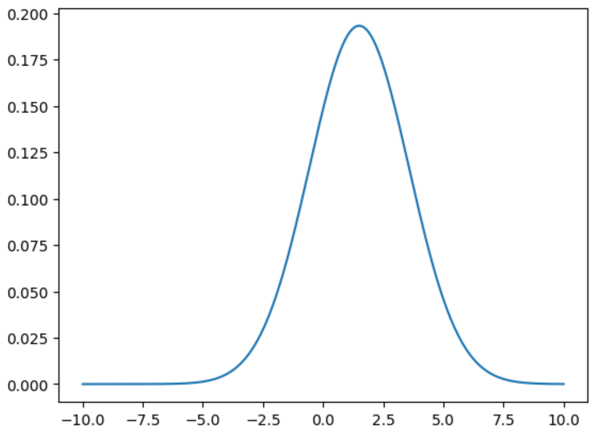

図1　1.1.(a) の描画結果

1.1 (a)

$$p(y) = \frac{1}{2} \cdot \frac{1}{\sqrt{2\pi \cdot 2^2}} \exp\{\frac{(y-1)^2}{2 \cdot 2^2}\} + \frac{1}{2} \cdot \frac{1}{\sqrt{2\pi \cdot 2^2}} \exp\{\frac{(y-2)^2}{2 \cdot 2^2}\} \tag{1}$$

```
#%%
import numpy as np
import matplotlib.pyplot as plt

def norm_dist(x: np.ndarray, mu: float, sigma: float) -> np.ndarray:
    return 1 / (sigma * np.sqrt(2 * np.pi)) * np.exp(- 1./2 * ((x - mu) / sigma) **2)

#%%
x = np.linspace(-10, 10, 1000)
y1 = norm_dist(x, 1, 2)
y2 = norm_dist(x, 2, 2)

y = 1/2 * y1 + 1/2 * y2
plt.plot(x, y)
plt.show()
```

1.1(b)

$$\Pr(\theta = 1|\mathrm{y} = 1) = \frac{\Pr(\theta = 1)\Pr(\mathrm{y} = 1|\theta = 1)}{\Pr(\mathrm{y} = 1)} \tag{2}$$
$$= \frac{\exp(-\frac{(1-1)^2}{2\cdot 2^2})}{\exp(-\frac{(1-1)^2}{2\cdot 2^2}) + \exp(-\frac{(1-2)^2}{2\cdot 2^2})} \tag{3}$$
$$= \frac{1}{1 + \exp(-0.125)} = 0.531 \tag{4}$$

1.1(c)

1.1(b)同様の式展開により、

$$\Pr(\theta = 1|\mathrm{y}) = \frac{\exp(-\frac{(\mathrm{y}-1)^2}{2\sigma^2})}{\exp(-\frac{(\mathrm{y}-1)^2}{2\sigma^2}) + \exp(-\frac{(\mathrm{y}-2)^2}{2\sigma^2})} \tag{5}$$
$$= \frac{1}{1 + \exp(-\frac{-2y+3}{2\sigma^2})} \tag{6}$$

$f(x) = 1/(1 + \exp(-\frac{x}{\sigma}))$ のグラフのスケールを変えて描画してみると、θの事後分布はσが小さくなるほど0か1の両極端な分布となり、大きくなるほど $\Pr(\theta = 1|\mathrm{y}) = \Pr(\theta = 2|\mathrm{y}) = 0.5$ と偏りのない分布に近づくことがわかる。

```
#%%
import numpy as np
import matplotlib.pyplot as plt

def func(x: np.ndarray, sigma: float) -> np.ndarray:
    return 1 / (1 + np.exp(-x / sigma))

#%%
x = np.linspace(-10, 10, 1000)
y1 = func(x, 0.01)
y2 = func(x, 0.1)
y3 = func(x, 10)
y4 = func(x, 100)

plt.plot(x, y1, label="sigma=0.01")
plt.plot(x, y2, label="sigma=0.1")
plt.plot(x, y3, label="sigma=10")
plt.plot(x, y4, label="sigma=100")
plt.legend()
plt.show()
```

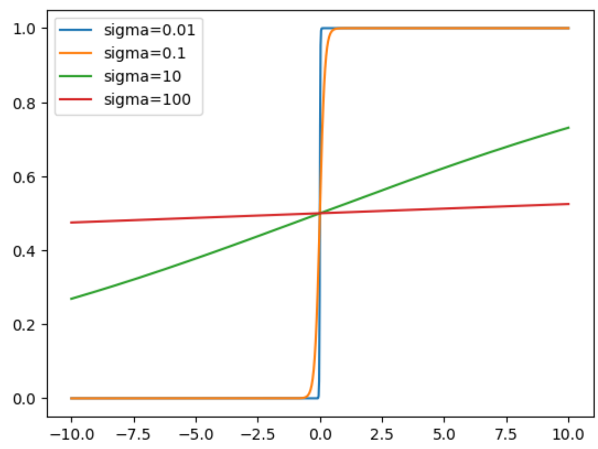
sigma=0.01
sigma=0.1
sigma=10
sigma=100
1.0
0.8
0.6
0.4
0.2
0.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

図2　1.1(c)の描画結果