Introduction of three-dimensionally expanded wave function

In the beginning, I propose to expand the wave function into three-dimensional. Due to this operation, we can express the width of a photon's orbital in it. The following is the method.

Firstly, we think how to rotate the wave function φ degrees (Fig. 2a). If we can find the value of point ψ', we can expand the wave function into three-dimensional since the point ψ is the wave function.

$$\psi = \mathrm{R}(\cos\theta + i\sin\theta)\ (\theta = \omega t - k_x x - k_y y - k_z z) \quad (1)$$

$$\begin{bmatrix} \mathrm{R}:\ \text{the existence probability of a photon,} \\ \omega:\ \text{the number of vibrations, } k_x, k_y, k_z: \text{the wave number,} \\ t: \text{the measurable time variable, } x,\ y,\ z: \text{the measurable space variables} \end{bmatrix}$$

Well, let us draw a perpendicular from ψ' to $\mathrm{O}\psi$ (origin at the point O). The length is the $\tilde{z}$-coordinate of ψ' and the coordinates of the intersection ψ'' are the $\tilde{x}$- and $\tilde{y}$ -coordinate of ψ' (Fig. 2b, Fig. 2c). Please note, however, that $\tilde{x}$, $\tilde{y}$ and $\tilde{z}$ are unmeasurable space variables in the spherical coordinate system.

$$\psi' = (\tilde{x},\ \tilde{y},\ \tilde{z}) = (\mathrm{R}\cos\varphi\cos\theta,\ \mathrm{R}\cos\varphi\sin\theta,\ \mathrm{R}\sin\varphi) \quad (2)$$

Secondly, we should find those numbers which take the place of an imaginary number i. In the polar form of a complex number ψ, the imaginary number i means that the existence probability R rotates through θ degrees about the origin. Based on hints from this fact, I propose to use Pauli matrices.

$$\psi' = \mathrm{R}\cos\varphi\cos\theta\sigma_{\tilde{x}} + \mathrm{R}\cos\varphi\sin\theta\sigma_{\tilde{y}} + \mathrm{R}\sin\varphi\sigma_{\tilde{z}}\ (\varphi = \omega t - k_x x - k_y y - k_z z) \quad (3)$$

$$\left[\theta:\ \text{the width of a photon's orbital, } \sigma_{\tilde{x}},\ \sigma_{\tilde{y}},\ \sigma_{\tilde{z}}:\ \text{the Pauli matrix}\right]$$

For the sake of simplicity, let us suppose that each Pauli matrix is handled as a number and they belong to non-Abelian group. This is hereinafter referred to as the Pauli coordinate system.

$$\begin{aligned} \left(\sigma_{\tilde{x}}\right)^2 &= 1, \\ \left(\sigma_{\tilde{y}}\right)^2 &= 1, \\ \left(\sigma_{\tilde{z}}\right)^2 &= 1 \end{aligned}$$

$$\begin{aligned} \sigma_{\tilde{x}}\sigma_{\tilde{y}} + \sigma_{\tilde{y}}\sigma_{\tilde{x}} &= 0, \\ \sigma_{\tilde{y}}\sigma_{\tilde{z}} + \sigma_{\tilde{z}}\sigma_{\tilde{y}} &= 0, \\ \sigma_{\tilde{x}}\sigma_{\tilde{z}} + \sigma_{\tilde{z}}\sigma_{\tilde{x}} &= 0 \end{aligned}$$

If we calculate with attention to the direction, the square of ψ' named "dice function

$\Xi(\mathrm{R},\ \varphi,\ \theta)$" is its own absolute value.

$$\Xi^2(\mathrm{R},\ \varphi,\ \theta) = \left(\mathrm{R}\cos\varphi\cos\theta\sigma_{\tilde{x}} + \mathrm{R}\cos\varphi\sin\theta\sigma_{\tilde{y}} + \mathrm{R}\sin\varphi\sigma_{\tilde{z}}\right)^2$$

$$\begin{aligned}
&= \mathrm{R}^2\cos^2\varphi\cos^2\theta\left(\sigma_{\tilde{x}}\right)^2 + \mathrm{R}^2\cos^2\varphi\sin^2\theta\left(\sigma_{\tilde{y}}\right)^2 + \mathrm{R}^2\sin^2\varphi\left(\sigma_{\tilde{z}}\right)^2 \\
&\quad + \mathrm{R}^2\cos^2\varphi\cos\theta\sin\theta\left(\sigma_{\tilde{x}}\sigma_{\tilde{y}} + \sigma_{\tilde{y}}\sigma_{\tilde{x}}\right) \\
&\quad + \mathrm{R}^2\cos\varphi\sin\varphi\cos\theta(\sigma_{\tilde{x}}\sigma_{\tilde{z}} + \sigma_{\tilde{z}}\sigma_{\tilde{x}}) \\
&\quad + \mathrm{R}^2\cos\varphi\sin\varphi\sin\theta(\sigma_{\tilde{y}}\sigma_{\tilde{z}} + \sigma_{\tilde{z}}\sigma_{\tilde{y}}) \\
&= \mathrm{R}^2\cos^2\varphi\cos^2\theta + \mathrm{R}^2\cos^2\varphi\sin^2\theta + \mathrm{R}^2\sin^2\varphi \\
&= \mathrm{R}^2\cos^2\varphi(\cos^2\theta + \sin^2\theta) + \mathrm{R}^2\sin^2\varphi \\
&= \mathrm{R}^2(\cos^2\varphi + \sin^2\varphi) \\
&= \mathrm{R}^2 \\
&= \left|\Xi(\mathrm{R},\ \varphi,\ \theta)\right|^2
\end{aligned}$$

$$(\because\ \left|\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)\right| = \sqrt{\left(\mathrm{R}\cos\varphi\cos\theta\right)^2 + \left(\mathrm{R}\cos\varphi\sin\theta\right)^2 + \left(\mathrm{R}\sin\varphi\right)^2})$$

Lastly, we should derive the relative equation $-k_x^2 - k_y^2 - k_z^2 + \frac{\omega^2}{c^2} = \frac{m^2c^2}{\hbar^2}$ from the dice function. For this purpose, let us introduce a new wave equation.

$$\frac{\partial^2\Xi(\mathrm{R},\ \varphi,\ \theta)}{\partial x^2} + \frac{\partial^2\Xi(\mathrm{R},\ \varphi,\ \theta)}{\partial y^2} + \frac{\partial^2\Xi(\mathrm{R},\ \varphi,\ \theta)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial t^2} = \frac{m^2c^2}{\hbar^2}\Xi\left(\mathrm{R},\ \varphi,\ \theta\right) \tag{4}$$

$$\left[m\text{: the mass of a particle, } c\text{: the velocity of light, } h\text{:the Planck constant } (\hbar = \frac{h}{2\pi})\right]$$

If we substitute equation (3) for $\Xi(\mathrm{R},\ \varphi,\ \theta)$ in equation (4), we can confirm that the wave equation was defined quantum-mechanically.

$$\frac{\partial\varphi}{\partial x}\frac{\partial\varphi}{\partial x}\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial\varphi^2} + \frac{\partial\varphi}{\partial y}\frac{\partial\varphi}{\partial y}\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial\varphi^2} + \frac{\partial\varphi}{\partial z}\frac{\partial\varphi}{\partial z}\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial\varphi^2} - \frac{1}{c^2}\frac{\partial\varphi}{\partial t}\frac{\partial\varphi}{\partial t}\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial\varphi^2} = \frac{m^2c^2}{\hbar^2}\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)$$

$$k_x^2\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial\varphi^2} + k_y^2\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial\varphi^2} + k_z^2\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial\varphi^2} - \frac{\omega^2}{c^2}\frac{\partial^2\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)}{\partial\varphi^2} = \frac{m^2c^2}{\hbar^2}\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)$$

$$(-k_x^2 - k_y^2 - k_z^2 + \frac{\omega^2}{c^2})\Xi\left(\mathrm{R},\ \varphi,\ \theta\right) = \frac{m^2c^2}{\hbar^2}\Xi\left(\mathrm{R},\ \varphi,\ \theta\right)$$

$$\therefore -k_x^2 - k_y^2 - k_z^2 + \frac{\omega^2}{c^2} = \frac{m^2c^2}{\hbar^2}$$