Class 12 Maths – 3D Geometry Study Guide

1️⃣ Theory in Simple Words with Visuals

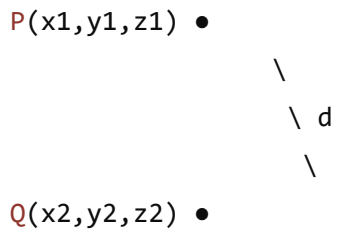
1.1 Distance Between Two Points in 3D

- If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are points in 3D, distance formula:

$$\text{Distance } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Analogy: Imagine a tiny drone flying from one point to another in space — it travels in a straight line, not along axes.

Visual:



1.2 Section Formula (3D)

- If a point divides the line joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in ratio $m : n$, then:

$$R\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Tip: "Weighted average" method – closer to the bigger weight.

1.3 Midpoint Formula

- Special case of section formula when $m = n = 1$:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

1.4 Direction Cosines & Ratios

- Direction cosines: $\cos\alpha, \cos\beta, \cos\gamma$ of a line along x, y, z axes.

$$l^2 + m^2 + n^2 = 1$$

- If line passes through origin: equation along direction ratios $a : b : c$

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$

Visual: Imagine a line shooting from the origin like a laser in 3D space.

1.5 Equation of a Line

- **Vector form:** $\vec{r} = \vec{a} + \lambda\vec{b}$
- **Cartesian form:**

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

- **Tip:** Use vector form for shortest distance problems; Cartesian for intersections.

1.6 Equation of a Plane

- If plane passes through point $P(x_0, y_0, z_0)$ with normal vector $\vec{n} = (A, B, C)$:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

- **Shortcut:** $Ax + By + Cz + D = 0$

Visual Tip: Plane = "flat sheet in space." Normal vector = arrow perpendicular to sheet.

1.7 Distance from Point to Plane

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

- **Tip:** Always take absolute value.

1.8 Angle Between Two Planes

$$\cos\theta = \frac{|A_1A_2 + B_1B_2 + C_1C_2|}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}}$$

- **Tip:** Use normal vectors dot product.

1.9 Line and Plane Angle

$$\cos\phi = \frac{|lA + mB + nC|}{\sqrt{l^2 + m^2 + n^2} \cdot \sqrt{A^2 + B^2 + C^2}}$$

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1. 🔟 Sphere

- Equation of sphere with center $C(a, b, c)$ and radius r:

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

- **Shortcut:** Center = $(-D/2, -E/2, -F/2)$ if general form $x^2 + y^2 + z^2 + Dx + Ey + Fz + G = 0$

2️ Key Concepts & Formulas

Concept	Formula	Tip / Mnemonic
Distance b/w 2 points	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$	"3D Pythagoras"
Midpoint	$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}$	Average = middle point
Section formula	$\frac{mx_2+nx_1}{m+n}, \ldots$	Weighted average
Direction ratios	$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$	Laser in 3D
Plane equation	$Ax + By + Cz + D = 0$	Normal vector trick
Distance point-plane	(d = \frac{	Ax+By+Cz+D
Sphere	$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$	Center-radius form

Mnemonic for Plane: "A, B, C → Arrow Perpendicular, D → Distance Shift"

3️ Solved Numerical Problems

Example 1: Distance Between Points

Q: Find distance between $P(1, 2, 3)$ and $Q(4, 6, 9)$.

Solution:

$$d = \sqrt{(4 - 1)^2 + (6 - 2)^2 + (9 - 3)^2} = \sqrt{3^2 + 4^2 + 6^2} = \sqrt{9 + 16 + 36} = \sqrt{61}$$

✅ **Tip:** Square first, then sum, avoid negative mistakes.

Example 2: Equation of Plane

Q: Plane passes through $P(1,2,3)$ with normal vector $\vec{n} = (2,-1,3)$.

$$2(x-1) - 1(y-2) + 3(z-3) = 0 \implies 2x - y + 3z - 9 = 0$$

✅ **Shortcut:** Always plug point once at the end to find D.

Example 3: Distance Point-Plane

Q: Distance from $P(1,1,1)$ to plane $2x - 3y + 6z - 4 = 0$

$$d = \frac{|2*1 - 3*1 + 6*1 - 4|}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{|1|}{\sqrt{4+9+36}} = \frac{1}{7}$$

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4 Previous Years' Board Questions

- **Recurring Types:**
  - Distance between points / planes
  - Equation of line/plane through points
  - Sphere problems (center, radius)
  - Angle between lines, line-plane, plane-plane
- **Pattern:** 2–3 questions carry high weight (6–8 marks each).

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5 Quick Revision Notes

- **Line:** $\vec{r} = \vec{a} + \lambda\vec{b}$
- **Plane:** $Ax + By + Cz + D = 0$
- **Distance P-P:** $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$
- **Distance P-Plane:** $\frac{|Ax+By+Cz+D|}{\sqrt{A^2+B^2+C^2}}$
- **Sphere:** $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$
- **Angles:** Use dot product of direction vectors / normal vectors

Visual Tip: Draw quick 3D axes and sketch line, plane, point for memory aid.

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6 Predicted / Likely Questions

- Find distance between skew lines.
- Equation of plane passing through 3 points.
- Angle between line and plane.
- Sphere: center and radius from general equation.
- Midpoint & section ratio problems.

7️⃣ Exam Tips & Tricks

- Always **label axes** in sketches.
- **Vector form** for shortest distance, **Cartesian form** for intersection.
- Check **units and signs** in distance formulas.
- Use **dot product** for angles; use **cross product** for perpendicular vectors.
- Keep a **small notebook** with formulas for last-minute revision.

8️⃣ Kid-Friendly Learning Style

- **Colors:** Use red for formulas, blue for diagrams, green for tips.
- **Symbols:** ✅ for correct method, ⚠️ for common mistakes.
- **Flowcharts:** Step-by-step for solving line-plane and distance problems.