🌈 CONTINUITY & DIFFERENTIABILITY – CLASS 12 SUPER STUDY GUIDE

🎨 1. THEORY IN SIMPLE WORDS (WITH VISUALS & ANALOGIES)

⭐ 1.1 What is Continuity?

A function is **continuous at x = a** if:

✔️ No breaks
✔️ No jumps
✔️ No holes
✔️ You can draw it *without lifting your pencil*

📘 Formal Definition

A function f(x) is continuous at x = a if:

$$\lim_{x \to a} f(x) = f(a)$$

🎨 Visual analogy

Think of a **smooth road** → continuous
A **broken bridge** → discontinuous

⭐ 1.2 Three Conditions for Continuity

For f(x) to be continuous at x = a:

1. f(a) exists
2. Left-hand limit exists = LHL
3. Right-hand limit exists = RHL
4. LHL = RHL = f(a)

🎨 Flowchart

```
Does f(a) exist?
    ↓ Yes
Are LHL & RHL finite?
    ↓ Yes
Is LHL = RHL = f(a)?
```

```
↓ Yes → Continuous
      No → Discontinuous
```

⭐ 1.3 Types of Discontinuity

Type	Meaning	Visual
Jump	LHL ≠ RHL	sudden step
Infinite	limit → ∞	vertical asymptote
Removable	hole at point	missing dot

⭐ 1.4 What is Differentiability?

A function is **differentiable at x = a** if the derivative exists there.

Derivative:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

📌 **Differentiability ⇒ Continuity, but continuity does NOT ⇒ differentiability**

Mnemonic:
👉 *"Smooth implies unbroken, but unbroken need not imply smooth."*

⭐ 1.5 Differentiability Conditions

Function f is differentiable at x = a if:

LHD = RHD
(where LHD and RHD are left-hand & right-hand derivatives)

🎨 Visual

Sharp corner / cusp → NOT differentiable
Smooth curve → differentiable

Example of non-differentiable point: $|x|$ at $x = 0$.

⭐ 1.6 Differentiability of Composite Functions

If:

- f is differentiable at g(x)
- g is differentiable at x

Then f(g(x)) is differentiable.

→ Chain Rule applies.

⭐ 1.7 Logarithmic Differentiation

Used when:
✔ functions like x^x
✔ products of many functions
✔ fractions of many functions

Steps:

1. Take ln on both sides
2. Differentiate
3. Solve for dy/dx

⭐ 1.8 Derivatives of Inverse Trigonometric Functions

(Needed in this chapter)

Function	Derivative
$\sin^{-1}x$	$1/\sqrt{(1-x^2)}$
$\cos^{-1}x$	$-1/\sqrt{(1-x^2)}$
$\tan^{-1}x$	$1/(1+x^2)$
$\cot^{-1}x$	$-1/(1+x^2)$
$\sec^{-1}x$	1/
$\text{cosec}^{-1}x$	−1/

🧠 2. KEY CONCEPTS & FORMULAS (QUICK TABLES)

⭐ Continuity Formulas

For continuity	Formula
Limit definition	LHL = RHL
Continuity at x=a	lim f(x) = f(a)
Algebra of continuous functions	Sum, product, quotient of continuous functions is continuous

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⭐ Differentiability Formulas

Concept	Formula
Differentiability	LHD = RHD
Chain rule	dy/dx = (dy/du)(du/dx)
Product rule	(uv)' = u'v + uv'
Quotient rule	(u/v)' = (u'v – uv')/v^2
Log diff	dy/dx = y × d/dx(ln y)

⭐ Memory Tricks

✔ For continuity: **"Limit = Value"**
✔ For differentiability: **"Left Derivative = Right Derivative"**
✔ For inverse trig derivatives:

- $\sin^{-1}x \rightarrow +$
- $\cos^{-1}x \rightarrow -$
- $\tan^{-1}x \rightarrow 1/(1 + x^2)$

✏️ 3. SOLVED NUMERICAL PROBLEMS

⭐ TYPE 1: Continuity Check

Q1. Check continuity of f(x) =

$$\begin{cases} x^2, & x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$$

Step 1: Compute LHL
LHL = 1^2 = 1

Step 2: Compute RHL
RHL = 2(1) − 1 = 1

Step 3: f(1)
= 2(1) −1 = 1

∴ Continuous at x=1.

⭐ TYPE 2: Differentiability Check at a Corner

Q2. f(x) = |x|

Check differentiability at 0.

f(x) =
x if x>0
−x if x<0

LHD = derivative of (−x) = −1
RHD = derivative of x = 1

Since LHD ≠ RHD → **Not differentiable** at 0.

⭐ TYPE 3: Derivative Using Chain Rule

Q3. y = $\sqrt{(1 + 3x^2)}$

dy/dx = $(1/(2\sqrt{(1+3x^2)})) \times (6x)$
= $3x / \sqrt{(1+3x^2)}$

⭐ TYPE 4: Logarithmic Differentiation

Q4. y = x^x

ln y = x ln x
Differentiate:
(1/y)(dy/dx) = ln x + 1
dy/dx = x^x(ln x + 1)

📒 4. PREVIOUS YEARS' BOARD QUESTIONS (SOLVED)

⭐ PYQ 1

Check continuity of:
f(x) =
x + 2, x < 1
3, x = 1
x^2, x > 1

Check LHL at x=1: 1 + 2 = 3
Check RHL: 1^2 = 1
Since LHL ≠ RHL → **Discontinuous**

⭐ PYQ 2

Check differentiability of f(x)=|x−3| at x=3.

RHD = +1
LHD = −1
Not differentiable.

⭐ PYQ 3

Find dy/dx of $\sin(\sin^{-1}x)$.

Ans: x.

⭐ PYQ 4

Differentiate $\tan^{-1}(2x/1-x^2)$

Use formula: $\tan^{-1}(f)$ derivative + chain rule.

⭐ Frequently Asked Patterns

✔ Continuity of piecewise functions
✔ Differentiability of modulus functions
✔ Derivatives of inverse trig functions
✔ Log differentiation
✔ Chain rule questions

⚡ 5. QUICK REVISION NOTES (1–2 pages)

⭐ Cont. Conditions

1. f(a) exists
2. LHL = RHL
3. limit = f(a)

⭐ Diff. Conditions

LHD = RHD

⭐ Smoothness Rule

Differentiability ⇒ Continuity

⭐ Not Differentiable At

✔ Corners
✔ Cusps
✔ Vertical tangent
✔ Breaks

⭐ Derivative Table

- $(x^n)' = nx^{n-1}$
- $(e^x)' = e^x$
- $(\log x)' = 1/x$
- $(\sin^{-1}x)' = 1/\sqrt{(1-x^2)}$
- $(\cos^{-1}x)' = -1/\sqrt{(1-x^2)}$

🔮 6. PREDICTED QUESTIONS

Short

1. Define continuity at x=a
2. State relation between continuity & differentiability
3. Evaluate: $\lim_{x\to 0} |x|/x$

Long

1. Check continuity & differentiability of a piecewise function
2. Use log-differentiation to find dy/dx
3. Differentiate inverse trigonometric composite functions

🧠 7. EXAM TIPS & TRICKS

⭐ MUST REMEMBER

✔ When checking differentiability → ALWAYS check continuity first
✔ For piecewise functions → compute LHL & RHL
✔ For |x| → sharp corner → not differentiable at 0
✔ For exams: always show steps clearly

⭐ Quick Flowchart

```
Start
 ↓
Check continuity at point
 ↓
LHL = RHL = f(a)?
 ↓ No → Not differentiable
 ↓ Yes
Compute LHD & RHD
 ↓
LHD = RHD → differentiable
```

🧸 8. VISUAL, FUN & KID-FRIENDLY MEMORY HACKS

🎨 Continuity = **No-break curve**
🎨 Differentiability = **Smooth curve**
🎨 Jump discontinuity = **Staircase jump**
🎨 Modulus graph = **Mountain peak (sharp!)**
🎨 Limit = value → *"Meeting point = function value"*