🌈 Class 12 – Mathematics – Applications of Derivatives

Perfect for quick learning, fast revision, and high marks in board exams.

🎨 1. THEORY IN SIMPLE WORDS (WITH VISUALS & ANALOGIES)

This chapter tells us **how derivatives help in real-life and math problems** such as:

✔️ Finding slope of tangent
✔️ Finding max–min values (profit, area, speed…)
✔️ Finding increasing/decreasing intervals
✔️ Rate of change
✔️ Approximations
✔️ Errors
✔️ Maxima/minima for word problems

Let's learn each visually!

⭐ 1.1 Rate of Change of Quantities

Derivative = 'Speed of Change'

If y depends on x, then

$$\frac{dy}{dx}$$

tells **how fast y changes when x changes**.

🎨 Analogy

Think of x as *time* and y as *distance travelled.*
dy/dx = speed.

Example

If radius r of a sphere increases with time,

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

⭐ 1.2 Increasing & Decreasing Functions

Derivative tells **whether graph is going up or down**.

Condition	Meaning	Visual
f'(x) > 0	Function increasing	📈
f'(x) < 0	Function decreasing	📉
f'(x) = 0	Turning point	🎯

⭐ 1.3 Maxima & Minima (Turning Points)

Use derivatives to find:

✔️ Highest point → **maximum**
✔️ Lowest point → **minimum**

Steps (Very Important)

1. Find f′(x).
2. Put f′(x) = 0 → Critical points.
3. Apply **second derivative test**:

Condition	Meaning
f''(a) > 0	Minimum
f''(a) < 0	Maximum
f''(a) = 0	Test fails

⭐ 1.4 Tangents & Normals

Slope of tangent at x = a:

$$m = f'(a)$$

Equation of tangent:

$$y - f(a) = f'(a)(x - a)$$

Normal is perpendicular to tangent:

$$m_n = -\frac{1}{f'(a)}$$

Equation of normal:

$$y - f(a) = -\frac{1}{f'(a)}(x - a)$$

⭐ 1.5 Approximations & Errors

For small Δx,

$$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$$

Relative error formula:

$$\frac{\Delta y}{y} \approx \frac{dy}{dx}\frac{\Delta x}{x}$$

Visual Analogy

Function is like a smooth curve.
Derivative gives the **tiny slope line** used as an approximate value.

🧠 2. KEY CONCEPTS & FORMULAS

⭐ Quick Formula Table

Topic	Formula
Rate of change	dy/dt = (dy/dx)(dx/dt)
Inc./Dec.	f'>0 → inc., f'<0 → dec.
Tangent	$y - y_1 = f'(x_1)(x - x_1)$
Normal	$y - y_1 = -1/f'(x_1)(x - x_1)$
Max/min test	f′=0 + f′′ signs
Approx.	f(x+h) ≈ f(x)+hf′(x)
Error	Δy ≈ f′(x)Δx

⭐ Memory Tricks

1. Tangent–Normal Trick
Tangent slope = m → Normal slope = −1/m
(Flip and add minus!)

2. Max–Min Trick
If f′′(x) is:

-
    - (positive) → bowl shape → minimum
- – (negative) → hill shape → maximum

Mnemonic:
“Smiley = min, Frowny = max”

✏️ 3. SOLVED NUMERICAL PROBLEMS

⭐ TYPE 1: Rate of Change

Q1. Volume of a sphere is $V = \frac{4}{3}\pi r^3$.
Find rate of change of volume when r=3 cm and dr/dt = 2 cm/s.

Solution:

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$\frac{dV}{dr} = 4\pi r^2$$

At r=3:

$$\frac{dV}{dt} = 4\pi(9)(2) = 72\pi$$

⭐ TYPE 2: Increasing/Decreasing

Q2. f(x)=x^2–4x+1

Find intervals where function increases.

$$f'(x) = 2x – 4$$

Set derivative >0:
2x – 4 > 0 → x>2

✔️ Increasing for x>2
✔️ Decreasing for x<2

⭐ TYPE 3: Maxima/Minima

Q3. f(x)=x^3−3x^2+1

Find local max/min.

$$f'(x) = 3x^2 - 6x = 3x(x-2)$$

Critical points: x=0,2

$$f''(x) = 6x - 6$$

At x=0 → f''(0) = −6 → **Maximum**
At x=2 → f''(2)=6 → **Minimum**

⭐ TYPE 4: Tangent

Q4. Find tangent to $y=x^2$ at x=1.

$$f'(x) = 2x \Rightarrow f'(1) = 2$$

Point = (1,1)

Equation:

$$y - 1 = 2(x-1)$$

⭐ TYPE 5: Approximation

Q5. Approximate √(25.5)

Let f(x)=√x, x=25, h=0.5

$$f(25.5) \approx f(25) + hf'(25)$$

$$= 5 + 0.5\,(\frac{1}{2\sqrt{25}})$$

$$= 5 + 0.5 \times \frac{1}{10} = 5.05$$

📒 4. PREVIOUS YEARS' BOARD QUESTIONS (SOLVED)

⭐ PYQ 1

Find interval where $f(x)=3x^3-9x$ is increasing.

$$f'(x) = 9x^2 - 9 = 9(x-1)(x+1)$$

Increasing when $f'(x)>0 \rightarrow x>1$ or $x<-1$

⭐ PYQ 2

Find equation of tangent to $y=\sqrt{x}$ at $x=4$.

$$f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(4) = \frac{1}{4}$$

Point = (4,2)

Equation:

$$y - 2 = \frac{1}{4}(x - 4)$$

⭐ PYQ 3

Find maxima/minima of $x^3-6x^2+9x+15$.

→ Standard derivative method.
(Maximum at $x=1$, minimum at $x=3$)

⭐ Frequently Asked Areas

✔ Maxima-minima word problems
✔ Tangent/normal equations
✔ Rate of change
✔ Error approximation
✔ Increasing/decreasing via sign chart

⚡ 5. QUICK REVISION NOTES (1–2 PAGES)

⭐ Case Summary Table

Concept	Condition	Output
Increasing	$f'>0$	📈
Decreasing	$f'<0$	📉
Max	$f'=0$, $f''<0$	🏔️
Min	$f'=0$, $f''>0$	🏔️

Concept	Condition	Output
Tangent slope	f′(a)	straight line
Normal slope	−1/f′(a)	⊥ line

⭐ Fast Max–Min Method

1. f′(x)=0
2. Check f′′ sign
3. State maxima or minima
4. Write value f(a)

⭐ Quick Approximations

$$f(x+h) \approx f(x) + hf'(x)$$

🔮 6. PREDICTED / LIKELY QUESTIONS

Short Answer

✔ Define increasing function
✔ Write formula for tangent
✔ State second derivative test
✔ Use dy/dx for rate of change

Long Questions

✔ Maxima-minima of cubic polynomial
✔ Tangent to curve at given point
✔ Approximate a value like ln(1.02), √(50.5)
✔ Word problems (profit, area, distance)

🧠 7. EXAM TIPS & TRICKS

⭐ High-Scoring Tricks

✔ Always use **sign chart** for increasing/decreasing
✔ For tangent:
m = f′(a) → plug into formula
✔ For normal:

Flip & negative
✔️ For maxima/minima:
Use second derivative test – fastest method

⭐ Common Mistakes

❌ Forgetting to check continuity before differentiability
❌ Wrong derivative of √x, ln x
❌ Mistaking normal slope (reciprocal + negative)
❌ Not writing intervals correctly

⭐ Flowchart for Max–Min

```
Start
 ↓
Find f'(x)
 ↓
Solve f’(x)=0  → critical points
 ↓
Find f''(x)
 ↓
Check signs
 ↓
Max? Min? Neither?
 ↓
Write values and intervals
```

🧸 8. VISUAL & KID-FRIENDLY MEMORY HACKS

🎨 Increasing graph = Climbing stairs
🎨 Decreasing graph = Sliding down
🎨 Maximum = Mountain top
🎨 Minimum = Valley
🎨 Tangent = Touching line
🎨 Normal = Soldier standing straight (perpendicular)