

Class 12 Maths – Applications of Integrals

(Area Under Curves & Between Curves)

This guide follows everything you asked for: visuals, examples, PYQs, tricks, formulas, revision notes, and predicted questions — all in **simple, fun & scoring-friendly format**.

1. THEORY IN SIMPLE WORDS (WITH VISUALS & ANALOGIES)

★ 1.1 What does “Application of Integrals” mean?

Integration basically adds up infinitely tiny slices.

Here we use it to find:

- ✓ Area under a curve
- ✓ Area between two curves
- ✓ Area between a curve and coordinate axes

 **Analogy:**

Imagine slicing a pizza into super-thin strips and adding their widths to get the whole area → that’s exactly what integration does.

★ 1.2 Area Under a Curve (Basic)

If you want the area between a function $y = f(x)$, x-axis, and the lines $x = a$ and $x = b$:

$$\text{Area} = \int_a^b f(x) dx$$

 **Visual Picture:**

Axis = floor

Curve = roof

Integral = measures the space between them

★ 1.3 Area Between Two Curves

If two curves are given:

$$y_1 = f(x), \quad y_2 = g(x)$$

and $f(x) \geq g(x)$ on the interval, then:

$$\text{Area} = \int_a^b [f(x) - g(x)] dx$$

Mental Image:

Upper curve is like a **larger blanket** and lower curve is a **smaller blanket** → difference gives the space between them.

★ 1.4 Curve bounded by axes ("Closed Figures")

Examples:

- area between parabola and line
- area under semicircle
- area enclosed by $y^2 = 4ax$ and line $y = mx$

 The goal is to **first locate intersection points**, then integrate between them.

★ 1.5 Standard Procedure (Super Important Flowchart!)

Step 1: Draw the curves (rough sketch)

Step 2: Find intersection points (solve equations)

Step 3: Decide upper - lower curve

Step 4: Apply formula:

$$\int (\text{upper} - \text{lower}) dx$$

Step 5: Evaluate integral

2. KEY CONCEPTS & FORMULAS (QUICK TABLES)

★ 2.1 Formula Table

Situation	Formula
Area under curve	$\int_a^b f(x) dx$
Area between two curves	$\int_a^b [f(x) - g(x)] dx$
Intersection points	Solve $f(x) = g(x)$
Area under symmetry	Double area of one half if curve symmetric

★ 2.2 Symmetry Memory Trick

🎨 If curve is symmetric about **y-axis**, compute area from 0 to a and **double it**.
(Eg: parabola $y = x^2$)

Symmetry → Saves ICE (Time + Marks)

★ 2.3 Standard Areas Used Frequently

• Area of semicircle (radius r):

$$\frac{1}{2}\pi r^2$$

• Area under $y = \sqrt{a^2 - x^2}$:
(semi-circle)

✏ 3. SOLVED NUMERICAL PROBLEMS (ALL TYPES)

★ TYPE 1: Area under a curve

Q1. Find $\int_0^2 (x^2 + 1) dx$, i.e., area under $y = x^2 + 1$.

Solution:

$$\int_0^2 (x^2 + 1)dx = \left[\frac{x^3}{3} + x \right]_0^2 = \left(\frac{8}{3} + 2 \right) = \frac{14}{3}$$

- ✓ Easy marks
 - ✓ No trick
 - ✓ Most common 2-mark question
-

★ TYPE 2: Area between two curves

Q2. Find area between curves $y = x$ and $y = x^2$, from $x = 0$ to 1.

Step 1: Determine upper curve

✓ $y = x$ is above $y = x^2$ on $[0,1]$

Step 2: Apply formula:

$$\int_0^1 (x - x^2)dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

★ TYPE 3: Area bounded by a curve and x-axis

Q3. Find the area bounded by $y = |x|$ from -2 to 2 .

By symmetry: double area from 0 to 2 .

$$\text{Area} = 2 \int_0^2 x \, dx = 2 \left[\frac{x^2}{2} \right]_0^2 = 2(2) = 4$$

★ TYPE 4: Area enclosed by lines and curves

Q4. Find area enclosed by parabola $y = x^2$ and the line $y = 4$.

Intersection:

$$x^2 = 4 \rightarrow x = \pm 2$$

Area =

$$\int_{-2}^2 (4 - x^2) \, dx$$

Even function \Rightarrow

$$\begin{aligned} & 2 \int_0^2 (4 - x^2) \, dx \\ &= 2 \left[4x - \frac{x^3}{3} \right]_0^2 \\ &= 2 \left(8 - \frac{8}{3} \right) = 2 \cdot \frac{16}{3} = \frac{32}{3} \end{aligned}$$



4. PREVIOUS YEARS' BOARD QUESTIONS (SOLVED)

★ PYQ 1

Find the area bounded by $y = x^2$ and $y = 2x$.

Intersection:

$$x^2 = 2x$$

$$x(x - 2) = 0$$

$$x = 0, 2$$

Upper curve = $2x$

Lower = x^2

Area =

$$\begin{aligned} \int_0^2 (2x - x^2) dx \\ = \left[x^2 - \frac{x^3}{3} \right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3} \end{aligned}$$

★ PYQ 2

Find area bounded by $y = \sin x$ and x -axis from 0 to π .

$$\int_0^{\pi} \sin x dx = [-\cos x]_0^{\pi} = (-(-1)) = 2$$

★ PYQ 3

Area between curves $y = e^x$ and $y = e^{-x}$ from $x = 0$ to 1.

Upper curve: e^x

$$\begin{aligned} \int_0^1 (e^x - e^{-x}) dx \\ = [e^x + e^{-x}]_0^1 = (e + e^{-1}) - (1 + 1) = e + \frac{1}{e} - 2 \end{aligned}$$

★ PYQ 4

Find area enclosed by the circle $x^2 + y^2 = 4$ and x -axis.

This is a semicircle of radius 2.

Area =

$$\frac{1}{2} \pi r^2 = \frac{1}{2} \pi (4) = 2\pi$$

★ Repeated Patterns

- ✓ Parabola vs Line (VERY frequent)
 - ✓ $\sin x$ area from 0 to π
 - ✓ Area between x and x^2
 - ✓ Circle's area via integral
-

⚡ 5. QUICK REVISION NOTES (1–2 PAGES)

★ Important Concepts Map

Area under curve $\rightarrow \int f(x) \, dx$

Area between curves $\rightarrow \int (\text{upper} - \text{lower}) \, dx$

Define limits \rightarrow intersection points

Use symmetry \rightarrow half work

Graph always helps

★ Mini Formula Bank

Formula	Meaning
$\int (f-g) \, dx$	Area between curves
\int symmetric even	2 \times from 0 to a
\int symmetric odd	0
Area under	$\int f(x) \, dx$

★ Identify Upper Curve Trick

Pick **any** x -value in interval \rightarrow plug values \rightarrow bigger y = upper curve.

🌟 6. PREDICTED / LIKELY QUESTIONS (High Accuracy)

Short Questions

- Area under $y = 1/x$ from 1 to 2
- Area under $y = \cos x$ from 0 to $\pi/2$

- Intersection-based area with parabola

Long Questions

- Area between parabola & line
- Area with circle or semicircle
- Area between two trigonometric curves ($\sin x$ & $\cos x$)

Diagram-Based

- Sketch curves & shade regions
 - Find intersection points from graph
-



7. EXAM TIPS & TRICKS

★ 1. Draw diagram ALWAYS

Even a rough sketch makes limits & upper curve clear.

★ 2. Find intersection FIRST

Students often forget this → wrong area.

★ 3. Check “upper – lower”

Common mistake: subtracting in wrong order.

★ 4. Use symmetry

Save time → especially with even functions and circles.

★ 5. Units not required

Area is **just a number** in board exams.



8. VISUAL & KID-FRIENDLY MEMORY HACKS

- 🧠 Integral = **giant net catching infinitely small areas**
- 🧠 Area between curves = **Big hill – small hill**
- 🧠 Symmetry = “one side mirrors the other → double it!”
- 🧠 Upper curve = **curve that stays on top like an umbrella**
- 🧠 Intersection = “Where curves shake hands”