🌈 INVERSE TRIGONOMETRIC FUNCTIONS – CLASS 12 SUPER STUDY GUIDE

🎨 1. THEORY IN SIMPLE WORDS (WITH VISUALS & ANALOGIES)

⭐ 1.1 What Are Inverse Trigonometric Functions?

Think of trigonometric functions (sin, cos, tan…) as **machines**:

```
Angle → sin machine → Ratio
(θ)              (sin θ)
```

Inverse trigonometric functions **reverse the machine**:

```
Ratio → sin⁻¹ machine → Angle
(½)                (sin⁻¹ ½ = π/6)
```

🌟 Super Simple Analogy

- Trigonometric function = **“What is the ratio for this angle?”**
- Inverse trigonometric function = **“What is the angle for this ratio?”**

⭐ 1.2 Why Do We Need Principal Values?

A single ratio (like √2/2) comes from **many angles**:
π/4, 7π/4, 9π/4 …

👉 Too many answers = confusion = not suitable for a function.
So mathematicians **restrict the domain** of trig functions
to make them **one-one**, and hence invertible.

🎨 Visual Memory Trick

Imagine cutting a **slice** from a big unit-circle “pizza” so that every ratio gives **only one unique angle**.

⭐ 1.3 Principal Value Branches (IMPORTANT!)

Function	Principal Value Range	Easy Visual
$\sin^{-1} x$	$[-\pi/2, \pi/2]$	Right/Left tilt, no full rotation

Function	Principal Value Range	Easy Visual
$\cos^{-1} x$	[0, π]	Top semicircle only
$\tan^{-1} x$	(-π/2, π/2)	Up–Down strip
$\cot^{-1} x$	((0, π)	Rightwards exclusive
$\sec^{-1} x$	([0, π] − {π/2}	Top semicircle skipping vertical
$\text{cosec}^{-1} x$	[-π/2, π/2] − {0}	Tilt up/down, no horizontal

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⭐ 1.4 How to Think of Inverse Trig Graphs

All inverse trig graphs:

- Are **smooth curves**
- Are **monotonic**
- Show **angles on y-axis** and **ratios on x-axis**

Example for $\sin^{-1}x$:
Starts at –π/2 and ends at π/2.

⭐ 1.5 Common Identity Patterns

Think of them as “angle-simplifiers”.

- **$\sin(\sin^{-1}x) = x$**
- **$\sin^{-1}(\sin x) =$ principal value of x**
  (Same rule for all others.)

🧮 2. KEY CONCEPTS & FORMULAS (QUICK REVISION TABLES)

⭐ Big Formula Table

Basic Identities

Expression	Simplified Value
$\sin(\sin^{-1} x)$	x
$\cos(\cos^{-1} x)$	x
$\tan(\tan^{-1} x)$	x

Expression	Simplified Value
$\sin^{-1}(\sin x)$	PV of x
$\cos^{-1}(\cos x)$	PV of x
$\tan^{-1}(\tan x)$	PV of x

⭐ Angle Transformation Formulas (VERY IMPORTANT FOR BOARDS)

Formula	Value
$\sin^{-1}x + \cos^{-1}x$	$\pi/2$
$\tan^{-1}x + \cot^{-1}x$	$\pi/2$
$\tan^{-1}x + \tan^{-1}y$	$\tan^{-1}((x+y) / (1-xy))$
$\tan^{-1}x - \tan^{-1}y$	$\tan^{-1}((x-y)/(1+xy))$

⭐ Sign & Quadrant Memory Trick

- **$\sin^{-1}$ x** → angle between –90° to 90°
- **$\cos^{-1}$ x** → angle between 0° to 180°
- **$\tan^{-1}$ x** → angle between –90° to 90°

🎯 **Mnemonic:**
"S & T stay small, C goes to the ceiling."

✏️ 3. SOLVED NUMERICAL PROBLEMS (STEP-BY-STEP)

⭐ TYPE 1: Evaluate Inverse Trig Values

Q1: Find $\sin^{-1}(½)$

Step 1: Think: "sin what = ½?"
= $\pi/6$
Step 2: Check if $\pi/6$ in principal range: YES
Final Answer: $\pi/6$

Q2: Calculate $\cos^{-1}(-\sqrt{3}/2)$

$\cos\theta = -\sqrt{3}/2 \rightarrow$ angle $= 5\pi/6$
($\because$ cos is negative in II quadrant, allowed in $[0,\pi]$)

⭐ TYPE 2: Simplify Expressions

Q3: Find $\sin(\sin^{-1} x + \cos^{-1} x)$

$\sin(A + B) = \sin(\pi/2) = 1$
✔ Final Ans: **1**

Q4: Show $\sin^{-1}x + \cos^{-1}x = \pi/2$

Let $\sin^{-1}x = \theta \rightarrow \sin\theta = x$
$\cos^{-1}x = \pi/2 - \theta$
Add: $\theta + (\pi/2 - \theta) = \pi/2$ ✔

⭐ TYPE 3: $\tan^{-1}$ Identities

Q5: $\tan^{-1}(1) + \tan^{-1}(2)$

Use formula:
$\tan^{-1}x + \tan^{-1}y = \tan^{-1}((x+y)/(1-xy))$

$= \tan^{-1}((1+2)/(1-2))$
$= \tan^{-1}(3/-1) = \tan^{-1}(-3)$

Angle lies in $(-\pi/2,\pi/2) \Rightarrow$ valid.

📝 4. PREVIOUS YEARS' BOARD QUESTIONS (SOLVED)

(Based on typical CBSE patterns)

⭐ PYQ 1: Evaluate $\sin(\sin^{-1}(3/5) + \cos^{-1}(4/5))$

$\cos^{-1}(4/5) = \theta \Rightarrow \cos\theta = 4/5 \Rightarrow \sin\theta = 3/5$
So $\sin^{-1}(3/5) + \cos^{-1}(4/5) = \pi/2$
Therefore $\sin(\pi/2) =$ **1**

⭐ PYQ 2: Find the value of $\tan^{-1}(1/2) + \tan^{-1}(1/3)$

Use identity:

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}((x+y)/(1-xy))$

$= \tan^{-1}((1/2 + 1/3)/(1-1/6))$
$= \tan^{-1}((5/6)/(5/6))$
$= \tan^{-1}(1)$
$= \pi/4$

⭐ PYQ 3: Prove $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{(1-y^2)} + y\sqrt{(1-x^2)})$

Standard identity. Proof involves sin(A + B) formula.

⭐ Frequently Asked Concepts (Based on trends)

✔ Principal values (VERY frequent)
✔ $\tan^{-1}$ identities
✔ Simplification of composite expressions
✔ Graph-based questions (rare but possible)

⚡ 5. QUICK REVISION NOTES (1–2 PAGES)

⭐ DEFINITIONS

- Inverse trig functions give **angle** for a **ratio**
- Need principal values → make function one-one
- Use identity $\sin^{-1}x + \cos^{-1}x = \pi/2$ everywhere

⭐ IMPORTANT VALUES

x	$\sin^{-1}x$	$\cos^{-1}x$	$\tan^{-1}x$
0	0	$\pi/2$	0
1	$\pi/2$	0	$\pi/4$
–1	$-\pi/2$	π	$-\pi/4$

⭐ GRAPH SHAPES

- $\sin^{-1} x$ → S-curve
- $\cos^{-1} x$ → decreasing curve

- $\tan^{-1} x$ → sigmoid curve

⭐ DO THESE ALWAYS

✔ Check principal value
✔ Use identity charts
✔ Convert to standard angles
✔ For negative inputs → use symmetry:

- $\sin^{-1}(-x) = -\sin^{-1}x$
- $\tan^{-1}(-x) = -\tan^{-1}x$
- $\cos^{-1}(-x) = \pi - \cos^{-1}x$

🔮 6. PREDICTED / LIKELY QUESTIONS FOR BOARD 2025

Short Answer

1. State principal value of $\tan^{-1}(-\sqrt{3})$.
2. Simplify $\sin^{-1}x + \cos^{-1}x$.
3. Plot rough graph of $\sin^{-1}x$.

Long/Numerical

1. Prove the identity involving $\sin^{-1}x + \sin^{-1}y$.
2. Evaluate $\tan^{-1}a + \tan^{-1}b$.
3. Solve composite expression:
   $\tan^{-1}(2) - \tan^{-1}(3)$

🧠 7. EXAM TIPS & TRICKS

⭐ Golden Rule

👉 **Always check the principal value range first.**
This removes 90% errors.

⭐ Common Mistakes

❌ Writing multiple answers (only one PV allowed)
❌ Using wrong quadrant
❌ Mixing up $\tan^{-1}$ identities
❌ Forgetting denominators in formulas

⭐ Time-Saving Flowchart

```
Start with given expression
        ↓
Convert values to standard angles
        ↓
Apply inverse trig identities
        ↓
Check principal value range
        ↓
Final simplified answer
```

🧸 8. SUPER KID-FRIENDLY VISUAL RECAP

- Inverse trig = **Angle Finder Machine**
- Principal value = **Special slice of circle**
- $\sin^{-1}x$ → **small tilt angles**
- $\cos^{-1}x$ → **top semi circle**
- $\tan^{-1}x$ → **slanted S-curve**

🎨 Memory Colors

- 🟢 $\sin^{-1}$ → green (gentle slope)
- 🔵 $\cos^{-1}$ → blue (cool upper region)
- 🟡 $\tan^{-1}$ → yellow (smooth curve)