

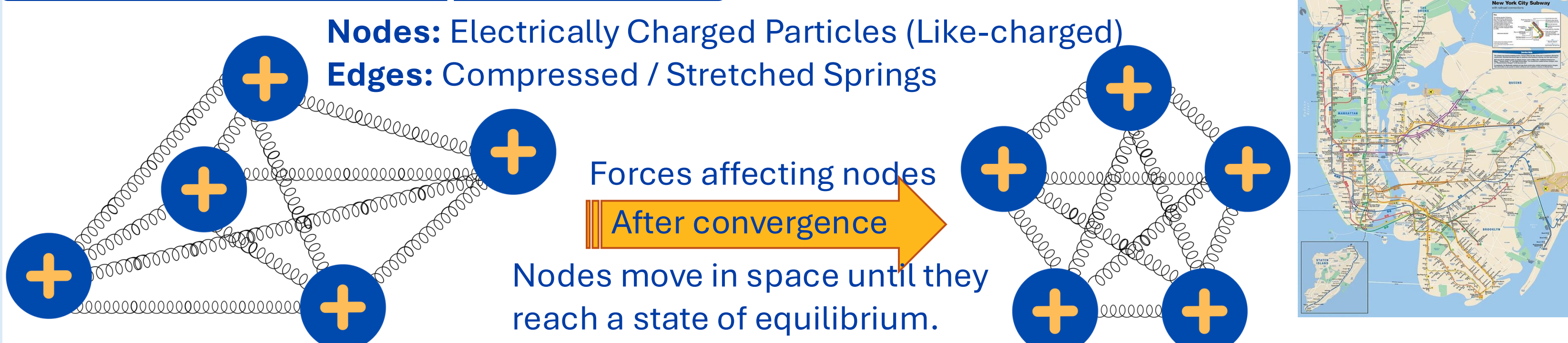
Background and Motivation



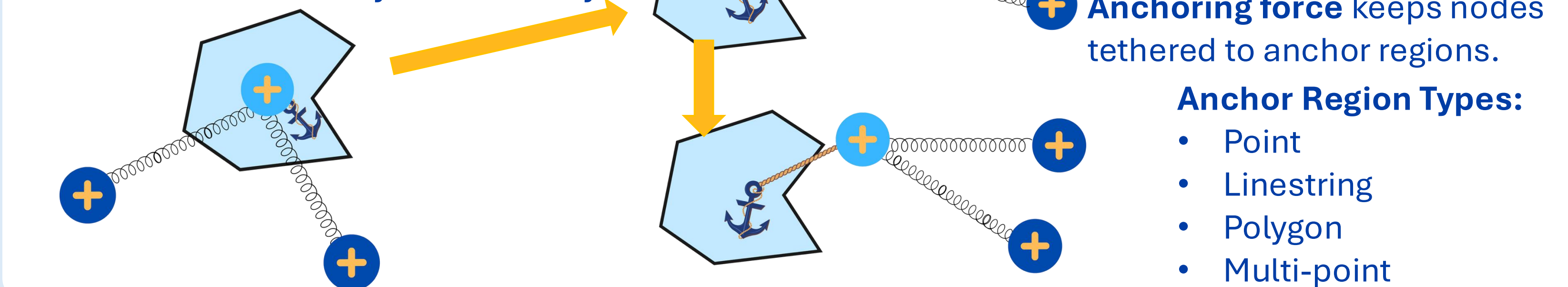
New York City
Subway Map

Collaboration
Network of
Geoscience in 2022

Force Directed Graph Model:



Light Blue Node: Stays tethered to anchoring region. Other nodes are not anchored so they move freely.



Method

Force Formulas:

Three types of forces between nodes:

Attractive (Spring) Forces:

$$\mathbf{F}_{u \leftarrow v}^{\text{att}} = -\frac{d}{L} \Delta \mathbf{r}_{uv},$$

Repulsive Forces:

$$d^2 = \max(\|\mathbf{r}_{uv}\|^2, \varepsilon^2), \quad \mathbf{F}_{u \leftarrow v}^{\text{rep}} = c_{\text{rep}} \frac{\mathbf{r}_{uv}}{d^2}.$$

$$\mathbf{F}_i^{\text{rep}} = \sum_{j \in \mathcal{N}_r(i)} c_{\text{rep}} \frac{\mathbf{p}_i - \mathbf{p}_j}{\max(\|\mathbf{p}_i - \mathbf{p}_j\|^2, \varepsilon^2)}.$$

Anchoring Forces:

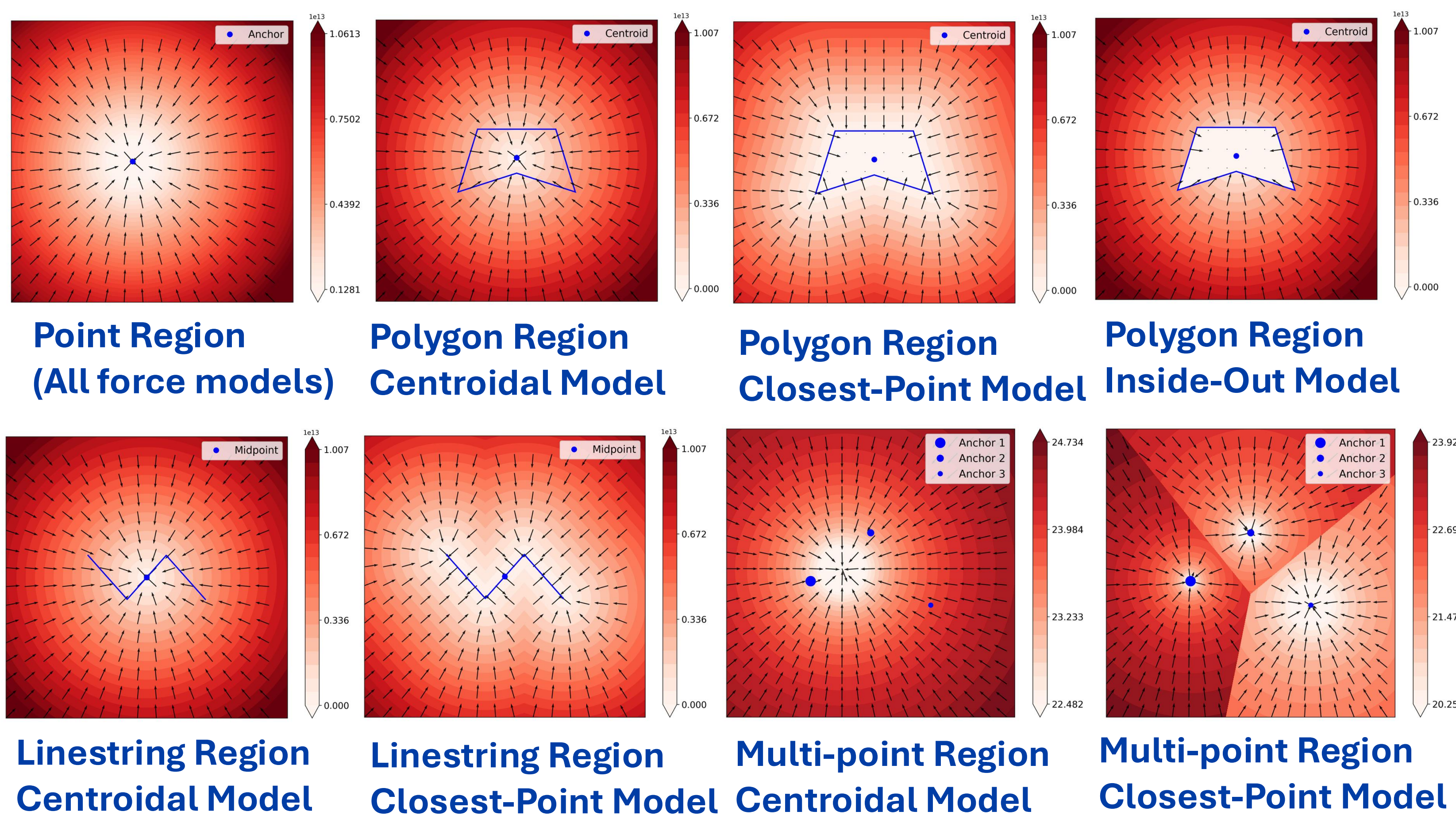
$$\mu(A_u) = \begin{cases} 1, & A_u \text{ is a point,} \\ |A_u|, & A_u \text{ is a finite set of points,} \\ \text{Length}(A_u), & A_u \text{ is a linestring,} \\ \text{Area}(A_u), & A_u \text{ is a polygon,} \end{cases}$$

$$\mathbf{c}_u = \frac{1}{\mu(A_u)} \int_{A_u} \mathbf{x} d\mu(\mathbf{x}), \quad \mathbf{p}_u = \arg \min_{\mathbf{y} \in A_u} \|\mathbf{r}_u - \mathbf{y}\|.$$

Anchoring Force Models:

- Centroidal: $\mathbf{F}_u^{\text{anchor}} = \alpha (\mathbf{c}_u - \mathbf{r}_u).$
- Inside-Out: $\mathbf{F}_u^{\text{anchor}} = \begin{cases} \alpha (\mathbf{c}_u - \mathbf{r}_u), & \mathbf{r}_u \notin A_u, \\ 0, & \mathbf{r}_u \in A_u. \end{cases}$
- Closest-Point: $\mathbf{F}_u^{\text{anchor}} = \begin{cases} \alpha (\mathbf{p}_u - \mathbf{r}_u), & \mathbf{r}_u \notin A_u, \\ 0, & \mathbf{r}_u \in A_u. \end{cases}$

Anchoring Force Fields:



Scalability:

Problem: Spatial Graphs can grow arbitrarily large

Naïve Calculation of Repulsive forces between all nodes: $O(N^2) \rightarrow$ Bottleneck

Spark SQL: Parallelize the computations in distributed form \rightarrow Increases Scalability

Future Work:

Use a Spatial Index (QuadTree partitioning of the space) and approximate effect of nodes that are further than the **distance threshold theta**. Based on well-separated pairs.

Related Work:

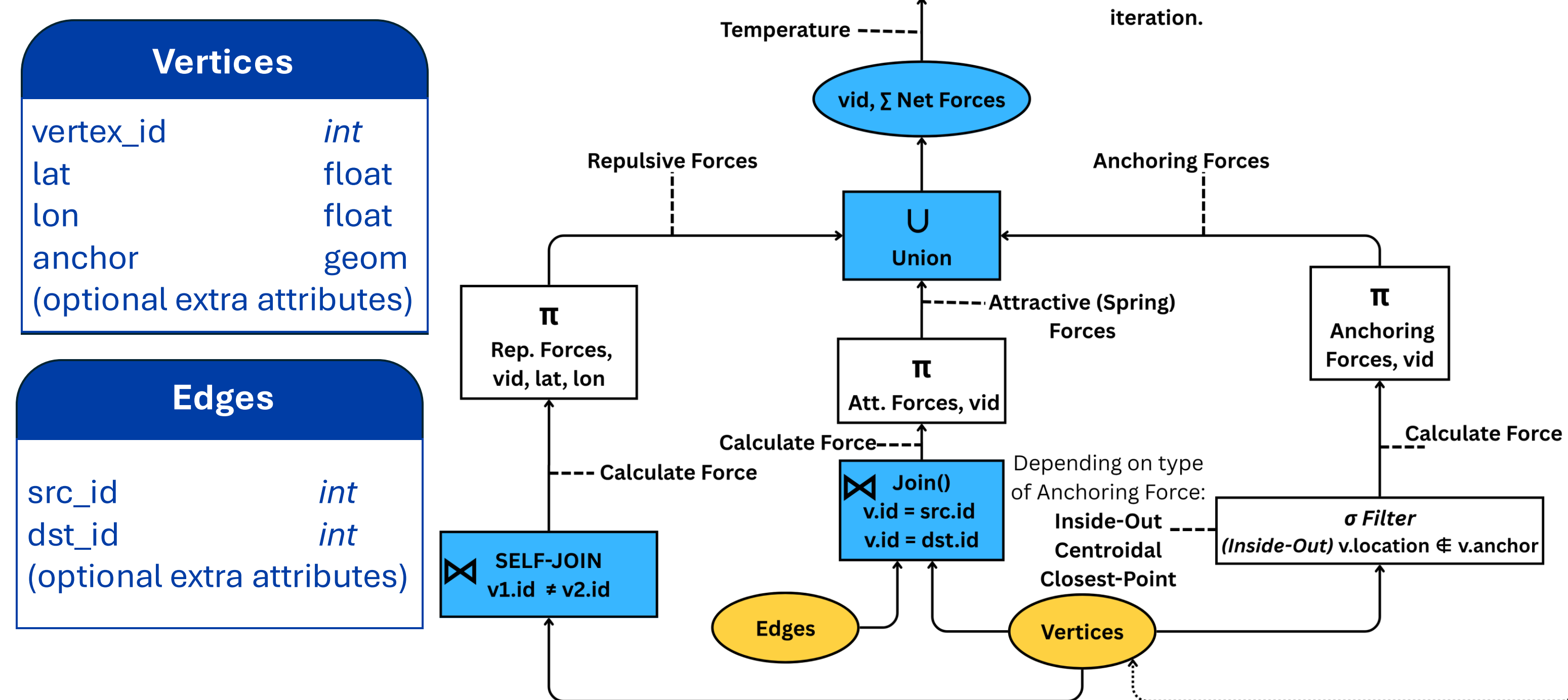
Polygonally Anchored Graph Drawing (Extended Abstract) (Chiu et al – 2024)

Well Separated Pair Decomposition (Chan - 2008)

A Potential-Field-Based Multilevel Algorithm for Drawing Large Graphs. (Hachul – 2005)

Relational Query Modeling

Relational Query DAG: (One Iteration)



Attractive Forces:

JOIN: Edges \bowtie Vertices \rightarrow Spring force αL (ideal spring length) applied to both dst and src vertices.

Repulsive Forces:

SPATIAL SELF-JOIN: Vertices \bowtie Vertices (within theta) \rightarrow Repulsive force applied to both vertices.

Anchoring Forces:

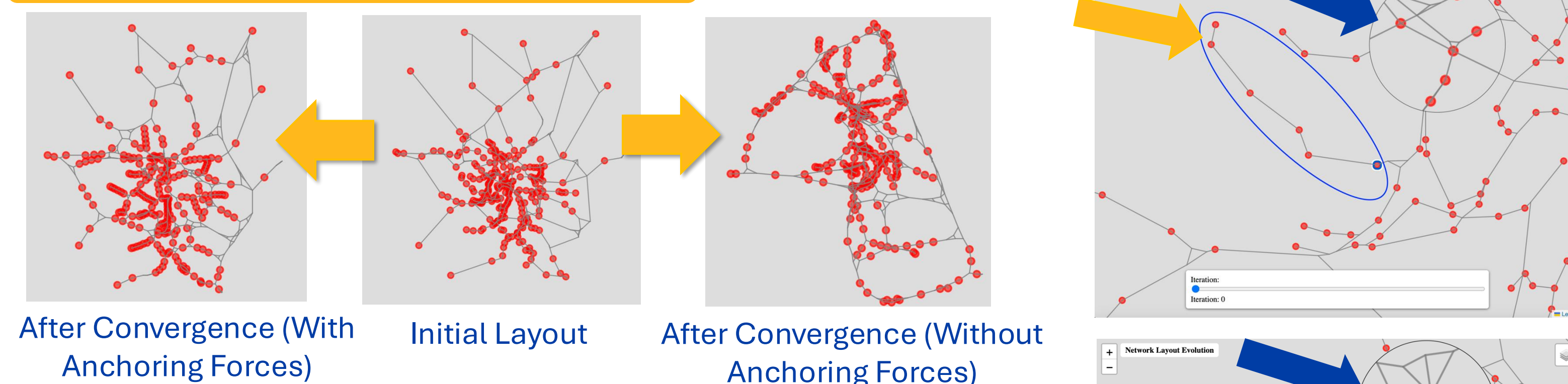
VERTICES \rightarrow pull-to-anchor (centroid / closest-point) (σ Filter inside-out: skip if inside region) \rightarrow per-vertex f

Net Force Aggregation:

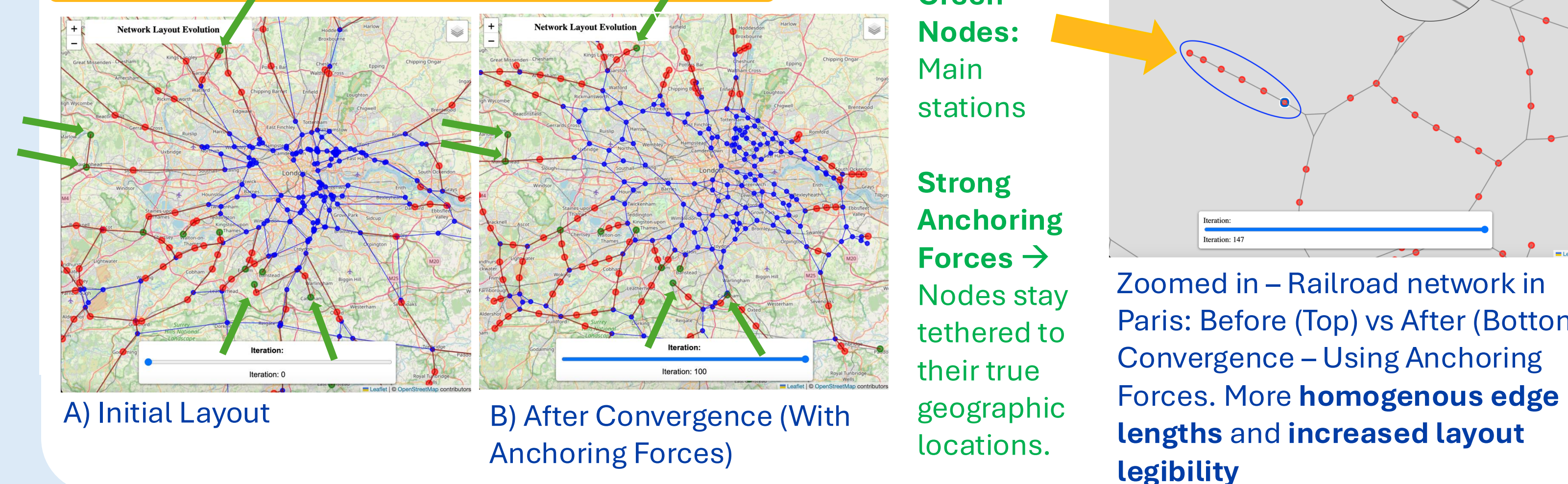
U UNION ALL (attractive, repulsive, anchoring) $\rightarrow \Sigma$ by vid $\rightarrow F_{\text{total}} \rightarrow$ Updater

Use Cases

Railroad Network in Paris Region:



Railroad Network in London Region:



Results and Conclusion

Experiment Setup:

We conducted experiments on a 12-node Spark cluster. The master node has two 8-core Intel Xeon E5-2609 v4 (1.7 GHz) CPUs and 128 GB RAM; each worker has two 6-core Xeon E5-2603 v4 CPUs and 64 GB RAM. All nodes run CentOS 7.5 with local SSDs for the OS and HDDs for data.

Dataset	# Vertices	# Edges
Gowalla Subset	22,803	381,384
Gowalla	107,092	913,660
Gowalla x2	214,184	1,827,320
Gowalla x5	535,460	4,568,300
Gowalla x10	1,070,920	9,136,600
Gowalla x 100	10,709,200	91,366,000

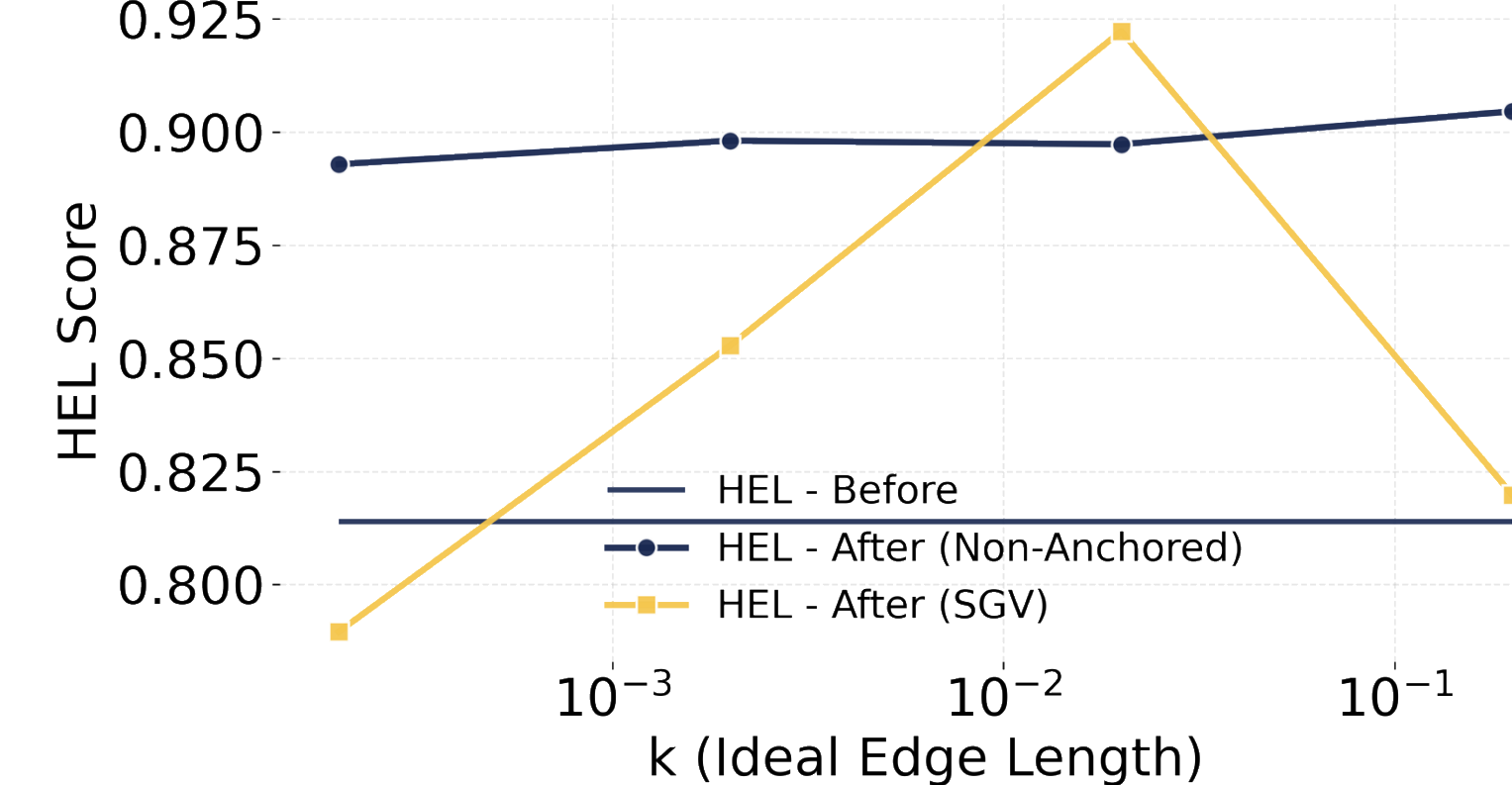
Goal: find a balance between **edge length homogeneity** and **average distance from anchor** of nodes in final layout.

Evaluation Metrics:

HEL: (Homogenous Edge Lengths)

Higher value is desired

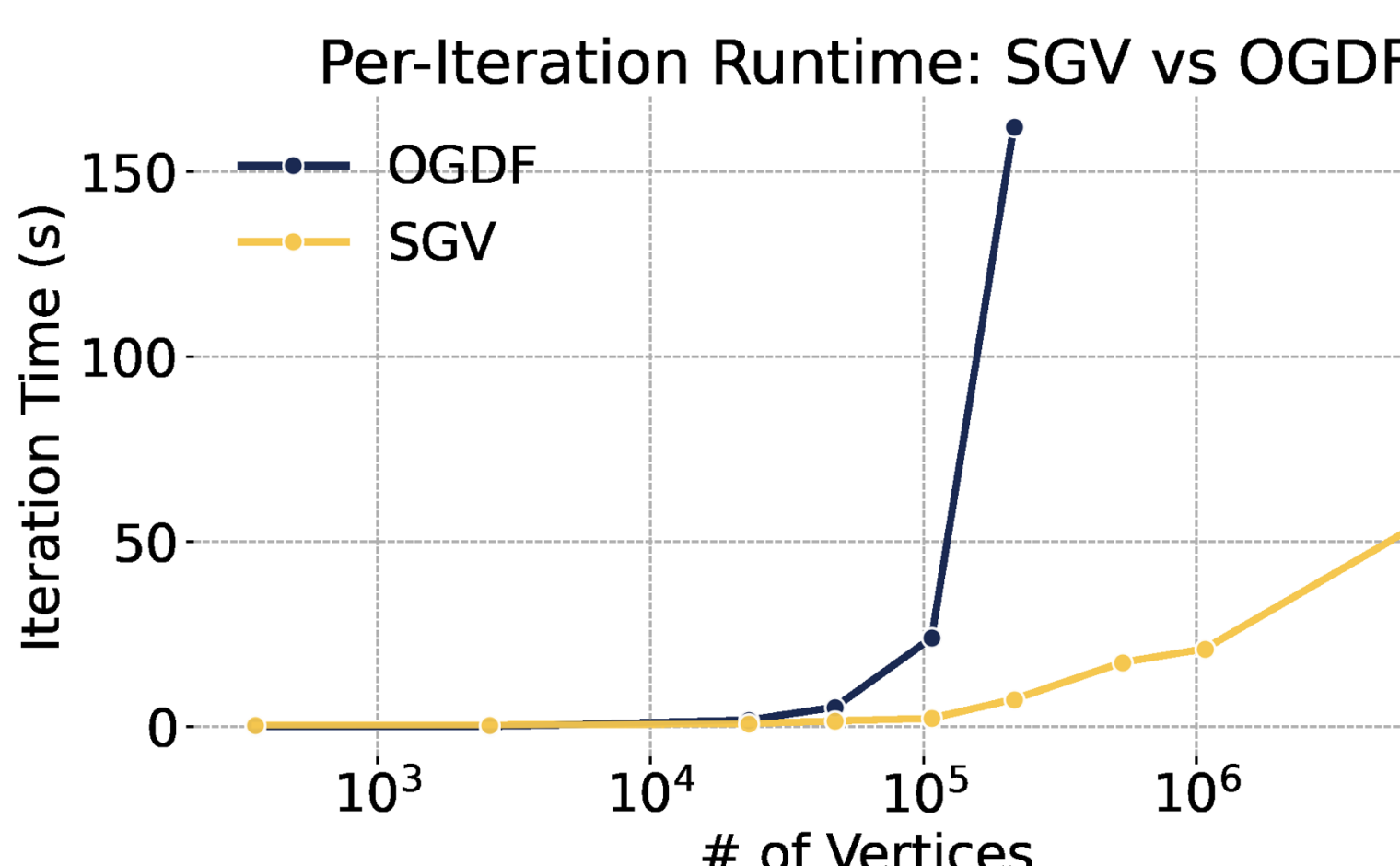
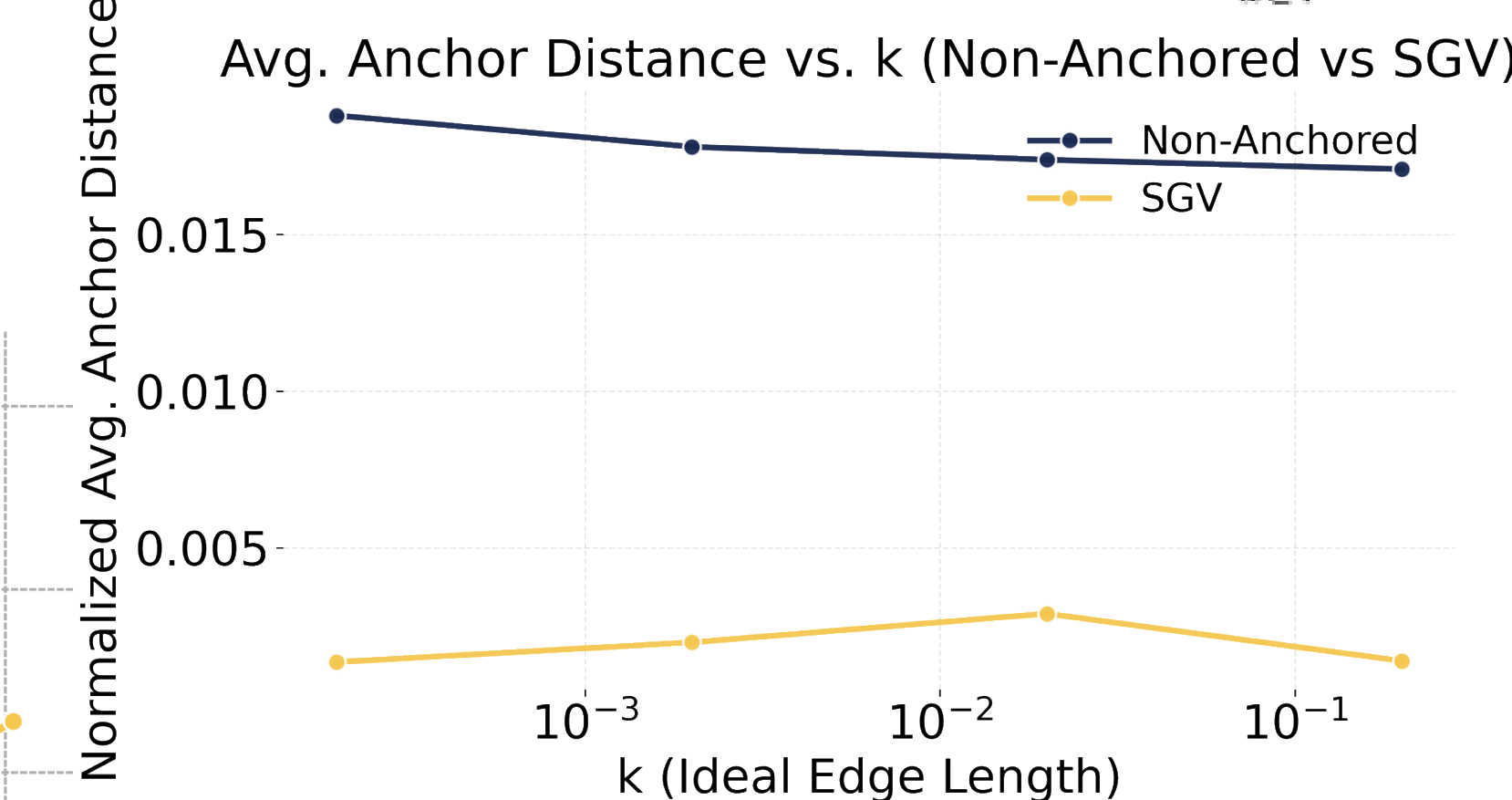
$$HEL = 1 - \frac{1}{m} \sum_{j=1}^m \left| \frac{\ell_j - \bar{\ell}}{\max(\bar{\ell}, \ell_{\max} - \bar{\ell})} \right|$$



NAD: (Normalized Anchor Distance)

Lower value is desired

$$d_A(u) = \min_{p \in A(u)} \|\mathbf{r}_u - p\|, \quad \tilde{d}_A = \frac{1}{|V| D_{\text{MBR}}} \sum_{u \in V} d_A(u)$$



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LEARN MORE:
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