
25     for (int i = 1, a, b; i <= k; ++i) {
26         scanf("%d%d", &a, &b);
27         add(a, b + n);
28     }
29     int maxMatch = 0;
30     for (int i = 1; i <= n; ++i) {
31         memset(st, 0, sizeof st);
32         if (find(i)) maxMatch++;
33     }
34     cout<<maxMatch<<endl;
35     return 0;
36 }
37

```

二分图最优匹配

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 const int maxn=110;
4 int n, m;
5 int a[maxn][maxn];
6 int lx[maxn], ly[maxn], link[maxn];
7 bool vx[maxn], vy[maxn];
8 int dfs(int x)
9 {
10     if(x==-1)return 0;
11     vx[x] = 1;
12     for (int i = 1; i <= n; i++)
13     {
14         if (!vy[i] && lx[x] + ly[i] == a[x][i])
15         {
16             vy[i] = 1;
17             if (link[i] == -1 || dfs(link[i]))
18             {
19                 link[i] = x;
20                 return 1;
21             }
22         }
23     }
24     return 0;
25 }

```

```

25 }
26 bool deal()
27 {
28     memset(ly, 0, sizeof(ly));
29     memset(lx, 0xf7, sizeof(lx));
30     memset(link, -1, sizeof(link));
31     for (int i = 1; i <= n; i++)
32     {
33         for (int j = 1; j <= n; j++)
34             lx[i] = max(lx[i], a[i][j]);
35     }
36     for(int i = 1; i <= n; i++)
37     {
38         while(1)
39         {
40             memset(vx, 0, sizeof(vx));
41             memset(vy, 0, sizeof(vy));
42             if (dfs(i)) break;
43             int delta = 0x7f7f7f7f;
44             for (int j= 1; j <= n; j++)
45             {
46                 if (vx[j] == 1)
47                     for(int k = 1; k <= n; k++)
48                         if (vy[k] == 0) delta = min(delta, lx[j]+ ly[k]- a[j][k]);
49             }
50             if (delta == 0x7f7f7f7f) return 0;
51             for (int j= 1; j <= n; j++)
52                 if (vx[j] == 1) lx[j] -= delta;
53             for(int k = 1; k <= n; k++)
54                 if (vy[k] == 1) ly[k] += delta;
55         }
56     }
57     return 1;
58 }
59 int main()
60 {
61     if (deal() == 1) {
62         int ans = 0;
63         for(int i = 1; i <= n; i++)
64         {
65             ans += a[link[i]][i];
66         }
67         cout << ans << '\n'; //取最小就把所有边权取负再跑
68     }
69     return 0;
70 }

```

二分图染色

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int maxn=1010;
4 int n,m;
5 vector<int>ve[maxn];
6 int col[maxn][maxn],ans[maxn*2];
7 void dfs(int x,int y,int c1,int c2)
8 {

```

```

9     if(col[y][c1]){
10         dfs(y,col[y][c1],c2,c1);
11         col[x][c1]=y;
12         col[y][c1]=x;
13     }
14     else {
15         col[x][c1]=y;
16         col[y][c1]=x;
17         col[y][c2]=0;
18     }
19 }
20 map<pair<int,int>,int>ma;
21 int main()
22 {
23     ios::sync_with_stdio(false);
24     cin.tie(0);
25     cin>>n>>m;
26     int anss=0;
27     for(int i=1;i<=m;i++){
28         int x,y;
29         cin>>x>>y;
30         ve[x].push_back(y);
31         ma[{x,y}]=i;
32         int c1=1,c2=1;
33         while(col[x][c1])c1++;
34         while(col[y][c2])c2++;
35         anss=max({c1,c2,anss});
36         if(c1>c2){
37             swap(x,y);swap(c1,c2);
38         }
39         if(c1==c2){
40             col[x][c1]=y;
41             col[y][c1]=x;
42         }
43         else{
44             dfs(x,y,c1,c2);
45         }
46     }
47     cout<<anss<<"\n";
48     for(int i=1;i<=n;i++){
49         for(int j=1;j<=anss;j++)if(col[i][j])ans[ma[{i,col[i][j]}]]=j;
50     }
51     for(int i=1;i<=m;i++)cout<<ans[i]<<"\n";
52     return 0;
53 }

```

圆方树

```

1     vector<vector<int>> e1(n);
2     int cnt = n;
3
4     int now = 0;
5     vector<int> dfn(n, -1), low(n);
6     vector<int> stk;
7     function<void(int)> tarjan = [&](int u) {
8         stk.push_back(u);
9         dfn[u] = low[u] = now++;

```

```

10     for (auto v : ve[u]) {
11         if (dfn[v] == -1) {
12             tarjan(v);
13             low[u] = min(low[u], low[v]);
14             if (low[v] == dfn[u]) {
15                 e1.push_back({});
16                 int x;
17                 do {
18                     x = stk.back();
19                     stk.pop_back();
20                     e1[cnt].push_back(x);
21                 } while (x != v);
22                 e1[u].push_back(cnt);
23                 ++cnt;
24             }
25         } else {
26             low[u] = min(low[u], dfn[v]);
27         }
28     }
29 };
30 tarjan(0);

```

基环树

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int maxn=2e5+10;
4  /*
5  1 init
6  2 addedge
7  3 Get
8  */
9  struct Graph{
10     vector<int>ve[maxn];
11     int base[maxn],id[maxn];
12     bool Incircle[maxn];
13     vector<int> Circle;
14     int len=0;
15     int dep[maxn],f[21][maxn];
16     int n;
17     void init(int _n){
18         n=_n;
19         for(int i=1;i<=n;i++){
20             for(int j=0;j<21;j++)f[j][i]=0;
21             ve[i].clear();
22             Incircle[i]=false;
23             id[i]=-1;
24             base[i]=i;
25             Circle.clear();
26             len=0;
27             dep[i]=0;
28         }
29     }
30     void addedge(int x,int y){
31         ve[x].push_back(y);
32         ve[y].push_back(x);
33     }

```

```

34 void dfs(int x,int fa)
35 {
36     base[x]=base[fa];
37     dep[x]=dep[fa]+1;
38     for(int i=0;i<=19;i++)
39         f[i+1][x]=f[i][f[i][x]];
40     for(auto it:ve[x])
41     {
42         if(it==fa) continue;
43         f[0][it]=x;
44         dfs(it,x);
45     }
46 }
47 void Get(){
48     vector<int> sta;
49     vector<bool> vis(n+1,false);
50     function<bool(int,int)> dfs2 = [&](int x,int h){
51         vis[x]=true;
52         sta.push_back(x);
53         for(auto it:ve[x])if(it!=h){
54             if(vis[it]){
55                 Circle.push_back(it);
56                 while(!sta.empty()&&sta.back()!=it){
57                     Circle.push_back(sta.back());
58                     sta.pop_back();
59                 }
60                 return true;
61             }
62             else{
63                 if(dfs2(it,x))return true;
64             }
65         }
66         sta.pop_back();
67         return false;
68     };
69     dfs2(1,0);
70     len=(int)Circle.size();
71     for(auto it:Circle)Incircle[it]=true;
72     for(auto it:Circle){
73         for(auto it2:ve[it])if(!Incircle[it2]){
74             f[0][it2]=it;
75             dfs(it2,it);
76         }
77     }
78     for(int i=0;i<len;i++)id[Circle[i]]=i;
79 }
80 int lca(int x,int y)
81 {
82     if(dep[x]<dep[y]) swap(x,y);
83     for(int i=20;i>=0;i--)
84     {
85         if(dep[f[i][x]]>=dep[y]) x=f[i][x];
86         if(x==y) return x;
87     }
88     for(int i=20;i>=0;i--)
89         if(f[i][x]!=f[i][y])

```

```

90         x=f[i][x],y=f[i][y];
91     return f[0][x];
92 }
93 int dis(int x,int y){
94     if(base[x]==base[y]){
95         int l=lca(x,y);
96         return dep[x]+dep[y]-2*dep[l];
97     }
98     else{
99         int g=(id[base[x]]-id[base[y]]+len)%len;
100        return dep[x]+dep[y]+min(g,len-g);
101    }
102 }
103 };
104 Graph g;

```

带权并查集 dsu

```

1  const int maxn=1e5+10;
2  int f[maxn],dis[maxn];
3  int getf(int x){
4      if(x==f[x])return x;
5      int z=getf(f[x]);
6      dis[x]+=dis[f[x]];
7      return f[x]=z;
8  }
9  void unit(int i,int j,int len){
10     int x=getf(i),y=getf(j);
11     f[x]=y;
12     //在赋值之前因为 x 是头节点所以 dis 一定等于 0
13     dis[x]=dis[j]-dis[i]+len;
14 }

```

带花树

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  struct blossom { // 0base
4      int n, vis_t;
5      vector<vector<int>> E;
6      vector<int> match, label, org, vis, parent;
7      queue<int> Q;
8      blossom(int _n) {
9          n = _n;
10         E = vector<vector<int>>(n, vector<int>());
11         match.assign(n, -1);
12         label.resize(n);
13         org.resize(n);
14         iota(org.begin(), org.end(), 0);
15         parent.assign(n, -1);
16         vis.assign(n, 0);
17         vis_t = 0;
18     }
19     void addEdge(int u, int v) {
20         E[u].emplace_back(v);
21         E[v].emplace_back(u);
22     }
23     auto lca(int v, int u) {

```

```

24     vis_t++;
25     while (true) {
26         if (v != -1) {
27             if (vis[v] == vis_t) {
28                 return v;
29             }
30             vis[v] = vis_t;
31             if (match[v] == -1) {
32                 v = -1;
33             } else {
34                 v = org[parent[match[v]]];
35             }
36         }
37         swap(v, u);
38     }
39 }
40 void agument(int v) {
41     while (v != -1) {
42         auto pv = parent[v];
43         auto nxt = match[pv];
44         match[v] = pv;
45         match[pv] = v;
46         v = nxt;
47     }
48 }
49 void flower(int v, int u, int a) {
50     while (org[v] != a) {
51         parent[v] = u;
52         u = match[v];
53         if (label[u] == 1) {
54             label[u] = 0;
55             Q.emplace(u);
56         }
57         org[v] = org[u] = a;
58         v = parent[u];
59     }
60 }
61 auto bfs(int root) {
62     fill(label.begin(), label.end(), -1);
63     iota(org.begin(), org.end(), 0);
64     while (!Q.empty()) {
65         Q.pop();
66     }
67     Q.emplace(root);
68     label[root] = 0;
69     while (!Q.empty()) {
70         auto u = Q.front();
71         Q.pop();
72         for (auto v : E[u]) {
73             if (label[v] == -1) {
74                 label[v] = 1;
75                 parent[v] = u;
76                 if (match[v] == -1) {
77                     agument(v);
78                     return true;
79                 }

```

```

80         label[match[v]] = 0;
81         Q.push(match[v]);
82         continue;
83     } else if (label[v] == 0 && org[v] != org[u]) {
84         auto a = lca(org[u], org[v]);
85         flower(v, u, a);
86         flower(u, v, a);
87     }
88 }
89 }
90 return false;
91 }
92 void solve() {
93     for (int i = 0; i < n; ++i) {
94         if (match[i] == -1) {
95             bfs(i);
96         }
97     }
98 }
99 };
100 int main()
101 {
102     blossom G(n);
103     for (int i = 0; i < n; ++i) {
104         for (int j = i + 1; j < n; ++j) {
105             auto [xi, yi] = stone[i];
106             auto [xj, yj] = stone[j];
107             if (abs(xi - xj) + abs(yi - yj) <= L) {
108                 G.addEdge(i, j);
109             }
110         }
111     }
112     G.solve();
113     int num = 0;
114     for (int i = 0; i < n; ++i) {
115         if (G.match[i] != -1) {
116             num++;
117         }
118     }
119 }

```

带负环最小费用最大流

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int N = 200 + 5, M = 2e4 + N;
4  struct flow {
5      int cnt = 1, hd[N], nxt[M << 1], to[M << 1], limit[M << 1], cst[M << 1];
6      void add(int u, int v, int w, int c) {
7          nxt[++cnt] = hd[u], hd[u] = cnt, to[cnt] = v, limit[cnt] = w, cst[cnt] = c;
8          nxt[++cnt] = hd[v], hd[v] = cnt, to[cnt] = u, limit[cnt] = 0, cst[cnt] = -c;
9      }
10     int fl[N], fr[N], dis[N], in[N];
11     pair<int, int> mincost(int s, int t) {
12         int flow = 0, cost = 0;
13         while (1) {
14             queue<int> q;

```



```

15     memset(dis, 0x3f, sizeof(dis));
16     q.push(s), fl[s] = 1e9, dis[s] = 0;
17     while (!q.empty()) {
18         int t = q.front();
19         q.pop(), in[t] = 0;
20         for (int i = hd[t]; i; i = nxt[i]) {
21             int it = to[i], d = dis[t] + cst[i];
22             if (limit[i] && d < dis[it]) {
23                 dis[it] = d, fl[it] = min(fl[t], limit[i]), fr[it] = i;
24                 if (!in[it]) in[it] = 1, q.push(it);
25             }
26         }
27     }
28     if (dis[t] > 1e9) return make_pair(flow, cost);
29     flow += fl[t], cost += dis[t] * fl[t];
30     for (int u = t; u != s; u = to[fr[u] ^ 1])
31         limit[fr[u]] -= fl[t], limit[fr[u] ^ 1] += fl[t];
32 }
33 }
34 };
35 struct bounded_flow {
36     int e, u[M], v[M], lo[M], hi[M], cst[M];
37     void add(int _u, int _v, int w, int c) {
38         if (c < 0) {
39             u[++e] = _u, v[e] = _v, lo[e] = w, hi[e] = w, cst[e] = c;
40             u[++e] = _v, v[e] = _u, lo[e] = 0, hi[e] = w, cst[e] = -c;
41         } else
42             u[++e] = _u, v[e] = _v, lo[e] = 0, hi[e] = w, cst[e] = c;
43     }
44     flow g;
45     pair<int, int> mincost(int n, int s, int t, int ss, int tt) {
46         static int w[N];
47         memset(w, 0, sizeof(w));
48         int flow = 0, cost = 0, tot = 0;
49         for (int i = 1; i <= e; i++) {
50             w[u[i]] -= lo[i], w[v[i]] += lo[i];
51             cost += lo[i] * cst[i];
52             g.add(u[i], v[i], hi[i] - lo[i], cst[i]);
53         }
54         for (int i = 1; i <= n; i++)
55             if (w[i] > 0)
56                 g.add(ss, i, w[i], 0), tot += w[i];
57             else if (w[i] < 0)
58                 g.add(i, tt, -w[i], 0);
59         g.add(t, s, 1e9, 0);
60         pair<int, int> res = g.mincost(ss, tt);
61         cost += res.second;
62         flow += g.limit[g.hd[s]];
63         g.hd[s] = g.nxt[g.hd[s]], g.hd[t] = g.nxt[g.hd[t]];
64         res = g.mincost(s, t);
65         return make_pair(flow + res.first, cost + res.second);
66     }
67 } f;
68 int n, m, s, t;
69 int main() {
70     cin >> n >> m >> s >> t;

```

```

71     for (int i = 1; i <= m; i++) {
72         int u, v, w, c;
73         cin >> u >> v >> w >> c, f.add(u, v, w, c);
74     }
75     pair<int, int> res = f.mincost(n, s, t, 0, n + 1);
76     cout << res.first << " " << res.second << endl;
77     return 0;
78 }
79

```

支配树

```

1  /*
2  lbase
3  注意 up 是数组需要外界导入
4  使用的时候直接 dtree::即可
5  */
6  namespace dtree{
7      const int MAXN = 200020;
8      vector<int> E[MAXN], RE[MAXN], rdom[MAXN];
9
10     int S[MAXN], RS[MAXN], cs;
11     int par[MAXN], val[MAXN], sdom[MAXN], rp[MAXN], dom[MAXN];
12
13     void clear(int n) {
14         cs = 0;
15         for(int i=0;i<=n;i++) {
16             par[i] = val[i] = sdom[i] = rp[i] = dom[i] = S[i] = RS[i] = 0;
17             E[i].clear(); RE[i].clear(); rdom[i].clear();
18         }
19     }
20     void add_edge(int x, int y) { E[x].push_back(y); }
21     void Union(int x, int y) { par[x] = y; }
22     int Find(int x, int c = 0) {
23         if(par[x] == x) return c ? -1 : x;
24         int p = Find(par[x], 1);
25         if(p == -1) return c ? par[x] : val[x];
26         if(sdom[val[x]] > sdom[val[par[x]]]) val[x] = val[par[x]];
27         par[x] = p;
28         return c ? p : val[x];
29     }
30     void dfs(int x) {
31         RS[ S[x] = ++cs ] = x;
32         par[cs] = sdom[cs] = val[cs] = cs;
33         for(int e : E[x]) {
34             if(S[e] == 0) dfs(e), rp[S[e]] = S[x];
35             RE[S[e]].push_back(S[x]);
36         }
37     }
38     int solve(int s, int *up) { //s 是起点
39         dfs(s);
40         for(int i=cs;i;i--) {
41             for(int e : RE[i]) sdom[i] = min(sdom[i], sdom[Find(e)]);
42             if(i > 1) rdom[sdom[i]].push_back(i);
43             for(int e : rdom[i]) {
44                 int p = Find(e);
45                 if(sdom[p] == i) dom[e] = i;
46             }
47         }
48     }
49 }

```

```

46     else dom[e] = p;
47 }
48 if(i > 1) Union(i, rp[i]);
49 }
50 for(int i=2;i<=cs;i++) if(sdom[i] != dom[i]) dom[i] = dom[dom[i]];
51 for(int i=2;i<=cs;i++) up[RS[i]] = RS[dom[i]];
52 return cs;
53 }
54 }

```

最小环

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int maxn=1e2+10;
4  const int inf=2e7+10;
5  int a[maxn][maxn],b[maxn][maxn];
6  int main()
7  {
8      int n;cin>>n;
9      int m;cin>>m;
10     for(int i=1;i<=n;i++){
11         for(int j=1;j<=n;j++){
12             a[i][j]=b[i][j]=inf;
13             a[i][i]=b[i][i]=0;
14         }
15     }
16     while(m--){
17         int x,y;cin>>x>>y;
18         int w;cin>>w;
19         a[x][y]=min(a[x][y],w);
20         a[y][x]=min(a[y][x],w);
21         b[x][y]=min(b[x][y],w);
22         b[y][x]=min(b[y][x],w);
23     }
24     int ans=inf;
25     for(int i=1;i<=n;i++){
26         for(int j=1;j<i;j++){
27             for(int k=j+1;k<i;k++){
28                 ans=min(ans,a[i][j]+a[i][k]+b[j][k]);
29             }
30         }
31     }
32     for(int j=1;j<=n;j++){
33         for(int k=1;k<=n;k++){
34             b[j][k]=min(b[j][i]+b[i][k],b[j][k]);
35         }
36     }
37     if(ans==inf)cout<<"No solution.";
38     else cout<<ans;
39 }

```

最小费用最大流

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int N=1e4,M=1e6;
4  struct SSP {
5      int cnt = 1, hd[N], nxt[M << 1], to[M << 1], limit[M << 1], cst[M << 1];
6      void init(){
7          memset(hd,0,sizeof(hd));
8          cnt=1;

```

```

9     }
10    // w limit c cost
11    void add(int u, int v, int w, int c) {
12        nxt[++cnt] = hd[u], hd[u] = cnt, to[cnt] = v, limit[cnt] = w, cst[cnt] = c;
13        nxt[++cnt] = hd[v], hd[v] = cnt, to[cnt] = u, limit[cnt] = 0, cst[cnt] = -c;
14    }
15
16    int fr[N], fl[N], in[N], dis[N];
17
18    pair<int, int> min_cost(int s, int t) {
19        int flow = 0, cost = 0;
20        while (true) { // SPFA
21            queue<int> q;
22            memset(dis, 0x3f, sizeof(dis));
23            memset(in, 0, sizeof(in));
24            fl[s] = 1e9, dis[s] = 0, q.push(s);
25            while (!q.empty()) {
26                int cur = q.front();
27                q.pop(), in[cur] = 0;
28                for (int i = hd[cur]; i; i = nxt[i]) {
29                    int it = to[i], d = dis[cur] + cst[i];
30                    if (limit[i] && d < dis[it]) {
31                        fl[it] = min(limit[i], fl[cur]), fr[it] = i, dis[it] = d;
32                        if (!in[it]) in[it] = 1, q.push(it);
33                    }
34                }
35            }
36            if (dis[t] > 1e9) return {flow, cost}; // 改成>0 就是可行流
37            flow += fl[t], cost += dis[t] * fl[t];
38            for (int u = t; u != s; u = to[fr[u] ^ 1]) limit[fr[u]] -= fl[t],
limit[fr[u] ^ 1] += fl[t];
39        }
40    }
41 } Sol;

```

欧拉回路

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  const int M=2333,N=666;
4  struct edge
5  {
6      int nxt,to;
7  }e[M<<1];
8  int head[N],tot=1;
9  int cut[M<<1];
10 void add(int u,int v)
11 {
12     e[++tot]=(edge){head[u],v},head[u]=tot;
13     e[++tot]=(edge){head[v],u},head[v]=tot;
14 }
15 vector<int> st;
16 void dfs(int u)//欧拉回路
17 {
18     for(int i=head[u];i!=0;i=e[i].nxt)
19     {
20         if(cut[i]) continue;

```

```

21     int v=e[i].to;
22     cut[i]=cut[i^1]=1;
23     dfs(v);
24 }
25 st.push(u);
26 }
27 int main()
28 {
29
30
31     return 0;
32 }
33

```

三四元环计数

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  const int mod=1e9+7;
4  void add(int &x,int y){
5      if((x+=y)>=mod)x-=mod;
6  }
7  void solve(){
8      int ans3=0,ans4=0;
9      int n,m;cin>>n>>m;
10     vector<pair<int,int>> edges(m);
11     vector<int>d(n+1),flg(n+1),id(n+1),rk(n+1),c(n+1);
12     vector<vector<int>> ve(n+1),f(n+1),g(n+1);
13     for(int i=0;i<m;i++){
14         int u,v;cin>>u>>v;
15         edges[i]=make_pair(u,v);
16         ve[u].push_back(v);
17         ve[v].push_back(u);
18     }
19     for(int i=1;i<=n;i++)
20         d[id[i]=i]=(int)ve[i].size();
21     for(auto [u,v]:edges){
22         if(d[u]>d[v]||(d[u]==d[v]&&u>v))g[u].push_back(v);
23         else g[v].push_back(u);
24     }
25     sort(id.begin()+1,id.end(),[&](int x,int y){
26         return d[x]<d[y];
27     });
28     for(int i=1;i<=n;i++)rk[id[i]]=i;
29     for(int u=1;u<=n;u++)for(auto v:ve[u])
30         if(rk[v]>rk[u])f[u].push_back(v);
31     for(int u=1,C=0;u<=n;u++){
32         for(auto v:ve[u])for(auto w:f[v])if(rk[w]>rk[u])add(ans4,c[w]),++c[w];
33         for(auto v:ve[u])for(auto w:f[v])if(rk[w]>rk[u])c[w]=0;
34         ++C;
35         for(auto v:g[u])flg[v]=C;
36         for(auto v:g[u])
37             for(auto w:g[v])add(ans3,(flg[w]==C));
38     }
39     cout<<ans3<<" "<<ans4<<endl;
40 }
41 /*

```

```
42 5 7
43 1 2
44 2 3
45 4 2
46 5 1
47 1 4
48 3 5
49 3 1
50
51 ans3=3 ans4=2
52 */
```

数据结构

SegmentSet

```
1  /*
2  every pair of pair<int,int> will not intersect
3  if i is true then it will be in the set
4  */
5  struct SegmentSet{
6      set<pair<int,int>>s;
7      void write(){
8          for(auto [x,y]:s)cout<<x<<"/"<<y<<" ";cout<<"\n";
9      }
10     //make l...r to be true
11     void insert(int l,int r)
12     {
13         int L=l,R=r;
14         auto it=s.lower_bound(make_pair(L,(int)-2e9));
15         while(it!=s.end()&&it->first<=R+1){
16             R=max(it->second,R);
17             it=s.erase(it);
18         }
19         if(it!=s.begin()){
20             it--;
21             if(it->second+1>=L){
22                 L=min(L,it->first);
23                 R=max(R,it->second);
24                 s.erase(it);
25             }
26         }
27         s.insert(make_pair(L,R));
28     }
29     // if l...r all true return false
30     bool query_no_full(int l,int r){
31         auto it=s.lower_bound(make_pair(l,(int)-2e9));
32         if(it!=s.end()){
33             if(l==it->first&&r<=it->second)return false;
34         }
35         if(it!=s.begin()){
36             it--;
37             if(it->second>=r)return false;
38         }
39         return true;
40     }
41     //make l...r to be false
42     void del(int l,int r){
43         auto it=s.lower_bound(make_pair(l,(int)-2e9));
44         while(it!=s.end()&&it->first<=r){
45             if(it->second<=r){
46                 it=s.erase(it);continue;
47             }
48             int R=it->second;
49             s.erase(it);
50             s.insert(make_pair(r+1,R));
51             break;
52         }
53         it=s.lower_bound(make_pair(l,(int)-2e9));
```

```

54     if(it!=s.begin()){
55         it--;
56         int L=it->first,R=it->second;
57         if(R>=l){
58             s.erase(it);
59             if(L<=l-1)s.insert(make_pair(L,l-1));
60             if(R>=r+1)s.insert(make_pair(r+1,R));
61         }
62     }
63 }
64 // if l...r all false return false
65 bool query_at_least_one(int l,int r){
66     auto it=s.lower_bound(make_pair(l,(int)-2e9));
67     if(it!=s.end()){
68         if(it->first<=r)return true;
69     }
70     if(it!=s.begin()){
71         it--;
72         if(it->second>=l)return true;
73     }
74     return false;
75 }
76 };

```

SegmentTree

```

1  //??????????? ???????>=?sum??????
2  #include <bits/stdc++.h>
3  using namespace std;
4  const int maxn=1e6+10;
5  struct Node{
6      int l,r,res,tag;
7  };
8  struct SegmentTree{
9      Node a[maxn*4];
10     void tag_init(int i){
11         a[i].tag=0;
12     }
13     void tag_union(int fa,int i){
14         a[i].tag+=a[fa].tag;
15     }
16     void tag_cal(int i){
17         a[i].res+=a[i].tag*(a[i].r-a[i].l+1);
18     }
19     void pushdown(int i){
20         tag_cal(i);
21         if(a[i].l!=a[i].r){
22             tag_union(i,i*2);
23             tag_union(i,i*2+1);
24         }
25         tag_init(i);
26     }
27     void pushup(int i){
28         if(a[i].l==a[i].r)return;
29         pushdown(i*2);
30         pushdown(i*2+1);
31         a[i].res=a[i*2].res+a[i*2+1].res;

```



```

32     }
33     void build(int i,int l,int r){
34         a[i].l=l,a[i].r=r;tag_init(i);
35         if(l>=r)return;
36         int mid=(l+r)/2;
37         build(i*2,l,mid);
38         build(i*2+1,mid+1,r);
39     }
40     void update(int i,int l,int r,int w){
41         pushdown(i);
42         if(a[i].r<l||a[i].l>r||l>r)return;
43         if(a[i].l>=l&&a[i].r<=r){
44             a[i].tag=w;
45             return;
46         }
47         update(i*2,l,r,w);
48         update(i*2+1,l,r,w);
49         pushup(i);
50     }
51     int query(int i,int l,int r){
52         pushdown(i);
53         if(a[i].r<l||a[i].l>r||l>r)return 0;
54         if(a[i].l>=l&&a[i].r<=r){
55             return a[i].res;
56         }
57         return query(i*2,l,r)+query(i*2+1,l,r);
58     }
59     int min_right(int qL, int& nowsum,int querysum, int i) {//???????>=sum???
60         pushdown(i);
61         if (a[i].r < qL)return -1;
62         if (qL <= a[i].l) {
63             int ss = nowsum+a[i].res;
64             if (ss<querysum) {
65                 nowsum = ss;
66                 return -1;
67             }
68             if (a[i].l == a[i].r)return a[i].l;
69         }
70         int pos = min_right(qL, nowsum,querysum,i*2);
71         if (pos != -1)return pos;
72         return min_right(qL, nowsum,querysum,2*i+1);
73     }
74     int max_left(int qR,int &nowsum,int querysum,int i){//???????>=sum???
75         pushdown(i);
76         if(a[i].l > qR)return -1;
77         if(qR>=a[i].r){
78             int ss=nowsum+a[i].res;
79             if(ss<querysum){
80                 nowsum=ss;
81                 return -1;
82             }
83             if(a[i].l==a[i].r)return a[i].r;
84         }
85         int pos=max_left(qR,nowsum,querysum,i*2+1);
86         if(pos!=-1)return pos;
87         return max_left(qR,nowsum,querysum,i*2);

```

```

88     }
89 };
90 SegmentTree tri;
91 int main()
92 {
93     ios::sync_with_stdio(false);
94     cin.tie(0);
95     int n,q;cin>>n>>q;
96     tri.build(1,1,n);
97     for(int i=1;i<=n;i++){
98         int x;cin>>x;tri.update(1,i,i,x);
99     }
100     while(q--){
101         int ops;cin>>ops;
102         if(ops==1){
103             int l,r,x;cin>>l>>r>>x;
104             tri.update(1,l,r,x);
105         }
106         if(ops==2){
107             int l,r;cin>>l>>r;
108             cout<<tri.query(1,l,r)<<"\n";
109         }
110         if(ops==3){
111             int x,sum;cin>>x>>sum;
112             int nowsum=0;
113             cout<<tri.min_right(x,nowsum,sum,1)<<"\n";
114         }
115         if(ops==4){
116             int x,sum;cin>>x>>sum;
117             int nowsum=0;
118             cout<<tri.max_left(x,nowsum,sum,1)<<"\n";
119         }
120     }
121 }
122

```

三维偏序 **cdq**

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int maxn=4e5+10;
4  struct Treearray{
5      int c[maxn],n;
6      void set_n(int _n){
7          n=_n;
8          for(int i=1;i<=n;i++)c[i]=0;
9      }
10     int lowbit(int x){
11         return x&(-x);
12     }
13     void update(int i,int k){    //在 i 位置加上 k
14         while(i <= n){
15             c[i] += k;
16             i += lowbit(i);
17         }
18     }
19     int getsum(int i){          //求 A[1 - i]的和

```

```

20     int res = 0;
21     while(i > 0){
22         res += c[i];
23         i -= lowbit(i);
24     }
25     return res;
26 }
27 };
28 Treearray tri;
29 int a[maxn],b[maxn],c[maxn];
30 int n,k;
31 int ans[maxn],id[maxn],cnt[maxn],t[maxn];
32 void subdiv(int l,int r){
33     if(l==r)return;
34     int mid=(l+r)/2;
35     subdiv(l,mid);
36     subdiv(mid+1,r);
37     sort(id+l,id+mid+1,[](int x,int y){
38         if(b[x]==b[y])return c[x]<c[y];
39         return b[x]<b[y];
40     });
41     sort(id+mid+1,id+r+1,[](int x,int y){
42         if(b[x]==b[y])return c[x]<c[y];
43         return b[x]<b[y];
44     });
45
46     // for(int i=l;i<=r;i++)cout<<id[i]<<" \n"[i==r];
47     assert(tri.getsum(tri.n)==0);
48     for(int i=l,j=mid+1;i<=mid||j<=r;){
49         if(j!=r+1&&(i==mid+1||b[id[i]]>b[id[j]])){
50             // cout<<"id="<<id[j]<<" +"<<tri.getsum(c[id[j]])<<"\n";
51             ans[id[j]]+=tri.getsum(c[id[j]]);
52             j++;
53         }
54         else{
55             tri.update(c[id[i]],t[id[i]]);
56             i++;
57         }
58     }
59     for(int i=l;i<=mid;i++)tri.update(c[id[i]],-t[id[i]]);
60 }
61 int main()
62 {
63     ios::sync_with_stdio(false);
64     cin.tie(0);
65     cin>>n>>k;
66     tri.set_n(200000);
67     vector<tuple<int,int,int>> v(n);
68     for(int i=0;i<n;i++){
69         int x,y,z;cin>>x>>y>>z;v[i]=make_tuple(x,y,z);
70     }
71     sort(v.begin(),v.end());
72     int g=0;
73     for(int i=0;i<n;i++){
74         int j=i;
75         while(j+1<n&&v[i]==v[j+1])j++;

```

```

76     g++;
77     a[g]=get<0>(v[i]);
78     b[g]=get<1>(v[i]);
79     c[g]=get<2>(v[i]);
80     t[g]=j-i+1;
81     i=j;
82     // cout<<"g="<<g<<" "<<a[g]<<" "<<b[g]<<" "<<c[g]<<"\n";
83 }
84 for(int i=1;i<=g;i++)id[i]=i;
85 subdiv(1,g);
86 for(int i=1;i<=g;i++){
87     cnt[ans[i]+t[i]-1]+=t[i];
88     // cout<<"i="<<i<<" ans="<<ans[i]<<"\n";
89 }
90 for(int i=0;i<n;i++)cout<<cnt[i]<<"\n";
91 return 0;
92 }
93

```

主席树

```

1 //注意 Sum 和 cnt 的区别
2 #include <bits/stdc++.h>
3 using namespace std;
4 const int maxn = 1e5; // 数据范围
5 int n, m;
6 struct Persistent_SegmentTree
7 {
8     int sum[(maxn << 5) + 10], root[maxn + 10], lch[(maxn << 5) + 10],
9         rch[(maxn << 5) + 10], cnt[(maxn<<5)+10];
10    int tot = 0;
11    void init()
12    {
13        tot = 0;
14    }
15    int a[maxn + 10];
16    void update(int &rot, int pr, int L, int R, int k)
17    { // 插入操作
18        rot = ++tot;
19        lch[rot] = lch[pr];
20        rch[rot] = rch[pr];
21        sum[rot] = sum[pr] + k;
22        cnt[rot] = cnt[pr] + 1;
23        if (L == R)
24            return;
25        int mid = (L + R) >> 1;
26        if (k <= mid)
27            update(lch[rot], lch[pr], L, mid, k);
28        else
29            update(rch[rot], rch[pr], mid + 1, R, k);
30    }
31    int getcnt(int s, int t, int L, int R, int l, int r) // s,t 为 root[l],root[r]的根
    节点 中所有大小在[l,r]之间数字出现次数
32    {
33        if (l <= L && R <= r)
34            return cnt[t] - cnt[s];
35        int res = 0;

```

```

36     int mid = (L + R) >> 1;
37     if (l <= mid)
38         res += getcnt(lch[s], lch[t], L, mid, l, r);
39     if (r > mid)
40         res += getcnt(rch[s], rch[t], mid + 1, R, l, r);
41     return res;
42 }
43
44 int getsum(int s, int t, int L, int R, int l, int r) // s,t为 root[l],root[r]的根
    节点 中所有大小在 [l,r] 之间数字的和
45 {
46     if (l <= L && R <= r)
47         return sum[t] - sum[s];
48     int res = 0;
49     int mid = (L + R) >> 1;
50     if (l <= mid)
51         res += getsum(lch[s], lch[t], L, mid, l, r);
52     if (r > mid)
53         res += getsum(rch[s], rch[t], mid + 1, R, l, r);
54     return res;
55 }
56 int get_Kth_min_Sum(int s,int t,int l,int r,int k,int &nowsum,int &ans){//return
    第 k 小的值
57     int ss = nowsum+cnt[t]-cnt[s];
58     if (ss<k) {
59         nowsum = ss;
60         ans+=sum[t]-sum[s];
61         return -1;
62     }
63     if (l == r){
64         ans+=(k-nowsum)*l;
65         return l;
66     }
67     int mid=(l+r)/2;
68     int pos = get_Kth_min_Sum(lch[s],lch[t],l,mid,k,nowsum,ans);
69     if (pos != -1)return pos;
70     return get_Kth_min_Sum(rch[s],rch[t],mid+1,r,k,nowsum,ans);
71 }
72 int get_Kth_max_Sum(int s,int t,int l,int r,int k,int &nowsum,int &ans){//return
    第 k 大的值
73     int ss = nowsum+cnt[t]-cnt[s];
74     if (ss<k) {
75         nowsum = ss;
76         ans+=sum[t]-sum[s];
77         return -1;
78     }
79     if (l == r){
80         ans+=(k-nowsum)*l;
81         return l;
82     }
83     int mid=(l+r)/2;
84     int pos = get_Kth_max_Sum(rch[s],rch[t],mid+1,r,k,nowsum,ans);
85     if (pos != -1)return pos;
86     return get_Kth_max_Sum(lch[s],lch[t],l,mid,k,nowsum,ans);
87 }
88 int get_upper(int s,int t,int l,int r,int x){//第一个大于等于 x 的数

```

```

89     if (r < x) return -1;
90     if (x <= l) {
91         int ss = cnt[t]-cnt[s];
92         if (ss==0) {
93             return -1;
94         }
95         if (l == r) return l;
96     }
97     int mid=(l+r)/2;
98     int pos = get_upper(lch[s],lch[t],l,mid,x);
99     if (pos != -1) return pos;
100    return get_upper(rch[s],rch[t],mid+1,r,x);
101 }
102 int get_lower(int s,int t,int l,int r,int x){//第一个小于等于 x 的数
103     if(l > x) return -1;
104     if(x>=r){
105         int ss=cnt[t]-cnt[s];
106         if(ss==0){
107             return -1;
108         }
109         if(l==r) return r;
110     }
111     int mid=(l+r)/2;
112     int pos=get_lower(rch[s],rch[t],mid+1,r,x);
113     if(pos!=-1) return pos;
114     return get_lower(lch[s],lch[t],l,mid,x);
115 }
116 };
117 Persistent_SegmentTree tri;
118 int main()
119 {
120     int n,q;
121     cin >> n >> q;
122
123     for (int i = 1; i <= n; i++)
124     {
125         int x;
126         cin >> x;
127         tri.update(tri.root[i], tri.root[i - 1], 1, n, x);
128         assert(1<=x&&x<=n);
129     }
130     while (q--)
131     {
132         int ops;cin>>ops;
133         int l,r,k,L,R;
134         if(ops==1){
135             cin>>l>>r>>L>>R;
136             cout<<tri.getsum(tri.root[l-1],tri.root[r],1,n,L,R)<<"\n";
137         }
138         if(ops==2){// get kth max no output -1
139             cin>>l>>r>>k;
140             int nowsum=0,ans=0;
141             cout<<tri.get_Kth_max_Sum(tri.root[l-1],tri.root[r],1,n,k,nowsum,ans)<<"\n";
142         }
143         if(ops==3){// get kth max sum siz<k return allsum
144             cin>>l>>r>>k;

```

```

145         int nowsum=0,ans=0;
146         tri.get_Kth_max_Sum(tri.root[l-1],tri.root[r],1,n,k,nowsum,ans);
147         cout<<ans<<"\n";
148     }
149     if(ops==4){// get kth min no output -1
150         cin>>l>>r>>k;
151         int nowsum=0,ans=0;
152         cout<<tri.get_Kth_min_Sum(tri.root[l-1],tri.root[r],1,n,k,nowsum,ans)<<"\n";
153     }
154     if(ops==5){// get kth min sum siz<k return allsum
155         cin>>l>>r>>k;
156         int nowsum=0,ans=0;
157         tri.get_Kth_min_Sum(tri.root[l-1],tri.root[r],1,n,k,nowsum,ans);
158         cout<<ans<<"\n";
159     }
160     if(ops==6){// get the min element >= k no return -1
161         cin>>l>>r>>k;
162         cout<<tri.get_upper(tri.root[l-1],tri.root[r],1,n,k)<<"\n";
163     }
164     if(ops==7){// get the max element <= k no return -1
165         cin>>l>>r>>k;
166         cout<<tri.get_lower(tri.root[l-1],tri.root[r],1,n,k)<<"\n";
167     }
168 }
169 }

```

二维树状数组

```

1  const int maxn=1010;
2  struct treearray{
3      int mkp1[maxn][maxn],mkp2[maxn][maxn],mkp3[maxn][maxn],mkp4[maxn][maxn];
4      inline int lowbit(int x)
5      {
6          return x&(-x);
7      }
8      inline void Update(int x,int y,int k)
9      {
10         for(int i=x;i<=n;i+=lowbit(i))
11         {
12             for(int j=y;j<=m;j+=lowbit(j))
13             {
14                 mkp1[i][j]+=k;
15                 mkp2[i][j]+=k*x;
16                 mkp3[i][j]+=k*y;
17                 mkp4[i][j]+=k*x*y;
18             }
19         }
20     }
21     inline void update(int a,int b,int x,int y,int k)
22     {
23         Update(a,b,k);
24         Update(a,y+1,-k);
25         Update(x+1,b,-k);
26         Update(x+1,y+1,k);
27     }
28     inline int Query(int x,int y)
29     {

```

```

30     int ans=0;
31     for(int i=x;i>=1;i-=lowbit(i))
32     {
33         for(int j=y;j>=1;j-=lowbit(j))
34         {
35             ans+=(x+1)*(y+1)*mkp1[i][j]
36             -(y+1)*mkp2[i][j]-(x+1)*mkp3[i][j]+mkp4[i][j];
37         }
38     }
39     return ans;
40 }
41 inline int query(int a,int b,int x,int y)
42 {
43     return Query(x,y)+Query(a-1,b-1)-Query(x,b-1)-Query(a-1,y);
44 }
45 };
46 treearray tri;
47 int main()
48 {
49
50 }
51

```

动态开点线段树

```

1 // root 表示整棵线段树的根结点; cnt 表示当前结点个数
2 const int maxn=1e5+10;
3 int n, cnt, root;
4 int sum[maxn*20], ls[maxn*20], rs[maxn*20];
5
6 // 用法: update(root, 1, n, x, f); 其中 x 为待修改节点的编号
7 void update(int& p, int L, int R, int x, int f) { // 引用传参
8     if (!p) p = ++cnt; // 当结点为空时, 创建一个新的结点
9     if (L == R) {
10         sum[p] += f;
11         return;
12     }
13     int m = L + ((R - L) >> 1);
14     if (x <= m)
15         update(ls[p], L, m, x, f);
16     else
17         update(rs[p], m + 1, R, x, f);
18     sum[p] = sum[ls[p]] + sum[rs[p]]; // pushup
19 }
20 // 用法: query(root, 1, n, l, r);
21 int query(int p, int L, int R, int l, int r) {
22     if (!p) return 0; // 如果结点为空, 返回 0
23     if (L >= l && R <= r) return sum[p];
24     int m = L + ((R - L) >> 1), ans = 0;
25     if (l <= m) ans += query(ls[p], L, m, l, r);
26     if (r > m) ans += query(rs[p], m + 1, R, l, r);
27     return ans;
28 }
29 int merge(int a, int b, int l, int r) {
30     if (!a) return b;
31     if (!b) return a;
32     if (l == r) {

```



```

33     sum[a]+=sum[b];
34     return a;
35 }
36 int mid = (l + r) >> 1;
37 ls[a] = merge(ls[a], ls[b], l, mid);
38 rs[a] = merge(rs[a], rs[b], mid + 1, r);
39 sum[a]=sum[ls[a]]+sum[rs[a]];
40 return a;
41 }
42 void split(int &p, int &q, int L, int R, int l, int r) { //p 原树 q 新树
43     if (R < l || r < L) return;
44     if (!p) return;
45     if (l <= L && R <= r) {
46         q = p;
47         p = 0;
48         return;
49     }
50     if (!q) q = ++cnt;
51     int m = L + R >> 1;
52     if (l <= m) split(ls[p], ls[q], L, m, l, r);
53     if (m < r) split(rs[p], rs[q], m + 1, R, l, r);
54     sum[p]=sum[ls[p]]+sum[rs[p]];
55     sum[q]=sum[ls[q]]+sum[rs[q]];
56 }
57 int query1(int& Sum,int Up, int rt, int l, int r) { //min_x of f(x)+x>Up
58     // cout<<"Sum="<<Sum<<" Up="<<Up<<" rt="<<rt<<" l="<<l<<" r="<<r<<"\n";
59     if(!rt){
60         if(r+Sum<=Up)return -1;
61         else return r-(r+Sum-Up)+1;
62     }
63     if (true) {
64         int ss = Sum+sum[rt];
65         if (ss+r<=Up) {
66             Sum = ss;
67             return -1;
68         }
69         if (l == r)return l;
70     }
71     int mid = (l + r) / 2;
72     int pos = query1(Sum,Up,ls[rt], l, mid);
73     if (pos != -1)return pos;
74     return query1(Sum,Up,rs[rt], mid + 1, r);
75 }

```

可持久化 01trie

```

1  const int maxn=1e5+10;
2  struct Persistent_Trie
3  {
4      int ch[maxn][2],tot,sum[maxn];
5      void clear()
6      {
7          tot=0;
8          return;
9      }
10     void cpy(int from,int to)
11     {

```

```

12     ch[to][0]=ch[from][0];
13     ch[to][1]=ch[from][1];
14     sum[to]=sum[from];
15     return;
16 }
17 void insert(int &root,int old,int num,int nowbit)
18 {
19     root=++tot;
20     cpy(old,root);
21     sum[root]++;
22     if(nowbit==-1)return;
23     if(num&(1<<nowbit))
24     {
25         insert(ch[root][1],ch[root][1],num,nowbit-1);
26     }
27     else
28     {
29         insert(ch[root][0],ch[root][0],num,nowbit-1);
30     }
31 }
32 int query(int s,int t,int x)
33 {
34     int ans=0;
35     for(int i=30;i>=0;--i)
36     {
37         if(sum[ch[t][!(x&(1<<i))]]-sum[ch[s][!(x&(1<<i))]])
38         {
39             t=ch[t][!(x&(1<<i))];
40             s=ch[s][!(x&(1<<i))];
41             ans|=(1<<i);
42         }
43         else
44         {
45             t=ch[t][!!(x&(1<<i))];
46             s=ch[s][!!(x&(1<<i))];
47         }
48     }
49     return ans;
50 }
51 };

```

最近公共祖先 LCA

```

1  const int maxn=1e5+10;
2  vector<int>ve[maxn];
3  int dep[maxn],f[21][maxn];
4  void dfs(int x,int fa)
5  {
6      dep[x]=dep[fa]+1;
7      for(int i=0;i<=19;i++)
8          f[i+1][x]=f[i][f[i][x]];
9      for(auto it:ve[x])
10     {
11         if(it==fa) continue;
12         f[0][it]=x;
13         dfs(it,x);
14     }

```

```

15 }
16 int lca(int x,int y)
17 {
18     if(dep[x]<dep[y]) swap(x,y);
19     for(int i=20;i>=0;i--)
20     {
21         if(dep[f[i][x]]>=dep[y]) x=f[i][x];
22         if(x==y) return x;
23     }
24     for(int i=20;i>=0;i--)
25         if(f[i][x]!=f[i][y])
26             x=f[i][x],y=f[i][y];
27     return f[0][x];
28 }

```

Lca(o1)

```

1 #define int long long
2 const int maxn=1e5+10;//注意开两倍大小的空间 在 dp 上
3 vector<pair<int,int>>ve[maxn];
4 int dep[maxn];
5 pair<int,int>dp[21][maxn*3];
6 int red[maxn],d[maxn];
7 int Dep[maxn];
8 int dfn[maxn];
9 void dfs(int x,int fa,int l,int dis)
10 {
11     if(red[x])dis=0;
12     d[x]=dis;
13     dep[x]=dep[fa]+1;
14     Dep[x]=Dep[fa]+l;
15     for(auto [it,len]:ve[x])
16     {
17         if(it==fa) continue;
18         dfs(it,x,len,dis+len);
19     }
20 }
21 vector<int> sp;
22 void dfs2(int u, int fa)
23 {
24
25     dfn[u] = sp.size();
26     sp.push_back(u);
27     for (auto& e : ve[u])if(e.first!=fa)
28     {
29         int& v = e.first;
30         dfs2(v, u);
31         sp.push_back(u);
32     }
33 }
34 void initrmq()
35 {
36     int n = sp.size();
37     for (int i = 0; i < n; i++) dp[0][i] = {dfn[sp[i]], sp[i]};
38     for (int i = 1; (1 << i) <= n; i++)
39         for (int j = 0; j + (1 << i) - 1 < n; j++)
40             dp[i][j] = min(dp[i - 1][j], dp[i - 1][j + (1 << (i - 1))]);

```

```

41 }
42 int lca(int u, int v)
43 {
44     int l = dfn[u], r = dfn[v];
45     if (l > r) swap(l, r);
46     int k = __lg(r-l+1);
47     return min(dp[k][l], dp[k][r - (1 << k) + 1]).second;
48 }

```

点分治

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int maxn = 100010;
4  const int inf = 2e9;
5  int n, siz[maxn], dist[maxn], vis[maxn], maxx[maxn];
6  vector<pair<int, int>> ve[maxn];
7
8  void add_edge(int x, int y, int z)
9  {
10     ve[x].emplace_back(y, z);
11 }
12
13 void calcsiz(int x, int fa, int sum, int &rt)
14 {
15     siz[x] = 1;
16     maxx[x] = 0;
17     for (auto &[to, len] : ve[x])
18         if (to != fa && !vis[to])
19             {
20                 calcsiz(to, x, sum, rt);
21                 maxx[x] = max(maxx[x], siz[to]);
22                 siz[x] += siz[to];
23             }
24     maxx[x] = max(maxx[x], sum - siz[x]);
25     if (maxx[x] < maxx[rt])
26         rt = x;
27 }
28
29
30 void calcdist(int x, int fa)
31 {
32     for (auto &[to, len] : ve[x])
33         if (to != fa && !vis[to])
34             {
35                 dist[to] = dist[x] + len, calcdist(to, x);
36             }
37 }
38 void dfs(int x, int fa)
39 {
40     vis[x] = true;
41     for (auto &[to, len] : ve[x])
42         if (to != fa && !vis[to])
43             {
44                 dist[to] = len;
45                 calcdist(to, x);
46             }

```

```

47
48
49     for (auto &[to, len] : ve[x])
50         if (to != fa && !vis[to])
51         {
52             int sum = siz[to];
53             int rt = 0;
54             calcsiz(to, x, sum, rt);
55             calcsiz(rt, -1, sum, rt);
56             dfs(rt, x);
57         }
58     }
59
60 int main()
61 {
62     ios::sync_with_stdio(false);
63     cin.tie(0);
64     cin >> n ;
65     for (int i = 1; i < n; i++)
66     {
67         int a, b,c;
68         cin >> a >> b>>c, add_edge(a, b, c), add_edge(b, a, c);
69     }
70     int rt = 0;
71     maxx[rt] = inf;
72     int sum = n;
73     calcsiz(1, -1, sum, rt);
74     calcsiz(rt, -1, sum, rt);
75     dfs(rt, -1);
76     return 0;
77 }
78

```

珂朵莉树

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  struct SegmentMap{
4      map<int,int> ma;
5      int sum=0;
6      int n;
7      SegmentMap(int _n=0){
8          n=_n;
9          ma[1]=0;
10         ma[n+1]=-1;
11     }
12     void del(int l,int r,int x){//减去当前 (l,r,x) 的贡献
13         sum-=(r-l+1)*x;
14     }
15     void add(int l,int r,int x){//加上当前 (l,r,x) 的贡献
16         sum+=(r-l+1)*x;
17     }
18     void split(int x){
19         auto t=prev(ma.upper_bound(x))->second;
20         ma[x]=t;
21     }
22     void update(int l,int r,auto&& T){

```

```

23     split(l);split(r+1);
24     auto it=prev(ma.upper_bound(l));
25     int pr=-1;
26     while(it->first<=r){
27         int nowl=it->first,nowr=next(it)->first-1;
28         int x=it->second;
29         del(nowl,nowr,x);
30         add(nowl,nowr,T(x));
31         if(l!=nowl&&pr==T(x))it=ma.erase(it);
32         else {
33             it->second=T(x);
34             it=next(it);
35         }
36         pr=T(x);
37     }
38     if(it->first!=n+1&&it->second==pr)ma.erase(it);
39     if(l!=1){
40         it=ma.lower_bound(l);
41         if(prev(it)->second==it->second)ma.erase(it);
42     }
43 }
44 };
45 int main()
46 {
47     ios::sync_with_stdio(false);
48     cin.tie(0);
49     int n,q;cin>>n>>q;
50     SegmentMap sol(n);
51     while(q--){
52         int op;cin>>op;
53         if(op==1){
54             int l,r;cin>>l>>r;
55             sol.update(l,r,[](int x){
56                 return (int)sqrt(x);
57             });
58         }
59         else{
60             int l,r,x;cin>>l>>r>>x;
61             sol.update(l,r,[x](int y){
62                 return x;
63             });
64         }
65         cout<<sol.sum<<"\n";
66     }
67 }

```

笛卡尔树

```

1  const int maxn=2e5+10;
2  vector<int> ve[maxn];
3  int a[maxn];
4  int n;
5  int build(){
6      int top=0;
7      vector<int> Stack(n+1,0);
8      for(int i=0;i<=n;i++){
9          ve[i].clear();

```

```

10     ve[i].resize(2,-1);
11 }
12 Stack[++top]=1;
13 for(int i=2;i<=n;i++){
14     while(top&&a[Stack[top]]>=a[i])top--;
15     if(!top)ve[i][0]=Stack[top+1];
16     else ve[i][0]=ve[Stack[top]][1],ve[Stack[top]][1]=i;
17     Stack[++top]=i;
18 }
19     return Stack[1];
20 }

```

线段树区间加区间历史最小值

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  #define int long long
4  const int maxn=1e6+10;
5  const int inf =2e9;
6  struct Node{
7      int l,r,res,tag,tag2,res2;
8  };
9  struct SegmentTree{
10     Node a[maxn*4];
11     void tag_init(int i){
12         a[i].tag=a[i].tag2=0;
13     }
14     void tag_union(int fa,int i){
15         if(a[fa].tag2<0)a[i].tag2=min(a[i].tag2,a[i].tag+a[fa].tag2);
16         a[i].tag+=a[fa].tag;
17     }
18     void tag_cal(int i){
19         if(a[i].tag2<0)a[i].res2=min(a[i].res2,a[i].res+a[i].tag2);
20         a[i].res+=a[i].tag;
21     }
22     void pushdown(int i){
23         tag_cal(i);
24         if(a[i].l!=a[i].r){
25             tag_union(i,i*2);
26             tag_union(i,i*2+1);
27         }
28         tag_init(i);
29     }
30     void pushup(int i){
31         if(a[i].l==a[i].r)return;
32         pushdown(i*2);
33         pushdown(i*2+1);
34         a[i].res=min(a[i*2].res,a[i*2+1].res);
35         a[i].res2=min(a[i*2].res2,a[i*2+1].res2);
36     }
37     void build(int i,int l,int r){
38         a[i].l=l,a[i].r=r;tag_init(i);
39         if(l>=r)return;
40         int mid=(l+r)/2;
41         build(i*2,l,mid);
42         build(i*2+1,mid+1,r);
43     }

```

```

44 void update(int i,int l,int r,int w){
45     if(a[i].r<l||a[i].l>r||l>r)return;
46     pushdown(i);
47     if(a[i].l>=l&&a[i].r<=r){
48         a[i].tag+=w;
49         a[i].tag2=min(a[i].tag2,a[i].tag);
50         return;
51     }
52     update(i*2,l,r,w);
53     update(i*2+1,l,r,w);
54     pushup(i);
55 }
56 int query(int i,int l,int r){
57     pushdown(i);
58     if(a[i].r<l||a[i].l>r||l>r)return inf;
59     if(a[i].l>=l&&a[i].r<=r){
60         return a[i].res2;
61     }
62     return min(query(i*2,l,r),query(i*2+1,l,r));
63 }
64 };
65 SegmentTree tri;
66 signed main()
67 {
68     int n,m;
69     cin>>n>>m;
70     tri.build(1,1,n);
71     for(int i=1;i<=m;i++){
72         int ops;cin>>ops;
73         if(ops==1){
74             int l,r,x;cin>>l>>r>>x;
75             tri.update(1,l,r,x);
76         }
77         else{
78             int l,r;cin>>l>>r;
79             cout<<tri.query(1,l,r)<<"\n";
80         }
81     }
82     return 0;
83 }

```

莫队

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 const int N=1e6+5;
4 int n,m,a[N],block,ans[N],now,cnt[N];
5 struct node
6 {
7     int l,r,i;
8     friend bool operator<(const node a,const node b)
9     {
10         if((a.l/block)==(b.l/block)) return ((a.l/block%2)?a.r>b.r:a.r<b.r);
11         else return a.l<b.l;
12     }
13 }q[N];
14 void add(int x)

```



```

15 {
16 }
17 void del(int x)
18 {
19 }
20 int main()
21 {
22     block=max(1,n/sqrt(m));
23     sort(q+1,q+1+m);
24     int l=1,r=0;
25     for(int i=1;i<=m;i++)
26     {
27         while(l<q[i].l) del(l++);
28         while(l>q[i].l) add(--l);
29         while(r<q[i].r) add(++r);
30         while(r>q[i].r) del(r--);
31         ans[q[i].i]=now;
32     }
33     for(int i=1;i<=m;i++)
34         printf("%d\n",ans[i]);
35     return 0;
36 }
37

```

虚树

```

1 //带边权的虚树
2 //ve2 是虚树,ve 是原树
3 //注意使用时 mlen 的意义需要修改 getlen 和 dfs 预处理的时候都要修改
4 //用的时候就 addintoxs(x) 丢进去,每次 init 就全部删除
5 //add 是加原边,ADD 不用管它
6 //solve 里面是必备的几步,不能去除
7 #include <bits/stdc++.h>
8 using namespace std;
9 #define int long long
10 const int maxn=1e5+10;
11 const int inf =1e18;
12 int n,m;
13 struct XS{
14     struct kkk{
15         int to,len;
16     };
17     int dep[maxn],f[maxn][21],id[maxn];
18     int mlen[maxn][21],ans[maxn][21];
19     int state[maxn];
20     int sum[maxn],all;
21     vector<kkk> ve[maxn],ve2[maxn];
22     int now;//统计 dfs 序
23     void dfs(int u,int fa)
24     {
25         id[u]=now++;
26         dep[u]=dep[fa]+1;
27         for(int i=0;i<=19;i++){
28             f[u][i+1]=f[f[u][i]][i];
29             mlen[u][i+1]=mlen[f[u][i]][i]+mlen[u][i];
30         }
31         for(auto v:ve[u])

```

```

32     {
33         if(v.to==fa) continue;
34         f[v.to][0]=u;
35         mlen[v.to][0]=v.len;
36         dfs(v.to,u);
37     }
38 }
39 int lca(int x,int y)
40 {
41     if(dep[x]<dep[y]) swap(x,y);
42     for(int i=20;i>=0;i--)
43     {
44         if(dep[f[x][i]]>=dep[y]) x=f[x][i];
45         if(x==y) return x;
46     }
47     for(int i=20;i>=0;i--)
48         if(f[x][i]!=f[y][i])
49             x=f[x][i],y=f[y][i];
50     return f[x][0];
51 }
52 int getlen(int x,int y)
53 {
54     if(x==y) return 0;
55     if(dep[x]<dep[y]) swap(x,y);
56     int res=0;
57     for(int i=20;i>=0;i--){
58         if(dep[f[x][i]]>=dep[y]){
59             res=res+mlen[x][i];
60             x=f[x][i];
61         }
62         if(x==y) return res;
63     }
64     return inf;
65 }
66 void add(int x,int y,int len){//加边
67     ve[x].push_back({y,len});
68     ve[y].push_back({x,len});
69 }
70 void ADD(int x,int y,int len){//虚树加边
71     ve2[x].push_back({y,len});
72     ve2[y].push_back({x,len});
73 }
74 vector<int> vis;//b vis 这两个用来初始化
75 bool b[maxn];
76 vector<int> v1;//用来表示加入虚树的点
77 int sta[maxn];
78 void init(){//删除虚树
79     {
80         for(auto it:vis){
81             ve2[it].clear();
82             b[it]=false;
83             sum[it]=0;
84         }
85         vis.clear();
86         v1.clear();
87     }

```

```

88     void makexs()
89     {
90         sort(v1.begin(),v1.end(),[&](int x,int y){
91             return id[x]<id[y];
92         });
93         sta[0]=0;
94         int top=1;
95         sta[top]=1;
96         vis.push_back(1);
97         for(auto it:v1){
98             int l=lca(it,sta[top]);
99             if(l!=sta[top]){
100                 while(id[l]<id[sta[top-1]]){
101                     int w=getlen(sta[top],sta[top-1]);
102                     ADD(sta[top-1],sta[top],w);
103                     vis.push_back(sta[top--]);
104                 }
105                 if(id[l]>id[sta[top-1]]){
106                     int w=getlen(sta[top],l);
107                     ADD(l,sta[top],w);
108                     vis.push_back(sta[top]);
109                     sta[top]=l;
110                 }
111                 else {
112                     int w=getlen(sta[top],sta[top-1]);
113                     ADD(sta[top],sta[top-1],w);
114                     vis.push_back(sta[top--]);
115                 }
116             }
117             sta[++top]=it;
118         }
119         for(int i=1;i<top;i++){
120             int w=getlen(sta[i],sta[i+1]);
121             ADD(sta[i],sta[i+1],w);
122             vis.push_back(sta[i+1]);
123         }
124     }
125     void addintoxs(int x){
126         if(x==1)return;
127         v1.push_back(x);
128         b[x]=true;
129     }
130     void solve(){
131         now=0;
132         dfs(1,0);
133         for(int i=1;i<=n;i++){
134             makexs();
135             init();
136         }
137     }
138 };
139 XS xs;

```

LCT 维护联通性

```

1  #include <bits/stdc++.h>
2  using namespace std;

```

```

3  const int maxn = 10010;
4
5  struct LCT {
6      int ch[maxn][2], fa[maxn], tag[maxn];
7
8      void clear(int x) { ch[x][0] = ch[x][1] = fa[x] = tag[x] = 0; }
9
10     int getch(int x) { return ch[fa[x]][1] == x; }
11
12     int isroot(int x) { return ch[fa[x]][0] != x && ch[fa[x]][1] != x; }
13
14     void pushdown(int x) {
15         if (tag[x]) {
16             if (ch[x][0]) swap(ch[ch[x][0]][0], ch[ch[x][0]][1]), tag[ch[x][0]] ^= 1;
17             if (ch[x][1]) swap(ch[ch[x][1]][0], ch[ch[x][1]][1]), tag[ch[x][1]] ^= 1;
18             tag[x] = 0;
19         }
20     }
21
22     void update(int x) {
23         if (!isroot(x)) update(fa[x]);
24         pushdown(x);
25     }
26
27     void rotate(int x) {
28         int y = fa[x], z = fa[y], chx = getch(x), chy = getch(y);
29         fa[x] = z;
30         if (!isroot(y)) ch[z][chy] = x;
31         ch[y][chx] = ch[x][chx ^ 1];
32         fa[ch[x][chx ^ 1]] = y;
33         ch[x][chx ^ 1] = y;
34         fa[y] = x;
35     }
36
37     void splay(int x) {
38         update(x);
39         for (int f = fa[x]; f = fa[x], !isroot(x); rotate(x))
40             if (!isroot(f)) rotate(getch(x) == getch(f) ? f : x);
41     }
42
43     void access(int x) {
44         for (int f = 0; x; f = x, x = fa[x]) splay(x), ch[x][1] = f;
45     }
46
47     void makeroot(int x) {
48         access(x);
49         splay(x);
50         swap(ch[x][0], ch[x][1]);
51         tag[x] ^= 1;
52     }
53
54     int find(int x) {
55         access(x);
56         splay(x);
57         while (ch[x][0]) x = ch[x][0];
58         splay(x);

```

```

59         return x;
60     }
61     /*-----*/
62     bool query(int x,int y){//查询是否为同一颗树
63         return find(x)==find(y);
64     }
65     void addedge(int x,int y){
66         if (find(x) != find(y)) makeroot(x), fa[x] = y;
67     }
68     void deledge(int x,int y){
69         makeroot(x);
70         access(y);
71         splay(y);
72         if (ch[y][0] == x && !ch[x][1]) ch[y][0] = fa[x] = 0;
73     }
74 } st;
75
76 int n, q, x, y;
77 char op[maxn];
78
79 int main() {
80     scanf("%d%d", &n, &q);
81     while (q--) {
82         scanf("%s%d%d", op, &x, &y);
83         if (op[0] == 'Q') {
84             if (st.query(x,y))
85                 printf("Yes\n");
86             else
87                 printf("No\n");
88         }
89         if (op[0] == 'C')st.addedge(x,y);
90         if (op[0] == 'D')st.deledge(x,y);
91     }
92     return 0;
93 }

```

树链剖分

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  typedef long long ll;
4  const int maxn=1e5+10;
5  int mod;
6  void add(int &x,int y){if((x+=y)>=mod)x-=mod;}
7  struct Node{
8      int l,r,res,tag;
9  };
10 struct SegmentTree{
11     Node a[maxn*4];
12     void tag_init(int i){
13         a[i].tag=0;
14     }
15     void tag_union(int fa,int i){
16         add(a[i].tag,a[fa].tag);
17     }
18     void tag_cal(int i){
19         add(a[i].res,(ll)a[i].tag*(a[i].r-a[i].l+1)%mod);

```

```

20     }
21     void pushdown(int i){
22         tag_cal(i);
23         if(a[i].l!=a[i].r){
24             tag_union(i,i*2);
25             tag_union(i,i*2+1);
26         }
27         tag_init(i);
28     }
29     void pushup(int i){
30         if(a[i].l==a[i].r) return;
31         pushdown(i*2);
32         pushdown(i*2+1);
33         a[i].res=(a[i*2].res+a[i*2+1].res)%mod;
34     }
35     void build(int i,int l,int r){
36         a[i].l=l,a[i].r=r;tag_init(i);a[i].res=0;
37         if(l==r) return;
38         int mid=(l+r)/2;
39         build(i*2,l,mid);
40         build(i*2+1,mid+1,r);
41     }
42     void update(int i,int l,int r,int w){
43         pushdown(i);
44         if(a[i].r<l||a[i].l>r||l>r) return;
45         if(a[i].l>=l&&a[i].r<=r){
46             a[i].tag=w;
47             return;
48         }
49         update(i*2,l,r,w);
50         update(i*2+1,l,r,w);
51         pushup(i);
52     }
53     int query(int i,int l,int r){
54         pushdown(i);
55         if(a[i].r<l||a[i].l>r||l>r) return 0;
56         if(a[i].l>=l&&a[i].r<=r){
57             return a[i].res;
58         }
59         return (query(i*2,l,r)+query(i*2+1,l,r))%mod;
60     }
61 };
62 SegmentTree tri;
63 vector<int> ve[maxn];
64 int L[maxn],R[maxn],tot,root=1;
65 int siz[maxn],top[maxn],dep[maxn],fa[maxn],dfn[maxn];
66 void dfs_sz(int x,int h){
67     if(x!=root)ve[x].erase(find(ve[x].begin(),ve[x].end(),h));
68     siz[x] = 1;
69     dep[x]=dep[h]+1;
70     fa[x]=h;
71     for(auto &it:ve[x]) {
72         dfs_sz(it,x);
73         siz[x] += siz[it];
74         if(siz[it] > siz[ve[x][0]]) {
75             swap(it, ve[x][0]);

```

```

76     }
77 }
78 }
79 void dfs_hld(int x) {
80     L[x] = ++tot;
81     dfn[tot] = x;
82     for(auto it: ve[x]) {
83         top[it] = (it == ve[x][0] ? top[x] : it);
84         dfs_hld(it);
85     }
86     R[x] = tot;
87 }
88 void chain_add(int x, int y, int w) { // chain add w
89     while(top[x] != top[y]) {
90         if(dep[top[x]] < dep[top[y]]) swap(x, y);
91         tri.update(1, L[top[x]], L[x], w);
92         x = fa[top[x]];
93     }
94     if(L[x] > L[y]) swap(x, y);
95     tri.update(1, L[x], L[y], w);
96 }
97 int chain_sum(int x, int y) { // query the length of chain
98     int sum = 0;
99     while(top[x] != top[y]) {
100         if(dep[top[x]] < dep[top[y]]) swap(x, y);
101         add(sum, tri.query(1, L[top[x]], L[x]));
102         x = fa[top[x]];
103     }
104     if(L[x] > L[y]) swap(x, y);
105     add(sum, tri.query(1, L[x], L[y]));
106     return sum;
107 }
108 int main()
109 {
110     ios::sync_with_stdio(false);
111     cin.tie(0);
112     int n, q; cin >> n >> q >> root >> mod;
113     vector<int> a(n+1);
114     for(int i=1; i<=n; i++) cin >> a[i], a[i] %= mod;
115     for(int i=1; i<=n; i++){
116         int x, y; cin >> x >> y;
117         ve[x].push_back(y);
118         ve[y].push_back(x);
119     }
120     top[root] = root;
121     tri.build(1, 1, n); dfs_sz(root, 0); dfs_hld(root);
122     for(int i=1; i<=n; i++){
123         tri.update(1, L[i], L[i], a[i]);
124     }
125     while(q--){
126         int op; cin >> op;
127         if(op == 1){
128             int x, y, w; cin >> x >> y >> w;
129             chain_add(x, y, w);
130         }
131         if(op == 2){

```

```

132         int x,y;cin>>x>>y;
133         cout<<chain_sum(x,y)<<"\n";
134     }
135     if(op==3){//update the subtree of root x +=y
136         int x,y;cin>>x>>y;
137         tri.update(1,L[x],R[x],y);
138     }
139     if(op==4){
140         int x;cin>>x;
141         cout<<tri.query(1,L[x],R[x])<<"\n";
142     }
143 }
144
145 }

```


数学

组合数

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  #define int long long
4  const int maxn=100010;
5  const int mod=1e9+7;
6  int ksm(int x,int k){
7      int res=1;
8      while(k){
9          if(k&1) res=res*x%mod;
10         x=x*x%mod;
11         k/=2;
12     }
13     return res;
14 }
15 int ny(int x){
16     return ksm(x,mod-2);
17 }
18 void add(int &x,int y){
19     if((x+=y)>=mod)x-=mod;
20 }
21 void del(int &x,int y){
22     if((x-=y)<0)x+=mod;
23 }
24 int inv[maxn],fac[maxn];
25 int C(int n,int m){return n==0?1:fac[n]*inv[n-m]%mod*inv[m]%mod;}
26 int A(int n,int m){return n==0?1:fac[n]*inv[n-m]%mod;}
27 void init(){
28     inv[0]=fac[0]=1;
29     inv[1]=1;
30     for(int i=1;i<maxn;i++){
31         fac[i]=fac[i-1]*i%mod;
32     }
33     inv[1]=1;
34     for(int i=2;i<maxn;i++){
35         inv[i]=(int)(mod-mod/i)*inv[mod%i]%mod;
36     }
37     inv[0]=1;
38     for(int i=1;i<maxn;i++){
39         inv[i]=inv[i-1]*inv[i]%mod;
40     }
41 }
42 signed main()
43 {
44     init();
45     if(mod==(int)(1e9+7))assert(C(2000,1000)==72475738);
46     if(mod==998244353)assert(C(2000,1000)==472799582);
47 }
```

BSGS 指数方程余数问题(求 $a^x=b\%p$)

```
1  #include <cstdio>
2  #include <cstring>
3  #include <cmath>
4  #include <algorithm>
```

```

5  #include <unordered_map>
6
7  using namespace std;
8
9  typedef long long LL;
10
11 const int INF = 0x3f3f3f3f;
12
13 int a, b, p;
14 unordered_map<int, int> hs;
15
16 int exgcd(int a, int b, int &x, int &y) {
17     if (!b) {
18         x = 1, y = 0;
19         return a;
20     }
21     int d = exgcd(b, a % b, y, x);
22     y -= a / b * x;
23     return d;
24 }
25
26 int BSGS(int a, int b, int p) {
27     if (1 % p == b % p) return 0;
28     int k = sqrt(p) + 1;
29     hs.clear();
30     for (int y = 0, r = b % p; y < k; y++) {
31         hs[r] = y;
32         r = (LL)r * a % p;
33     }
34     int ak = 1;
35     for (int i = 1; i <= k; i++) ak = (LL)ak * a % p;
36     for (int x = 1, l = ak; x <= k; x++) {
37         if (hs.count(l)) return k * x - hs[l];
38         l = (LL)l * ak % p;
39     }
40     return -INF;
41 }
42
43 int exBSGS(int a, int b, int p) {
44     b = (b % p + p) % p;
45     if (1 % p == b % p) return 0;
46     int x, y;
47     int d = exgcd(a, p, x, y);
48     if (d > 1) {
49         if (b % d) return -INF;
50         exgcd(a / d, p / d, x, y);
51         return exBSGS(a, (LL)b / d * x % (p / d), p / d) + 1;
52     }
53     return BSGS(a, b, p);
54 }
55
56 int main() {
57     while (~scanf("%d%d%d", &a, &p, &b), a || b || p) {
58         int res = exBSGS(a, b, p);
59         if (res < 0) puts("No Solution");
60         else printf("%d\n", res);

```

```

61     }
62     return 0;
63 }

```

EXGCD

```

1  int exgcd(int a, int b, int &x, int &y){/* ax+by=gcd(a,b)  !(a==0&&b==0)
2      if(b==0){
3          x=1;
4          y=0;
5          return a;
6      }
7      int d=exgcd(b,a%b,x,y);
8      int t=x;
9      x=y;
10     y=t-(a/b)*y;
11     return d;
12 }

```

FFT

```

1  //当 vector 用就可以了
2  #include <bits/stdc++.h>
3  #define fp(i, a, b) for (int i = (a), i##_ = (b) + 1; i < i##_; ++i)
4  #define fd(i, a, b) for (int i = (a), i##_ = (b) - 1; i > i##_; --i)
5  using namespace std;
6  using ll = int64_t;
7  using db = double;
8  /*-----*/
9  struct cp {
10     db x, y;
11     cp(db real = 0, db imag = 0) : x(real), y(imag){};
12     cp operator+(cp b) const { return {x + b.x, y + b.y}; }
13     cp operator-(cp b) const { return {x - b.x, y - b.y}; }
14     cp operator*(cp b) const { return {x * b.x - y * b.y, x * b.y + y * b.x}; }
15 };
16 using vcp = vector<cp>;
17 using Poly = vector<int>;
18 namespace FFT {
19     const db pi = acos(-1);
20     vcp Omega(int L) {
21         vcp w(L); w[1] = 1;
22         for (int i = 2; i < L; i <= 1) {
23             auto w0 = w.begin() + i / 2, w1 = w.begin() + i;
24             cp wn(cos(pi / i), sin(pi / i));
25             for (int j = 0; j < i; j += 2)
26                 w1[j] = w0[j >> 1], w1[j + 1] = w1[j] * wn;
27         }
28         return w;
29     }
30     auto W = Omega(1 << 21); // NOLINT
31     void DIF(cp *a, int n) {
32         cp x, y;
33         for (int k = n >> 1; k >= 1)
34             for (int i = 0; i < n; i += k << 1)
35                 for (int j = 0; j < k; ++j)
36                     x = a[i + j], y = a[i + j + k],
37                     a[i + j + k] = (a[i + j] - y) * W[k + j], a[i + j] = x + y;

```

```

38     }
39     void IDIT(cp *a, int n) {
40         cp x, y;
41         for (int k = 1; k < n; k <= 1)
42             for (int i = 0; i < n; i += k < 1)
43                 for (int j = 0; j < k; ++j)
44                     x = a[i + j], y = a[i + j + k] * W[k + j],
45                     a[i + j + k] = x - y, a[i + j] = x + y;
46         const db Inv = 1. / n;
47         fp(i, 0, n - 1) a[i].x *= Inv, a[i].y *= Inv;
48         reverse(a + 1, a + n);
49     }
50 }
51
52 namespace Polynomial {
53     // basic operator
54     void DFT(vcp &a) { FFT::DIF(a.data(), a.size()); }
55     void IDFT(vcp &a) { FFT::IDIT(a.data(), a.size()); }
56     int norm(int n) { return 1 << (__lg(n - 1) + 1); }
57
58     // Poly mul
59     vcp &dot(vcp &a, vcp &b) { fp(i, 0, a.size() - 1) a[i] = a[i] * b[i]; return a; }
60     Poly operator+(Poly a, Poly b) {
61         int maxlen = max(a.size(), b.size());
62         Poly ans(maxlen + 1);
63         a.resize(maxlen + 1), b.resize(maxlen + 1);
64         for (int i = 0; i < maxlen; i++)
65             ans[i] = a[i] + b[i];
66         return ans;
67     }
68     Poly operator*(ll k, Poly a) {
69         Poly ans;
70         for(auto i:a)
71             ans.push_back(k * i);
72         return ans;
73     }
74     Poly operator*(Poly a, Poly b) {
75         int n = a.size() + b.size() - 1;
76         vcp c(norm(n));
77         fp(i, 0, a.size() - 1) c[i].x = a[i];
78         fp(i, 0, b.size() - 1) c[i].y = b[i];
79         DFT(c), dot(c, c), IDFT(c), a.resize(n);
80         fp(i, 0, n - 1) a[i] = int(c[i].y * .5 + .5);
81         return a;
82     }
83 }
84 /*-----*/
85 using namespace Polynomial;

```

FWT

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int mod = 998244353;
4 void add(int &x, int y) {
5     if ((x += y) >= mod) x -= mod;
6 }

```

```

7 void del(int &x, int y) {
8     if ((x -= y) < 0) x += mod;
9 }
10 void fwtor(int a[], int m, int opt) //(1,-1)
11 {
12     for (int len = 2; len <= m; len <= 1)
13         for (int p = len >> 1, i = 0; i < m; i += len)
14             for (int j = i; j < i + p; j++)
15                 if (opt > 0)
16                     add(a[j + p], a[j]);
17                 else
18                     del(a[j + p], a[j]);
19 }
20 void fwtand(int a[], int m, int opt) //(1,-1)
21 {
22     for (int len = 2; len <= m; len <= 1)
23         for (int p = len >> 1, i = 0; i < m; i += len)
24             for (int j = i; j < i + p; j++)
25                 if (opt > 0)
26                     add(a[j], a[j + p]);
27                 else
28                     del(a[j], a[j + p]);
29 }
30 void fwtxor(int a[], int m, int opt) //(1,1/2)
31 {
32     for (int len = 2; len <= m; len <= 1)
33         for (int p = len >> 1, i = 0; i < m; i += len)
34             for (int j = i; j < i + p; j++) {
35                 add(a[j], a[j + p]);
36                 a[j + p] = (a[j] - 2ll * a[j + p] % mod + mod) % mod;
37                 a[j] = 1ll * a[j] * opt % mod;
38                 a[j + p] = 1ll * a[j + p] * opt % mod;
39             }
40 }
41 int a[1 << 17], b[1 << 17], c[1 << 17];
42 void mul(int a[], int b[], int c[], int m) {
43     for (int i = 0; i < m; i++) c[i] = 1ll * a[i] * b[i] % mod;
44 }
45 void print(int a[], int m) {
46     for (int i = 0; i < m; i++) cout << a[i] << " \n"[i == m - 1];
47 }
48 int main() {
49     int n;
50     cin >> n;
51     int m = 1 << n;
52     for (int i = 0; i < m; i++) cin >> a[i];
53     for (int i = 0; i < m; i++) cin >> b[i];
54
55     fwtor(a, m, 1), fwtor(b, m, 1), mul(a, b, c, m);
56     fwtor(a, m, -1), fwtor(b, m, -1), fwtor(c, m, -1), print(c, m);
57
58     fwtand(a, m, 1), fwtand(b, m, 1), mul(a, b, c, m);
59     fwtand(a, m, -1), fwtand(b, m, -1), fwtand(c, m, -1), print(c, m);
60
61     fwtxor(a, m, 1), fwtxor(b, m, 1), mul(a, b, c, m);

```

```

62     fwtxor(c, m, (mod + 1) / 2), print(c, m);
63 }

```

莫比乌斯反演

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const int maxn=1e5+10;
4  int pr[maxn],tot,mul[maxn],phi[maxn];
5  bool vis[maxn];
6  void init(int n)
7  {
8      mul[1]=phi[1]=1;
9      for(int i=2;i<=n;i++)
10     {
11         if(!vis[i])
12         {
13             mul[i]=-1;
14             pr[++tot]=i;
15             phi[i]=i-1;
16         }
17         for(int j=1;j<=tot && (long long)pr[j]*i<=n;j++)
18         {
19             int num=pr[j]*i;
20             vis[num]=1;
21             mul[num]=-mul[i];
22             phi[num]=phi[i]*phi[pr[j]];
23             if(i%pr[j]==0)
24             {
25                 phi[num]=pr[j]*phi[i];
26                 mul[num]=0;
27                 break;
28             }
29         }
30     }
31 }

```

NTT

```

1  #include <bits/stdc++.h>
2
3  #define fp(i, a, b) for (int i = (a), i##_ = (b) + 1; i < i##_; ++i)
4  #define fd(i, a, b) for (int i = (a), i##_ = (b) - 1; i > i##_; --i)
5  #define file(s) freopen(s".in", "r", stdin), freopen(s".out", "w", stdout)
6  using namespace std;
7  const int maxn = 2e5 + 5, P = 998244353;
8  using arr = int[maxn];
9  using ll = int64_t;
10 /*-----*/
11 class Cipolla {
12     int P, I2{};
13     using pll = pair<ll, ll>;
14     #define X first
15     #define Y second
16     ll mul(ll a, ll b) const { return a * b % P; }
17     pll mul(pll a, pll b) const { return {(a.X * b.X + I2 * a.Y % P * b.Y) % P, (a.X
18 * b.Y + a.Y * b.X) % P}; }

```

```

18     template<class T> T POW(T a, int b, T x) { for (; b >= 1, a = mul(a, a)) if
    (b & 1) x = mul(x, a); return x; }
19 public:
20     Cipolla(int p = 0) : P(p) {}
21     pair<int, int> sqrt(int n) {
22         int a = rand(), x;
23         if (!(n % P)) return {0, 0};
24         if (POW(n, (P - 1) >> 1, (int)1) == P - 1) return {-1, -1};
25         while (POW(I2 = ((ll) a * a - n + P) % P, (P - 1) >> 1, (int)1) == 1) a
    = rand();
26         x = (int) POW(pll{a, 1}, (P + 1) >> 1, {1, 0}).X;
27         if (2 * x > P) x = P - x;
28         return {x, P - x};
29     }
30 #undef X
31 #undef Y
32 };
33 /*-----*/
34 #define ADD(a, b) (((a) += (b)) >= P ? (a) -= P : 0) // (a += b) %= P
35 #define SUB(a, b) (((a) -= (b)) < 0 ? (a) += P : 0) // ((a -= b) += P) %= P
36 #define MUL(a, b) ((ll) (a) * (b) % P)
37 //vector<int> getInv(int L) {
38 //    vector<int> inv(L); inv[1] = 1;
39 //    fp(i, 1, L - 1) inv[i] = MUL((P - P / i), inv[P % i]);
40 //    return inv;
41 //}
42 //auto inv = getInv(maxn); // NOLINT
43 int POW(ll a, int b = P - 2, ll x = 1) { for (; b >= 1, a = a * a % P) if (b & 1)
    x = x * a % P; return x; }
44 //int INV(int a) { return a < maxn ? inv[a] : POW(a); }
45 namespace NTT {
46     const int g = 3;
47     vector<int> Omega(int L) {
48         int wn = POW(g, P / L);
49         vector<int> w(L); w[L >> 1] = 1;
50         fp(i, L / 2 + 1, L - 1) w[i] = MUL(w[i - 1], wn);
51         fd(i, L / 2 - 1, 1) w[i] = w[i << 1];
52         return w;
53     }
54     auto W = Omega(1 << 21); // NOLINT
55     void DIF(int *a, int n) {
56         for (int k = n >> 1; k; k >>= 1)
57             for (int i = 0, y; i < n; i += k << 1)
58                 fp(j, 0, k - 1)
59                     y = a[i + j + k], a[i + j + k] = MUL(a[i + j] - y + P, W[k + j]),
    ADD(a[i + j], y);
60     }
61     void IDIT(int *a, int n) {
62         for (int k = 1; k < n; k <= 1)
63             for (int i = 0, x, y; i < n; i += k << 1)
64                 fp(j, 0, k - 1)
65                     x = a[i + j], y = MUL(a[i + j + k], W[k + j]),
66                     a[i + j + k] = x - y < 0 ? x - y + P : x - y, ADD(a[i + j], y);
67         int Inv = P - (P - 1) / n;
68         fp(i, 0, n - 1) a[i] = MUL(a[i], Inv);
69         reverse(a + 1, a + n);

```

```

70     }
71 }
72 namespace Polynomial {
73     using Poly = std::vector<int>;
74
75     // mul/div int
76     Poly &operator*=(Poly &a, int b) { for (auto &x : a) x = MUL(x, b); return a; }
77     Poly operator*(Poly a, int b) { return a *= b; }
78     Poly operator*(int a, Poly b) { return b * a; }
79     Poly &operator/=(Poly &a, int b) { return a *= POW(b); }
80     Poly operator/(Poly a, int b) { return a /= b; }
81
82     // Poly add/sub
83     Poly &operator+=(Poly &a, Poly b) {
84         a.resize(max(a.size(), b.size()));
85         fp(i, 0, b.size() - 1) ADD(a[i], b[i]);
86         return a;
87     }
88     Poly operator+(Poly a, Poly b) { return a += b; }
89     Poly &operator-=(Poly &a, Poly b) {
90         a.resize(max(a.size(), b.size()));
91         fp(i, 0, b.size() - 1) SUB(a[i], b[i]);
92         return a;
93     }
94     Poly operator-(Poly a, Poly b) { return a -= b; }
95
96     // Poly mul
97     void DFT(Poly &a) { NTT::DIF(a.data(), a.size()); }
98     void IDFT(Poly &a) { NTT::IDIT(a.data(), a.size()); }
99     int norm(int n) { return 1 << (32 - __builtin_clz(n - 1)); }
100    void norm(Poly &a) { if (!a.empty()) a.resize(norm(a.size()), 0); }
101    Poly &dot(Poly &a, Poly &b) {
102        fp(i, 0, a.size() - 1) a[i] = MUL(a[i], b[i]);
103        return a;
104    }
105    Poly operator*(Poly a, Poly b) {
106        int n = a.size() + b.size() - 1, L = norm(n);
107        if (a.size() <= 8 || b.size() <= 8) {
108            Poly c(n);
109            fp(i, 0, a.size() - 1) fp(j, 0, b.size() - 1)
110                c[i + j] = (c[i + j] + (ll) a[i] * b[j]) % P;
111            return c;
112        }
113        a.resize(L), b.resize(L);
114        DFT(a), DFT(b), dot(a, b), IDFT(a);
115        return a.resize(n), a;
116    }
117
118    // Poly inv
119    Poly Inv2k(Poly a) { // a.size() = 2^k
120        int n = a.size(), m = n >> 1;
121        if (n == 1) return {POW(a[0])};
122        Poly b = Inv2k(Poly(a.begin(), a.begin() + m)), c = b;
123        b.resize(n), DFT(a), DFT(b), dot(a, b), IDFT(a);
124        fp(i, 0, n - 1) a[i] = i < m ? 0 : P - a[i];
125        DFT(a), dot(a, b), IDFT(a);

```



```

126         return move(c.begin(), c.end(), a.begin()), a;
127     }
128     Poly Inv(Poly a) {
129         int n = a.size();
130         norm(a), a = Inv2k(a);
131         return a.resize(n), a;
132     }
133
134     // Poly div/mod
135     Poly operator/(Poly a, Poly b){
136         int k = a.size() - b.size() + 1;
137         if (k < 0) return {0};
138         reverse(a.begin(), a.end());
139         reverse(b.begin(), b.end());
140         b.resize(k), a = a * Inv(b);
141         a.resize(k), reverse(a.begin(), a.end());
142         return a;
143     }
144     pair<Poly, Poly> operator%(Poly a, const Poly& b) {
145         Poly c = a / b;
146         a -= b * c, a.resize(b.size() - 1);
147         return {c, a};
148     }
149
150     // Poly sqrt
151     Poly Sqrt(Poly a) {
152         int n = a.size(), k = norm(n);
153         Poly b = {(new Cipolla(P))->sqrt(a[0]).first}, c;
154         a.resize(k * 2, 0);
155         for (int L = 2; L <= k; L <= 1) {
156             b.resize(2 * L, 0), c = Poly(a.begin(), a.begin() + L) * Inv(b);
157             fp(i, 0, 2 * L - 1) b[i] = MUL(b[i] + c[i], (P + 1) / 2);
158         }
159         return b.resize(n), b;
160     }
161
162     // Poly calculus
163     void Derivative(Poly &a) {
164         fp(i, 1, a.size() - 1) a[i - 1] = MUL(i, a[i]);
165         a.pop_back();
166     }
167 }

```

任意模数 NTT

```

1  const long long mod = 1e18;
2  namespace polynomial {
3      typedef complex<long double> cplx;
4      const long double pi = acos((long double)-1.0);
5      const int len = 15, mask = (1 << len) - 1;
6      struct UnitRoot {
7          static vector<cplx> w;
8          static vector<cplx> get_root(int n) {
9              n = 1 << 32 - __builtin_clz(n);
10             if (n > w.size()) {
11                 w.resize(n);
12                 for (int i = 0; i < n; i++)

```

```

13         w[i] = cplx(cos(2 * i * pi / n), sin(2 * i * pi / n));
14     }
15     int m = w.size() / n;
16     vector<cplx> res(n);
17     for (int i = 0, j = 0; i < n; i++, j += m) res[i] = w[j];
18     return res;
19 }
20 };
21 vector<cplx> UnitRoot::w;
22
23 void fft(vector<cplx> &p, const vector<cplx> &w) {
24     int n = w.size();
25     for (int i = 1, j = 0; i < n - 1; ++i) {
26         int s = n;
27         do {
28             s >>= 1;
29             j ^= s;
30         } while (~j & s);
31         if (i < j) {
32             swap(p[i], p[j]);
33         }
34     }
35     for (int d = 0; (1 << d) < n; ++d) {
36         int m = 1 << d, m2 = m * 2, rm = n >> (d + 1);
37         for (int i = 0; i < n; i += m2) {
38             for (int j = 0; j < m; ++j) {
39                 auto &p1 = p[i + j + m], &p2 = p[i + j];
40                 auto t = w[rm * j] * p1;
41                 p1 = p2 - t;
42                 p2 = p2 + t;
43             }
44         }
45     }
46 }
47 vector<long long> conv(const vector<long long> &a, const vector<long long> &b) {
48     vector<cplx> w = UnitRoot::get_root(a.size() + b.size() - 1);
49     int n = w.size();
50     vector<cplx> A(n), B(n), C(n), D(n);
51     for (int i = 0; i < a.size(); ++i) A[i] = cplx(a[i] >> len, a[i] & mask);
52     for (int i = 0; i < b.size(); ++i) B[i] = cplx(b[i] >> len, b[i] & mask);
53     fft(A, w), fft(B, w);
54     for (int i = 0; i < n; ++i) {
55         int j = (n - i) % n;
56         cplx da = (A[i] - conj(A[j])) * cplx(0, -0.5), db = (A[i] + conj(A[j]))
57 * cplx(0.5, 0), dc = (B[i] - conj(B[j])) * cplx(0, -0.5), dd = (B[i] + conj(B[j])) *
58 cplx(0.5, 0);
59         C[j] = da * dd + da * dc * cplx(0, 1); D[j] = db * dd + db * dc * cplx(0, 1);
60     }
61     fft(C, w), fft(D, w);
62     vector<long long> res(a.size() + b.size() - 1);
63     for (int i = 0; i < res.size(); ++i) {
64         long long da = (long long)(C[i].imag() / n + 0.5) % mod, db = (long long)
65 (C[i].real() / n + 0.5) % mod, dc = (long long)(D[i].imag() / n + 0.5) % mod, dd = (long
66 long)(D[i].real() / n + 0.5) % mod;
67         res[i] = ((dd << (len * 2)) + ((db + dc) << len) + da) % mod;
68     }
69 }

```

```

65         return res;
66     }
67 };
68 using namespace polynomial;

```

Pollard_Rho

```

1  typedef long long ll;
2  map<ll, bool>P;
3  mt19937_64 rnd(time(0));
4  namespace Pollard_Rho
5  {
6  #define ldb long double
7  ll mul(ll x, ll y, ll mod)
8  {
9      return ((x * y - (ll)((ldb)x / mod * y) * mod) + mod) % mod;
10 }
11 ll gcd(ll a, ll b)
12 {
13     return (b == 0 ? a : gcd(b, a % b));
14 }
15 ll ksm(ll a, ll b, ll mod)
16 {
17     ll ans = 1; a %= mod;
18     while (b) {if (b & 1)ans = mul(ans, a, mod); b >>= 1; a = mul(a, a, mod);}
19     return ans;
20 }
21 int pr[15] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
22 bool Miller_Rabin(ll n)
23 {
24     if (n == 2 || n == 3)return 1;
25     if (n % 2 == 0 || n == 1)return 0;
26     ll d = n - 1;
27     int s = 0;
28     while (d % 2 == 0)s ++, d >>= 1;
29     for (int i = 0; i <= 11; i ++ )
30     {
31         if (pr[i] >= n)break;
32         ll a = pr[i];
33         ll x = ksm(a, d, n);
34         ll y = 0;
35         for (int j = 0; j <= s - 1; j ++ )
36         {
37             y = mul(x, x, n);
38             if (y == 1 && x != 1 && x != (n - 1))return 0;
39             x = y;
40         }
41         if (y != 1)return 0;
42     }
43     return 1;
44 }
45 ll Pollard_Rho(ll n)
46 {
47     ll now, pre, g;
48     while (true)
49     {
50         now = pre = rnd() % (n - 1) + 1;

```

```

51     g = 1;
52     ll c = rnd() % (n - 1) + 1;
53     for (int i = 1, fst = 1 ;; i++)
54     {
55         now = (mul(now, now, n) + c) % n;
56         g = mul(g, abs(now - pre), n);
57         if (now == pre || !g) break;
58         if (!(i & 127) || i == fst)
59         {
60             g = gcd(g, n);
61             if (g > 1) return g;
62             if (i == fst) pre = now, fst <= 1;
63         }
64     }
65 }
66 }
67 void Find(ll n)
68 {
69     if (n == 1) return ;
70     if (Miller_Rabin(n))
71     {
72         P[n] = 1;
73         return ;
74     }
75     ll p = Pollard_Rho(n);
76     int c = 0;
77     while (!(n % p))
78     {
79         n /= p, c++;
80     }
81     Find(p);
82     Find(n);
83 }
84 }
85 void solve(int x, set<int> &s){
86     Pollard_Rho :: Find(x);
87     for (auto [x, _] : P) s.insert(x);
88     P.clear();
89 }

```

扩展中国剩余定理

```

1  #define int long long
2  int mul(int a,int b,int mod){//0(1)取模快速乘, 不会爆 long long
3      return (a*b-(int)((long double)a/mod*b)*mod+mod)%mod;
4  }
5  int exgcd(int a, int b, int& x, int& y){
6      if(!b){
7          x = 1, y = 0;
8          return a;
9      }
10     int d = exgcd(b,a%b,y,x);
11     y -= a/b*x;
12     return d;
13 }
14 int solve(int n,vector<int>&mo,vector<int>&res){
15     int a1,m1;

```

```

16     a1=res[0],m1=mo[0];
17     bool ok = 1;
18     for(int i=1;i<n;i++){
19         int a2,m2,k1,k2;
20         m2=mo[i],a2=res[i];
21         int d = exgcd(m1,m2,k1,k2);
22         if((a2-a1)%d) ok = 0;
23         else{
24             k1=mul(k1,(a2-a1)/d,m2/d);//这个地方必须要用取模快速乘
25             a1=a1+k1*m1;
26             m1=abs(m1/d*m2);
27         }
28     }
29     if(ok) return (a1%m1+m1)%m1;
30     else return -1;
31 }

```

拉格朗日插值

```

1  #define int long long
2  const int N=1e6+10,mod=998244353;
3  int ksm(int a,int n,int m=mod){int s=1;while(n){if(n&1) s=s*a%m;a=a*a%m;n>>=1;}return s;}
4  int fac[N+5],facinv[N+5],inv[N+5];
5  struct LR{
6      int Inv(int n){return ksm(n,mod-2);}
7      void init(){ //预处理阶乘和阶乘逆元,逆元.
8          fac[0]=inv[0]=inv[1]=1;
9          for(int i=1;i<=N;i++)
10             fac[i]=fac[i-1]*i%mod;
11         facinv[N]=Inv(fac[N]);
12         for(int i=N-1;~i;i--)
13             facinv[i]=facinv[i+1]*(i+1)%mod;
14         for(int i=2;i<N+5;i++)
15             inv[i]=(mod-mod/i)*inv[mod%i]%mod;
16     }
17     int cal(vector<int>&x,vector<int>&y,int k){ //离散点 n 个点[0,n-1] x[i],y[i] 插 f(k)
18         int n=x.size();
19         int s=0;
20         for(int i=0;i<n;i++)if(x[i]==k) return y[i];
21         for(int i=0;i<n;i++){
22             int p=y[i]%mod,q=1;
23             for(int j=0;j<n;j++){
24                 if(i==j) continue;
25                 p=p*((k-x[j])%mod+mod)%mod;
26                 q=q*((x[i]-x[j])%mod+mod)%mod;
27             }
28             s=(s+p*Inv(q)%mod)%mod;
29         }return (s%mod+mod)%mod;
30     }
31     int inpo(vector<int>&f,int x){ //给定 连续 i 属于[0,n] f(i) 拉插 f(x)
32         int n=f.size()-1;
33         if(x>=0&&x<=n) return f[x];
34         int p,s=0;
35         vector<int>pre(n+1),suf(n+1);
36         pre[0]=x-0;
37         for(int i=1;i<=n;i++)pre[i]=pre[i-1]*(x-i)%mod;

```

```

38     suf[n]=x-n;
39     for(int i=n-1;i>=0;i--)suf[i]=suf[i+1]*(x-i)%mod;
40     for(int i=0;i<=n;i++){
41         p=facinv[n-i]%mod*facinv[i]%mod;
42         if(i>0)p=p*pre[i-1]%mod;
43         if(i<n)p=p*suf[i+1]%mod;
44         if((n-i)&1) s=(s-p*f[i]%mod+mod)%mod;
45         else s=(s+p*f[i]%mod)%mod;
46     }
47     return (s%mod+mod)%mod;
48 }
49 }sol;

```

拉格朗日插值没有模数

```

1 struct LR{
2     int inter(std::vector<int> vec, int x){
3         int n = vec.size() - 1;
4         int ans = 0;
5         for (int i = 0; i <= n; ++ i){
6             int div = 1;
7             for (int j = 0; j <= n; ++ j){
8                 if (i != j) div *= (i - j);
9             }
10            bool flag = div < 0;
11            div = std::abs(div);
12            int prod = vec[i];
13            for (int j = 0; j <= n; ++ j){
14                if (i == j) continue;
15                int gcd = std::abs(std::__gcd(x - j, div));
16                prod *= (x - j) / gcd;
17                div /= gcd;
18            }
19            ans += flag ? -prod : prod;
20        }
21        return ans;
22    }

```

杜教筛

```

1 const int maxn=3e6+10;
2 int sumf[maxn];
3 int Sum(int n){// 这是 f * g 的 n 项前缀和
4
5 }
6 int Sumg(int n){// g 的 n 项前缀和
7
8 }
9 map<int,int> f;
10 int F (int n) {
11     if (n <= 3000'000) return sumf[n]; // 预处理出 n 较小时的前缀和
12     if (f.find(n)!=f.end()) return f[n]; // 记忆化, 如果求过这个值, 就不需要再递归一遍了
13     int ans = Sum(n);
14     for (int l = 2, r; l <= n; l = r + 1) // 整除分块
15         r = n / (n / l), ans -= (Sumg(r) - Sumg(l-1)) * F (n / l);
16         // [l,r] 的 F (n / l) 是一样的, 对 g(x) 求个和即可
17     return f[n] = ans / Sumg(1); // 别忘了除上 g(1)

```

```
18 }
19
```

线性筛质数

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 const int maxn=1e6+10;
4 bool v[maxn];
5 int n,pr;
6 vector<int> p;
7 void init()
8 {
9     v[1]=true;
10    for(int i=2;i<maxn;i++)
11    {
12        if(!v[i])p.push_back(i);
13        for(int j=0;j<p.size()&&i*p[j]<maxn;++j){v[i*p[j]]=true;if(i%p[j]==0)break;}
14    }
15 }
16
```

线性递推

```
1 #include<bits/stdc++.h>
2 using namespace std;
3 #define rep(i,a,n) for (int i=a;i<n;i++)
4 #define per(i,a,n) for (int i=n-1;i>=a;i--)
5 #define all(x) (x).begin(),(x).end()
6 #define siz(x) ((int)(x).size())
7 typedef vector<int> VI;
8 typedef long long ll;
9 typedef pair<int,int> PII;
10 const ll mod=1000000007;
11 ll powmod(ll a,ll b) {ll res=1;a%=mod; assert(b>=0); for(;b>=>1)
12     {if(b&1)res=res*a%mod;a=a*a%mod;}return res;}
13 ll n;
14 namespace linear_seq {
15     const int N=10010;
16     ll res[N],base[N],_c[N],_md[N];
17     vector<int> Md;
18     void mul(ll *a,ll *b,int k) {
19         rep(i,0,k+k) _c[i]=0;
20         rep(i,0,k) if (a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
21         for (int i=k+k-1;i>=k;i--) if (_c[i])
22             rep(j,0,siz(Md)) _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
23         rep(i,0,k) a[i]=_c[i];
24     }
25     int solve(ll n,VI a,VI b) {
26         ll ans=0,pnt=0;
27         int k=siz(a);
28         assert(siz(a)==siz(b));
29         rep(i,0,k) _md[k-1-i]=-a[i];_md[k]=1;
30         Md.clear();
31         rep(i,0,k) if (_md[i]!=0) Md.push_back(i);
32         rep(i,0,k) res[i]=base[i]=0;
33         res[0]=1;
```

```

34     while ((1ll<<pnt)<=n) pnt++;
35     for (int p=pnt;p>=0;p--) {
36         mul(res,res,k);
37         if ((n>>p)&1) {
38             for (int i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;
39             rep(j,0,siz(Md)) res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
40         }
41     }
42     rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
43     if (ans<0) ans+=mod;
44     return ans;
45 }
46 VI BM(VI s) {
47     VI C(1,1),B(1,1);
48     int L=0,m=1,b=1;
49     rep(n,0,siz(s)) {
50         ll d=0;
51         rep(i,0,L+1) d=(d+(ll)C[i]*s[n-i])%mod;
52         if (d==0) ++m;
53         else if (2*L<=n) {
54             VI T=C;
55             ll c=mod-d*powmod(b,mod-2)%mod;
56             while (siz(C)<siz(B)+m) C.push_back(0);
57             rep(i,0,siz(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
58             L=n+1-L; B=T; b=d; m=1;
59         } else {
60             ll c=mod-d*powmod(b,mod-2)%mod;
61             while (siz(C)<siz(B)+m) C.push_back(0);
62             rep(i,0,siz(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
63             ++m;
64         }
65     }
66     return C;
67 }
68 int gao(VI a,ll n) {
69     VI c=BM(a);
70     c.erase(c.begin());
71     rep(i,0,siz(c)) c[i]=(mod-c[i])%mod;
72     return solve(n,c,VI(a.begin(),a.begin()+siz(c)));
73 }
74 };
75
76 int main() {
77     vector<int>v;
78     v.push_back(2);
79     v.push_back(24);
80     v.push_back(96);
81     v.push_back(416);
82     v.push_back(1536);
83     v.push_back(5504);
84     v.push_back(18944);
85     v.push_back(64000);
86     v.push_back(212992);
87     v.push_back(702464);
88     scanf("%lld", &n);
89     printf("%lld\n",1LL * linear_seq::gao(v,n-1) % mod);

```



```
90 }
91
```

辛普森积分

```
1 double simpson(double l, double r) {
2     double mid = (l + r) / 2;
3     return (r - l) * (f(l) + 4 * f(mid) + f(r)) / 6; // 辛普森公式
4 }
5
6 double asr(double l, double r, double eps, double ans, int step) { // step 是递归的下限
7     double mid = (l + r) / 2;
8     double fl = simpson(l, mid), fr = simpson(mid, r);
9     if (abs(fl + fr - ans) <= 15 * eps && step < 0)
10         return fl + fr + (fl + fr - ans) / 15; // 足够相似的话就直接返回
11     return asr(l, mid, eps / 2, fl, step - 1) +
12           asr(mid, r, eps / 2, fr, step - 1); // 否则分割成两段递归求解
13 }
14
15 double calc(double l, double r, double eps) {
16     return asr(l, r, eps, simpson(l, r), 12);
17 }
18
```

高斯消元(模意义)

```
1 #define int long long
2 const int eps=0;
3 const int maxn=220;
4 const int mod=1e6+3;
5 int ksm(int x,int k){
6     int res=1;
7     while(k){
8         if(k&1)res=res*x%mod;
9         x=x*x%mod;
10        k/=2;
11    }
12    return res;
13 }
14 int ny(int x){
15     return ksm(x,mod-2);
16 }
17 void add(int &x,int y){
18     if((x+=y)>=mod)x-=mod;
19 }
20 void del(int &x,int y){
21     if((x-=y)<0)x+=mod;
22 }
23 int a[maxn][maxn],x[maxn]; // 方程左边的矩阵和方程右边的值, 求解之后 x 存的就是结果
24 int Gauss(int equ,int var){ // equ 方程数 var 未知数个数 return 1 表示有解
25     int i,j,k,col,max_r;
26     for(k=0,col=0;k<equ&&col<var;k++,col++){
27         max_r=k;
28         for(i=k+1;i<equ;i++)
29             if((a[i][col])>(a[max_r][col]))
30                 max_r=i;
31         if((a[max_r][col])==0)return 0;
32         if(k!=max_r){
```

```

33         for(j=col;j<var;j++)
34             swap(a[k][j],a[max_r][j]);
35         swap(x[k],x[max_r]);
36     }
37     x[k]=x[k]*ny(a[k][col])%mod;
38     for(j=col+1;j<var;j++)a[k][j]=a[k][j]*ny(a[k][col])%mod;
39     a[k][col]=1;
40     for(i=0;i<equ;i++)
41         if(i!=k){
42             del(x[i],x[k]*a[i][col]%mod);
43             for(j=col+1;j<var;j++)del(a[i][j],a[k][j]*a[i][col]%mod);
44             a[i][col]=0;
45         }
46     }
47     return 1;
48 }

```

高斯消元(浮点数)

```

1  #define lf double
2  //0 base
3  const lf eps=1e-9;
4  const int maxn=220;
5  lf a[maxn][maxn],x[maxn]; //方程左边的矩阵和方程右边的值，求解之后 x 存的就是结果
6  int Gauss(int equ,int var){ //equ 方程数 var 未知数个数 return 1 表示有解
7      int i,j,k,col,max_r;
8      for(k=0,col=0;k<equ&&col<var;k++,col++){
9          max_r=k;
10         for(i=k+1;i<equ;i++)
11             if(fabs(a[i][col])>fabs(a[max_r][col]))
12                 max_r=i;
13         if(fabs(a[max_r][col])<eps) return 0;
14         if(k!=max_r){
15             for(j=col;j<var;j++){
16                 swap(a[k][j],a[max_r][j]);
17                 swap(x[k],x[max_r]);
18             }
19         }
20         x[k]/=a[k][col];
21         for(j=col+1;j<var;j++)a[k][j]/=a[k][col];
22         a[k][col]=1;
23         for(i=0;i<equ;i++){
24             if(i!=k){
25                 x[i]-=x[k]*a[i][col];
26                 for(j=col+1;j<var;j++)a[i][j]-=a[k][j]*a[i][col];
27                 a[i][col]=0;
28             }
29         }
30     }
31     return 1;
32 }

```

计算几何

开头

```
1 #include <bits/stdc++.h>
2 using namespace std;
3
4 using point_t=long double; //全局数据类型, 可修改为 long long 等
5
6 constexpr point_t eps=1e-8;
7 constexpr long double PI=3.14159265358979323841;
8
```

点与向量

```
1 // 点与向量
2 template<typename T> struct point
3 {
4     T x,y;
5
6     bool operator==(const point &a) const {return (abs(x-a.x)<=eps && abs(y-a.y)<=eps);}
7     bool operator<(const point &a) const {if (abs(x-a.x)<=eps) return y<a.y-eps;
8     return x<a.x-eps;}
9     bool operator>(const point &a) const {return !(*this<a || *this==a);}
10    point operator+(const point &a) const {return {x+a.x,y+a.y};}
11    point operator-(const point &a) const {return {x-a.x,y-a.y};}
12    point operator-() const {return {-x,-y};}
13    point operator*(const T k) const {return {k*x,k*y};}
14    point operator/(const T k) const {return {x/k,y/k};}
15    T operator*(const point &a) const {return x*a.x+y*a.y;} // 点积
16    T operator^(const point &a) const {return x*a.y-y*a.x;} // 叉积, 注意优先级
17    int toleft(const point &a) const {const auto t=(*this)^a; return (t>eps)-(t<-eps);} // to-left 测试
18    T len2() const {return (*this)*(*this);} // 向量长度的平方
19    T dis2(const point &a) const {return (a-(*this)).len2();} // 两点距离的平方
20
21    // 涉及浮点数
22    long double len() const {return sqrtl(len2());} // 向量长度
23    long double dis(const point &a) const {return sqrtl(dis2(a));} // 两点距离
24    long double ang(const point &a) const {return acosl(max(-1.0l,min(1.0l,((*this)*a)/(len()*a.len()))));} // 向量夹角
25    point rot(const long double rad) const {return {x*cos(rad)-y*sin(rad),x*sin(rad)+y*cos(rad)};} // 逆时针旋转 (给定角度)
26    point rot(const long double cosr,const long double sinr) const {return {x*cosr-y*sinr,x*sinr+y*cosr};} // 逆时针旋转 (给定角度的正弦与余弦)
27 };
28 using Point=point<point_t>;
```

极角排序

```
1 // 极角排序
2 struct argcmp
3 {
4     bool operator()(const Point &a,const Point &b) const
5     {
6         const auto quad=[](const Point &a)
```

```

7      {
8          if (a.y<-eps) return 1;
9          if (a.y>eps) return 4;
10         if (a.x<-eps) return 5;
11         if (a.x>eps) return 3;
12         return 2;
13     };
14     const int qa=quad(a),qb=quad(b);
15     if (qa!=qb) return qa<qb;
16     const auto t=a^b;
17     // if (abs(t)<=eps) return a*a<b*b-eps; // 不同长度的向量需要分开
18     return t>eps;
19 }
20 };

```

直线

```

1 // 直线
2 template<typename T> struct line
3 {
4     point<T> p,v; // p 为直线上一点, v 为方向向量
5
6     bool operator==(const line &a) const {return v.toleft(a.v)==0 && v.toleft(p-
    a.p)==0;}
7     int toleft(const point<T> &a) const {return v.toleft(a-p);} // to-left 测试
8     bool operator<(const line &a) const // 半平面交算法定义的排序
9     {
10         if (abs(v^a.v)<=eps && v*a.v>=-eps) return toleft(a.p)==-1;
11         return argcmp()(v,a.v);
12     }
13
14     // 涉及浮点数
15     point<T> inter(const line &a) const {return p+v*((a.v^(p-a.p))/(v^a.v));} // 直
    线交点
16     long double dis(const point<T> &a) const {return abs(v^(a-p))/v.len();} // 点到
    直线距离
17     point<T> proj(const point<T> &a) const {return p+v*((v*(a-p))/(v*v));} // 点在直
    线上的投影
18 };
19
20 using Line=line<point_t>;

```

线段

```

1 // 线段
2 template<typename T> struct segment
3 {
4     point<T> a,b;
5
6     bool operator<(const segment &s) const {return make_pair(a,b)<make_pair(s.a,s.b);}
7
8     // 判定性函数建议在整数域使用
9
10    // 判断点是否在线段上
11    // -1 点在线段端点 | 0 点不在线段上 | 1 点严格在线段上
12    int is_on(const point<T> &p) const
13    {

```

```

14     if (p==a || p==b) return -1;
15     return (p-a).topleft(p-b)==0 && (p-a)*(p-b)<-eps;
16 }
17
18 // 判断线段直线是否相交
19 // -1 直线经过线段端点 | 0 线段和直线不相交 | 1 线段和直线严格相交
20 int is_inter(const line<T> &l) const
21 {
22     if (l.topleft(a)==0 || l.topleft(b)==0) return -1;
23     return l.topleft(a)!=l.topleft(b);
24 }
25
26 // 判断两线段是否相交
27 // -1 在某一线段端点处相交 | 0 两线段不相交 | 1 两线段严格相交
28 int is_inter(const segment<T> &s) const
29 {
30     if (is_on(s.a) || is_on(s.b) || s.is_on(a) || s.is_on(b)) return -1;
31     const line<T> l{a,b-a},ls{s.a,s.b-s.a};
32     return l.topleft(s.a)*l.topleft(s.b)==-1 && ls.topleft(a)*ls.topleft(b)==-1;
33 }
34
35 // 点到线段距离
36 long double dis(const point<T> &p) const
37 {
38     if ((p-a)*(b-a)<-eps || (p-b)*(a-b)<-eps) return min(p.dis(a),p.dis(b));
39     const line<T> l{a,b-a};
40     return l.dis(p);
41 }
42
43 // 两线段间距离
44 long double dis(const segment<T> &s) const
45 {
46     if (is_inter(s)) return 0;
47     return min({dis(s.a),dis(s.b),s.dis(a),s.dis(b)});
48 }
49 };
50
51 using Segment=segment<point_t>;

```

多边形

```

1 // 多边形
2 template<typename T> struct polygon
3 {
4     vector<point<T>> p; // 以逆时针顺序存储
5
6     size_t nxt(const size_t i) const {return i==p.size()-1?0:i+1;}
7     size_t pre(const size_t i) const {return i==0?p.size()-1:i-1;}
8
9     // 回转数
10    // 返回值第一项表示点是否在多边形边上
11    // 对于狭义多边形, 回转数为 0 表示点在多边形外, 否则点在多边形内
12    pair<bool,int> winding(const point<T> &a) const
13    {
14        int cnt=0;
15        for (size_t i=0;i<p.size();i++)

```

```

16     {
17         const point<T> u=p[i],v=p[nxt(i)];
18         if (abs((a-u)^(a-v))<=eps && (a-u)*(a-v)<=eps) return {true,0};
19         if (abs(u.y-v.y)<=eps) continue;
20         const Line uv={u,v-u};
21         if (u.y<v.y-eps && uv.toleft(a)<=0) continue;
22         if (u.y>v.y+eps && uv.toleft(a)>=0) continue;
23         if (u.y<a.y-eps && v.y>=a.y-eps) cnt++;
24         if (u.y>=a.y-eps && v.y<a.y-eps) cnt--;
25     }
26     return {false,cnt};
27 }
28
29 // 多边形面积的两倍
30 // 可用于判断点的存储顺序是顺时针或逆时针
31 T area() const
32 {
33     T sum=0;
34     for (size_t i=0;i<p.size();i++) sum+=p[i]^p[nxt(i)];
35     return sum;
36 }
37
38 // 多边形的周长
39 long double circ() const
40 {
41     long double sum=0;
42     for (size_t i=0;i<p.size();i++) sum+=p[i].dis(p[nxt(i)]);
43     return sum;
44 }
45 };
46
47 using Polygon=polygon<point_t>;

```

凸多边形

```

1 //凸多边形
2 template<typename T> struct convex: polygon<T>
3 {
4     // 闵可夫斯基和
5     convex operator+(const convex &c) const
6     {
7         const auto &p=this->p;
8         vector<Segment> e1(p.size()),e2(c.p.size()),edge(p.size()+c.p.size());
9         vector<point<T>> res; res.reserve(p.size()+c.p.size());
10        const auto cmp=[](const Segment &u,const Segment &v) {return argcmp()(u.b-
u.a,v.b-v.a);};
11        for (size_t i=0;i<p.size();i++) e1[i]={p[i],p[this->nxt(i)]};
12        for (size_t i=0;i<c.p.size();i++) e2[i]={c.p[i],c.p[c.nxt(i)]};
13        rotate(e1.begin(),min_element(e1.begin(),e1.end(),cmp),e1.end());
14        rotate(e2.begin(),min_element(e2.begin(),e2.end(),cmp),e2.end());
15        merge(e1.begin(),e1.end(),e2.begin(),e2.end(),edge.begin(),cmp);
16        const auto check=[](const vector<point<T>> &res,const point<T> &u)
17        {
18            const auto back1=res.back(),back2=*prev(res.end(),2);
19            return (back1-back2).toleft(u-back1)==0 && (back1-back2)*(u-back1)>=-eps;
20        };
21        auto u=e1[0].a+e2[0].a;

```

```

22     for (const auto &v:edge)
23     {
24         while (res.size()>1 && check(res,u)) res.pop_back();
25         res.push_back(u);
26         u=u+v.b-v.a;
27     }
28     if (res.size()>1 && check(res,res[0])) res.pop_back();
29     return {res};
30 }
31
32 // 旋转卡壳
33 // func 为更新答案的函数, 可以根据题目调整位置
34 template<typename F> void rotcaliper(const F &func) const
35 {
36     const auto &p=this->p;
37     const auto area=[](const point<T> &u,const point<T> &v,const point<T> &w)
{return (w-u)^(w-v)};};
38     for (size_t i=0,j=1;i<p.size();i++)
39     {
40         const auto nxti=this->nxt(i);
41         func(p[i],p[nxti],p[j]);
42         while (area(p[this->nxt(j)],p[i],p[nxti])>=area(p[j],p[i],p[nxti]))
43         {
44             j=this->nxt(j);
45             func(p[i],p[nxti],p[j]);
46         }
47     }
48 }
49
50 // 凸多边形的直径的平方
51 T diameter2() const
52 {
53     const auto &p=this->p;
54     if (p.size()==1) return 0;
55     if (p.size()==2) return p[0].dis2(p[1]);
56     T ans=0;
57     auto func=[&](const point<T> &u,const point<T> &v,const point<T> &w)
{return max({ans,w.dis2(u),w.dis2(v)});};
58     rotcaliper(func);
59     return ans;
60 }
61
62 // 判断点是否在凸多边形内
63 // 复杂度 O(logn)
64 // -1 点在多边形边上 | 0 点在多边形外 | 1 点在多边形内
65 int is_in(const point<T> &a) const
66 {
67     const auto &p=this->p;
68     if (p.size()==1) return a==p[0]?-1:0;
69     if (p.size()==2) return segment<T>{p[0],p[1]}.is_on(a)?-1:0;
70     if (a==p[0]) return -1;
71     if ((p[1]-p[0]).toleft(a-p[0])==-1 || (p.back()-p[0]).toleft(a-p[0])==1)
return 0;
72     const auto cmp=[&](const Point &u,const Point &v){return (u-p[0]).toleft(v-
p[0])==1;};
73     const size_t i=lower_bound(p.begin()+1,p.end(),a,cmp)-p.begin();

```

```

74     if (i==1) return segment<T>{p[0],p[i]}.is_on(a)?-1:0;
75     if (i==p.size()-1 && segment<T>{p[0],p[i]}.is_on(a)) return -1;
76     if (segment<T>{p[i-1],p[i]}.is_on(a)) return -1;
77     return (p[i]-p[i-1]).toleft(a-p[i-1])>0;
78 }
79
80 // 凸多边形关于某一方向的极点
81 // 复杂度 O(logn)
82 // 参考资料: https://codeforces.com/blog/entry/48868
83 template<typename F> size_t extreme(const F &dir) const
84 {
85     const auto &p=this->p;
86     const auto check=[&](const size_t i){return dir(p[i]).toleft(p[this->nxt(i)]-
p[i])>=0;};
87     const auto dir0=dir(p[0]); const auto check0=check(0);
88     if (!check0 && check(p.size()-1)) return 0;
89     const auto cmp=[&](const Point &v)
90     {
91         const size_t vi=&v-p.data();
92         if (vi==0) return 1;
93         const auto checkv=check(vi);
94         const auto t=dir0.toleft(v-p[0]);
95         if (vi==1 && checkv==check0 && t==0) return 1;
96         return checkv^(checkv==check0 && t==0);
97     };
98     return partition_point(p.begin(),p.end(),cmp)-p.begin();
99 }
100
101 // 过凸多边形外一点求凸多边形的切线, 返回切点下标
102 // 复杂度 O(logn)
103 // 必须保证点在多边形外
104 pair<size_t,size_t> tangent(const point<T> &a) const
105 {
106     const size_t i=extreme([&](const point<T> &u){return u-a;});
107     const size_t j=extreme([&](const point<T> &u){return a-u;});
108     return {i,j};
109 }
110
111 // 求平行于给定直线的凸多边形的切线, 返回切点下标
112 // 复杂度 O(logn)
113 pair<size_t,size_t> tangent(const line<T> &a) const
114 {
115     const size_t i=extreme([&](...){return a.v;});
116     const size_t j=extreme([&](...){return -a.v;});
117     return {i,j};
118 }
119 };
120
121 using Convex=convex<point_t>;

```

圆

```

1 // 圆
2 struct Circle
3 {
4     Point c;
5     long double r;

```



```

6
7     bool operator==(const Circle &a) const {return c==a.c && abs(r-a.r)<=eps;}
8     long double circ() const {return 2*PI*r;} // 周长
9     long double area() const {return PI*r*r;} // 面积
10
11     // 点与圆的关系
12     // -1 圆上 | 0 圆外 | 1 圆内
13     int is_in(const Point &p) const {const long double d=p.dis(c); return abs(d-
14     r)<=eps?-1:d<r-eps;}
15
16     // 直线与圆关系
17     // 0 相离 | 1 相切 | 2 相交
18     int relation(const Line &l) const
19     {
20         const long double d=l.dis(c);
21         if (d>r+eps) return 0;
22         if (abs(d-r)<=eps) return 1;
23         return 2;
24     }
25
26     // 圆与圆关系
27     // -1 相同 | 0 相离 | 1 外切 | 2 相交 | 3 内切 | 4 内含
28     int relation(const Circle &a) const
29     {
30         if (*this==a) return -1;
31         const long double d=c.dis(a.c);
32         if (d>r+a.r+eps) return 0;
33         if (abs(d-r-a.r)<=eps) return 1;
34         if (abs(d-abs(r-a.r))<=eps) return 3;
35         if (d<abs(r-a.r)-eps) return 4;
36         return 2;
37     }
38
39     // 直线与圆的交点
40     vector<Point> inter(const Line &l) const
41     {
42         const long double d=l.dis(c);
43         const Point p=l.proj(c);
44         const int t=relation(l);
45         if (t==0) return vector<Point>();
46         if (t==1) return vector<Point>{p};
47         const long double k=sqrt(r*r-d*d);
48         return vector<Point>{p-(l.v/l.v.len())*k,p+(l.v/l.v.len())*k};
49     }
50
51     // 圆与圆交点
52     vector<Point> inter(const Circle &a) const
53     {
54         const long double d=c.dis(a.c);
55         const int t=relation(a);
56         if (t==-1 || t==0 || t==4) return vector<Point>();
57         Point e=a.c-c; e=e/e.len()*r;
58         if (t==1 || t==3)
59         {
60             if (r*r+d*d-a.r*a.r>=eps) return vector<Point>{c+e};
61             return vector<Point>{c-e};
62         }
63     }

```

```

61     }
62     const long double costh=(r*r+d*d-a.r*a.r)/(2*r*d),sinh=sqrt(1-costh*costh);
63     return vector<Point>{c+e.rot(costh,-sinh),c+e.rot(costh,sinh)};
64 }
65
66 // 圆与圆交面积
67 long double inter_area(const Circle &a) const
68 {
69     const long double d=c.dis(a.c);
70     const int t=relation(a);
71     if (t==-1) return area();
72     if (t<2) return 0;
73     if (t>2) return min(area(),a.area());
74     const long double costh1=(r*r+d*d-a.r*a.r)/(2*r*d),costh2=(a.r*a.r+d*d-r*r)/
(2*a.r*d);
75     const long double sinh1=sqrt(1-costh1*costh1),sinh2=sqrt(1-costh2*costh2);
76     const long double th1=acos(costh1),th2=acos(costh2);
77     return r*r*(th1-costh1*sinh1)+a.r*a.r*(th2-costh2*sinh2);
78 }
79
80 // 过圆外一点圆的切线
81 vector<Line> tangent(const Point &a) const
82 {
83     const int t=is_in(a);
84     if (t==1) return vector<Line>();
85     if (t==-1)
86     {
87         const Point v={-(a-c).y,(a-c).x};
88         return vector<Line>{{a,v}};
89     }
90     Point e=a-c; e=e/e.len()*r;
91     const long double costh=r/c.dis(a),sinh=sqrt(1-costh*costh);
92     const Point t1=c+e.rot(costh,-sinh),t2=c+e.rot(costh,sinh);
93     return vector<Line>{{a,t1-a},{a,t2-a}};
94 }
95
96 // 两圆的公切线
97 vector<Line> tangent(const Circle &a) const
98 {
99     const int t=relation(a);
100     vector<Line> lines;
101     if (t==-1 || t==4) return lines;
102     if (t==1 || t==3)
103     {
104         const Point p=inter(a)[0],v={-(a.c-c).y,(a.c-c).x};
105         lines.push_back({p,v});
106     }
107     const long double d=c.dis(a.c);
108     const Point e=(a.c-c)/(a.c-c).len();
109     if (t<=2)
110     {
111         const long double costh=(r-a.r)/d,sinh=sqrt(1-costh*costh);
112         const Point d1=e.rot(costh,-sinh),d2=e.rot(costh,sinh);
113         const Point u1=c+d1*r,u2=c+d2*r,v1=a.c+d1*a.r,v2=a.c+d2*a.r;
114         lines.push_back({u1,v1-u1}); lines.push_back({u2,v2-u2});
115     }

```

```

116     if (t==0)
117     {
118         const long double costh=(r+a.r)/d,sinth=sqrt(1-costh*costh);
119         const Point d1=e.rot(costh,-sinth),d2=e.rot(costh,sinth);
120         const Point u1=c+d1*r,u2=c+d2*r,v1=a.c-d1*a.r,v2=a.c-d2*a.r;
121         lines.push_back({u1,v1-u1}); lines.push_back({u2,v2-u2});
122     }
123     return lines;
124 }
125
126 // 圆的反演
127 tuple<int,Circle,Line> inverse(const Line &l) const
128 {
129     const Circle null_c={{0.0,0.0},0.0};
130     const Line null_l={{0.0,0.0},{0.0,0.0}};
131     if (l.toleft(c)==0) return {2,null_c,l};
132     const Point v=l.toleft(c)==1?Point{l.v.y,-l.v.x}:Point{-l.v.y,l.v.x};
133     const long double d=r*r/l.dis(c);
134     const Point p=c+v/v.len()*d;
135     return {1,{(c+p)/2,d/2},null_l};
136 }
137
138 tuple<int,Circle,Line> inverse(const Circle &a) const
139 {
140     const Circle null_c={{0.0,0.0},0.0};
141     const Line null_l={{0.0,0.0},{0.0,0.0}};
142     const Point v=a.c-c;
143     if (a.is_in(c)==-1)
144     {
145         const long double d=r*r/(a.r+a.r);
146         const Point p=c+v/v.len()*d;
147         return {2,null_c,{p,{-v.y,v.x}}};
148     }
149     if (c==a.c) return {1,{c,r*r/a.r},null_l};
150     const long double d1=r*r/(c.dis(a.c)-a.r),d2=r*r/(c.dis(a.c)+a.r);
151     const Point p=c+v/v.len()*d1,q=c+v/v.len()*d2;
152     return {1,{(p+q)/2,p.dis(q)/2},null_l};
153 }
154 };
155
156 // 圆与多边形面积交
157 long double area_inter(const Circle &circ,const Polygon &poly)
158 {
159     const auto cal=[](const Circle &circ,const Point &a,const Point &b)
160     {
161         if ((a-circ.c).toleft(b-circ.c)==0) return 0.0l;
162         const auto ina=circ.is_in(a),inb=circ.is_in(b);
163         const Line ab={a,b-a};
164         if (ina && inb) return ((a-circ.c)^(b-circ.c))/2;
165         if (ina && !inb)
166         {
167             const auto t=circ.inter(ab);
168             const Point p=t.size()==1?t[0]:t[1];
169             const long double ans=((a-circ.c)^(p-circ.c))/2;
170             const long double th=(p-circ.c).ang(b-circ.c);
171             const long double d=circ.r*circ.r*th/2;

```

```

172         if ((a-circ.c).topleft(b-circ.c)==1) return ans+d;
173         return ans-d;
174     }
175     if (!ina && inb)
176     {
177         const Point p=circ.inter(ab)[0];
178         const long double ans=((p-circ.c)^(b-circ.c))/2;
179         const long double th=(a-circ.c).ang(p-circ.c);
180         const long double d=circ.r*circ.r*th/2;
181         if ((a-circ.c).topleft(b-circ.c)==1) return ans+d;
182         return ans-d;
183     }
184     const auto p=circ.inter(ab);
185     if (p.size()==2 && Segment{a,b}.dis(circ.c)<=circ.r+eps)
186     {
187         const long double ans=((p[0]-circ.c)^(p[1]-circ.c))/2;
188         const long double th1=(a-circ.c).ang(p[0]-circ.c),th2=(b-circ.c).ang(p[1]-
circ.c);
189         const long double d1=circ.r*circ.r*th1/2,d2=circ.r*circ.r*th2/2;
190         if ((a-circ.c).topleft(b-circ.c)==1) return ans+d1+d2;
191         return ans-d1-d2;
192     }
193     const long double th=(a-circ.c).ang(b-circ.c);
194     if ((a-circ.c).topleft(b-circ.c)==1) return circ.r*circ.r*th/2;
195     return -circ.r*circ.r*th/2;
196 };
197
198 long double ans=0;
199 for (size_t i=0;i<poly.p.size();i++)
200 {
201     const Point a=poly.p[i],b=poly.p[poly.nxt(i)];
202     ans+=cal(circ,a,b);
203 }
204 return ans;
205 }

```

判断多条线段是否有交点

```

1 // 判断多条线段是否有交点
2 // 扫描线, 复杂度 O(nlogn)
3 bool segs_inter(const vector<Segment> &segs)
4 {
5     if (segs.empty()) return false;
6     using seq_t=tuple<point_t,int,Segment>;
7     const auto seqcmp=[](const seq_t &u, const seq_t &v)
8     {
9         const auto [u0,u1,u2]=u;
10        const auto [v0,v1,v2]=v;
11        if (abs(u0-v0)<=eps) return make_pair(u1,u2)<make_pair(v1,v2);
12        return u0<v0-eps;
13    };
14    vector<seq_t> seq;
15    for (auto seg:segs)
16    {
17        if (seg.a.x>seg.b.x+eps) swap(seg.a,seg.b);
18        seq.push_back({seg.a.x,0,seg});
19        seq.push_back({seg.b.x,1,seg});

```

```

20     }
21     sort(seq.begin(), seq.end(), seqcmp);
22     point_t x_now;
23     auto cmp=[&](const Segment &u, const Segment &v)
24     {
25         if (abs(u.a.x-u.b.x)<=eps || abs(v.a.x-v.b.x)<=eps) return u.a.y<v.a.y-eps;
26         return ((x_now-u.a.x)*(u.b.y-u.a.y)+u.a.y*(u.b.x-u.a.x))*(v.b.x-v.a.x)<((x_now-
v.a.x)*(v.b.y-v.a.y)+v.a.y*(v.b.x-v.a.x))*(u.b.x-u.a.x)-eps;
27     };
28     multiset<Segment, decltype(cmp)> s{cmp};
29     for (const auto [x,o,seg]:seq)
30     {
31         x_now=x;
32         const auto it=s.lower_bound(seg);
33         if (o==0)
34         {
35             if (it!=s.end() && seg.is_inter(*it)) return true;
36             if (it!=s.begin() && seg.is_inter(*prev(it))) return true;
37             s.insert(seg);
38         }
39         else
40         {
41             if (next(it)!=s.end() && it!=s.begin() && (*prev(it)).is_inter(*next(it)))
return true;
42             s.erase(it);
43         }
44     }
45     return false;
46 }

```

半平面交

```

1 // 半平面交
2 // 排序增量法, 复杂度 O(nlogn)
3 // 输入与返回值都是用直线表示的半平面集合
4 vector<Line> halfinter(vector<Line> l, const point_t lim=1e9)
5 {
6     const auto check=[](const Line &a,const Line &b,const Line &c){return
a.toleft(b.inter(c))<0;};
7     // 无精度误差的方法, 但注意取值范围会扩大到三次方
8     /*const auto check=[](const Line &a,const Line &b,const Line &c)
9     {
10         const Point p=a.v*(b.v^c.v),q=b.p*(b.v^c.v)+b.v*(c.v^(b.p-c.p))-a.p*(b.v^c.v);
11         return p.toleft(q)<0;
12     };*/
13     l.push_back({{-lim,0},{0,-1}}); l.push_back({{0,-lim},{1,0}});
14     l.push_back({{lim,0},{0,1}}); l.push_back({{0,lim},{-1,0}});
15     sort(l.begin(),l.end());
16     deque<Line> q;
17     for (size_t i=0;i<l.size();i++)
18     {
19         if (i>0 && l[i-1].v.toleft(l[i].v)==0 && l[i-1].v*l[i].v>eps) continue;
20         while (q.size()>1 && check(l[i],q.back(),q[q.size()-2])) q.pop_back();
21         while (q.size()>1 && check(l[i],q[0],q[1])) q.pop_front();
22         if (!q.empty() && q.back().v.toleft(l[i].v)<=0) return vector<Line>();
23         q.push_back(l[i]);
24     }

```

```

25     while (q.size()>1 && check(q[0],q.back(),q[q.size()-2])) q.pop_back();
26     while (q.size()>1 && check(q.back(),q[0],q[1])) q.pop_front();
27     return vector<Line>(q.begin(),q.end());
28 }

```

圆面积并

```

1  // 圆面积并
2  // 轮廓积分, 复杂度  $O(n^2 \log n)$ 
3  // ans[i] 表示被至少覆盖了 i+1 次的区域的面积
4  vector<long double> area_union(const vector<Circle> &circs)
5  {
6      const size_t siz=circs.size();
7      using arc_t=tuple<Point,long double,long double,long double>;
8      vector<vector<arc_t>> arcs(siz);
9      const auto eq=[](const arc_t &u,const arc_t &v)
10     {
11         const auto [u1,u2,u3,u4]=u;
12         const auto [v1,v2,v3,v4]=v;
13         return u1==v1 && abs(u2-v2)<=eps && abs(u3-v3)<=eps && abs(u4-v4)<=eps;
14     };
15
16     auto cut_circ=[](const Circle &ci,const size_t i)
17     {
18         vector<pair<long double,int>> evt;
19         evt.push_back({-PI,0}); evt.push_back({PI,0});
20         int init=0;
21         for (size_t j=0;j<circs.size();j++)
22         {
23             if (i==j) continue;
24             const Circle &cj=circs[j];
25             if (ci.r<cj.r-eps && ci.relation(cj)>=3) init++;
26             const auto inters=ci.inter(cj);
27             if (inters.size()==1) evt.push_back({atan2l((inters[0]-ci.c).y,(inters[0]-
ci.c).x),0});
28             if (inters.size()==2)
29             {
30                 const Point dl=inters[0]-ci.c,dr=inters[1]-ci.c;
31                 long double argl=atan2l(dl.y,dl.x),argr=atan2l(dr.y,dr.x);
32                 if (abs(argl+PI)<=eps) argl=PI;
33                 if (abs(argr+PI)<=eps) argr=PI;
34                 if (argl>argr+eps)
35                 {
36                     evt.push_back({argl,1}); evt.push_back({PI,-1});
37                     evt.push_back({-PI,1}); evt.push_back({argr,-1});
38                 }
39                 else
40                 {
41                     evt.push_back({argl,1});
42                     evt.push_back({argr,-1});
43                 }
44             }
45         }
46         sort(evt.begin(),evt.end());
47         int sum=init;
48         for (size_t i=0;i<evt.size();i++)
49         {

```

```

50         sum+=evt[i].second;
51         if (abs(evt[i].first-evt[i+1].first)>eps)
arcs[sum].push_back({ci.c,ci.r,evt[i].first,evt[i+1].first});
52         if (abs(evt[i+1].first-PI)<=eps) break;
53     }
54 };
55
56     const auto oint=[](const arc_t &arc)
57     {
58         const auto [cc,cr,l,r]=arc;
59         if (abs(r-l-PI)<=eps) return 2.0l*PI*cr*cr;
60         return cr*cr*(r-l)+cc.x*cr*(sin(r)-sin(l))-cc.y*cr*(cos(r)-cos(l));
61     };
62
63     for (size_t i=0;i<circs.size();i++)
64     {
65         const auto &ci=circs[i];
66         cut_circ(ci,i);
67     }
68     vector<long double> ans(siz);
69     for (size_t i=0;i<siz;i++)
70     {
71         long double sum=0;
72         sort(arcs[i].begin(),arcs[i].end());
73         int cnt=0;
74         for (size_t j=0;j<arcs[i].size();j++)
75         {
76             if (j>0 && eq(arcs[i][j],arcs[i][j-1])) arcs[i+(++cnt)].push_back(arcs[i]
[j]);
77             else cnt=0,sum+=oint(arcs[i][j]);
78         }
79         ans[i]=sum/2;
80     }
81     return ans;
82 }

```

多边形面积并

```

1 // 多边形面积并
2 // 轮廓积分, 复杂度  $O(n^2 \log n)$ ,  $n$  为边数
3 // ans[i] 表示被至少覆盖了  $i+1$  次的区域的面积
4 vector<long double> area_union(const vector<Polygon> &polys)
5 {
6     const size_t siz=polys.size();
7     vector<vector<pair<Point,Point>>> segs(siz);
8     const auto check=[](const Point &u,const Segment &e){return !((u<e.a && u<e.b) ||
(u>e.a && u>e.b));};
9
10    auto cut_edge=[&](const Segment &e,const size_t i)
11    {
12        const Line le{e.a,e.b-e.a};
13        vector<pair<Point,int>> evt;
14        evt.push_back({e.a,0}); evt.push_back({e.b,0});
15        for (size_t j=0;j<polys.size();j++)
16        {
17            if (i==j) continue;
18            const auto &pj=polys[j];

```

```

19     for (size_t k=0;k<pj.p.size();k++)
20     {
21         const Segment s={pj.p[k],pj.p[pj.nxt(k)]};
22         if (le.toleft(s.a)==0 && le.toleft(s.b)==0)
23         {
24             evt.push_back({s.a,0});
25             evt.push_back({s.b,0});
26         }
27         else if (s.is_inter(le))
28         {
29             const Line ls{s.a,s.b-s.a};
30             const Point u=le.inter(ls);
31             if (le.toleft(s.a)<0 && le.toleft(s.b)>=0) evt.push_back({u,-1});
32             else if (le.toleft(s.a)>=0 && le.toleft(s.b)<0) evt.push_back({u,1});
33         }
34     }
35 }
36 sort(evt.begin(),evt.end());
37 if (e.a>e.b) reverse(evt.begin(),evt.end());
38 int sum=0;
39 for (size_t i=0;i<evt.size();i++)
40 {
41     sum+=evt[i].second;
42     const Point u=evt[i].first,v=evt[i+1].first;
43     if (!(u==v) && check(u,e) && check(v,e)) segs[sum].push_back({u,v});
44     if (v==e.b) break;
45 }
46 };
47
48 for (size_t i=0;i<polys.size();i++)
49 {
50     const auto &pi=polys[i];
51     for (size_t k=0;k<pi.p.size();k++)
52     {
53         const Segment ei={pi.p[k],pi.p[pi.nxt(k)]};
54         cut_edge(ei,i);
55     }
56 }
57 vector<long double> ans(siz);
58 for (size_t i=0;i<siz;i++)
59 {
60     long double sum=0;
61     sort(segs[i].begin(),segs[i].end());
62     int cnt=0;
63     for (size_t j=0;j<segs[i].size();j++)
64     {
65         if (j>0 && segs[i][j]==segs[i][j-1]) segs[i+(++cnt)].push_back(segs[i][j]);
66         else cnt=0,sum+=segs[i][j].first^segs[i][j].second;
67     }
68     ans[i]=sum/2;
69 }
70 return ans;
71 }

```

点集形成的最小最大三角形


```

1 // 点集形成的最小最大三角形
2 // 极角序扫描线, 复杂度  $O(n^2 \log n)$ 
3 // 最大三角形问题可以使用凸包与旋转卡壳做到  $O(n^2)$ 
4 pair<point_t, point_t> minmax_triangle(const vector<Point> &vec)
5 {
6     if (vec.size() <= 2) return {0, 0};
7     vector<pair<int, int>> evt;
8     evt.reserve(vec.size() * vec.size());
9     point_t maxans = 0, minans = numeric_limits<point_t>::max();
10    for (size_t i = 0; i < vec.size(); i++)
11    {
12        for (size_t j = 0; j < vec.size(); j++)
13        {
14            if (i == j) continue;
15            if (vec[i] == vec[j]) minans = 0;
16            else evt.push_back({i, j});
17        }
18    }
19    sort(evt.begin(), evt.end(), [&](const pair<int, int> &u, const pair<int, int> &v)
20    {
21        const Point du = vec[u.second] - vec[u.first], dv = vec[v.second] - vec[v.first];
22        return argcmp()({du.y, -du.x}, {dv.y, -dv.x});
23    });
24    vector<size_t> vx(vec.size()), pos(vec.size());
25    for (size_t i = 0; i < vec.size(); i++) vx[i] = i;
26    sort(vx.begin(), vx.end(), [&](int x, int y){return vec[x] < vec[y];});
27    for (size_t i = 0; i < vx.size(); i++) pos[vx[i]] = i;
28    for (auto [u, v]: evt)
29    {
30        const size_t i = pos[u], j = pos[v];
31        const size_t l = min(i, j), r = max(i, j);
32        const Point vecu = vec[u], vecv = vec[v];
33        if (l > 0) minans = min(minans, abs((vec[vx[l-1]] - vecu)^(vec[vx[l-1]] - vecv)));
34        if (r < vx.size() - 1) minans = min(minans, abs((vec[vx[r+1]] - vecu)^(vec[vx[r+1]] -
35        vecv)));
36        maxans = max({maxans, abs((vec[vx[0]] - vecu)^(vec[vx[0]] -
37        vecv)), abs((vec[vx.back()] - vecu)^(vec[vx.back()] - vecv))});
38        if (i < j) swap(vx[i], vx[j]), pos[u] = j, pos[v] = i;
39    }
40    return {minans, maxans};
41 }

```

点集的凸包

```

1 // 点集的凸包
2 // Andrew 算法, 复杂度  $O(n \log n)$ 
3 Convex convexhull(vector<Point> p)
4 {
5     vector<Point> st;
6     if (p.empty()) return Convex{st};
7     sort(p.begin(), p.end());
8     const auto check = [&](const vector<Point> &st, const Point &u)
9     {
10        const auto back1 = st.back(), back2 = *prev(st.end(), 2);
11        return (back1 - back2).toleft(u - back1) <= 0;
12    };
13    for (const Point &u: p)

```

```

14     {
15         while (st.size()>1 && check(st,u)) st.pop_back();
16         st.push_back(u);
17     }
18     size_t k=st.size();
19     p.pop_back(); reverse(p.begin(),p.end());
20     for (const Point &u:p)
21     {
22         while (st.size()>k && check(st,u)) st.pop_back();
23         st.push_back(u);
24     }
25     st.pop_back();
26     return Convex{st};
27 }

```

给定三点求圆心

```

1 void cal(Point& a, Point& b, Point& c){ // 圆上三点定圆心
2     double a1=b.x-a.x, b1=b.y-a.y, c1=(a1*a1+b1*b1)/2;
3     double a2=c.x-a.x, b2=c.y-a.y, c2=(a2*a2+b2*b2)/2;
4     double d =a1*b2-a2*b1;
5     X =a.x+(c1*b2-c2*b1)/d;
6     Y =a.y+(a1*c2-a2*c1)/d;
7     r=dis2(a);
8 }
9

```