✅ 1. Understanding the Problem

You are given a function with two input-output pairs:

- $f(1) = 3$
- $f(2) = 5$

A **function rule** is an algebraic expression (like $f(x) = mx + b$) that gives the output for any valid input.

Since only two points are given and they form a straight-line pattern, we look for a **linear function**:

$$f(x) = mx + b$$

✅ 2. Find the Function Rule

We need to determine m (slope) and b (y-intercept).

Step 1: Find the slope m

Use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute the points $(1, 3)$ and $(2, 5)$:

$$m = \frac{5 - 3}{2 - 1} = \frac{2}{1} = 2$$

So,

$$m = 2$$

Step 2: Find the intercept b

Use the form:

$$f(x) = 2x + b$$

Plug in any point. Let's use $f(1) = 3$:

$$3 = 2(1) + b$$

$$3 = 2 + b$$

$$b = 1$$

✅ 3. Final Function Rule

$$\boxed{f(x) = 2x + 1}$$

✨ 4. Check the Rule

For $x = 1$:

$$f(1) = 2(1) + 1 = 3 \quad \checkmark$$

For $x = 2$:

$$f(2) = 2(2) + 1 = 5 \quad \checkmark$$

Works perfectly.

📘 5. Theory Behind It

1️⃣ Linear Functions

A linear function has the form:

$$f(x) = mx + b$$

- m = slope → rate of change
- b = y-intercept → value when $x = 0$

2️⃣ Slope

Slope measures the steepness of the line:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

3️⃣ Why Linear?

Two points always define exactly **one unique line**.
Thus, with two mappings, the best fit is a linear function.

4️⃣ Function Mapping

A function matches each input x to exactly one output y.
A mapping like $1 \longrightarrow 3$ means:

$$f(1) = 3$$

🧮 6. General Method (Formula)

If you have two points (x_1, y_1) and (x_2, y_2):

Step 1: Find slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 2: Find intercept

$$b = y_1 - mx_1$$

Step 3: Write rule

$$f(x) = mx + b$$

🎯 Final Answer

$$\boxed{f(x) = 2x + 1}$$