🌈 DETERMINANTS – CLASS 12 SUPER STUDY GUIDE

🎨 1. THEORY IN SIMPLE WORDS (WITH VISUALS & ANALOGIES)

⭐ 1.1 What is a Determinant?

A **determinant** is a *special number* we get from a **square matrix** only.

Matrix → grid of numbers
Determinant → one number that tells you how the matrix "behaves".

Think of a determinant as a **"matrix power meter"**—it tells how powerful a matrix is (e.g., whether it has an inverse).

🎨 Visual

Matrix A:

```
[ a   b ]
[ c   d ]
```

Determinant |A| = ad − bc

⭐ 1.2 For Which Matrices Do Determinants Exist?

✔️ Only **Square Matrices** (2×2, 3×3, 4×4...)
❌ Not for rectangular matrices.

⭐ 1.3 Geometric Meaning (Easy Analogy)

For a 2×2 matrix, |A| gives the **area scaling factor**.
For a 3×3 matrix, |A| gives **volume scaling factor**.

If |A| = 0 → No scaling → Matrix is "flat", not invertible.

→ Zero determinant means area collapses to a line!

⭐ 1.4 How to Find a Determinant

📘 2×2 Matrix

```
| a  b |
| c  d | = ad − bc
```

📕 3×3 Matrix (Expansion Method)

We expand along a row/column.

For matrix:

```
| a  b  c |
| d  e  f |
| g  h  i |
```

Expand along first row:

|A| =
a × (ei – fh)
– b × (di – fg)

- c × (dh – eg)

🎨 Pattern: + – +

Signs alternate.
Memory Trick: "**Happy-Sad-Happy**"

⭐ 1.5 Minors and Cofactors (Very Important)

For element a_{ij}:

- **Minor M_{ij}** = determinant of matrix after removing row i & column j
- **Cofactor C_{ij} = $(-1)^{(i+j)} \times M_{ij}$**

Easy Analogy

Minor = "*matrix after deleting a row & column*"
Cofactor = Minor with a "sign badge"

⭐ 1.6 Properties of Determinants (Super Important for Boards)

No.	Property	Meaning
1	If two rows/columns are equal →	A
2	If any row/column is zero →	A

No.	Property	Meaning
3	Interchanging rows/columns → sign changes	
4	Multiply row/column by k → determinant multiplies by k	
5	Add multiple of one row to another → determinant same	
6	Triangular matrix → determinant = product of diagonal	

⭐ Shortcut Memory Trick:
“SAME → ZERO” (identical rows/columns → 0)
“SWAP → SIGN change”
"MULTIPLY → MULTIPLY"
"ADD → NO CHANGE"

🧠 2. KEY CONCEPTS & FORMULAS (QUICK TABLES)

⭐ Determinant Formulas

Type	Formula
2×2	ad – bc
3×3 expansion	a(ei−fh) − b(di−fg) + c(dh−eg)
Minor	delete row & column
Cofactor	(–1)^(i+j) × Minor

⭐ Adjoint of a Matrix

Adj(A) = transpose of cofactor matrix.

⭐ Inverse of Matrix

$A^{-1} = adj(A) / |A|$
(If $|A| \neq 0$)

⭐ Condition for Matrix Inverse

$|A| \neq 0$
(If $|A| = 0$ → matrix is singular → no inverse)

⭐ Mnemonics

- **For signs in 3×3 expansion:**
  → "+ − +" for first row
- **For cofactors:**
  → Chessboard pattern:

```
+  −  +
−  +  −
+  −  +
```

✏️ 3. SOLVED NUMERICAL PROBLEMS (STEP-BY-STEP)

⭐ TYPE 1: 2×2 Determinant

Q1

```
| 3   4 |
| 2   5 |
```

$|A| = (3)(5) - (4)(2)$
$= 15 - 8$
$= \mathbf{7}$

⭐ TYPE 2: 3×3 Determinant (Expansion)

Q2

```
| 1   2   3 |
| 4   5   6 |
| 7   8   9 |
```

Expand along row 1:

$|A| =$
$1(5\times9 - 6\times8) -$
$2(4\times9 - 6\times7) +$
$3(4\times8 - 5\times7)$

$= 1(45-48) - 2(36-42) + 3(32-35)$
$= -3 - 2(-6) + 3(-3)$
$= -3 + 12 - 9$
$=$ **0**

⭐ Observation: Rows are in AP → determinant zero.

⭐ TYPE 3: Minor & Cofactor

Q3

Find M_{12} and C_{12} of

```
|1  2  3|
|4  5  6|
|7  8  9|
```

Delete row 1, column 2 →

```
|4  6|
|7  9|
```

$M_{12} = 4\times9 - 6\times7 = 36 - 42 = -6$
$C_{12} = (-1)^{(1+2)} M_{12} = -(-6) =$ **6**

⭐ TYPE 4: Using Properties (Fast!)

Q4

```
|1   2   3|
|2   4   6|
|1   2   3|
```

Row 1 and Row 3 identical → $|A| =$ **0**

⭐ TYPE 5: Inverse Using Determinant

Given

```
A = |4 7; 2 6|
```

$|A| = 4\times6 - 7\times2 = 24 - 14 = 10$

Cofactor matrix:
$= [6\ -7;\ -2\ 4]$

Adj(A) = transpose =
$[6\ -2;\ -7\ 4]$

$A^{-1} = 1/|A| \times \text{Adj}(A)$
$= (1/10)[6\ -2;\ -7\ 4]$

📒 4. PREVIOUS YEARS' BOARD QUESTIONS (SOLVED)

(Representative CBSE-style)

⭐ PYQ 1

Evaluate:

```
|1  2  3|
|2  3  4|
|3  4  5|
```

Rows in AP → determinant = **0**

⭐ PYQ 2

If A is

```
|a  b|
|c  d|
```

Show $|A| = |A^T|$.

$\text{Det}(A) = ad - bc$
$\text{Det}(A^T) = ad - cb$
Same ⇒ proved.

⭐ PYQ 3

Find minor and cofactor of 5 in

$$\begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix}$$

M_{12} = determinant of [2] = 2
$C_{12} = (-1)^{(1+2)} \times 2 = -2$

⭐ PYQ 4

Find A^{-1} for

$$\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$|A| = 2\times1 - 1\times1 = 1$
Inverse = adj(A)

⭐ Frequently Asked Areas:

✔ Determinant evaluation (2×2 and 3×3)
✔ Minors & Cofactors
✔ Use of determinant properties
✔ Inverse using adjoint
✔ Prove properties
✔ Show A is singular

⚡ 5. QUICK REVISION NOTES (1–2 PAGES)

⭐ DETERMINANT RULES

- Only for square matrices
- $|A|=0$ → singular → no inverse
- $|AB| = |A||B|$
- $|A^T| = |A|$

⭐ PROPERTIES SHORT TABLE

Action	Determinant Effect
Swap rows	sign changes
Multiply row by k	det × k

Action	Determinant Effect
Add multiple of another row	no change
Identical rows	det = 0
Triangular matrix	product of diagonal

⭐ SIGNS FOR COFACTORS

```
+ − +
− + −
+ − +
```

⭐ SHORTCUTS

- If two rows proportional → det = 0
- Use row/column with maximum zeros for expansion
- Apply $R_2 \rightarrow R_2 - R_1$ to create zeros

🔮 6. PREDICTED / LIKELY QUESTIONS

Short Answer

1. Define minor and cofactor
2. Write cofactor matrix
3. Use properties to find determinant

Long Answer

1. Evaluate 3×3 determinant
2. Find A^{-1} using adj(A)
3. Prove determinant property (swap/multiply)

Numerical

1. Expand along first row
2. Use $R_3 \rightarrow R_3 - R_1$ to simplify
3. Find |AB| using |A| and |B|

🧠 7. EXAM TIPS & TRICKS

⭐ Smart Tips

✔ Always simplify using properties FIRST
✔ Prefer row with most zeros
✔ Make zeros using $R_1 \to R_1 - R_2$ patterns
✔ Keep sign patterns clear (+ − +)

⭐ Common Mistakes

❌ Expanding with wrong signs
❌ Forgetting determinant changes after row operations
❌ Writing A^{-1} when $|A|=0$ (not possible!)

⭐ Flowchart (Must Learn!)

```
Start
 ↓
Check for zero/identical rows?
 ↓ Yes
|A| = 0
 ↓ No
Use determinant properties to simplify
 ↓
Expand using row with max zeros
 ↓
Find minors & cofactors if needed
 ↓
Compute |A| or A⁻¹
```

🧸 8. VISUAL & KID-FRIENDLY SUMMARY

🎨 Determinant = **Matrix Power Number**
🎨 Minor = **Cut & Solve**
🎨 Cofactor = **Cut + Sign**
🎨 Inverse = **Adjoint ÷ Determinant**
🎨 Properties save **time + marks**