🌈 Class 12 Mathematics – Differential Equations

This guide is designed exactly as you asked—**simple language, diagrams, flowcharts, formulas, solved examples, PYQs, revision notes, and exam tricks.**

🎨 1. THEORY IN SIMPLE WORDS WITH VISUALS

⭐ 1.1 What is a Differential Equation?

A **Differential Equation (DE)** is an equation that contains:

- A function (like y)
- Its derivatives (dy/dx, d^2y/dx^2, etc.)

📦 **Analogy:**
Imagine a recipe that tells you not the final dish directly, but **how the dish is changing** at every step.
That "change rule" = differential equation.

⭐ 1.2 Order & Degree (Super Important!)

Concept	Meaning	Easy Trick
Order	Highest derivative present	Count highest derivative (dy/dx, d^2y/dx^2...)
Degree	Power of highest derivative (after removing roots & fractions)	Only consider powers

🎨 **Example:**
$(d^2y/dx^2)^3 + (dy/dx)^2 = 5x$

- Order = 2
- Degree = 3

⭐ 1.3 General & Particular Solutions

Type	Meaning	Example
General Solution	Contains constant C	y = 2x + C
Particular Solution	Found using given condition	y(0) = 5

🎨 Think like:
General = "Family of curves"

Particular = "One member of family"

⭐ 1.4 Formation of Differential Equation

To form a DE, **differentiate** and **eliminate constants**.

🎨 Steps:

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Step 1: Start with equation involving constants (C1, C2)
Step 2: Differentiate as many times as number of constants
Step 3: Eliminate constants
→ Result is differential equation
```

⭐ 1.5 Types of Differential Equations (Class 12)

1. **Variable Separable**
2. **Homogeneous Differential Equations**
3. **Linear Differential Equations**

We solve these three only.

⭐ 1.6 Variable Separable Method

If you can write:

$$\frac{dy}{dx} = f(x)\, g(y)$$

Then separate variables:

$$\frac{dy}{g(y)} = f(x)dx$$

Integrate both sides.

🎨 Visual: Think of splitting two angry kids into separate rooms (x-room & y-room).

⭐ 1.7 Homogeneous Differential Equation

Form:

$$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$$

Use substitution:

$$y = vx \quad \Rightarrow \quad dy/dx = v + xdv/dx$$

⭐ 1.8 Linear Differential Equation

Form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Solution uses **Integrating Factor (IF)**:

$$IF = e^{\int P(x)dx}$$

Final solution:

$$y(IF) = \int Q(x)(IF)dx + C$$

🎨 Memory Trick:
"IF → Multiply → Integrate → Divide"

🧠 2. KEY CONCEPTS & FORMULAS (EASY TABLES)

⭐ Table: Summary of All Types

Type of DE	Standard Form	Method
Variable Separable	dy/dx = f(x) g(y)	Separate + integrate
Homogeneous	dy/dx = F(y/x)	Put y = vx
Linear	dy/dx + P(x)y = Q(x)	IF method

⭐ Integrating Factor Formula

$$IF = e^{\int P(x)dx}$$

⭐ Linear DE Final Formula

$$y \cdot IF = \int (Q \cdot IF)dx + C$$

🎨 Mnemonic:
“Multiply → Integrate → Divide → Done!”

🏏 3. SOLVED NUMERICAL PROBLEMS

⭐ TYPE 1: Variable Separable

Q1. Solve:

$$\frac{dy}{dx} = xy$$

Step 1: Separate variables

$$\frac{dy}{y} = xdx$$

Step 2: Integrate

$$\ln|y| = \frac{x^2}{2} + C$$

Step 3: Final answer

$$y = Ce^{x^2/2}$$

⭐ TYPE 2: Homogeneous

Q2. Solve:

$$\frac{dy}{dx} = \frac{x+y}{x}$$
$$= 1 + \frac{y}{x}$$

Put y = vx
dy/dx = v + x dv/dx

$$v + xdv/dx = 1 + v$$

Cancel v:

$$xdv/dx = 1$$

Integrate:

$$v = \ln x + C$$

Substitute v = y/x:

$$\frac{y}{x} = \ln x + C$$

Final:

$$y = x(\ln x + C)$$

⭐ TYPE 3: Linear Differential Equation

Q3. Solve:

$$\frac{dy}{dx} + y = e^x$$

Here P(x) = 1

$$IF = e^{\int 1 dx} = e^x$$

Multiply equation by IF:

$$e^x \frac{dy}{dx} + e^x y = e^{2x}$$

LHS = derivative of ye^x

$$\frac{d}{dx}(ye^x) = e^{2x}$$

Integrate:

$$ye^x = \frac{e^{2x}}{2} + C$$

Final:

$$y = \frac{e^x}{2} + Ce^{-x}$$

📒 4. PREVIOUS YEARS' BOARD QUESTIONS (SOLVED)

⭐ PYQ 1:

Solve:

$$\frac{dy}{dx} = \frac{y}{x}$$

Separable:

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrate:

$$\ln y = \ln x + C$$

$$y = Cx$$

⭐ PYQ 2:

Solve the linear DE:

$$\frac{dy}{dx} + 2y = 1$$

IF = e^{2x}

$$ye^{2x} = \frac{e^{2x}}{2} + C$$

$$y = \frac{1}{2} + Ce^{-2x}$$

⭐ PYQ 3:

Solve:

$$\frac{dy}{dx} = \frac{x^2}{y}$$

Separable:

$$y\,dy = x^2 dx$$

Integrate:

$$\frac{y^2}{2} = \frac{x^3}{3} + C$$

⭐ PYQ 4: Form the differential equation of

$y = Ae^{2x} + B$

Differentiate twice → eliminate A, B.

Final answer:

$$\frac{d^2y}{dx^2} - 4y = 0$$

⚡ 5. QUICK REVISION NOTES (1–2 Pages)

⭐ Must-Know Quick Concepts

- DE = equation with derivatives
- Order = highest derivative
- Degree = power of highest derivative
- Variable separable → split x & y
- Homogeneous → substitute y = vx
- Linear → IF method

⭐ Formula Flashcards

🎴 **IF = e^(∫P dx)**
🎴 **Linear DE: yIF = ∫QIF dx + C**
🎴 **Homogeneous: y = vx**

⭐ Exam-Ready Flowchart

```
Given DE →
Check if separable?
  Yes → separate & integrate
No → check if homogeneous?
  Yes → y=vx
No → check if linear?
  Yes → IF method
Else → Not in Class 12 syllabus
```

🔮 6. PREDICTED / LIKELY BOARD QUESTIONS

2–3 Mark Questions

- Solve separable DE like $dy/dx = (x^2 + y^2)/xy$
- Homogeneous DE manipulation
- Simple forming DE from given family

5–6 Mark Questions

- Linear DE with IF
- Homogeneous + initial condition
- Formation of DE by eliminating constants (order 2 mostly)

Diagram-Based

- Identify type from graph / pattern
- Show family of curves

🧠 7. EXAM TIPS & TRICKS

⭐ Magic Trick 1: Identify Type in 5 Seconds

Look for patterns:

Clue	Type
Multiplying x & y terms	Separable
y/x appears	Homogeneous
dy/dx + P(x)y	Linear

⭐ Magic Trick 2: Always Write IF Formula

Even if you forget, you get partial marks.

⭐ Magic Trick 3: Don’t forget "+ C"

5–10% marks lost by this mistake!

⭐ Magic Trick 4: Use substitution neatly

Better handwriting = better marks in DE.

🧸 8. VISUAL & KID-FRIENDLY MEMORY AIDS

🎨 **Separable = Two rooms (x-room & y-room).**
🎨 **Homogeneous = Replace y by vx (v rides on x’s shoulder!)**

🎨 Linear = Magic IF lamp (rub → solution appears).
🎨 Order = How many floors (derivatives)
🎨 Degree = How many power-ups (powers).