Chapter 1

Introduction

In this chapter we introduce two of the most famous problems in analyzing the frictional force as problems of statistical mechanics. We give recent developments for solving these problems. We then pose another problem regarding *manipulation* of the sliding friction which occurs in highly lubricated solids. To this end we simplify our problem into a dimensional crossover in lattice systems.

The sliding friction in solids is a very complicated problem to analyze, despite the fact that our daily lives are linked with it in various forms. One reason of the difficulty is that there is no general theory which determines the most important microscopic degree of freedom to describe the macroscopic phenomenon of the sliding friction.

One may think that with the skill of statistical mechanics we can deal with the problem in a systematic manner. However, there are several essential problems including the following: (i) The sliding friction is essentially a non-equilibrium phenomenon (see Section 1.1); (ii) We cannot directly observe the sliding surface (see Section 1.2).

1.1 Sliding Frictions as Non-Equilibrium Problems

We can regard the sliding friction as the following elementary problem. We consider an object O and a substrate S, and let S slide against O by applying an external force $f_{\rm ext}$ to O. When O and S interact with each other, the kinetic energy of O given by the external force is expected to be lost through the interaction, and then the entire system $O + S$ heats up (if the system is closed) or the energy dissipates from the system to an external environment (if the system is open). In the latter case, when we control the external force $f_{\rm ext}$ to balance it with the frictional force $f_{\rm fric}$ the sliding velocity v becomes constant and the dissipation process becomes stationary. Then the frictional force $f_{\rm fric}$ can be considered as a function of the sliding velocity.

The standard method of the statistical mechanics called the linear-response theory appears to solve the problem if the velocity is much smaller than the rate ξ/τ, where ξ and τ are the characteristic length and time of the system, respectively. However, we already know well that the static frictional force is non-zero for several systems, for which the frictional force has a non-linear velocity dependence. This shows us that the complexity of the problem is beyond the linear-response theory.

Several model calculations and experiments have solved these problems, directlyCite! or indirectlyCite!. Phononic contributions to sliding frictions were analyzed with the Frenkel-Kontorova model [1–7] and simulated with cold atoms [8–10], whereas eletronic contributions were discovered [11] and explained within an image potential theory [12]. As a quantum phenomena, sliding friction between adsorbed helium atoms and its substrates [13–15] is recently getting attention, but their theoretical description is still not satisfactory and remains to be an open problem.

1.2 Impossibility of the Observation of the Sliding Surface

The dimensionality of the sliding surface is up to two, if that of the whole system is three. Sliding surfaces are different in many ways from the standard two-dimensional surfaces on three-dimensional solids which have been investigated for many years. In particular we cannot perform a direct observation of the sliding surface by apparatuses such as microscopes. The difficulty prevents us from clarifying non-equilibrium properties of the sliding friction.

The frictional-force microscope have played important roles is investigating such sliding surfaces. This apparatus sees the roughness of the substrate by observing frictional force on its tips, by which we can recover the macroscopic frictional force as its integration over the area of surface. By the use of frictional-force microscopes a local lateral force was measured by Ternes *et al.* [16] and *superlubricity* was observed by Urbakh [17] as an intriguing phenomena that the frictional force is strongly dependent on the sliding direction and almost vanishes in several directions.

1.3 Manipulating the Friction

As mentioned above, recent researches well revealed the nature of sliding frictions. This gave rise to new problems about the friction in atomically microscopic systems.

Ordinary frictions in solids are mostly governed by excitations of phonon degrees of freedom, because the sliding surface is almost always rougher than the scale of the atom. Once we lubricate the sliding surface highly, other degrees of freedom such as the orbital and the spin angular momenta of electrons emerge as additional contributions to the friction.

We are already familiar with the most remarkable example of such a system in our daily

lives, namely *micro electric mechanical systems* (**MEMS**). MEMS plays important roles in the printing head of inkjet printers, the accelerometer in smartphones and so on. MEMS has clean planes sliding with an accuracy of a micrometer or a nanometer [18–20], because of which they experience the friction with many kinds of degrees of freedom including not only phonon excitations but also other contributionsConcrete Exmaples!. The small size of these systems results in a large friction, which lessens the working efficiency or disables the operation of MEMS, because the ratio of the surface area over the volume of the system becomes larger with the smaller size and a fixed geometry. In order to tackle with the issue of manipulation of the friction in small systems, we need to obtain more fundamental knowledge of the friction.

1.4 Magnetic Friction

The way to manipulate the friction in small systems is less understood than its nature. We here consider manipulation of magnetic materials by numerically simulating lattice models.

The nature of magnetic frictions was first revealed by Kadau *et al.* [21] by the use of the Ising model. Just after it, Fusco *et al.* investigated magnetic frictions in Heisenberg spin systems and a moving dipole moment on them. A crossover of the velocity dependence on the magnetic frictional force was revealed by Magiera *et al.* [22]. A direct observation of magnetic frictions by a scanning probe microscopy was then performed by Wolter *et al.* [23].

In the present work we consider two strips of the quasi-one-dimensional Ising model sliding against each other with a fixed velocity. We discuss the difference of the frictional forces for the two boundary conditions, the anti-parallel and the parallel, and its dependence on the distance between the two boundaries.

Based on the results in Ref. [24], we consider a dimensional crossover from one dimension to two in Ising models with two fixed boundary conditions. In the one-dimensional limit

the boundary conditions seem to have the maximum effect on the friction, whereas in the two-dimensional limit there seems to be no effects. Behaviors in the both limits for the free boundary conditions correspond to the results in Ref. [24]. We find a method of manipulating the friction using different boundary conditions. These boundary conditions can be realized in experiments by aligning the boundary spins of sliding magnets.

Frictions in such models were considered first in a numerical study by Kadau *et al.* [21]. They revealed by Monte Carlo simulations that two square lattices of the Ising model which slide against each other experience the friction, depending on the temperature and the sliding velocity. Immediately after the research, it was revealed [24] that the Ising model goes under a non-trivial non-equilibrium phase transition (**NEPT**) in the high-velocity limit, where the two square lattices are decoupled in terms of the correlation between the two lattices and feel an effective mean field depending on the magnetization of each other [24]. This analysis showed that a novel critical point which is located at a generally higher temperature than the ordinal critical point for models in arbitrary dimensions and geometries (see Chapter 2). They also developed a new dynamics [24] which enables an analytical treatment for finite sliding velocities, obtaining a non-equilibrium critical line.

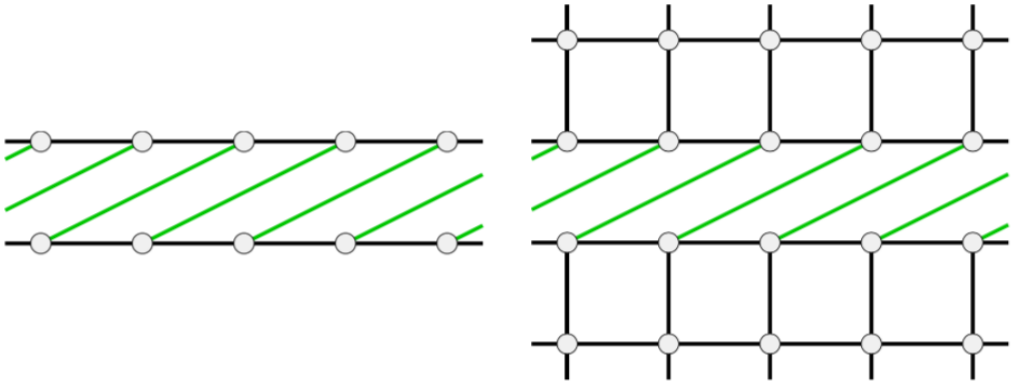
Chapter 2

Velocity-driven Non-equilibrium Phase Transition in Ising Models

To discuss the non-equilibrium crossover between two different dimensionalities, we make a brief review of the exact results [24] by Hucht. His analysis is based on the fact that two Ising cylinders with relative motion make a novel mean field, which leads the system to a non-trivial phase transition.

Let us consider two equivalent square lattices of the Ising model each of which contacts the other by one of its one-dimensional boundaries (see Fig. 2.1). We make one lattice slide along the contact plane against the other lattice with a constant velocity v. The entire system thereby goes into a non-equilibrium stationary state instead of equilibration. The non-equilibrium stationary state well describes the behavior of two magnetic materials with a friction. This setup is explained in detail in Chapter 3.

As a well known fact, the ordinary two-dimensional Ising model has an equilibrium phase transition at the critical temperature $T_{\mathrm{c,eq}} = 2/(\log\left[1+\sqrt{2}\right])$ in the thermodynamical limit. The system with the friction becomes equivalent to the equilibrium case in the limit of $v \to 0$. In



(a) (b)

Figure 2.1: Sketches of the models considered in Ref. [24]. Both cases depict a schematic view after the sliding by twice a lattice constant. (a) Two chains of the one-dimensional model. (b) Two lattices of the two-dimensional model.

addition to the ordinary phase transition, a novel phase transition in which the magnetization grows on the sliding boundary (see Fig. 2.2). Now we denote the velocity dependent non-equilibrium critical point by $T_{\mathrm{c}}(v)$ apart from the equilibrium critical point $T_{\mathrm{eq,c}}$. Hucht [24] claims that the critical temperature $T_{\mathrm{c}}(v)$ *deviates* from $T_{\mathrm{c,eq}}$ at the point $v = 0$ towards the limit $v = \infty$.

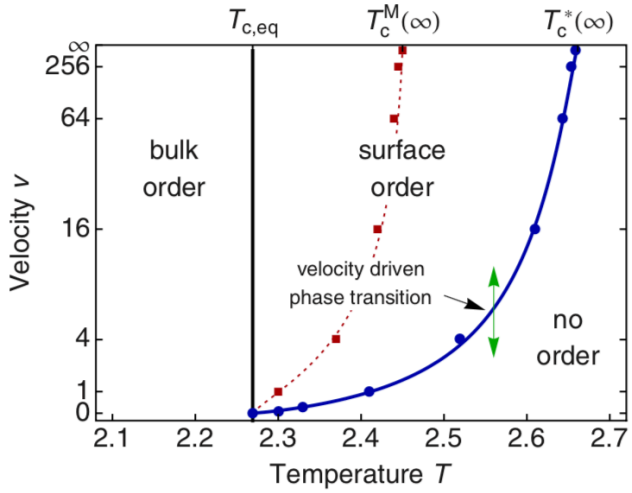


Figure 2.2: The phase diagram of the two-dimensional non-equilibrium Ising model [24]. The black solid line, the red dashed line and the blue solid line indicate the ordinary bulk phase transition, a non-equilibrium boundary phase transition for the Metropolis rate and the multiplicative rate, respectively. From right to left across the non-equilibrium phase boundary the system acquires non-zero expectation value of magnetization on the sliding boundary.

This phenomenon wes first reported in the numerical results [21] by Kadau *et al.* using Monte

Carlo simulations both on the Metropolis and the Glauber algorithms in a two-dimensional model, and then was investigated in a more analytic manner [24] by Hucht in several dimensionalities and model geometries. One of the important points of the latter result is that in the limit $v \to \infty$ we can write down a closed exact equation for the *second* critical temperature $T_{\rm c}^*(\infty)$. It is also important that a novel algorithm called *multipricative rate* enabled us to give an equation of $T_{\rm c}^*(v)$, which depends on the flip rate and the sliding velocity $v > 0$.

If the velocity v is much less than the rate $\xi_x^{(\rm eq)}(\beta)/\tau_x^{(\rm eq)}(\beta)$, we can expect the system to behave similarly to its equilibrium state, where $\xi_x^{(\rm eq)}(\beta)$ and $\tau_x^{(\rm eq)}(\beta)$ are the correlation length along the direction parallel to the sliding surface and the correlation time, respectively, for the equilibrium state at an inverse temperature $\beta := (k_{\rm B}T)^{-1}$. This corresponds to the case in which the pumped energy by the constant sliding quickly relaxes into the heat bath and the structure of domain walls near the sliding boundary is well sustained. On the other hand, the velocity v much greater than the rate $\xi_x^{(\rm eq)}(\beta)/\tau_x^{(\rm eq)}(\beta)$ should lead the system to a stationary state far from equilibrium, so that the structure near the sliding boundary is destroyed.

In the latter case a mean field picture well describes the behavior of the system; a set of the moving spins along the contact plane act on the other set of *relatively* moving spins as a spatially averaged effective field. This view enables us to write a self-consistent equation for the temperature $T_{\rm c}(\infty)$.

We summarize the result for one-dimensional chains and two-dimensional planes in order to discuss the crossover from one dimension to two dimensions in our models in Chapter 5. We first give a general Hamiltonian of the Ising model as follows.

$$\beta\mathcal{H}_\mu := -K\sum_{i<j}\sigma_i\sigma_j - h^{\rm ext}\sum_i \sigma_i - \sum_i k_i\mu_i\sigma_i, \tag{2.1}$$

where K, $h^{\rm ext}$ and $k_i\mu_i$ denote the exchange interaction, the external field and the stochastic field on the ith spin ($\mu_i = \pm 1$), respectively. The geometry of the model is either Fig. 2.1 (a) or (b). Now we assume that μ_i obeys a probability distribution $p_i(\mu_i)$ such that $\langle \mu_i \rangle := \sum_{\mu_i=\pm 1} p_i(\mu_i)\mu_i = m_i$ for a given value of m_i. The form $p_i(\mu_i) := (1+\mu_i m_i)/2$ actually satisfies the condition.

If we decompose the Hamiltonian into the contribution of the stochastic field and the rest as

$$\beta\mathcal{H}_\mu = \beta\mathcal{H}_0 - \sum_i k_i\mu_i\sigma_i, \tag{2.2}$$

where

$$\beta\mathcal{H}_0 := -K\sum_{i<j}\sigma_i\sigma_j - \sum_i h_i^{\rm ext}\sigma_i, \tag{2.3}$$

the partition function of the system is written in the form

$$\mathcal{Z} = \left\langle {\rm Tr}_\sigma \left[{\rm e}^{-\beta\mathcal{H}_\mu} \right] \right\rangle = {\rm Tr}_\sigma \left[{\rm e}^{-\beta\mathcal{H}_0} \left\langle \prod_i {\rm e}^{k_i\mu_i\sigma_i} \right\rangle \right] \tag{2.4}$$

$$= \prod_j \cosh k_j \ {\rm Tr}_\sigma \left[{\rm e}^{-\beta\mathcal{H}_0} \prod_j (1+\sigma_i m_i \tanh k_i) \right]. \tag{2.5}$$

Owing to the translation invariance along the sliding direction, we assume a homogeneous boundary magnetization $m_i = m_{\rm b}$ for all boundary sites i. Each boundary magnetization acts as an effective field $h_{\rm b}$ on the other boundary magnetization.

The model therefore reduces to

$$\beta\mathcal{H}_{\rm eq} := -K\sum_{i<j}\sigma_i\sigma_j - h_{\rm b}\sum_i \sigma_i = \beta\mathcal{H}_0 - \sum_i b\sigma_i, \tag{2.6}$$

with $b := h_{\rm b} - h^{\rm ext}$, which relaxes toward the equilibrium state. Its partition function is written as

$$\mathcal{Z}_{\rm eq} = {\rm Tr}_\sigma\left[{\rm e}^{-\beta\mathcal{H}_{\rm eq}}\right] = {\rm Tr}_\sigma\left[{\rm e}^{\beta\mathcal{H}_0}\sum_i {\rm e}^{b\sigma_i}\right] \tag{2.7}$$

$$= \prod_i \cosh b\ {\rm Tr}_\sigma\left[{\rm e}^{-\beta\mathcal{H}_0}\prod_i(1+\sigma_i\tanh b)\right]. \tag{2.8}$$

Comparing the right-hand sides of Eqs. (2.5) and (2.8), we have $\mathcal{Z} \propto \mathcal{Z}_{\rm eq}$ if it holds that $\tanh b = m_{\rm b}\tanh k_i$.

Our model gives the strength of the stochastic field k_i as the exchange interaction across the sliding surface K, and thus it holds that $k_i = K$ and

$$h_{\rm b} = \tanh^{-1}\left(m_{\rm b}\tanh K\right), \tag{2.9}$$

in the limit $h^{\rm ext} \to 0$. The boundary magnetization under a static field $h_{\rm b}$ has a form of

$$m_{\rm b,eq}(K, h_{\rm b}) := \frac{\partial}{\partial h_{\rm b}}\mathcal{Z}_{\rm eq}, \tag{2.10}$$

and thus we have

$$m_{\rm b,eq}\left(K, \tanh^{-1}\left(m_b\tanh K\right)\right) = m_b \tag{2.11}$$

as a self-consistent relation for m_b. The critical point is given by

$$1 = \left. \frac{\partial m_{\mathrm{b,eq}}}{\partial m_{\mathrm{b}}} \right|_{m_{\mathrm{b}}=0}. \tag{2.12}$$

Expanding the left-hand side of Eq. (2.11) to the first order of m_{b} and using the condition (2.12) we have

$$m_{\mathrm{b,eq}}\left(K,0\right) + m_b \tanh\left(K\right) \left. \frac{\partial m_{\mathrm{b,eq}}}{\partial h_{\mathrm{b}}} \right|_{h_{\mathrm{b}}=0} = m_b. \tag{2.13}$$

With the definition of the equilibrium susceptibility $\chi_{\mathrm{b,eq}}(K) := \partial m_{\mathrm{b,eq}}/\partial h_{\mathrm{b}}|_{h_{\mathrm{b}}=0}$ and the fact $m_{\mathrm{b,eq}}\left(K,0\right) = 0$, we finally have

$$\tanh\left(K\right) \chi^{(0)}_{\mathrm{b,eq}}(K) = 1. \tag{2.14}$$

The condition (2.14) determines the non-equilibrium critical temperature $K = K_{\mathrm{c}}$. Using the expression of the magnetization for the one-dimensional Ising model and the boudnary magnetization for the two-dimensional model, we have the numerical solutions of the condition (2.14) as

$${K_{\mathrm{c}}}^{-1} = \begin{cases} 2.2691853\ldots & \text{in one dimension,} \\ 2.6614725\ldots & \text{in two dimensions.} \end{cases} \tag{2.15}$$

Note that the temperature K_{c} corresponds to the non-equilibrium critical temperature in the limit $v \to \infty$, and is consistent to the extrapolation from the results for two dimensions with the multipricative rate (see Fig. 2.2).

Chapter 3

Numerical Simulations

3.1 Setup of the Model

Sliding friction is a form of energy dissipation on the surface between a moving object and its substrate. The dissipated energy is originated in the kinetic energy of the moving object. We here consider a constantly moving case in which an external force maintains the motion of the object with endless supply of its kinetic energy. This view leads to its *non-equilibrium stationary state*. When the system is in a non-equilibrium stationary state, it is often easy to calculate *energy currents* such as the frictional heat, its power and so on. Applying the view to our case in which two square lattices of the Ising model slide against each other, we can formulate the problem as follows; see Fig. 3.1.

1. We prepare a square lattice of the Ising model of size $L_x \times L_z$ and impose periodic boundary conditions in the transverse (x) direction. We first set the system in the equilibrium state of a temperature T, whereas we set the open boundary conditions in the longitudinal (z) direction for the moment.

2. We cut the system along the x-direction into two parts, maintaining interactions on the

cut.

3. We slide two parts along the cut plane with relative velocity v. In other words, we shift the upper half by a lattice constant every $1/v$ unit time.

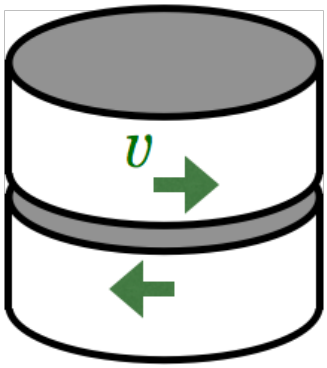


Figure 3.1: Two cylinders of the Ising model sliding with the velocity v.

The Hamiltonian of the system is given by

$$\hat{H} = \hat{H}_{\text{upper}} + \hat{H}_{\text{lower}} + \hat{H}_{\text{slip}}(t), \tag{3.1}$$

where

$$\hat{H}_{\text{upper}} := -J \sum_{\langle i,j \rangle \in \text{upper}} \hat{\sigma}_i \hat{\sigma}_j, \tag{3.2}$$

$$\hat{H}_{\text{lower}} := -J \sum_{\langle i,j \rangle \in \text{lower}} \hat{\sigma}_i \hat{\sigma}_j, \tag{3.3}$$

$$\hat{H}_{\text{slip}}(t) := -J \sum_{\langle i,j(t) \rangle \in \text{slip}} \hat{\sigma}_i \hat{\sigma}_{j(t)} \tag{3.4}$$

upper, lower and slip representing the set of interacting spin pairs on the upper half, the lower half and the slip plane, respectively. Shift operations lead the system to repeated *pumping* and *dissipation* processes as follows:

1. **Shift**: A shift operation excites the energy on the slip plane by the amount $\langle \hat{H}_{\text{slip}}(t') -$

$\hat{H}_{\rm slip}(t)\rangle_{\rm st}$. The letter t' denotes the time just after the shift operation at time t.

2. **Relax-1**: The excited energy on the slip plane $\langle \hat{H}_{\rm slip}(t') - \hat{H}_{\rm slip}(t)\rangle_{\rm st}$ dissipates to the entire system.

3. **Relax-2**: The excited entire system relaxes towards the equilibrium.

We defined the stationary state average $\langle \hat{A}\rangle_{\rm st} := \sum_i A_i p_i^{\rm (st)}$ for an arbitrary observable $\hat{A}$, where $\{A\}_i$ are eigenvalues of $\hat{A}$ and $\{p_i^{\rm (st)}\}$ is the stationary-state probability distribution, which is different from the equilibrium (canonical) probability distribution $p_i^{\rm (eq)} \propto \exp\left[-E_i/k_{\rm B}T\right]$. Note that the distribution $\{p_i^{\rm (st)}\}$ depends on the sliding velocity v.

The excited and relaxed amounts of energy per unit time correspond to the energy pumping and dissipation, respectively. The energy pumping $P(t)$ and dissipation $D(t)$ are given by

$$P(t) := \sum_{i_v=0}^{v-1} \left\langle \hat{H}_{\rm slip}\left(t' - 1 + \frac{i_v}{v}\right) - \hat{H}_{\rm slip}\left(t - 1 + \frac{i_v}{v}\right)\right\rangle_{\rm st}, \tag{3.5}$$

$$D(t) := \sum_{i_v=0}^{v-1} \left\langle \hat{H}_{\rm slip}\left(t - 1 + \frac{i_v+1}{v}\right) - \hat{H}_{\rm slip}\left(t' - 1 + \frac{i_v}{v}\right)\right\rangle_{\rm st}, \tag{3.6}$$

respectively. $P(t)$ and $D(t)$ correspond to the energy difference due to the **Shift** and the **Relax** processed, respectively. Note that absolute values of $P(t)$ and $D(t)$ become equal to each other in the non-equilibrium stationary state, by its definition.

3.2 Definitions of Physical Quantities

We now consider the case in which the system is in a non-equilibrium stationary state. We denote by $P(L_x, L_z, T)$ and $D(L_x, L_z, T)$ the long-time limit of energy pumping $P(t)$ and dissipation $D(t)$ for a system of size $L_x \times L_z$ at the temperature T. We define the frictional force

density $f(L_z, T)$ by

$$f(L_z, T) := \lim_{L_x \to \infty} \frac{F(L_x, L_z, T)}{L_x}. \tag{3.7}$$

In numerical simulations, we calculate the frictional force $F(L_x, L_z, T)$ using its power $D(L_x, L_z, T)$ by the formula

$$F(L_x, L_z, T) = \frac{D(L_x, L_z, T)}{v}. \tag{3.8}$$

We can easily verify the formula (3.8) by considering general cases in which the frictional force and its power are both time dependent. Denoting the frictional force $F(x)$ at the position x, it holds that

$$\int_{t_0}^{t_1} dt\ D(t) = \int_{x(t_0)}^{x(t_1)} dx\ F(x) = \int_{t_0}^{t_1} \frac{dx}{dt} dt\ F(x(t)) = v \int_{t_0}^{t_1} dt\ F(x(t)) \tag{3.9}$$

for a time dependent $D(t)$, because $dx/dt = v$. Under the assumption of a non-equilibrium stationary state, the integrands in both-hand sides of the relation (3.9) are still equal to each other in the long-time limit, and hence

$$D(L_x, L_z, T) = vF(L_x, L_z, T). \tag{3.10}$$

From now we call the quantity $D(L_x, L_z, T)$ the *energy dissipation*.

Our model always reaches a non-equilibrium stationary states in the long-time limit $t \to \infty$, which depends on the temperature T and the sliding velocity v; We will prove it in App. A. We use the fact that $\lim_{t\to\infty} |D(t)| = \lim_{t\to\infty} |P(t)|$ in order to estimate the average $\bar{D}$; the average

$\bar{P}$ has less statistical fluctuation [22, 25, 26]. We therefore have

$$P(L_x, L_z, T) = vF(L_x, L_z, T). \tag{3.11}$$

We also define the bulk energy density $\epsilon_{\rm b}(L_z, T)$ as follows:

$$\epsilon_{\rm b}(L_z, T) := \lim_{L_x \to \infty} \frac{E_{\rm b}(L_x, L_z, T)}{L_x L_z}, \tag{3.12}$$

where $E_{\rm b}(L_x, L_z, T)$ is the energy of the entire system. From this, we define the bulk heat capacity $c_{\rm b}(L_z, T)$ as follows:

$$c_{\rm b}(L_z, T) := \frac{\partial \epsilon_{\rm b}(L_z, T)}{\partial T}. \tag{3.13}$$

3.3 Non-equilibrium Monte Carlo Simulation

The dissipation process towards the heat bath occurs via a spin flip. This fundamental processes do not only describe equilibrium states but also non-equilibrium stationary states at a fixed temperature T [27]. Using Monte Carlo method, we simulate this process.

3.3.1 Introduction the Time Scale to Ising Models

In order to calculate dynamical observables such as the frictional power (3.11) and its dissipation rate (3.8), we have to define *a unit time* for finite size systems.

For the equilibrium Monte Carlo simulation, the most naive approach for the equilibrium state is the single-spin-flip algorithm, where we perform the sequence of a random selection of a spin and its flip with a temperature dependent probability $p(T)$. Whatever we use as

the probability for the Monte Caro simulation should satisfy a good property, called *detailed balanced condition*, which certainly leads the system towards the true equilibrium state with enough repetition of the sequence. For example, we often use the Metropolis probability $p_{\mathrm{M}}(T) := \min\{1, \mathrm{e}^{-\frac{\Delta E}{k_{\mathrm{B}}T}}\}$ as the probability $p(T)$, where ΔE is the energy difference due to the flip. We often call a *Monte Carlo step* a single process of the algorithm, and define a *Monte Carlo sweep* by N Monte Carlo steps, where N is the number of spins.

Which do we have to define a unit time by a Monte Carlo step or a Monte Carlo sweep? Its answer can be seen in the following manner: We often assume that a statistical mechanical model are coupling to a heat bath by each local degree of freedom. The temperature of the system is kept constant by the heat bath and exchanges its energy with the heat bath through local degrees of freedom. It is most natural to assume that the frequency of the exchanging process is dependent only on the temperature of the heat bath. Thus the total number of times exchanged are proportional to the number of degrees of freedom of the system. It is justified to define a unit time by a Monte Carlo sweep. Note that the more spins the system contains, the higher time resolution we can simulate with.

3.3.2 Slip Plane with the Velocity v

Using the introduced time scale, we can also introduce the slip plane with the velocity v to the system with N spins. Corresponding to the setup in Sec. 3.1, we perform an extended single-spin-flip algorithm as follows:

1. **Shift**: We shift the upper half of the lattice by a lattice constant.

2. **Flip**: We perform ordinary single flips for N/v times.

3. We repeat the processes 1 and 2 for v times.

In the extended algorithm, the upper half slides with the velocity v in a unit time at regular intervals. We proved the fact that this algorithm leads the system of any size to a non-equilibrium stationary state depending on the temperature T and the velocity v (see App. A for details).

3.3.3 Calculation Method

The observables that we are interested in are the frictional power $P(t)$ and its dissipation rate $D(t)$. In Monte Carlo simulations, they are the energy difference for a unit time due to the shift and the flip operation, respectively. Both observables have the same absolute value in the long-time limit.