🌈 RELATIONS & FUNCTIONS – CLASS 12 SUPER STUDY GUIDE

🎨 1. THEORY IN SIMPLE WORDS (With Visuals & Analogies)

⭐ 1.1 What is a Relation?

Think of a *relation* as a **matching game** between two sets.

🎨 Visual:

```
Set A → people
Set B → their favourite fruits

A = {Riya, Aarav, Sana}
B = {Mango, Apple, Grapes}

Relation R:
(Riya, Mango), (Aarav, Apple), (Sana, Mango)
```

⭐ Formal Definition:

A **relation** from set A to B is a **subset of A × B** (the Cartesian product).

⭐ 1.2 Types of Relations

Use this **traffic-light coding** for memory:

- 🟢 Reflexive
- 🟡 Symmetric
- 🔵 Transitive
- 🔴 Equivalence

✔️ Reflexive Relation

A relation R on set A is **reflexive** if
every element relates to itself
→ (a, a) ∈ R for all a ∈ A.

✔️ Symmetric Relation

If (a, b) ∈ R → (b, a) must also be in R.

✔️ Transitive Relation

If (a, b) ∈ R and (b, c) ∈ R → (a, c) must be in R.

⭐ Equivalence Relation = (Reflexive + Symmetric + Transitive)

⭐ 1.3 What is a Function?

A **function** is a *special relation* where:

✨ Every element of A has **exactly one** arrow going out to B.

🎨 Visual:

```
A: 1 → 2      (OK)
A: 1 → 2 and 1 → 3  (NOT OK ❌)
```

Simple analogy:

A function is like a **vending machine**.
You put one code → you get exactly **one snack**, never two.

⭐ 1.4 Types of Functions

Type	Meaning	Visual
One-One (Injective)	Different inputs → different outputs	No two arrows meet at same point
Onto (Surjective)	Every element of B is used	All elements in B have an arrow
Bijective	One-One + Onto	Perfect pairing

⭐ 1.5 Composite Function (fog, gof)

Composition = **joining two machines**.

```
A --f--> B --g--> C

(g∘f)(x) = g(f(x))
```

⭐ 1.6 Inverse of a Function

A function has an inverse only if it is **bijective**.

Inverse reverses the arrows.

🧮 2. KEY CONCEPTS & FORMULAS (Quick Revision Tables)

⭐ Table: Types of Relations

Relation	Condition
Reflexive	$(a, a) \in R$
Symmetric	$(a, b) \in R \rightarrow (b, a) \in R$
Transitive	(a, b) & $(b, c) \rightarrow (a, c) \in R$
Equivalence	All 3 above

⭐ Table: Function Types

Function Type	Condition
One-one	$f(a_1) = f(a_2) \rightarrow a_1 = a_2$
Onto	Range = Codomain
Bijective	One-one + Onto
Constant	$f(x) = k$
Identity	$f(x) = x$

⭐ Composite & Inverse

Concept	Formula
Composite	$(g \circ f)(x) = g(f(x))$
Inverse	$f^{-1}(f(x)) = x$

⭐ Mnemonics

- **"OST"** → Onto = Surjective = Total coverage
- **"BOB"** → Bijective = One-one + Onto = Best Of Both
- **"CST"** → Check Symmetry → then Transitivity

✏️ 3. SOLVED NUMERICAL PROBLEMS

⭐ 3.1 Check if relation is Reflexive/Symmetric/Transitive

Q: A = {1,2,3}, R = {(1,1),(2,2),(3,3),(1,2),(2,1)}
Check properties.

Solution:
✔️ Reflexive? YES (all (a,a) present)
✔️ Symmetric? (1,2) → (2,1) (Yes)
✔️ Transitive?
(1,2) & (2,1) → (1,1) ✓
(2,1) & (1,2) → (2,2) ✓
→ YES

⭐ **R is an equivalence relation.**

⭐ 3.2 Show a function is Injective

Q: f(x)=3x+5. Show it is one-one.

Solution:
Assume $f(a_1)=f(a_2)$
→ $3a_1+5=3a_2+5$
→ $a_1 = a_2$
✔️ One-one.

⭐ 3.3 Composite Function

Q: f(x)=x+3, g(x)=2x. Find g∘f.

g(f(x)) = g(x+3)=2(x+3)=2x+6 ✔️

⭐ 3.4 Inverse Function

Q: f(x)=3x–4. Find f^{-1}.

Let y = 3x – 4
x = (y+4)/3
Swap x & y → $f^{-1}(x)=(x+4)/3$ ✔️

📝 4. PREVIOUS YEARS' BOARD QUESTIONS (Solved)

(General CBSE-style)

⭐ Q1: Show relation R = {(x,y): x–y is even} is an equivalence relation.

✔ Reflexive: x–x=0 (even)
✔ Symmetric: if x–y even → y–x also even
✔ Transitive: even+even = even

Ans: Equivalence relation.

⭐ Q2: Find inverse of f(x)=2x+7.

Solution: $f^{-1}(x) = (x-7)/2$

⭐ Q3: If $f(x)=x^2+1$, $g(x)=x-1$, find f(g(x)).

$f(g(x)) = f(x-1) = (x-1)^2 + 1$

📌 **Frequently Asked Patterns:**
✔ Find fog, gof → Every year
✔ Check relation properties
✔ Find inverse of function
✔ One-one/onto proofs
✔ Domain & range questions

⚡ 5. QUICK REVISION NOTES (1–2 Pages)

⭐ Definitions

- Relation = subset of A×B
- Function = each element of A has **one unique** output
- Types: Injective, Surjective, Bijective
- Composite function = g(f(x))
- Inverse exists only for bijective functions

⭐ Diagrams

📌 Function:
A → B (one arrow from each)

📌 Not a function:
A → B (two arrows from same element)

⭐ Steps to Check Function Type

1. For One-one → assume $f(a_1)=f(a_2)$
2. For Onto → solve $f(x)=y$
3. For Bijective → both above

🔮 6. PREDICTED / LIKELY QUESTIONS

Short Answer

1. Define one-one and onto.
2. Check if relation xRy: $x^2=y^2$ is symmetric.

Long Answer

1. Show $(g \circ f)$ is bijective if both f and g are bijective.
2. Find composite function fog and gof for given f, g.

Numerical

1. Find inverse of $f(x)=ax+b$.
2. Check function is injective and/or surjective.

🧠 7. EXAM TIPS & TRICKS

⭐ Super Tips

✔️ Always check **dom–range** first
✔️ For composite → go from **inside to outside**
✔️ For inverse →
Step 1: Write $y=f(x)$
Step 2: Solve for x
Step 3: Swap variables

⭐ Common Mistakes

❌ Using wrong order: $f \circ g \neq g \circ f$
❌ Forgetting to check domain
❌ Missing symmetry/transitivity pairs

⭐ Time-Saving Flowchart

```
Is it a function?
         ↓
Check one-one
         ↓
Check onto
```

```
          ↓
Bijective?
          ↓
Find inverse
```

🧸 8. VISUAL & KID-FRIENDLY STYLE RECAP

Memory Tricks

- **Relation = Matching Game**
- **Function = Vending Machine**
- **Composite = Two Machines Connected**
- **Inverse = Arrows Reversed**

Color Code

- 🟢 Equivalence → All 3 properties
- 🔵 Injective → Inputs stay unique
- 🟡 Surjective → Outputs filled
- 🔴 Inverse → Only if bijective