# Notes concerning article

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# 1 Role of $\gamma$

## 1.1 Long version

In the article the chiral mass $\chi$ transformed as $\chi \to \lambda^y \chi$. The exponent $y$ is related to the mass anomalous dimension $\gamma_m$ which we will show here.

We will work with the renormalized and bare mass $m_r$ and $m_0$. the renormalization scale is $\mu$. The renormalized mass is defined through

$$m_0(\lambda) \equiv m_r(\lambda) Z_m(\epsilon, g_r) \tag{1.1}$$

The mass anamolous dimension $\gamma_m$ is defined by

$$-\gamma_m \equiv \frac{\partial \log m_r(\lambda)}{\partial \log \mu} = -\frac{\partial \log Z_m}{\partial \log \mu} = \frac{\partial \log Z_m}{\partial \log \mu_0} \tag{1.2}$$

where $Z_m = Z_m(\frac{\mu}{\mu_0})$. The bare mass and renormalization scale are

$$m_0(\lambda) = \lambda m_0, \quad \mu_0(\lambda) = \lambda \mu_0$$

We then want to find how $m_r(\lambda)$ transforms. Differentiating equation 1.1 wrt $\log \lambda$ gives

$$\frac{\partial m_0(\lambda)}{\partial \log \lambda} = \frac{\partial}{\partial \log \lambda}(m_r(\lambda) Z_m) = Z_m \frac{\partial m_r(\lambda)}{\partial \log \lambda} + m_r(\lambda) \frac{\partial Z_m}{\partial \log \lambda}$$

Dividing through by $Z_m$ we get

$$\frac{1}{Z_m}\frac{\partial m_0(\lambda)}{\partial \log \lambda} = \frac{\partial m_r(\lambda)}{\partial \log \lambda} + \frac{m_r(\lambda)}{Z_m}\frac{\partial Z_m}{\partial \mu_0}\frac{\partial \mu_0}{\partial \log \lambda} \tag{1.3}$$

Both the $m$ and $\mu_0$ logarithmic derivatives go as:

$$\frac{\partial m_0(\lambda)}{\partial \log \lambda} = \frac{\partial m_0(\lambda)}{\partial \lambda}\left(\frac{\partial \log \lambda}{\partial \lambda}\right)^{-1} = m_0 \lambda = m_0(\lambda)$$

Using this on both sides of 1.3 we are left with

$$\frac{m_0(\lambda)}{Z_m} = \frac{\partial m_r(\lambda)}{\partial \log \lambda} + \frac{m_r(\lambda)}{Z_m}\frac{\partial Z_m}{\partial \mu_0}\mu_0(\lambda) \tag{1.4}$$

We want something of the form in 1.2, so the $Z$ derivative is rewritten to include logarithms:

$$\frac{\partial Z_m}{\partial \mu_0} = \frac{\partial \log Z_m}{\partial \log \mu_0}\frac{\partial \log \mu_0}{\partial \mu_0}\left(\frac{\partial \log Z_m}{\partial Z_m}\right)^{-1} = -\gamma_m \frac{Z_m}{\mu_0}$$

By plugging this into 1.4 and using the relation between the renormalized and bare masses we get the differentialequation

$$\frac{m_0(\lambda)}{Z_m} = m_r(\lambda) = \frac{\partial m_r(\lambda)}{\partial \log \lambda} - \gamma_m m_r(\lambda)$$

Which can be rewritten in the simpler form:

$$\frac{\partial m_r(\lambda)}{\partial \log \lambda} = (1+\gamma_m) m_r(\lambda) \tag{1.5}$$

Solving this is done by rewriting the logarithmic derivative

$$\frac{\partial m_r(\lambda)}{\partial \lambda} = \frac{(1+\gamma_m)}{\lambda} m_r(\lambda) \Rightarrow m_r(\lambda) = \lambda^{1+\gamma_m} m_r \tag{1.6}$$

which means that the the renormalized mass transforms as

$$m_r \to \lambda^{1+\gamma_m} m_r \tag{1.7}$$

## 1.2 Short version

By using a renormalization scale of $\mu$ and the usual way of defining a renormalized mass $m_r = \frac{1}{Z_m} m_0$ we define the mass anomalous dimension by

$$\gamma_m \equiv \frac{\partial \log m_r}{\partial \mu} \tag{1.8}$$

If the bare mass transforms like its canonical dimension according to

$$m \rightarrow \lambda m$$

this leads to the renormalized mass transforming as

$$m_r \rightarrow \lambda^{1+\gamma_m} m_r$$

since it also tranforms as

$$m_r \rightarrow \lambda^{4-y} m_r$$

$\gamma_m$ is related to $y$ by

$$y = 3 - \gamma_m \tag{1.9}$$