Notes concerning article

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1 Role of γ

1.1 Introduction

μ is the renormalization scale,

$$m_0\left(\lambda\right) \equiv m_r\left(\lambda\right) Z_m(\epsilon, g_r) \tag{1.1}$$

RG equations

$$-\gamma_m \equiv \frac{\partial \log m_r\left(\lambda\right)}{\partial \log \mu} = -\frac{\partial \log Z_m}{\partial \log \mu} = \frac{\partial \log Z_m}{\partial \log \mu_0} \tag{1.2}$$

$$m_0(\lambda) = \lambda m_0, \quad \mu_0(\lambda) = \lambda \mu_0$$

We want to find how $m_r(\lambda)$ transforms. Differentiating equation 1.1

$$\frac{\partial m_0\left(\lambda\right)}{\partial \log \lambda} = \frac{\partial}{\partial \log \lambda}\left(m_r\left(\lambda\right) Z_m\right) = Z_m \frac{\partial m_r\left(\lambda\right)}{\partial \log \lambda} + m_r\left(\lambda\right) \frac{\partial Z_m}{\partial \log \lambda}$$

$$\frac{1}{Z_m}\frac{\partial m_0\left(\lambda\right)}{\partial \log \lambda} = \frac{\partial m_r\left(\lambda\right)}{\partial \log \lambda} + \frac{m_r\left(\lambda\right)}{Z_m}\frac{\partial Z_m}{\partial \mu_0}\frac{\partial \mu_0}{\partial \log \lambda}$$

$$\frac{\partial m_0(\lambda)}{\partial \log \lambda} = \frac{\partial m_0(\lambda)}{\partial \lambda}\left(\frac{\partial \log \lambda}{\partial \lambda}\right)^{-1} = m_0 \lambda = m_0(\lambda)$$

$$\frac{m_0(\lambda)}{Z_m} = \frac{\partial m_r\left(\lambda\right)}{\partial \log \lambda} + \frac{m_r\left(\lambda\right)}{Z_m}\frac{\partial Z_m}{\partial \mu_0}\mu_0(\lambda)$$

$$\frac{\partial Z_m}{\partial \mu_0} = \frac{\partial \log Z_m}{\partial \log \mu_0}\frac{\partial \log \mu_0}{\partial \mu_0}\left(\frac{\partial \log Z_m}{\partial Z_m}\right)^{-1} = -\gamma_m \frac{Z_m}{\mu_0}$$

$$\frac{m_0(\lambda)}{Z_m} = m_r(\lambda) = \frac{\partial m_r\left(\lambda\right)}{\partial \log \lambda} - \gamma_m m_r(\lambda)$$

$$\frac{\partial m_r(\lambda)}{\partial \log \lambda} = \left(1 + \gamma_m\right) m_r(\lambda) \tag{1.3}$$

$$\frac{\partial m_r(\lambda)}{\partial \lambda} = \frac{\left(1 + \gamma_m\right)}{\lambda} m_r(\lambda) \Rightarrow m_r(\lambda) = \lambda^{1+\gamma_m} m_r \tag{1.4}$$