Numerical Calculation of 1-d Dimensional Heat Equation by explicit method

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Heat Equation

Explicit method

$$\frac{\partial\theta}{\partial t} = \kappa\frac{\partial^2\theta}{\partial x^2} \tag{1}$$

Recurrence formula is ...

$$\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} = \kappa\frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta x)^2} \tag{2}$$

Heat equation
Explicit method

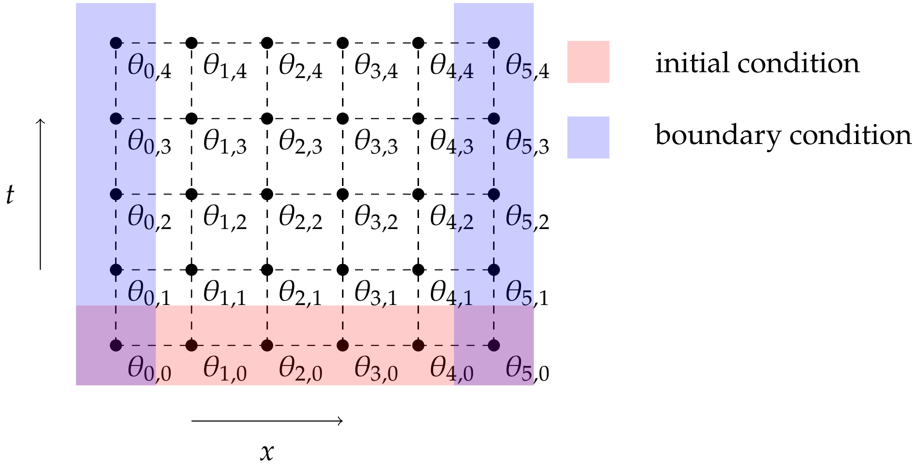
Transforming equation 2, we get

$$\theta_{i,J+i} = r\theta_{i-1,j} + (1-2r)\theta_{i,j} + r\theta_{i-1,j} \tag{3}$$

Where $r = \kappa \dfrac{\Delta t}{(\Delta x)^2}$

Heat equation

Explicit method



Heat equation

Initial condition and boundary condition

Initial condition

$\theta_{i,0}$: initial temperature of the conductor

Boundary condition

$\theta_{0,j}$, $\theta_{n,j}$: boundary temperature of the conductor (n: last element)

Heat equation

Initial condition and boundary condition

Example

Put 300°C soldering iron on left side of the metal at room temperature (20°C) , and the right side was released.

- Initial condition $\theta_{i,0}$: 20 °C
- Boundary condition (left side) $\theta_{0,j}$: 300 °C
- Boundary condition (right side) $\theta_{n,j}$: 20 °C

Heat equation

$j = 0$

$i = 1:$

$$\theta_{1,1} = r\theta_{0,0} + (1-2r)\theta_{1,0} + r\theta_{2,0}$$

$i = 2:$

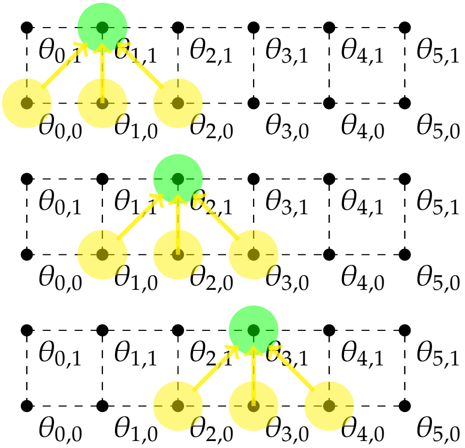
$$\theta_{2,1} = r\theta_{1,0} + (1-2r)\theta_{2,0} + r\theta_{3,0}$$

$i = 3:$

$$\theta_{3,1} = r\theta_{2,0} + (1-2r)\theta_{3,0} + r\theta_{4,0}$$

$i = 4:$

$$\theta_{4,1} = r\theta_{3,0} + (1-2r)\theta_{4,0} + r\theta_{5,0}$$



Heat equation

Matrix

$$
\begin{pmatrix} \boldsymbol{\theta_{0,k+1}} \\ \boldsymbol{\theta_{1,k+1}} \\ \boldsymbol{\theta_{2,k+1}} \\ \boldsymbol{\theta_{3,k+1}} \\ \vdots \\ \boldsymbol{\theta_{n,k+1}} \end{pmatrix} = \begin{pmatrix} 1-2r & r & 0 & 0 & \cdots & 0 \\ r & 1-2r & r & 0 & \cdots & 0 \\ 0 & r & 1-2r & r & \cdots & 0 \\ 0 & 0 & r & 1-2r & r & \vdots \\ \vdots & & & & \ddots & \\ 0 & \cdots & & & r & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{\theta_{0,k}} \\ \boldsymbol{\theta_{1,k}} \\ \boldsymbol{\theta_{2,k}} \\ \boldsymbol{\theta_{3,k}} \\ \vdots \\ \boldsymbol{\theta_{n,k}} \end{pmatrix} \tag{4}
$$

Calculate temperature at next step using temperature at current step