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Magnitude Homology of Graphs and the Magnitude as its Categorification

2264257 Kensho Yachi

https://taro-ken.com

Supervisor : Yuta Nozaki Associate Professor

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Abstract

Sample Abstract

Contents

1 Introduction

Lamport's guide to LaTeX [1].

2 The definition

In this section, we define the magnitude of a graph G and the magnitude homology of a graph G, give some very basic examples and properties. By a *graph* we mean a finite undirected graph with no loops or multiple edges. The set of vertices of a graph G is denoted by $V(G)$, and the set of edges of G is denoted by $E(G)$. If x and y are vertices of a graph G, then the *distance* $d_G(x, y)$ between x and y is defined to be the length of a shortest edge path from x to y. If x and y lie in different components of G then $d(x, y) = \infty$.

2.1 The Definition of The Magnitude and Some Examples, Properties

Here, we difine the magnitude of a graph, which can be expressed as either a rational function over $\mathbb{Q}$ or a formal power series over $\mathbb{Z}$. Write $\mathbb{Z}[q]$ for the polynomial ring over the integers in one variable q.

Definition 2.1.1. Let G be a graph. Let $Z_G = Z_G(q)$ be the square matrix over $\mathbb{Z}[q]$ whose rows and columns are indexed by the vertices of G, and whose (x, y)-entry is given by

$$Z_G(q)(x, y) = q^{d(x,y)} \quad (x, y \in V(G))$$

where by convention $q^\infty = 0$.

2.2 The Definition of The Magnitude Homology and Some Examples, Properties

Magnitude homology

References

[1] Leslie Lamport. *LaTeX: A Document Preparation System*. Addison-Wesley, 2nd edition, 1994.

[2] Donald E. Knuth. *The TeXbook*. Addison-Wesley, 1984.