

Class 12 Maths – Vector Algebra

Complete Exam-Focused Guide

THEORY IN SIMPLE WORDS WITH VISUALS

1.1 What is a Vector?

A **vector** is a quantity that has both **magnitude** (size) and **direction**.

- Examples: Displacement, Velocity, Force
- Analogy: A flying arrow \rightarrow points in a direction, has length = magnitude

 **Notation:**

- Bold: \mathbf{a}
 - Arrow on top: \vec{a}
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1.2 Types of Vectors

Type	Meaning	Visual Idea
Zero Vector	No magnitude, no direction	Dot at origin
Unit Vector	Magnitude = 1	Arrow with length 1
Position Vector	From origin to a point	Arrow from origin to point P(x,y,z)
Equal Vectors	Same magnitude & direction	Two arrows identical
Negative Vector	Same magnitude, opposite direction	Arrow flips backward

1.3 Vector Operations

1.3.1 Addition & Subtraction

- **Tip-to-tail method:** Place tail of second vector at tip of first.
- **Parallelogram law:** Vectors form a parallelogram; diagonal = sum.

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad (\text{Commutative})$$

- **Subtraction:** $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

1.3.2 Scalar Multiplication

- Multiply vector by a number k :

$k\vec{a}$ = changes magnitude, same or opposite direction

1.3.3 Dot Product (Scalar Product)

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$$

- Gives a number (scalar)
 - Tip: "Dot = Angle Cosine"
 - Properties: Commutative, distributive
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1.3.4 Cross Product (Vector Product)

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin \theta \hat{n}$$

- Gives a vector perpendicular to both
 - Use **Right-Hand Rule**: Curl fingers from $\vec{a} \rightarrow \vec{b}$, thumb = direction of $\vec{a} \times \vec{b}$
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1.3.5 Triple Product

- Scalar Triple Product:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \text{volume of parallelepiped}$$

- Vector Triple Product:

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

2 KEY CONCEPTS & FORMULAS

★ Quick Table of Vector Formulas

Operation	Formula	Notes / Mnemonics
Addition	$\vec{a} + \vec{b}$	Tip-to-tail or components

Operation	Formula	Notes / Mnemonics
Subtraction	$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$	Flip direction of b
Dot Product	$(\vec{a} \cdot \vec{b}) =$	\vec{a}
Cross Product	$(\vec{a} \times \vec{b}) =$	\vec{a}
Scalar Triple	$\vec{a} \cdot (\vec{b} \times \vec{c})$	Volume of parallelepiped
Vector Triple	$\vec{a} \times (\vec{b} \times \vec{c})$	BAC-CAB formula

★ Component Formulas

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$:

- **Dot:** $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$
- **Cross:** Determinant:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

3 SOLVED NUMERICAL PROBLEMS

◆ Example 1 – Dot Product

Q: Find the angle between $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j}$.

Solution:

1. Compute dot product:
 $\vec{a} \cdot \vec{b} = 1 * 2 + 2 * 1 = 4$
2. Magnitudes:
 $|\vec{a}| = \sqrt{1^2 + 2^2} = \sqrt{5}$
 $|\vec{b}| = \sqrt{2^2 + 1^2} = \sqrt{5}$
3. $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{4}{5}$
4. $\theta = \cos^{-1}(4/5)$

◆ Example 2 – Cross Product

Q: $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = 7\hat{i} + 5\hat{j} - 5\hat{k}$$

◆ Example 3 – Scalar Triple Product

Q: Find volume of parallelepiped for $\vec{a}, \vec{b}, \vec{c}$.

$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

Compute cross first, then dot \rightarrow absolute value.

4 PREVIOUS YEARS' BOARD QUESTIONS (SOLVED)

1. Find angle between two vectors \rightarrow **Dot Product**
2. Find vector perpendicular to two given vectors \rightarrow **Cross Product**
3. Volume of parallelepiped \rightarrow **Scalar Triple**
4. Verify vector identities \rightarrow **Vector Triple Product / Algebra**

Pattern: Dot and cross products are frequently repeated; always check component form.

5 QUICK REVISION NOTES

- Vectors have **magnitude + direction**
- **Dot product** \rightarrow scalar $\rightarrow \cos \theta$
- **Cross product** \rightarrow vector $\rightarrow \sin \theta$, RHR
- **Triple product** \rightarrow volume or BAC-CAB
- **Addition/subtraction** \rightarrow tip-to-tail or components
- Components: i, j, k

6 PREDICTED / LIKELY QUESTIONS

- Angle between two vectors (2–3 marks)
- Vector perpendicular to two vectors (cross product)
- Volume of parallelepiped (scalar triple product)
- Solve vector equations in components
- Verify identities like $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a})$

7 EXAM TIPS & TRICKS

- **Shortcut:** Dot \rightarrow cos, Cross \rightarrow sin, Triple \rightarrow volume
 - **Common mistake:** Forget RHR for cross product
 - **Time-saving:** Always use components if vectors are in i, j, k form
 - **Check:** Magnitude positive, angles within $0-180^\circ$
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8 VISUAL & KID-FRIENDLY LEARNING

- 🧠 **Tip-to-Tail:** Like connecting arrows
- 🧠 **Cross Product:** Thumb rule for vector direction
- 🧠 **Dot Product:** Measures "alignment" of vectors
- 🧠 **Scalar Triple:** Volume \rightarrow parallelepiped box

Flowchart to solve vector problems:

Given Vectors

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├ Dot Product? $\rightarrow \cos \theta$ / scalar

├ Cross Product? \rightarrow vector perpendicular / $\sin \theta$

├ Triple Product? \rightarrow volume or verify identities