Notes concerning article

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1 Role of γ

1.1 Introduction

In the article the chiral mass χ transformed as $\chi \to \lambda^y \chi$. The exponent y is related to the mass anomalous dimension γ_m which we will show here.

We will work with the renormalized and bare mass m_r and m_0. the renormalization scale is μ. The renormalized mass is defined through

$$m_0(\lambda) \equiv m_r(\lambda) Z_m(\epsilon, g_r) \tag{1.1}$$

The mass anamolous dimension γ_m is defined by

$$-\gamma_m \equiv \frac{\partial \log m_r(\lambda)}{\partial \log \mu} = -\frac{\partial \log Z_m}{\partial \log \mu} = \frac{\partial \log Z_m}{\partial \log \mu_0} \tag{1.2}$$

where $Z_m = Z_m(\frac{\mu}{\mu_0})$. The bare mass and renormalization scale are

$$m_0(\lambda) = \lambda m_0, \quad \mu_0(\lambda) = \lambda \mu_0$$

We then want to find how $m_r(\lambda)$ transforms. Differentiating equation 1.1 wrt $\log \lambda$ gives

$$\frac{\partial m_0(\lambda)}{\partial \log \lambda} = \frac{\partial}{\partial \log \lambda}(m_r(\lambda) Z_m) = Z_m \frac{\partial m_r(\lambda)}{\partial \log \lambda} + m_r(\lambda) \frac{\partial Z_m}{\partial \log \lambda}$$

Dividing through by Z_m we get

$$\frac{1}{Z_m} \frac{\partial m_0(\lambda)}{\partial \log \lambda} = \frac{\partial m_r(\lambda)}{\partial \log \lambda} + \frac{m_r(\lambda)}{Z_m} \frac{\partial Z_m}{\partial \mu_0} \frac{\partial \mu_0}{\partial \log \lambda} \tag{1.3}$$

Now both the m and μ_0 derivatives go as:

$$\frac{\partial m_0(\lambda)}{\partial \log \lambda} = \frac{\partial m_0(\lambda)}{\partial \lambda} \left(\frac{\partial \log \lambda}{\partial \lambda} \right)^{-1} = m_0 \lambda = m_0(\lambda)$$

Using this on both sides of 1.3 we are left with

$$\frac{m_0(\lambda)}{Z_m} = \frac{\partial m_r(\lambda)}{\partial \log \lambda} + \frac{m_r(\lambda)}{Z_m} \frac{\partial Z_m}{\partial \mu_0} \mu_0(\lambda) \tag{1.4}$$

We want something of the form in 1.2, so the Z derivative is rewritten to include logarithms:

$$\frac{\partial Z_m}{\partial \mu_0} = \frac{\partial \log Z_m}{\partial \log \mu_0} \frac{\partial \log \mu_0}{\partial \mu_0} \left(\frac{\partial \log Z_m}{\partial Z_m} \right)^{-1} = -\gamma_m \frac{Z_m}{\mu_0}$$

By plugging this into 1.4 and using the relation between the renormalized and bare masses we get the differentialequation

$$\frac{m_0(\lambda)}{Z_m} = m_r(\lambda) = \frac{\partial m_r(\lambda)}{\partial \log \lambda} - \gamma_m m_r(\lambda)$$

Which can be rewritten in the simpler form:

$$\frac{\partial m_r(\lambda)}{\partial \log \lambda} = (1 + \gamma_m) m_r(\lambda) \tag{1.5}$$

Solving this is done by rewriting the logarithmic derivative

$$\frac{\partial m_r(\lambda)}{\partial \lambda} = \frac{(1 + \gamma_m)}{\lambda} m_r(\lambda) \Rightarrow m_r(\lambda) = \lambda^{1+\gamma_m} m_r \tag{1.6}$$

which means that the the renormalized mass transforms as

$$m_r \to \lambda^{1+\gamma_m} m_r \tag{1.7}$$