Black Hole Information Paradox

An introduction

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Outline

- Hawking's three points
- Hawking radiation
- Page Curve
- Suggested resolutions

References include

- Hawking's original paper [Phys. Rev. D 14, 2460]
- Reviews by
  - Polchinski [1609.04036]
  - Raju [2012.05770]
  - Hartman [`hartmanhep.net/topics2015/gravity-lectures.pdf`]

- Hawking's paradox:
  Evolving an initial state to a final state in a BH background turns a pure state into a mixed state.
  From unitarity of quantum mechanics this is not allowed and points to information being lost.
- We will look at Hawking's arguments and then see other versions of the paradox, e.g. Page curve.

Hawking's Paradox

Intuition

- First point:
  Black holes are thermodynamic systems, with temperature, entropy etc.

"...black holes create and emit particles at a steady rate with a thermal spectrum. Because this radiation carries away energy, the black holes must presumably lose mass and eventually disappear."

- Can be seen even classically. Take Schwartzhild black hole

$$\frac{\mathrm{d}M_{BH}}{\mathrm{d}t} \propto -A\,T^{d+1} \tag{1}$$

Using dimensional analysis $m \propto r_h^{d-2}$, $T \propto r_h^{-1}$, and $A \propto r_h^{d-1}$:

$$\frac{\mathrm{d}r_h}{\mathrm{d}t} \propto -r_h^{-d+1} \tag{2}$$

- i.e. BH evaporates in time $t \propto r_h^d \propto \frac{A}{T}$

- Second point:
  We need date from both H and $\mathscr{I}^+$ to determine state initial on $\mathscr{I}^-$

- Third point:
  This is also true quantum mechanically through hawking radiation.
- For simplicity, take Scwhartzhild metric in 1+1 dimensions (i.e. suppressing angular directions), a long time after the BH was formed, so no transient effects:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}, \qquad f(r) \equiv 1 - \frac{r_h}{r} \tag{3}$$

- Transform coordinates to

$$\mathrm{d}s^2 = -f(r)\mathrm{d}u\mathrm{d}v = -\frac{4r_h^2}{r}e^{-r/r_h}\mathrm{d}U\mathrm{d}V \tag{4}$$

with

$$\begin{gathered} r_* = r + r_h \log(r - r_h), \\ u = t - r^* = -2r_h \log(-U/r_h), \quad v = t + r^* = 2r_h \log(V/r_h). \end{gathered} \tag{5}$$

- U, V are global coordinates. Smooth across horizon, so natural for an in-falling observer.
- u, v are conformally related to U, V and are only defined outside horizon.

- Then consider a massless scalar field ϕ, satisfying KG equation

$$\Box\phi = 0 \tag{6}$$

Will focus on the outgoing modes

$$\begin{aligned} \phi_{out} &= \int_0^\infty \frac{d\nu}{2\pi(2\nu)^{1/2}} \left(a_\nu e^{-i\nu U} + a_\nu^\dagger e^{i\nu U}\right) \\ &= \int_0^\infty \frac{d\omega}{2\pi(2\omega)^{1/2}} \left(b_\omega e^{-i\omega u} + b_\omega^\dagger e^{i\omega u}\right) . \end{aligned} \tag{7}$$

The nonzero canonical commutators are

$$[a_\nu, a_{\nu'}] = 2\pi\delta(\nu - \nu') , \quad [b_\omega, b_{\omega'}] = 2\pi\delta(\omega - \omega') . \tag{8}$$

- From the mode expansion

$$b_\omega = \int_0^\infty \frac{d\nu}{2\pi} \left(\alpha_{\omega\nu} a_\nu + \beta_{\omega\nu} a_\nu^\dagger \right) , \tag{9}$$

where

$$\begin{aligned} \alpha_{\omega\nu} &= 2r_h(\omega/\nu)^{1/2}(2r_h\nu)^{2ir_h\omega} e^{\pi r_h \omega}\Gamma(-2ir_h\omega) , \\ \beta_{\omega\nu} &= 2r_h(\omega/\nu)^{1/2}(2r_h\nu)^{2ir_h\omega} e^{-\pi r_h \omega}\Gamma(-2ir_h\omega) . \end{aligned} \tag{10}$$

- **Reminder:** Adiabatic Principle. An in-falling observer sees the bh modes as empty, i.e.

$$a_\nu |\psi\rangle = 0 \tag{11}$$

- It follow from this that the occupation number for the b_ν modes is given by

$$\langle\psi| b^\dagger_\omega b_{\omega'} |\psi\rangle = \frac{2\pi\delta(\omega - \omega')}{e^{\omega/T_H} - 1} \tag{12}$$

- I.e. we have a blackbody spectrum.

- If one accounts for the modes incoming modes as well (labeled by $\tilde{b}, \tilde{b}^\dagger$) one finds the final ground state

$$|\psi\rangle = \mathcal{N} \exp\left(\int_0^\infty \frac{d\omega}{2\pi}\, e^{-\omega/2T_H} b_\omega^\dagger \tilde{b}_\omega^\dagger \right) |0\rangle_{b,\tilde{b}}, \tag{13}$$

This is not a pure state!

Thought experiment.

- Start with a pure state outside the BH consisting of n entangled qubit pairs (EPR pairs).
- One part of the state goes into the BH and the other state reaching future null infinity.
- ρ_{out} is mixed and so the entropy $S_{out} \sim n \log(2)$.
- After BH is completely evaporated, the entanglement doesn't decrease because of causality and so we are left with

$$S_E \sim n \log(2) \tag{14}$$

Hawking's Paradox

Unitarity

- We went from a pure state to a mixed state under time evolution. Not coherent with quantum mechanics.

$$\rho^{final}_{mm'} = \not{S}_{mm',nn'} \rho^{initial}_{nn'} \,, \tag{15}$$

- Hawking argued that since "information ... requires energy", all the information about the initial state cannot emerge in the final stages of black hole evaporation.

Page curve

- Another way to view the paradox, using the Page curve.

Page curve

Second law of thermodynamics

- Beckenstein-Hawking entropy

$$S_{BH} = \frac{A_h}{4} \tag{16}$$

- Generalized (coarse-grained) entropy

$$S_{gen} = S_{BH} + S_{outside} \tag{17}$$

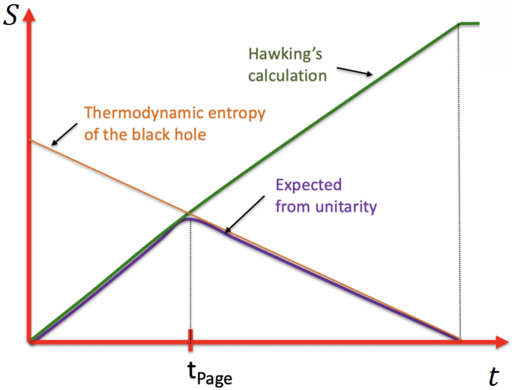
- Fine-grained entropy

$$S_{fine} = -\operatorname{Tr}[\rho \log \rho] \tag{18}$$

- Further

$$S_{coarse} \geq S_{fine} \tag{19}$$

Page curve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Hawking's
calculation
Thermodynamic entropy
of the black hole
Expected
from unitarity
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Resolutions

Three broad possibilities

- Calculation is correct and information is lost.
  This breaks quantum mechanics/unitarity.
- Hawking radiation contains all the information.
  It seems to point at our EFT breaking down.
- Remnants, i.e. BH doesn't shrink all the way, containing information that is still entangled.
  We could have started with an arbitrarily large black hole, so the number of states that must be available to this Planck-sized remnant is unbounded above

Problems

Some problems

- Calculation of ρ is not exact

$$\rho = \rho^{thermal} + \text{ loops } + \mathcal{O}\left(e^{-S}\right) \tag{20}$$

- If the number of terms in entropy is of size e^S, then these corrections could correct this and make the final state pure.
- Probably can not answer this in low energy EFT, so we need some UV theory.
- Hartman and collaborators argue that one can get a unitary entropy curve using low energy EFT.

Thank you for your attention.

$$b_\omega = R_\omega c_\omega + T_\omega \int_0^\infty \frac{d\nu}{2\pi} \left(\alpha_{\omega\nu} a_\nu + \beta_{\omega\nu} a_\nu^\dagger \right) . \tag{21}$$

c_ω are left-moving modes, coming in from spatial infinity $\mathscr{I}^-$.
R_ω is the amplitude for them to reflect before reaching the horizon and T_ω is the transmission amplitude, with $|R_\omega|^2 + |T_\omega|^2 = 1$.

$$\langle \psi | b_\omega^\dagger b_{\omega'} | \psi \rangle = |T_\omega|^2 \frac{2\pi\delta(\omega - \omega')}{e^{\omega/T_H} - 1} . \tag{22}$$

Types of entropy

- Von Neumann/fine-grained entropy

$$S(\rho) = -\operatorname{Tr}(\rho \log \rho) \tag{23}$$

  Vanishes in a pure state and constant under unitary evolution.

- Coarse-grained entropy. Measure a subset of observables. Consider all density matrices which gives the same result as ρ for our observables, and then maximize over the entropy

$$S(\tilde{\rho}; E, \dots) = \max_{\tilde{\rho}; E, \cdots = \rho} [S(\tilde{\rho})] \geq S(\rho) \tag{24}$$

- Captured in spirit by

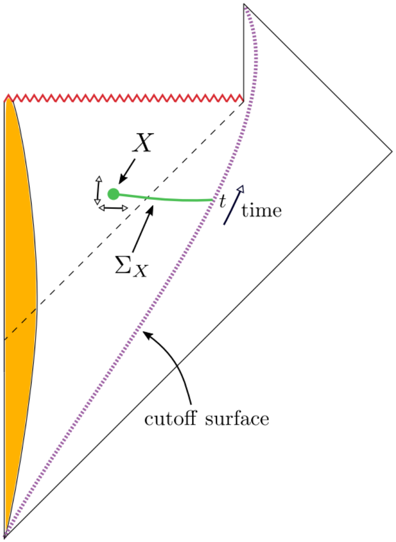
$$S \sim \min[\frac{A_h}{4} + S_{outside}] \tag{25}$$

- Rather, take a Cauchy spatial slice and find a minimal surface. Then take the maximum of all the slices found, if there are multiple. Then

$$S = \min_X[\text{ext}_X[\frac{A(X)}{4} + S_{semi-cl}(\Sigma_X)]] \tag{26}$$

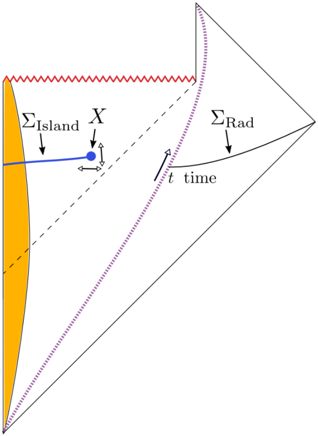
Page curve

Second law of thermodynamics



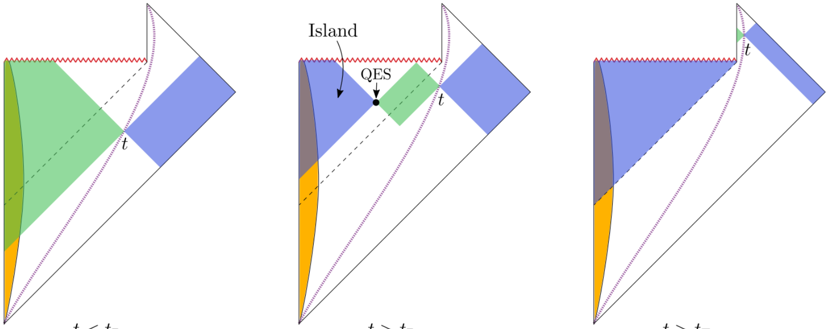
Page curve

Second law of thermodynamics



Page curve

Second law of thermodynamics



$t < t_{\text{Page}}$
(a)

$t > t_{\text{Page}}$
(b)

$t > t_{\text{Evap}}$
(c)

$$S = \min_X[\mathsf{ext}_X[\frac{A(X)}{4} + S_{semi-cl}(\Sigma_{Rad} \cup \Sigma_{Island})]] \tag{27}$$