Magnitude Homology of Graphs and the Magnitude as its Categorification

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１．研究背景・動機

The concept of magnitude is introduced by Leinster and it is defined for enriched categories of finite objects, for example, generalized finite metric spaces such as finite graphs. Then, Leinster focuses on the magnitude of graphs in using his idea of magnitude of a metric space, which is one of a family of cardinality-like invariants extending across mathematics; it is a cousin to Euler characteristic and geometric measure. Among its cardinality-like properties are multiplicativity with respect to cartesian product and an inclusion-exclusion formula for the magnitude of a union under mild hypotheses. Formally, the magnitude of a graph is both a rational function over $\mathbb{Q}$ and a power series over $\mathbb{Z}$.

Richard and Simon introduced a bigraded homology theory for graphs which has the magnitude as its graded Euler characteristic and showed how properties of magnitude proved by Leinster categorify to properties such as a Kunneth Theorem and a Mayer-Vietoris Theorem.

Here, we first review the definition of the magnitude of graphs, the magnitude homology of graphs, and their properties. Then we focus on the magnitude of enriched categories and discuss how the magnitude of graphs is introduced from that of enriched categories. We denote the magnitude of a graph G by $\#G$ and the magnitude homology of G by $\mathrm{MH}_{*,*}(G)$.

２．主結果

The first main result is a certain property that we would like graph invariants to satisfy, the inclusion-exclusion formula;

$$\#(G \cup H) = \#G + \#H - \#(G \cap H).$$

For this we must impose some hypotheses. Indeed, Leinster shows that there is no nontrivial graph invariant that is fully cardinality-like in the sense of satisfying both multiplication and inclusion-exclusion formula without restriction. However, the hypotheses we impose are mild enough to cover a wide range of examples, including trees, forests, wedge sums, and graphs containing a cycle of length at least 4. For example, let G be a graph shown below.

The second main result is confirming that the magnitude defined in the context of enriched categories coincides with that defined in the context of graphs.

$\#G$ は monoidal category で $\mid x \mid = e^x = q$ としたもの...

３．意義・証明のアイデアや方法

主結果の意義，証明 (アイデアや方法) を説明してください．とくに，新しいアイデアや，証明のポイントを簡潔に説明してください．また，得られた主結果にどのような意義があるのか，専門外の人にわかるように，明確に説明してください．○○○

４．今後の課題

今後，研究を続けるとしたら，どのような課題があるのか，また，どのような問題を研究したいか，説明してください．さらに，研究を発展させる計画や，予想などがあれば，明確に説明してください．○○○

参考文献

[1] 著者名，論文題目，雑誌名，巻，号 (年号)，ページ番号.
[2] 著者名，論文題目，雑誌名，巻，号 (年号)，ページ番号.