🌈 Class 12 – Mathematics – Integrals (INDÉFINITE + DEFINITE)

Designed with **simple language**, **visual metaphors**, **flowcharts**, **formulas**, **solved numericals**, **PYQs**, **quick notes**, and **exam-focused tips**.

🎨 1. THEORY IN SIMPLE WORDS (WITH VISUALS & ANALOGIES)

⭐ 1.1 What is an Integral?

Derivative = rate of change (breaking into tiny pieces)
Integral = putting tiny pieces back together

📦 **Analogy:**
Imagine cutting chocolate into tiny squares (differentiation),
Then adding all squares to reconstruct chocolate (integration).
That "adding up" = **integration**.

⭐ 1.2 Types of Integrals

Type	Meaning	Symbol
Indefinite	Family of functions + C	∫ f(x) dx
Definite	Area under curve	$\int_a^b f(x)\,dx$

🎨 Mental Image:

- **Indefinite integral = treasure box** (∞ many answers due to +C)
- **Definite integral = shaded area** under curve

⭐ 1.3 Integration as Reverse Differentiation

If

$$\frac{d}{dx}(F(x)) = f(x)$$

then

$$\int f(x)dx = F(x) + C$$

⭐ 1.4 Geometric Meaning of Definite Integral

$$\int_a^b f(x)\,dx = \text{Area between curve and x-axis from a to b}$$

🎨 Visual: Shaded region between curve & axis.

⭐ 1.5 Properties of Definite Integrals (Very Important!)

1. $\int_a^a f(x)dx = 0$
2. $\int_a^b = -\int_b^a$
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
4. If f(x) is even:

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$$

5. If f(x) is odd:

$$\int_{-a}^{a} f(x)dx = 0$$

🎨 Memory Trick:

- **Even → 2 Same Areas**
- **Odd → Cancel (Zero)**

⭐ 1.6 Methods of Integration

Method	Used When	Idea
Standard formulas	Basic functions	Reverse derivatives
Substitution	Composite functions	Make inside = new variable
Integration by parts	Product of functions	ILATE rule
Partial fractions	Rational expressions	Split into simple fractions

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🧠 2. KEY CONCEPTS & FORMULAS (QUICK TABLES)

⭐ 2.1 Basic Integration Formulas (Super Important)

$$\int k\,dx = kx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

⭐ 2.2 Substitution Formula

If x = g(t):

$$\int f(g(x))g'(x)dx = \int f(t)dt$$

🎨 Trick: “Spot inside function → make it t”

⭐ 2.3 Integration by Parts

$$\int uv\,dx = u\int v dx - \int \left(\frac{du}{dx}\int v\,dx\right) dx$$

Use **ILATE rule** to choose u:

I – Inverse Trig
L – Logarithm
A – Algebra
T – Trigonometric
E – Exponential

⭐ 2.4 Partial Fractions

Use when the form is:

$$\frac{P(x)}{Q(x)} \quad \text{where degree(P)} < \text{degree(Q)}$$

Split denominator into smaller parts.

🧨 3. SOLVED NUMERICAL PROBLEMS

⭐ TYPE 1: Basic Standard Integration

Q1. $\int (3x^2 + 5)dx$

Solution:
$= 3 \cdot \frac{x^3}{3} + 5x + C$
$= x^3 + 5x + C$

⭐ TYPE 2: Substitution Method

Q2. $\int \frac{1}{1+x^2} dx$

Recognize derivative of $\tan^{-1}x$.
Answer = **$\tan^{-1}x$ + C**

Q3. $\int \sin(3x)dx$

Let t = 3x → dt = 3 dx
Then
= -cos(3x)/3 + C

⭐ TYPE 3: Integration by Parts

Q4. $\int xe^x dx$

Choose u = x (A from ILATE)
dv = e^x dx

u' = 1
v = e^x

Apply formula:
= x·e^x − ∫e^x dx
= x e^x − e^x + C

⭐ TYPE 4: Partial Fractions

Q5. $\int \frac{1}{x^2-1} dx$

$x^2-1 = (x-1)(x+1)$

$$\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

Integrate:

$$\frac{1}{2}(\ln|x-1| - \ln|x+1|) + C$$

⭐ TYPE 5: Definite Integrals

Q6. $\int_0^1 (x^2+1) dx$

$= \left[\frac{x^3}{3} + x\right]_0^1$
= (1/3 + 1) − 0
= **4/3**

⭐ TYPE 6: Even/Odd Function Trick

Q7. $\int_{-3}^{3} x^3 \, dx$

x^3 is **odd**, so answer = **0**

📒 4. PREVIOUS YEARS' BOARD QUESTIONS (SOLVED)

⭐ PYQ 1

$\int \frac{\cos x}{1+\sin x} dx$

Let t = sin x → dt = cos x dx
Integral becomes:

$$\int \frac{1}{1+t} dt = \ln|1+t| + C$$

⭐ PYQ 2

$\int_0^{\pi/2} \sin x dx$

$= [-\cos x]_0^{\pi/2} = (0) - (-1) = 1$

⭐ PYQ 3

$\int x \ln x dx$

By parts:
u = ln x
dv = x dx
Ans: $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$

⭐ PYQ 4

$\int \frac{dx}{x^2+4}$

$= \frac{1}{2} \tan^{-1} \frac{x}{2} + C$

⭐ Frequently Asked Patterns

- ✔ Definite integrals (easy scoring)
- ✔ Substitution problems
- ✔ Partial fractions
- ✔ Integration by parts (ln x · x^n, x·e^x, x·sin x)
- ✔ Standard integrals

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5. QUICK REVISION NOTES (1–2 PAGES)

⭐ Master Strategy

Step	What to Check
1	Try direct formula?
2	Is substitution possible?
3	Is it product → by parts?
4	Rational expression → partial fractions?
5	Definite → use properties?

⭐ Important Mini-Chart

🎨 Standard +C

Always add +C for indefinite integrals.

🎨 ILATE Ladder

Choose u from highest priority:
Inverse > Log > Algebra > Trig > Exp

⭐ DEFINITE Integral Properties Quick Map

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Same limits → 0
Swap limits → negative
Break interval → sum
Even → double
Odd → zero
```

🔮 6. PREDICTED / LIKELY QUESTIONS

Short Answer (2–3 Marks)

- Evaluate $\int \sin ax \, dx$
- Evaluate $\int 1/(x^2+a^2) \, dx$
- Evaluate $\int_0^a f(x) \, dx$ for odd/even

Long (5–6 Marks)

- By parts: x ln x, x sin x, x e^x
- Partial fraction integrals
- Definite integrals with substitution
- Application of properties

🧠 7. EXAM TIPS & TRICKS

⭐ Ultimate Tricks

✔️ Try substitution FIRST — often easiest
✔️ Check for derivative hidden inside function
✔️ For polynomials × exponential/trig → ALWAYS use by parts
✔️ For rational functions → partial fractions
✔️ For definite integrals → use even/odd shortcut

⭐ Common Mistakes

❌ Forgetting +C
❌ Incorrect substitution limits
❌ Wrong ILATE order
❌ Poor simplification → messy answers

🧸 8. VISUAL & KID-FRIENDLY MEMORY HACKS

🎨 Integral sign ∫ looks like a **curvy ribbon collecting small pieces** → area
🎨 Even function graph = **mirror image** → double area
🎨 Odd function graph = **symmetric flip** → cancellation
🎨 ILATE = **I Love A Tall Elephant** (fun mnemonic!)
🎨 By parts = "**Give one, take one**" (u gives derivative, v takes integral)