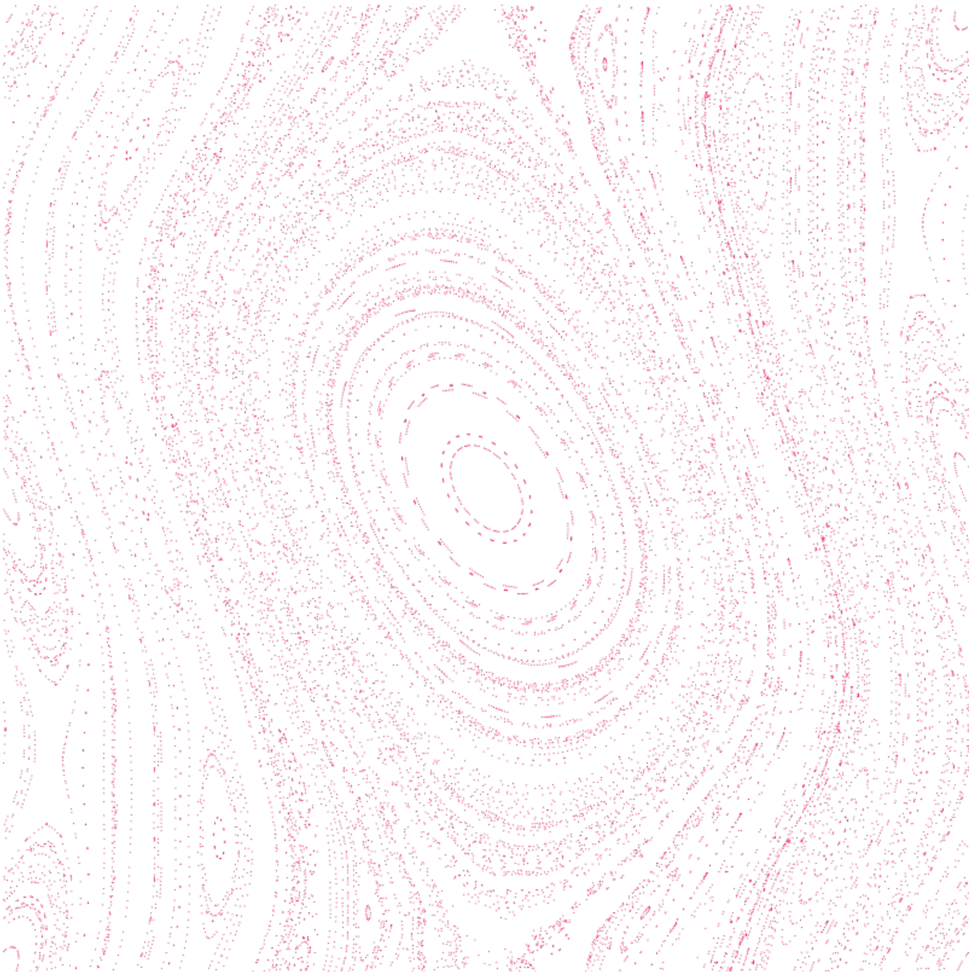
# ASSIGNMENT 1

Esteve Tarrago Sanchis

October 4, 2018

## 1 Simulation Results



The image on top shows the orbit generated by 250 random seeds in the domain with 100 iterations of the map.

## 2 Exact 2-periodic points

In order to find the exact solution we will proceed analytically. Being F the dynamical map:

$F(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x + a * sin(x+y) \\ x+y \end{bmatrix}$

The equation a point has to fulfill to be two periodic is:

$F(F(\begin{bmatrix} x \\ y \end{bmatrix})) = \begin{bmatrix} x \\ y \end{bmatrix}$

By substitution:

$F(\begin{bmatrix} x + a * sin(x+y) \\ x+y \end{bmatrix}) = \begin{bmatrix} x \\ y \end{bmatrix}$

And again:

$\begin{bmatrix} x + a * sin(x+y) + a * sin(x + a * sin(x+y) + x + y) \\ x + a * sin(x+y) + x + y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

Using the second row we obtain:

$a * sin(x+y) = -2 * x$

By substitution in the first row:

$-2 * x + a * sin(y) = 0$

Isolating x:

$x = a * sin(y)/2$

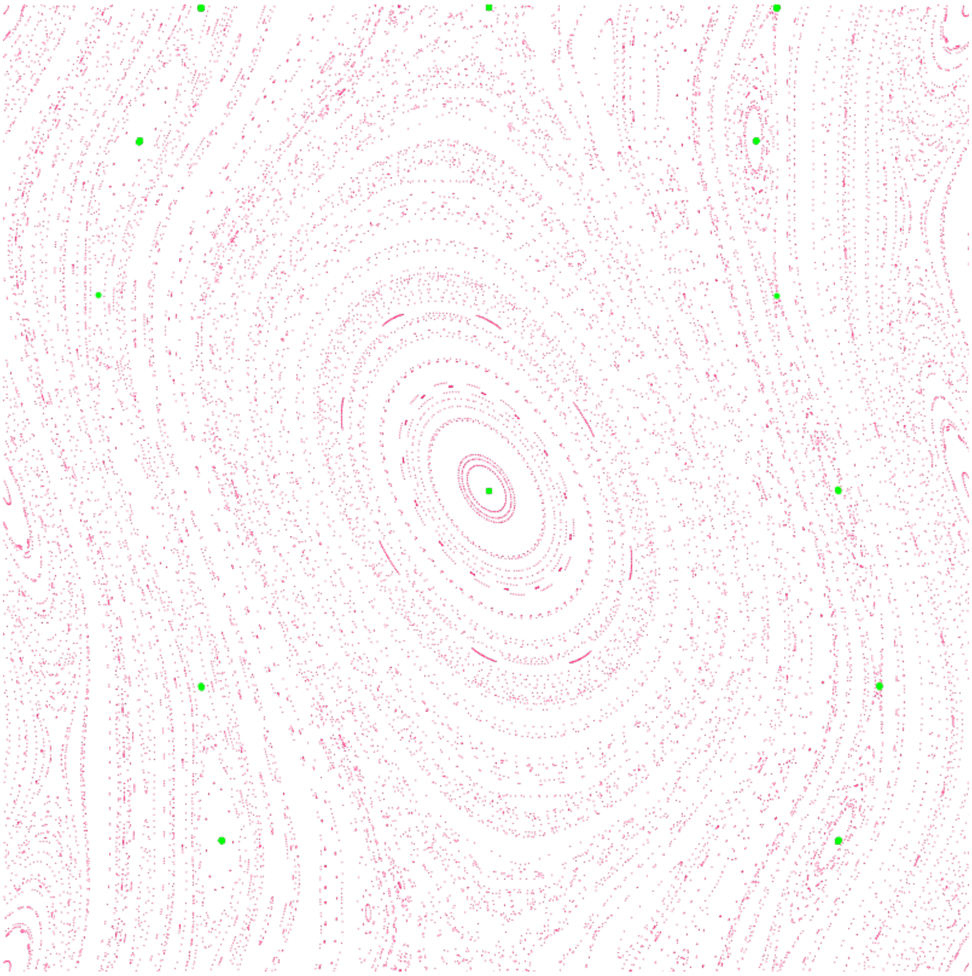
We can express the solution as:

$\begin{bmatrix} a * sin(y)/2 \\ y \end{bmatrix}$

(0,0) and (0,$\pi$) are solutions but we discard them cause they are equilibrium points.

One example of 2-periodic point would be (a/2,$\pi$/2).

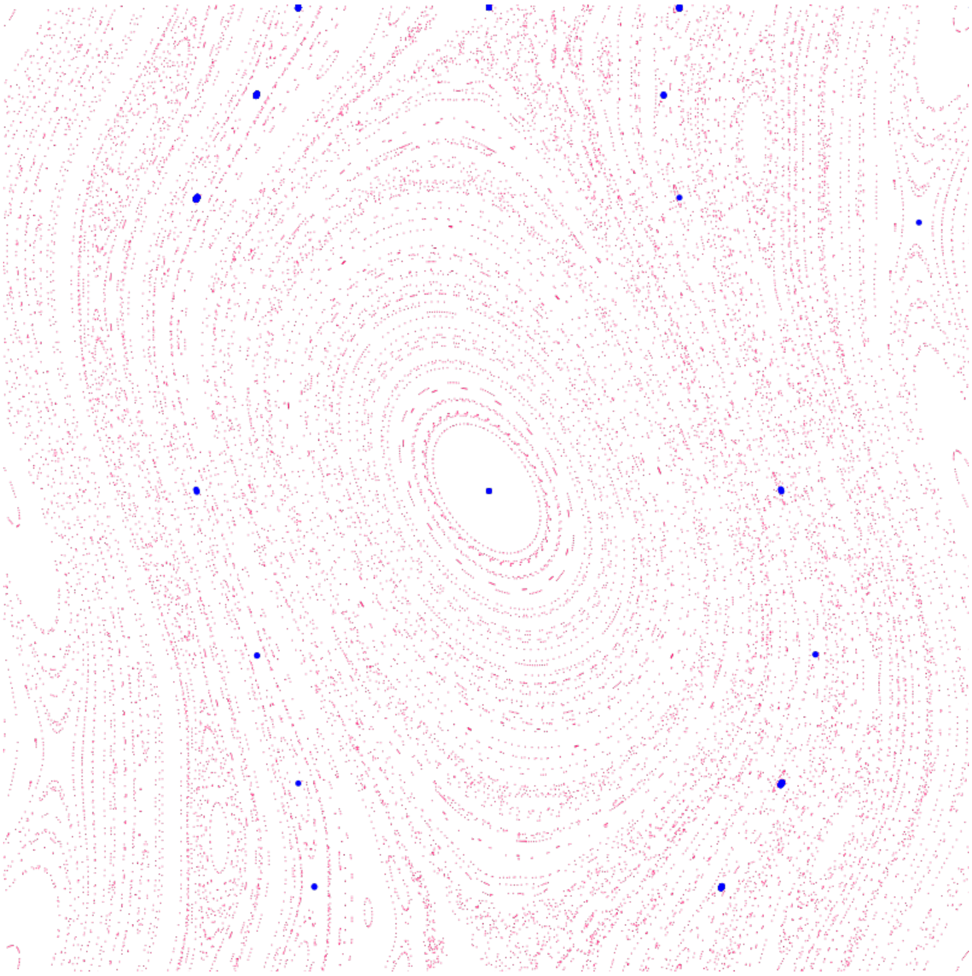
## 3 Approximate 3-periodic points



In green you can see the numerical some numerical solutions obtained for $F^3(X) = X$. You can see we find again equilibrium points also plotted in green.

Some numeric values are:
1.7380918413444433, 2.272693953960068,
1.8712855525053995, 3.139591126041121
2.2739822843670643, -2.2704212354909648

## 4 Approximate 4-periodic points



In blue you can see the numerical some numerical solutions obtained for $F^4(X) = X$. You can see we find again equilibrium points also plotted in blue.

Some numeric values are:
-1.508663922159468, 2.5807669229866903
1.9065899112674964,-1.8934780387244492
1.23878287680494, 3.1394713507992167

## 5 Chaos

There clearly exist Chaos in some regions of the map. We can see this in the regions between two nearby stable equilibrium points. A slight difference in the beginning of the trajectory lead us two completely different orbits.

## 6 Invariant curves

Around the equilibrium points we can see periodic elliptic orbits that define invariant curves.

## 7 Dynamic comments

Most of points in the system are stable points but not asymptotically stable which generate a lot of periodic orbits in the plot.

Also the fact that the domain is a torus generate two more equilibrium points + periodic orbits.