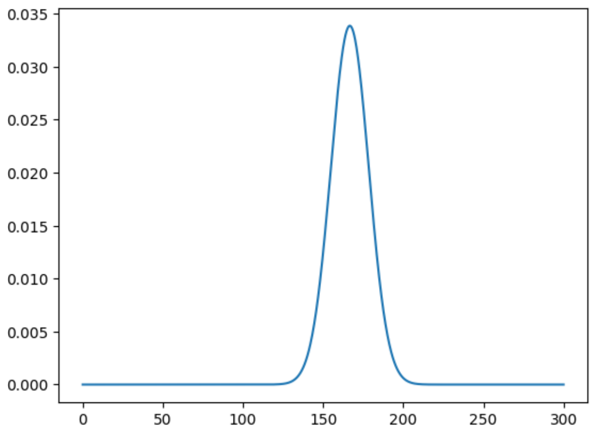

図1　2.3の描画結果

2.3

(a)

$y \sim \mathrm{Bin}(1000, 1/6)$であるため $y \sim \mathcal{N}(1000/6, 1000/6 \cdot (1 - 1/6))$ と近似できる。

```
import numpy as np
import matplotlib.pyplot as plt

def norm_dist(x: np.ndarray, mu: float, sigma: float) -> np.ndarray:
    return 1 / (sigma * np.sqrt(2 * np.pi)) * np.exp(- 1./2 * ((x - mu) / sigma) **2)

#%%
x = np.linspace(0, 300, 1000)
y = norm_dist(x, 1000/6, np.sqrt(1000/6 * 5/6))

plt.plot(x, y)
plt.show()
```

(b)

```
# %%
from scipy.stats import norm
l = [norm.ppf(x)*np.sqrt(1000/6 * 5/6) + 1000/6 for x in [0.05, 0.25, 0.5, 0.75, 0.95]]
print([float(x) for x in l])
```

答えはそれぞれ [147.3, 158.7, 166.7, 174.6, 186.0]。

2.15

$$
\begin{aligned}
\mathbb{E}[Z^m(1-Z)^n] &= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 z^m (1-z)^n z^{\alpha-1}(1-z)^{\beta-1} dz && (1)\\
&= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 z^{m+\alpha-1}(1-z)^{n+\beta-1} dz && (2)\\
&= \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+m)\Gamma(\beta+n)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta+m+n)} && (3)\\
&= \frac{(\alpha+m-1)\cdots\alpha\Gamma(\alpha)(\beta+n-1)\cdots\beta\Gamma(\beta)\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)(\alpha+\beta+m+n-1)\cdots(\alpha+\beta)\Gamma(\alpha+\beta)} && (4)\\
&= \frac{(\alpha+m-1)\cdots\alpha(\beta+n-1)\cdots\beta}{(\alpha+\beta+m+n-1)\cdots(\alpha+\beta)} && (5)
\end{aligned}
$$

よって、平均は

$$
\mathbb{E}[Z^1(1-Z)^0] = \frac{\alpha}{\alpha+\beta} \tag{6}
$$

分散は

$$
\begin{aligned}
\mathbb{E}[Z^2(1-Z)^0] - (\mathbb{E}[Z])^2 &= \frac{(\alpha+1)\alpha}{(\alpha+\beta+1)(\alpha+\beta)} - \frac{\alpha^2}{(\alpha+\beta)^2} && (7)\\
&= \frac{(\alpha^2+\alpha)(\alpha+\beta) - \alpha^3 - \alpha^2\beta - \alpha^2}{(\alpha+\beta+1)(\alpha+\beta)^2} && (8)\\
&= \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2} && (9)
\end{aligned}
$$