Non-Radar Localization Methods

Tarlan Ahadli, 2023

1 Introduction

1.1 About the Instructor

- **From:** Azerbaijan
- **Bachelor's Degree:** Automation Engineering
- **Master's Degree:** Computer Science for Autonomous Systems from ELTE
- **Current Status:** Pursuing PhD
- **Email:** tarlanahad@gmail.com
- **Mobile:** +36 70 599 2557

1.2 Motivational Quote

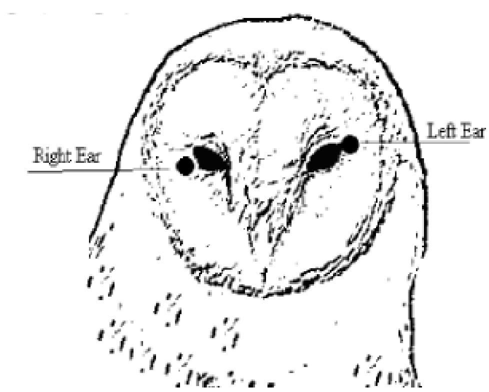
Albert Einstein (1951) — "Look deep into nature, and then you will understand everything better."

1.3 Inspiration from Nature

- Birds vs. Airplanes
  - Bird-Wing Structure
  - Landing Gear Inspired by Bird Legs and Talons
- Neural Networks vs. Human Brain

1.4 Unique Traits of Owls

- Can hunt in complete darkness
- Asymmetric ear openings



1.5 Why Study Localization?

Understanding localization is critical for a variety of applications including robotics, wildlife tracking, and emergency services.

2 Hypothesis

Owl locates a sound source due to the Time Difference of Arrival (TDOA) to its ears, calculated using a specialized mechanism evolved over millions of years.

2.1 Implications

- Understanding this mechanism can lead to advancements in localization technology.
- Can be used in various fields such as robotics, surveillance, and healthcare.

3 Technical Explanation

3.1 Objective

To determine the Time Difference of Arrival (TDOA) for each microphone in an array.

3.2 Emulation of Owl's Auditory System

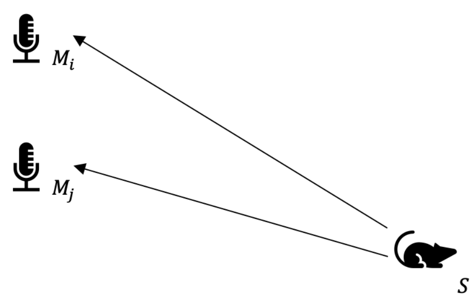
- Owl has 2 ears but can rotate its head by 270 degrees.
- Use N microphones $M_1, M_2, \ldots, M_N$ to emulate this system.

3.2.1 Advantages of Using Multiple Microphones

- Increases the accuracy of TDOA calculation.
- Can cover a larger area for sound source localization.

3.3 Terminology

- S : Sound Source (x, y, z)
- M_i : Microphone i (x_i, y_i, z_i)
- M_j : Microphone j (x_j, y_j, z_j)



4 Calculating TDOA: Cross-Correlation

4.1 Conceptual Framework

1. Given N signals $x_i[n]$ received by N microphones.
2. Objective: find the time difference t for each pair of signals.

4.2 Mathematical Details

- Use dot product to measure the correlation between signals.
- Formula: $A \cdot B = |A| \times |B| \times \cos(\theta)$

4.2.1 Signal Matching

Given two signals $x[n]$ and $x[n+t]$, we aim to find the value of t that represents the shift between the two signals. To do this, we can consider two cases:

1. When the signals are shifted.
2. When the signals are not shifted.

4.2.2 Correlation and Dot Product

The dot product serves as a good measure to find how correlated or agreed two signals are. The higher the dot product value, the more the signals match. For example, consider two signals X and Y:

$$X = [0, 1, -2, 3, -4, 5]$$
$$Y = [0, 1, -2, 3, -4, 5]$$

Their dot product would yield $1 + 4 + 9 + 16 + 25 = 55$.

Now consider two other signals x and y:

$$x = [0, 1, -2, 3, -4, 5]$$
$$y = [0, 0, 1, -2, 3, -4]$$

Their dot product would yield $0 + 0 + (-2) + (-6) + (-12) + (-20) = -40$.

The dot product formula $A \cdot B = |A| \times |B| \times \cos(\theta)$ shows that as θ approaches zero, $\cos(\theta)$ approaches 1. This means the more matching the signals are, the greater the value of $A \cdot B$.

5 Cross Correlation

To find the time shift t between two signals $x[n]$ and $x'[n+t]$, we can use cross-correlation. Below is the Python implementation for the same.

```python
import numpy as np

def find_shift(x, x_shifted):
    N = len(x)
    max_correlation = -np.inf
    optimal_shift = 0

    for t in range(N):
        current_correlation = 0
        for n in range(N - t):
            current_correlation += x[n] * x_shifted[n + t]

        if current_correlation > max_correlation:
            max_correlation = current_correlation
            optimal_shift = t

    return optimal_shift

# Test the function
x = np.array([0, 1, -2, 3, -4, 5])
x_shifted = np.array([0, 0, 1, -2, 3, -4])  # shifted by 1 sample
print(find_shift(x, x_shifted))  # Should return 1
```

5.1 Possible Improvements

- Parallelization: Cross-correlation calculations can be parallelized for improved speed.
- Optimization Algorithms: Advanced algorithms like Fast Fourier Transform (FFT) can make the process faster.

6 Problem Formulation

6.1 Equations for Distance and Time

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}$$
$$t_i = \frac{d_i}{c} \quad \text{where} \quad c \approx 343\,\mathrm{m/s}$$

6.2 Optimization Problem

Minimize the difference between predicted and real TDOA:

$$\arg\min_{(x,y,z)} \left(\sum_{i=1}^{N} \sum_{j=i}^{N} (\Delta T_{ij}^{\text{pred}} - \Delta T_{ij}^{\text{real}})^2 \right)$$

6.3 Solution Methods

- Least Squares Minimization
- Nelder-Mead Optimization
- Genetic Algorithms: Useful for multi-objective optimization problems.