Universitat Politècnica de Catalunya
Facultat de Matemàtiques i Estadística

Master in Advanced Mathematics and Mathematical Engineering
Master's thesis

A Remotely-driven Hoverboard With Platform Leaning Control

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Abstract

This thesis is centered around the design, construction and control of a remote controlled hoverboard. The main challenge of this project is the study of the leaning control of the hoverboard platform and the restrictions it induce.

Nowadays segways are a popular way of human transportation and there also exist segway robots. Both, human-driven and autonomous segway robots have their speed determined by their inclination. (e.i. If you wish to go forward in a segway then you should put your weight in the same direction). So in these systems the inclination is not a degree of freedom but a compulsory consequence from moving. Unblocking this degree of freedom may help two-wheel robots perform new task as measurements, taking images or samples from other inclinations, avoiding obstacles, etc.

We have studied different mechanisms to control the leaning while allowing the movement of the robot. Then we studied the dynamics of our system in order to determine its dimensions and create an optimal design accordingly. We have also run simulations with different policies to see how our system would evolve.

Finally, we build and programmed our robot so it can be remotely operated from anywhere.

Keywords

Dynamic, System, Control, Design

Contents

Figure 1: Picture of Tibi and Dabo, *two segway robots*

1. Introduction

In the IRI lab we have two segway robots, Tibi and Dabo show in figure 1. Both of them move the same way. They must incline their body to compensate the wheel reaction torque. We will get deeper in why these phenomena happens in section 4.

This movement restriction may cause some problems when trying to avoid obstacles, as for example going through low roof path. Another limitation is the acceleration that the robot can achieve which is directly related with the inclination that it's limited to 90 degrees in the best case. Also, the amount of uphill is limited with an additional problem: the safety system stops the robot if it detects something near it (in this case the ground).

So we decided to build a prototype of segway robot that could solve this problem by controlling its inclination independently. The chosen robot is inspired in a *segway hover-board*, similar to the one appearing in Figure 2. The two wheels are controlled with classic control algorithms and the inclination of the central body is controlled with a flywheel mechanism that we will discuss in section 2.1.

Figure 2: Picture of a commercial *segway hover-board*

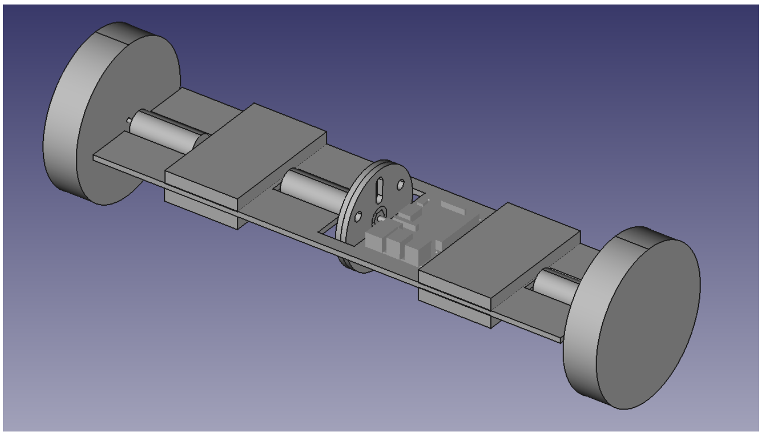


Figure 3: Isometric render view

2. Initial design considerations

The first thing we decided was the number of actuators. Most of segway robot include two motors for the motion control, but we added a third on in order to control the inclination. So we ended up with three motors in total because we want to control three degrees of freedom (inclination and speed of both wheels).

In order to control the inclination of the platform we needed to produce an external torque to the platform. We considered three methods: accelerating a flywheel, holding a pendulum in a non-vertical position and air friction with a fan. We discarded the last one due to the high speeds we needed to obtain a reasonable torque on the platform.

Both methods have strengths in different situations, so we decided to build a mixed piece that could combine both. The flywheel mode allows to deliver the maximum torque from the beginning but fails to deliver a continuous torque due to the motor achieving max speed. In the other hand the pendulum allows the system to apply an amount of torque over time.

We took two more restriction in our design. The first one symmetry along the motors/inclination axis in order to have an equilibrium in all possible inclinations without the need of external forces. We also took in consideration that some experiments may start clumsy so none of the configurations should touch the ground to avoid crashes. Figure 6 illustrates this last restriction.

The design of the robot was done with the 3D design software Free-cad and most of the parts were 3D printed. 3D printing has also its own restrictions, for example not being able to print parts bigger than 25 cm. All part files are uploaded to the GitHub repository `https://github.com/tarragoesteve/TFM` under the hardware folder. So anyone can build this robot.

You can see the main views of an initial design on Figure 3, 4, 5 and 6.

2.1 Flywheel design

To control the inclination of the body two strategies are taken in to account. Creating torque by a pendulum or accelerating the flywheel. In order to experiment with both of them we designed a part to allow both configuration by placing weights in different spots, see figure 7.

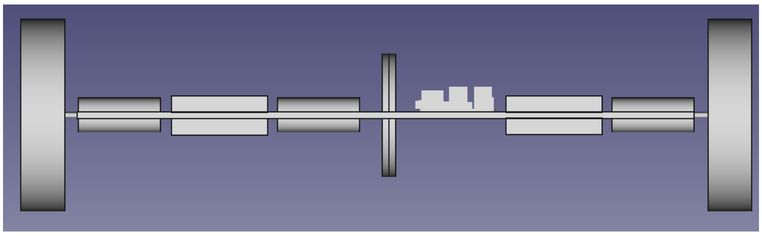


Figure 4: Front render view

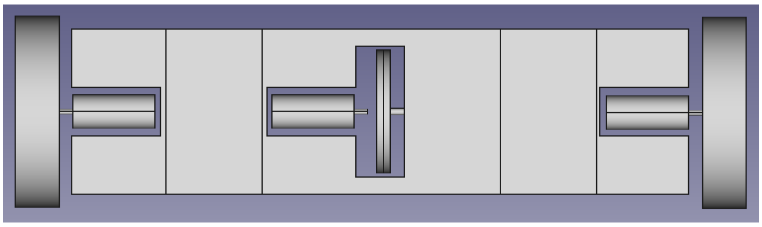


Figure 5: Top render view.

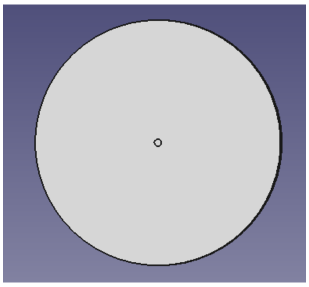


Figure 6: Side render view.

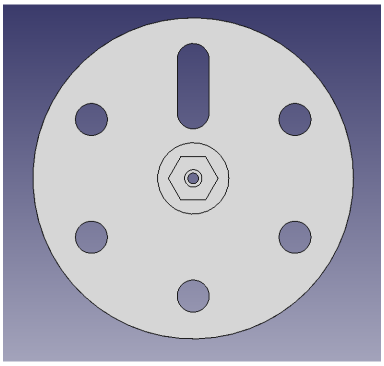


Figure 7: Flywheel side render view.

In order to create a configuration with maximum pendulum torque we have done the following computation. We denote the torque pendulum torque by τ, consider the masses are cylinders of mass $m_{cylinder}$ with radius $r_{cylinder}$ and width w and the radius of the flywheel is $r_{flywheel}$. We want to place all N masses at the same distance of the center of the flywheel (r_{max}) in a regular polygon except for one mass that will be at r_{min}. See figure 7.

Each mass weights:

$$m_{cylinder} = \rho \cdot w \cdot \pi \cdot r_{cylinder}^2$$

We neglect the mass of the flywheel structure versus the mass of the cylinders. All the gravitational torque is created by the cylinder masses and all of them are compensated with the opposite weight except for the two masses with different radius.

One of the weight can be placed along a rail. The distance to the center will vary from $r_{min} = r_{cylinder} + r_{motor-axis} \approx r_{cylinder}$ to $r_{max} = r_{flywheel} - r_{cylinder}$.

The maximum torque takes place when these two masses are aligned horizontal with respect the ground and the movable weight is at distance r_{min} from the center.

$$\tau_{max}(r_{cylinder}) = m_{cylinder} \cdot g \cdot r_{max} - m_{cylinder} \cdot g \cdot r_{min} = m_{cylinder} \cdot g \cdot (r_{flywheel} - 2 \cdot r_{cylinder})$$

In order to maximize τ it we first compute the derivative:

$$\frac{\partial \tau_{max}(r_{cylinder})}{\partial r_c} = g \cdot (\frac{\partial m}{\partial r_c} \cdot (r_f - 2 \cdot r_c) - m_{cylinder} \cdot 2)$$

$$\frac{\partial m_{cylinder}}{\partial r_{cylinder}} = 2 \cdot \rho \cdot w \cdot \pi \cdot r_{cylinder}$$

And make it zero to find the maximum:

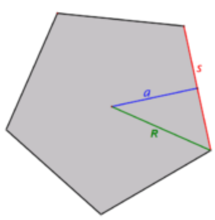
$$\frac{\partial \tau_{max}(r_{cylinder})}{\partial r_{cylinder}} = 0$$

Substituting and simplifying we get:

$$\frac{\partial m}{\partial r_{cylinder}} \cdot (r_{flywheel} - 2 \cdot r_{cylinder}) = m_{cylinder} \cdot 2 \Rightarrow 2 \cdot \rho \cdot w \cdot \pi \cdot r_{cylinder} \cdot (r_{flywheel} - 2 \cdot r_{cylinder}) = \rho \cdot w \cdot \pi \cdot r_c^2 \cdot 2$$

$$\Rightarrow r_c \cdot (r_{flywheel} - 2 \cdot r_{cylinder}) = r_{cylinder}^2 \Rightarrow (r_{flywheel} - 2 \cdot r_{cylinder}) = r_{cylinder} \Rightarrow \boxed{r_f = 3 \cdot r_{cylinder}}$$

The circumradius R from the center of a regular polygon to one of the vertices is related to the side length s by:



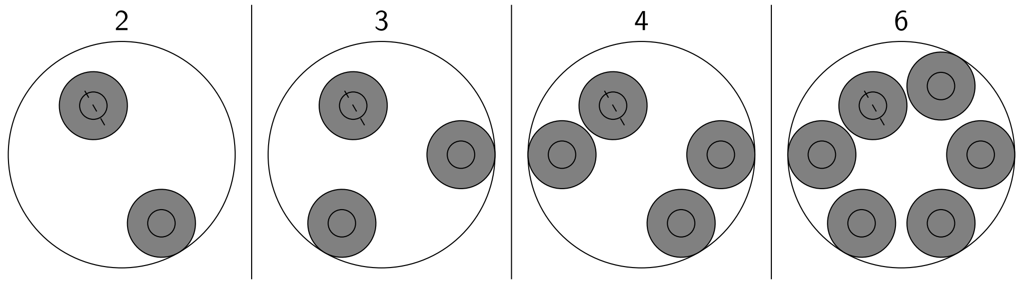
$$R = \frac{s}{2 \cdot \sin \frac{\pi}{n}}$$

In our case:

$$R = r_{flywheel} - r_{cylinder};$$

$$s = 2 \cdot r_{cylinder}$$

Substituting in the circumradius equation we get n = 6, so we will use up 6 masses in our flywheel. We will have a variable number of masses N that we will be able to add to the flywheel as shown in the following table:



3. Optimization Setup

In this section we will set up the requirements and the cost function we want to optimize.

3.1 Restrictions

The restrictions are a list of inequalities that our system has to fulfill. The first restrictions is due to the initial design requirements. Finally, the other three are somehow arbitrary but will help us to reduce the size of the robot.

1. We will place our flywheel in a hole on our robot. We don't want to touch the ground in any configuration so:

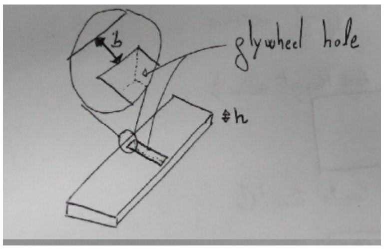


Figure 8: Flywheel hole diagram

$$r_{wheel} > \sqrt{(r_{flywheel} + b)^2 + (\frac{h}{2})^2}$$

2. Being able to insert the robot in to a wheel of diameter 0.5 m so:

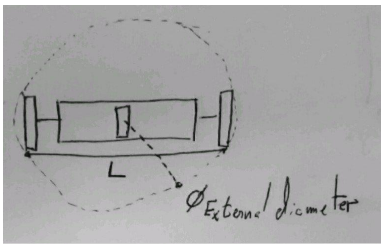


Figure 9: External diameter diagram

$$0.25m > \sqrt{r_{wheel}^2 + L^2/4}$$

3. We can place all electronic the devices:

$$L > 0.3m + w$$

4. Maximum weight of the robot: 5 kg

3.2 Requirements

We would like our robot to reach some mechanical specifications. These are related to mechanical equations that we will develop in the next section. They refer to the max speed, acceleration and inclination the robot may achieve while controlling its inclination. We have divided our specifications in two blocks according to the two mechanisms.

Flywheel mode

1. $\dot{y}_{max}$ (equation 10) $> 0.1 m/s$.
2. $\ddot{y}_{max}$ (equation 9) $> 1 m/s^2$.
3. $sin(\alpha_{max})$ (equation 13) > 0.2.

Pendulum mode

1. $\dot{y}_{max}$ (equation 12) $> 1 m/s$.
2. $\ddot{y}_{max}$ (equation 11) $> 0.1 m/s^2$.
3. $sin(\alpha_{max})$ (equation 14) > 0.02.

3.3 Cost function

In addition to fulfilling the previous inequalities we will minimize a cost function.

We will maximize the maximum sinus in the pendulum mode (equation 14) because it gives the robot the capacity to deliver force in a permanent state.

And we will also maximize the square of the max speed the robot can achieve in flywheel mode (equation 10) because it is proportional to the energy the robot can deliver using the flywheel at a certain moment.

$$cost(r_{flywheel}, r_{wheel}, w, N) = -sin(\alpha_{max})_{pendulum} - \dot{y}^2_{max-flywheel} \tag{1}$$

In the next section we will find out what are the values of these equations with the construction parameters. For reference these terms are equal to:

$$m_{cylinder} = \rho * w * \pi * (\frac{r_{flywheel}}{3})^2$$

$$sin(\alpha_{max})_{pendulum} = \frac{m_{cylinder} \cdot (r_{max} - r_{min})}{m_{total} \cdot r_{wheel}} = \frac{m_{cylinder} \cdot (\frac{r_{flywheel}}{3})}{(m_{rest} + N \cdot m_{cylinder}) \cdot r_{wheel}}$$

$$\dot{y}_{max} = r_{wheel} \cdot R \cdot \dot{\theta}_{max} = r_{wheel} \cdot \frac{N \cdot m_{cylinder} \cdot (\frac{2 \cdot r_{flywheel}}{3})^2}{r^2_{wheel} \cdot (m_{rest} + N \cdot m_{cylinder}) + 2 \cdot I_{wheel}} \cdot \dot{\theta}_{max}$$

4. Mechanical analysis

In this section we will analyze and understand the key dynamics of our robot, so we can choose the design parameters based on performance indicators. All the analysis is made supposing the inclination is fixed. As parameters, we have the with of the flywheel masses w, the number of masses N, the radius of the flywheel $r_{flywheel}$, and the radius of the wheels r_{wheel}.

4.1 Inclination control

In order to keep the inclination of the platform at a certain angle ϕ we must be able to compensate all the torque being applied to the platform.

$$\ddot{\phi} \cdot I_{platform} = \tau_{platform}$$

Assuming that the platform is well-balanced (the center of masses is located at the rotation axis by our design restriction) and neglecting the torque generated by the friction with air, the sum of all the torques in the motor axis applied to the platform is equal to the sum of the torque applied by the motors:

$$\tau_{platform} = \sum \tau_{motors}$$

The torque that the motors deliver to the wheels and to the flywheel create a reaction in the platform in the opposite direction.

$$\tau_{platform} = -\tau_{motor-right-wheel} - \tau_{motor-left-wheel} - \tau_{motor-flywheel}$$

If we want keep the inclination ϕ, we must be able to cancel $\tau_{platform}$. Observe that the angular acceleration $\ddot{\phi}$ of the platform is linearly dependent with the torque it receives.

$$\begin{gathered} 0 = -\tau_{motor-right-wheel} - \tau_{motor-left-wheel} - \tau_{motor-flywheel} \Rightarrow \\ \tau_{motor-right-wheel} + \tau_{motor-left-wheel} = -\tau_{motor-flywheel} \end{gathered} \tag{2}$$

In other words, we must overpass the torque of the wheels with the torque of the flywheel if we want to control the inclination.

4.2 Wheels torque

The wheel torque we can induce is limited by the motor specifications. Note that the maximum torque of the motor is a function of velocity and in particular at max speed the torque is zero.

$$\tau_{motor-wheel}(\omega_{wheel})$$

We assume that the wheels just roll and do no slip. The robot is pushed by the wheels that make a force $F_{friction}$ against the ground in the contact point. See figure 11.

We can express the torque at the center of the wheel as:

$$\tau_{motor-wheel} + F_{friction} \cdot r_{wheel} = I_{wheel} \cdot \dot{\omega}_{wheel}$$

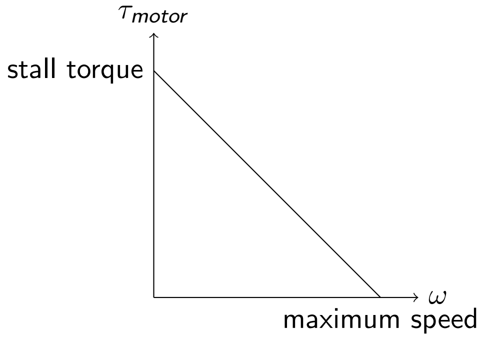


Figure 10: Motor torque.

$$\tau_{motor-wheel} = I_{wheel} \cdot \dot{w}_{wheel} - F_{friction} \cdot r_{wheel} \tag{3}$$

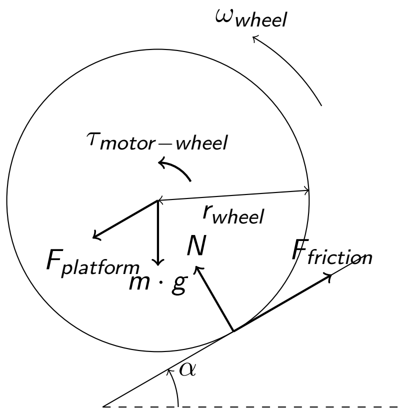


Figure 11: Wheel force diagram.

4.3 Flywheel torque

The flywheel torque we can induce is also limited by the motor specifications.

Assuming a general configuration of the flywheel where the moving mass is at distance r from the axis and angle θ, see figure 12. We formulate its torque the following way:

$$\tau_{motor-flywheel} + m_{cylinder} \cdot g \cdot (r - r_{max}) \cdot \sin\theta = \ddot{\theta} \cdot I_{flywheel}(r)$$

$$\tau_{motor-flywheel} = \ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta \tag{4}$$

Note that in equation 4 the two terms correspond to the two methods: accelerating and non vertical position.

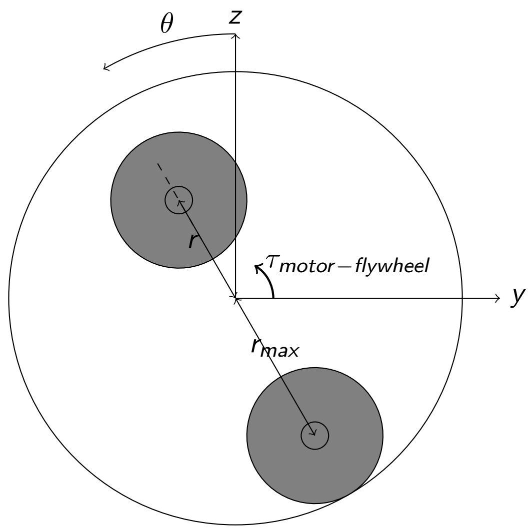


Figure 12: Flywheel diagram for $N = 2$

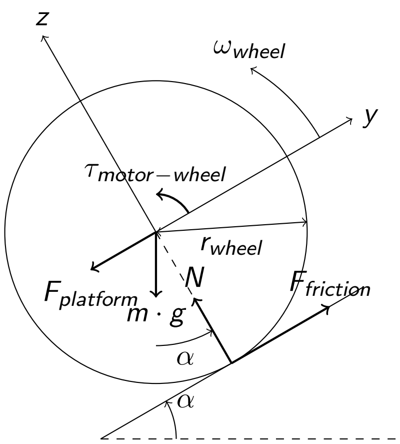


Figure 13: Wheel forward force diagram.

4.4 Maximum speed, acceleration and inclination

In this subsection we would like to study the maximum speed and acceleration the robot may obtain in straight direction facing and inclination α.

We will assume both wheels turn at the same speed, have the same $F_{friction}$ and the same τ_{wheel}:

$$\omega_{wheel-left} = \omega_{wheel-right} = \omega_{wheel}$$

Applying Newton's first law in the y-axis of figure 13:

$$\ddot{y} \cdot m_{total} = 2 \cdot F_{friction} - m_{total} \cdot g \cdot sin(\alpha)$$

Substituting $F_{friction}$ taking in to account equation 3:

$$\ddot{y} \cdot m_{total} = 2 \cdot \frac{I_{wheel} \cdot \dot{w}_{wheel} - \tau_{motor-wheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha)$$

Using equation 2:

$$\Rightarrow \ddot{y} \cdot m_{total} = \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}} + \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha) \tag{5}$$

We will now study different cases to better understand this equation.

4.4.1 Speed and acceleration with no inclination ($\alpha = 0$)

The objective here is to obtain the maximum speed and acceleration we can get starting from rest in a plain surface.

The equation we get by substituting $\alpha = 0$ in equation 5:

$$\ddot{y} \cdot m_{total} = \frac{\tau_{motor-flywheel}}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

Substituting equation 4:

$$\ddot{y} \cdot m_{total} = \frac{\ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{\omega}_{wheel}}{r_{wheel}} \tag{6}$$

We will now split the study in two cases:

1. Flywheel case: r is fixed to $r = r_{max}$

   Then:

   $$\ddot{y} \cdot m_{total} = -\frac{\ddot{\theta} \cdot I_{flywheel}(r_{max})}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{\omega}_{wheel}}{r_{wheel}}$$

   Taking in to account the following relation:

   $$-\omega_{wheel} \cdot r_{wheel} = \dot{y} \Rightarrow -\dot{\omega}_{wheel} \cdot r_{wheel} = \ddot{y} \tag{7}$$

   Substituting in equation 6:

   $$-\dot{\omega}_{wheel} \cdot r_{wheel} \cdot m_{total} = \frac{\ddot{\theta} \cdot I_{flywheel}}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{\omega}_{wheel}}{r_{wheel}}$$

   $$-\dot{\omega}_{wheel} \cdot (r_{wheel} \cdot m_{total} + \frac{2 \cdot I_{wheel}}{r_{wheel}}) = \frac{\ddot{\theta} \cdot I_{flywheel}}{r_{wheel}}$$

   We now define R as a non dimensional constant being the quotient between $\dot{w}_{wheel}$ and $-\ddot{\theta}$.

   $$R = \frac{\dot{\omega}_{wheel}}{-\ddot{\theta}} = \frac{I_{flywheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}} \tag{8}$$

   The moments of inertia are:

   $$I_{wheel} \approx \frac{1}{2} \cdot m_{wheel} \cdot r_{wheel}^2$$

   $$I_{flywheel} \approx N \cdot m_{cylinder} \cdot r_{max}^2$$

Substituting those in equation 8 we get:

$$R \approx \frac{N \cdot m_{cylinder} \cdot r_{max}^2}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}} < 1$$

We can see that R will always be smaller than 1 because $N \cdot m_{cylinder} < m_{total}$ and $r_{max} < r_{wheel}$.

This means that we will the maximum acceleration of the wheels will be limited by the acceleration of the flywheel. The same is true for the speed.

In order to get the forward acceleration we can use equation 7

$$\ddot{y} = -\dot{\omega}_{wheel} \cdot r_{wheel} = R \cdot \ddot{\theta} \cdot r_{wheel}$$

And using equation 4 we get that the maximum is:

$$\tau_{motor}(\omega) = \ddot{\theta} \cdot I_{flywheel}(r) \Rightarrow \ddot{\theta} = \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)}$$

$$\ddot{y}_{max} = R \cdot \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} \cdot r_{wheel}$$

$$\Rightarrow \ddot{y}_{max} = \frac{I_{flywheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}} \cdot \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} \cdot r_{wheel}$$

$$\boxed{\ddot{y}_{max} = \frac{\tau_{motor}(\dot{\theta}) \cdot r_{wheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}}} \tag{9}$$

In order to compute the maximum speed we assume that the initial conditions are $\dot{\theta} = 0$ and $\omega_{wheel} = 0$

$$\omega_{wheel-max} = \int_{t=0}^{t=t_{max}} \dot{\omega}_{wheel} \cdot dt$$

Now we will proceed to do a change of variables in the integral.

$$\frac{\partial\dot{\theta}}{\partial t} = \ddot{\theta} \Rightarrow dt = \frac{d\dot{\theta}}{\ddot{\theta}}$$

$$\omega_{wheel-max} = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} \frac{\dot{w}_{wheel}}{\ddot{\theta}} \cdot d\dot{\theta} = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} -R \cdot d\dot{\theta} = -R \cdot \dot{\theta}_{max}$$

$$\boxed{\dot{y}_{max} = r_{wheel} \cdot R \cdot \dot{\theta}_{max}} \tag{10}$$

And $\dot{\theta}_{max}$ is a limitation imposed by the motor specifications. Note that this is the maximum speed we can get using the flywheel system starting from rest.

2. Pendulum: $\dot{\theta} = 0$, and r is fixed to $r = r_{min}$

   Using equation 6 and $\ddot{\theta} = 0$

$$\ddot{y} \cdot m_{total} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

   Multiplying by r_{wheel} both sides of the equation we get:

$$\ddot{y} \cdot m_{total} \cdot r_{wheel} = m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta + 2 \cdot I_{wheel} \cdot \dot{w}_{wheel}$$

   And using $\dot{w}_{wheel} = -\frac{\ddot{y}}{r_{wheel}}$

$$\ddot{y} \cdot m_{total} \cdot r_{wheel} + 2 \cdot I_{wheel} \cdot \frac{\ddot{y}}{r_{wheel}} = m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta$$

   With some manipulation:

$$\ddot{y} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta}{m_{total} \cdot r_{wheel} + \frac{2 \cdot I_{wheel}}{r_{wheel}}}$$

   Which is maximum when $\sin\theta = 1$

$$\boxed{\ddot{y}_{max} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot}{m_{total} \cdot r_{wheel} + \frac{2 \cdot I_{wheel}}{r_{wheel}}}} \qquad (11)$$

   We need to take into consideration air friction to see the speed limitation in the pendulum case.

$$F_{drag} = \frac{1}{2} \cdot \rho \cdot C_D \cdot A \cdot \dot{y}^2$$

   Adding this term to equation 5 we get:

$$\ddot{y} \cdot m_{total} = \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}} + \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha) - F_{drag}$$

   And making $\ddot{y} = 0$ and $\alpha = 0$.

$$F_{drag} = \frac{\tau_{motor-flywheel}}{r_{wheel}}$$

$$\frac{1}{2} \cdot \rho \cdot C_D \cdot A \cdot \dot{y}^2 = m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot sin(\theta)/r_{wheel}$$

   The maximum $\dot{y}$ is then obtained when $\theta = \frac{\pi}{2}$:

$$\dot{y}_{max} = \sqrt{\frac{2 \cdot m_{cylinder} \cdot g \cdot (r_{max} - r_{min})}{\rho \cdot C_D \cdot A \cdot r_{wheel}}}$$

   We will pick $C_D = 1$, $\rho = 1.2 kg/m^3$ and $A = 0.01 m^2$ for our computations.

   Also, note that the speed is also limited by maximum speed a motor can get $\dot{\theta}_{max}$.

$$\boxed{\dot{y}_{max} = min(\dot{\theta}_{max} \cdot r_{wheel}, \sqrt{\frac{2 \cdot m_{cylinder} \cdot g \cdot (r_{max} - r_{min})}{\rho \cdot C_D \cdot A \cdot r_{wheel}}})} \qquad (12)$$

4.4.2 Inclination ($\alpha > 0$)

The goal of this subsection is to study which is the maximum inclination α_{max} the robot can overpass.

Substituting $\ddot{y} = 0$ and $\dot{\omega}_{wheel} = 0$ in equation 5

$$0 = \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha)$$

$$m_{total} \cdot g \cdot sin(\alpha) = \frac{\tau_{motor-flywheel}}{r_{wheel}}$$

$$sin(\alpha) = min(1, \frac{\tau_{motor-flywheel}}{m_{total} \cdot g \cdot r_{wheel}})$$

Substituting equation 4

$$sin(\alpha) = min(1, \frac{\ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta}{m_{total} \cdot g \cdot r_{wheel}})$$

We are going to distinguish the same two cases as in the previous section:

1. Flywheel case: r is fixed to $r = r_{max}$ The maximum inclination at a certain moment:

$$\boxed{sin(\alpha_{max}) = min(1, \frac{\ddot{\theta} \cdot I_{flywheel}}{m_{total} \cdot g \cdot r_{wheel}}) = min(1, \frac{\tau_{motor-flywheel}(\dot{\theta})}{m_{total} \cdot g \cdot r_{wheel}})} \tag{13}$$

   This angle doesn't give us a lot of information because it may not be fulfilled in a permanent state. That's why in addition we would like to compute the maximum height our robot can achieve start from rest position.

2. Pendulum: $\dot{\theta} = 0$, and r is fixed to $r = r_{min}$

$$sin(\alpha) = \frac{min(\tau_{motor-flywheel}, m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta)}{m_{total} \cdot g \cdot r_{wheel}}$$

   Which is maximum when $\sin\theta = 1$

$$\boxed{sin(\alpha_{max}) = \frac{min(\tau_{motor-flywheel}, m_{cylinder} \cdot g \cdot (r_{max} - r_{min}))}{m_{total} \cdot g \cdot r_{wheel}}} \tag{14}$$

   Note that the $sin(\alpha_{max})$ will not be more than 1.

5. Optimization Results

Our procedure has been making a grid with the four parameters of the robot design: w (width of the cylinders), N(number of cylinders), r_{wheel} and $r_{flywheel}$. We have fixed $r_{flywheel}$, iterated over the other three parameters and kept the best configuration in therms of our cost function (eq. 1) that fulfilled the requirements and restrictions.

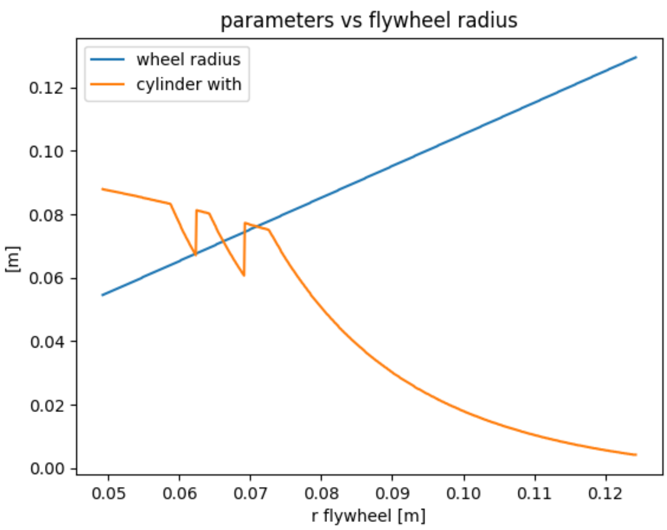


Figure 14: Plot of the wheel radius and the width of the cylinder that was optimal for each flywheel radius.

We can observe that the relation between the flywheel radius and the wheel radius is linear. This is because we are always on the edge of the restriction number one.

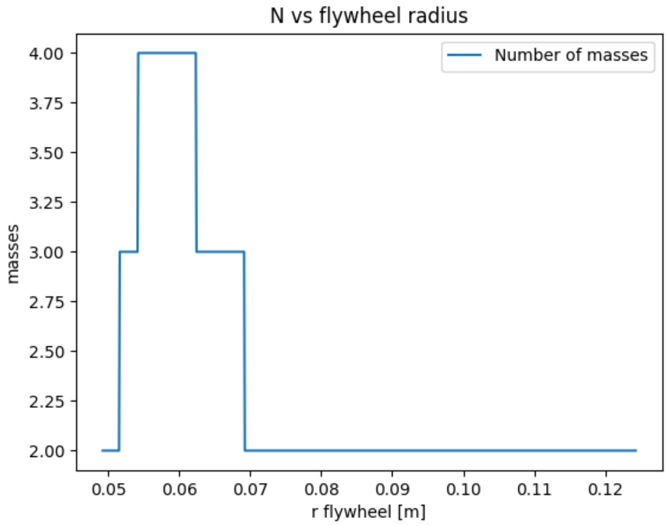


Figure 15: Plot of the N that minimize the cost function.

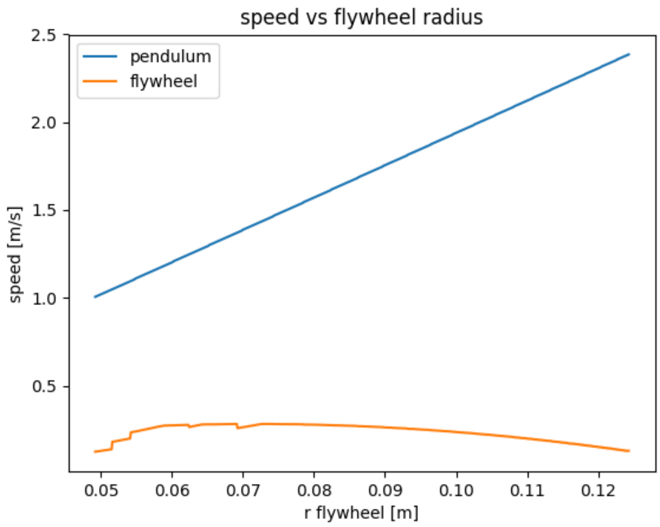


Figure 16: Plot of the equations 10 and 12 at the parameters that minimize the cost function and fullfil the requirements and restrictions

In this figure we can observe that the speed of in the pendulum mode increases linearly with the flywheel radius. This is because it is linearly related with the wheel radius.

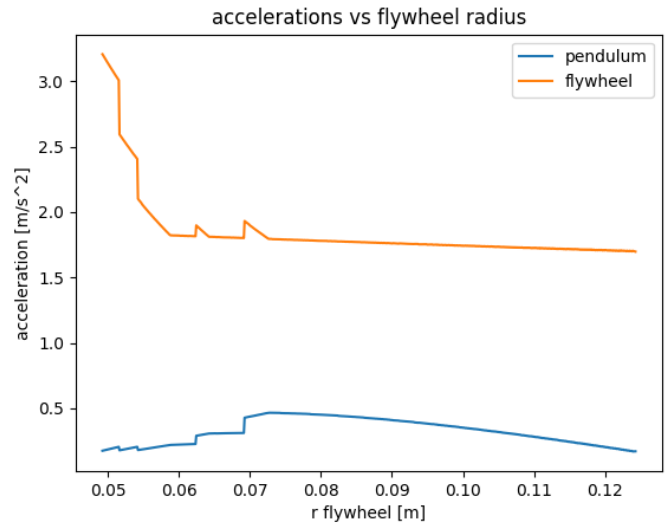


Figure 17: Plot of the equations 9 and 11 at the parameters that minimize the cost and fulfill the requirements and restrictions

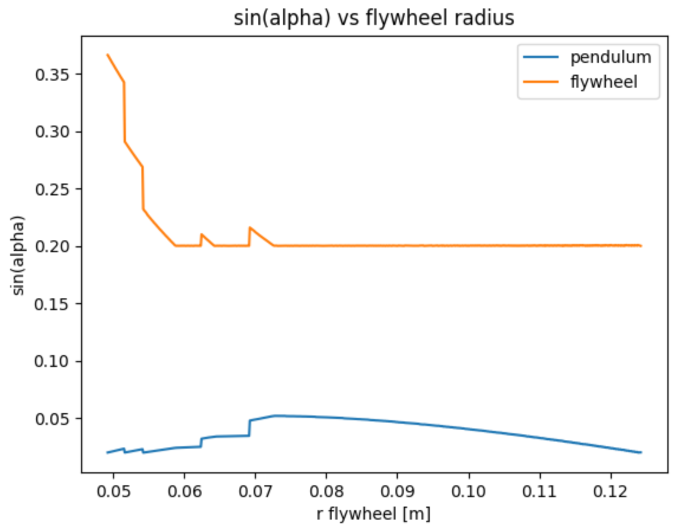


Figure 18: Plot of the equations 13 and 14 at the parameters that minimize the cost and fulfill the requirements and restrictions

We can see in figure 18 that when the flywheel radius exceeds 0.07 m the requirement of the flywheel sinus becomes active.

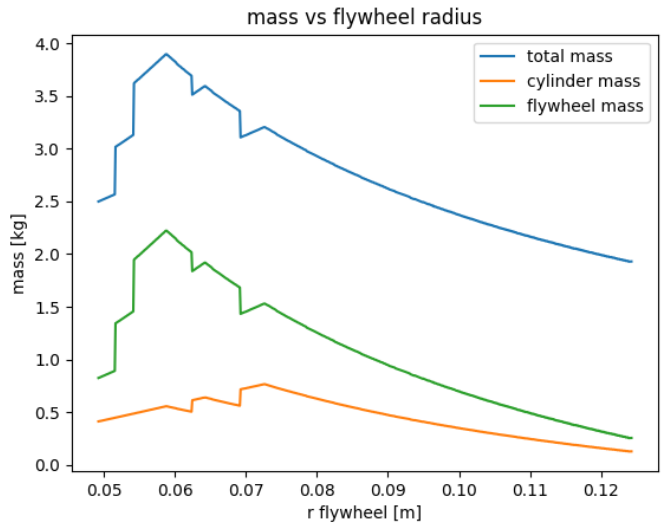


Figure 19: Plot of the mass for each configuration.

We can see in figure 19 that the total mass never exceeds the 5 kg limit.

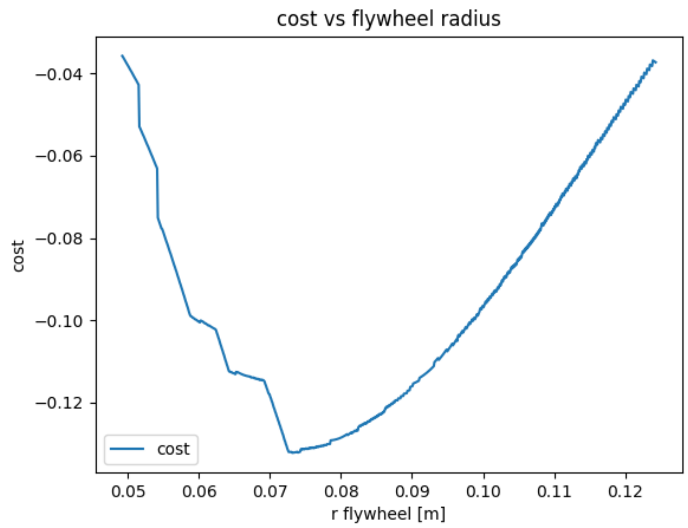


Figure 20: Plot of the equation 1 for each configuration.

Our selected parameters are then:

$r_{flywheel}$	r_{wheel}	w	N
8cm	10cm	5cm	2

With these parameters we get the following specifications:

Total mass	2,91 kg
Pendulum	
Maximum sinus	0,042
Maximum speed horizontal	1,84 m/s
Maximum acceleration horizontal	0,38 m/s^2
Flywheel	
Maximum sinus	0,171
Maximum speed horizontal	0,24 m/s
Maximum acceleration horizontal	1.52 m/s^2

6. Rectilinear movement dynamics with $r_{flywheel}$ fixed

In order to study the dynamics of the robot we will use Lagrange mechanics. To reduce the number of variables we will study the case of rectilinear movement by imposing that both wheels turn at the same speed. We will also set the radius of the free weight to $r_{flywheel}$. Note that with these equations the platform can rotate.

The generalized coordinates (q) will be:

1. $\phi_{ground-wheel}$: rotation of the wheel respect the ground.
2. $\phi_{wheel-platform}$: rotation of the platform respect the wheel.
3. $\phi_{platform-flywheel}$: rotation of the flywheel respect the platform.

We will use two auxiliary variables:

1. $\phi_{ground-platform} = \phi_{ground-wheel} + \phi_{wheel-platform}$: rotation of the platform respect the ground.
2. $\phi_{ground-flywheel} = \phi_{ground-platform} + \phi_{platform-flywheel}$: rotation of the flywheel respect the ground.

The total potential energy:

$$V = m_{cylinder} \cdot (r_{flywheel} - r_{flywheel-max}) \cdot cos(\phi_{ground-flywheel}) \cdot g \tag{15}$$

The total kinetic energy:

$$T = \frac{1}{2} \cdot [\dot{\phi}^2_{ground-wheel} \cdot I_{wheel} + \dot{\phi}^2_{ground-platform} \cdot I_{platform} + \dot{\phi}^2_{ground-flywheel} \cdot I_{flywheel} + \dot{\phi}^2_{ground-wheel} \cdot r^2_{wheel} \cdot m_{total}] \tag{16}$$

The Lagrangian is defined as:

$$L = T - V \tag{17}$$

Lagrange's equation is:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_j}) = \frac{\partial L}{\partial q_j} + F_j \tag{18}$$

So in our case:

$$\frac{\partial L}{\partial \dot{\phi}_{ground-wheel}} = \dot{\phi}_{ground-wheel} \cdot I_{wheel} + \dot{\phi}_{ground-platform} \cdot I_{platform} + \dot{\phi}_{ground-flywheel} \cdot I_{flywheel} + \dot{\phi}_{ground-wheel} \cdot r^2_{wheel} \cdot m_{total} \tag{19}$$

$$\frac{\partial L}{\partial \dot{\phi}_{wheel-platform}} = \dot{\phi}_{ground-platform} \cdot I_{platform} + \dot{\phi}_{ground-flywheel} \cdot I_{flywheel} \tag{20}$$

$$\frac{\partial L}{\partial \dot{\phi}_{platform-flywheel}} = \dot{\phi}_{ground-flywheel} \cdot I_{flywheel} \tag{21}$$

We define M as the following matrix:

$$\begin{pmatrix} I_{wheel} + I_{platform} + I_{flywheel} + r^2_{wheel} \cdot m_{total} & I_{platform} + I_{flywheel} & I_{flywheel} \\ I_{platform} + I_{flywheel} & I_{platform} + I_{flywheel} & I_{flywheel} \\ I_{flywheel} & I_{flywheel} & I_{flywheel} \end{pmatrix} \tag{22}$$

In matrix form and with our generalized coordinates:

$$\frac{\partial L}{\partial \dot{q}} = M \cdot \dot{q} \tag{23}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = M \cdot \ddot{q} \tag{24}$$

Let a be a constant:

$$a = m_{cylinder} \cdot (r_{flywheel} - r_{flywheel-max}) \cdot g \tag{25}$$

$$\frac{\partial L}{\partial q} = a \cdot sin(\phi_{ground-flywheel}) \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{26}$$

So using Lagrange's equation we get:

$$\boxed{M \cdot \ddot{q} = a \cdot sin(\phi_{ground-flywheel}) \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + F} \tag{27}$$

6.1 Simulations

In this subsection we study different policies for the external forces (F) applied to the robot in rectilinear movement case.

The robot has 2 actuators, the motors between the wheel and the platform and the motor between the flywheel and the platform. All motors in the systems have a limited torque related with the speed as you may see in Figure 10.

$$\boxed{F = \begin{pmatrix} 0 \\ \tau_{wheel-platform} \\ \tau_{platform-flywheel} \end{pmatrix}} \tag{28}$$

In order to compare the different policies all of them have the same target. The robot must travel 30 radians (3 meters) and stop at the end. All the parameters from the simulation are set equal to the values found in section 5. Keep in mind that the wheel radius is 10 cm.

Here you can see a table that summarize the results:

Experiment	**Total time(s)**	**Time to max speed(s)**	**Max speed(rad/s)**	**Time to brake(s)**
Controlling inclination				
Flywheel	12,45	0,7	2,45	0,15
Pendulum	5,59	2,69	10,36	2,90
Waitress	8,24	4,12	5,49	4,12
Free inclination				
Double flywheel	3,9	1	8,51	0.27
Compose mechanism	3,82	2,3	10,2	2,04

6.1.1 Controlling the platform inclination

In the first two experiments we want to keep the platform in a horizontal position. That's the reason why we force the $\tau_{wheel-platform}$ equal to $\tau_{platform-flywheel}$ in the experiments flywheel and pendulum.

1. Flywheel

   The movable weight is fixed at r_{max}. In this experiment we consign the platform-flywheel motor to output the maximum possible torque. Then the platform-wheels motors deliver the same torque. The platform-flywheel motor reaches in less than one second the maximum speed. This then limits the wheel-platform speed because no more torque can be applied one that point is reached.

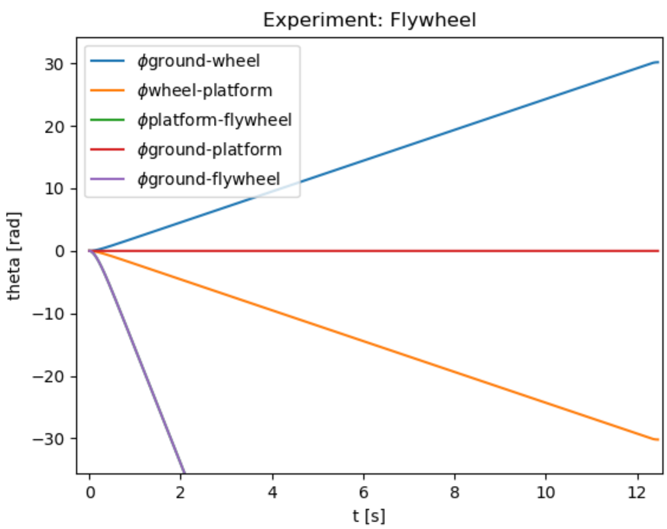


Figure 21: Plot of the angles for the flywheel experiment.

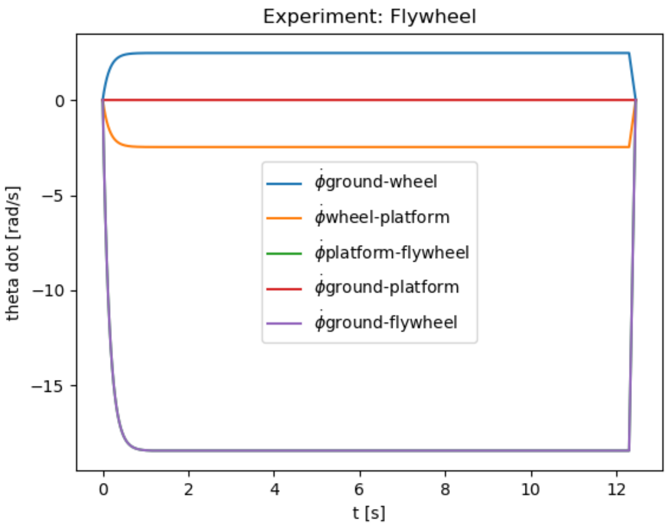


Figure 22: Plot of the angular velocities for the flywheel experiment.

2. Pendulum

The movable weight is fixed at r_{min}. In this experiment we consign the platform-flywheel motor to get to 90 with a PID controller. Then the platform-wheels motors deliver the same torque. The robot does not accelerate as fast as in the flywheel experiment but reaches a higher speed that allows it to travel the distance in less time. Note that the robot could have kept accelerating but had to brake before reaching the maximum speed.

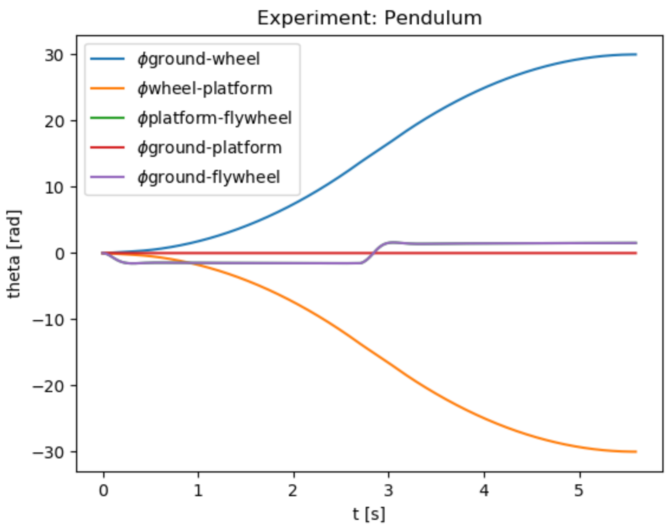


Figure 23: Plot of the angles for the pendulum experiment.

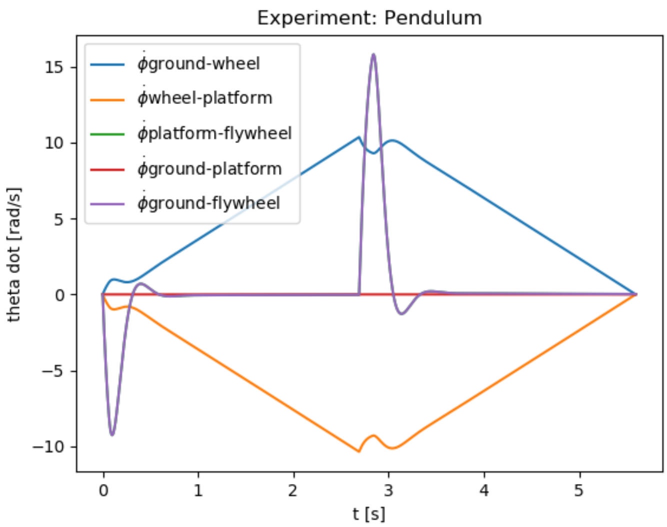


Figure 24: Plot of the angular velocities for the pendulum experiment.

3. Waitress

The movable weight is fixed at r_{min}. This experiment is different that the previous two and it's aim is to illustrate how the robot could follow and inclination consign. The experiment is designed the following way. A set point torque is given to the wheel-platform and the flywheel-motor PID has an inclination consign too. The inclination consign is such that the sum of accelerations received from and object in the platform is perpendicular to the platform.

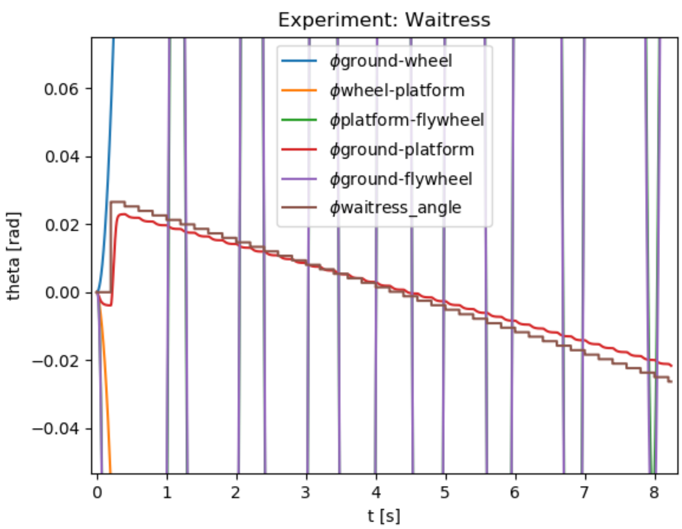


Figure 25: Plot of the angles for the waitress experiment.

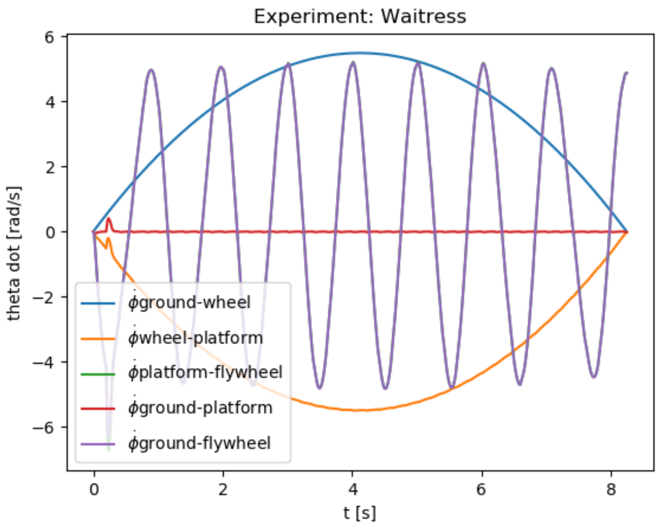


Figure 26: Plot of the angular velocities for the waitress experiment.

6.1.2 Letting the platform turn

In this experiment we take advantage of the fact that the platform may not be needed to stay horizontal in some displacement. The robot uses the platform as an additional flywheel to help the robot accelerate and brake.

1. Double Flywheel

   The movable weight is fixed at r_{max}. In this experiment we add the flywheel turn over the platform turn to create more torque to accelerate our wheels. We always do the maximum possible torque with all the motors.

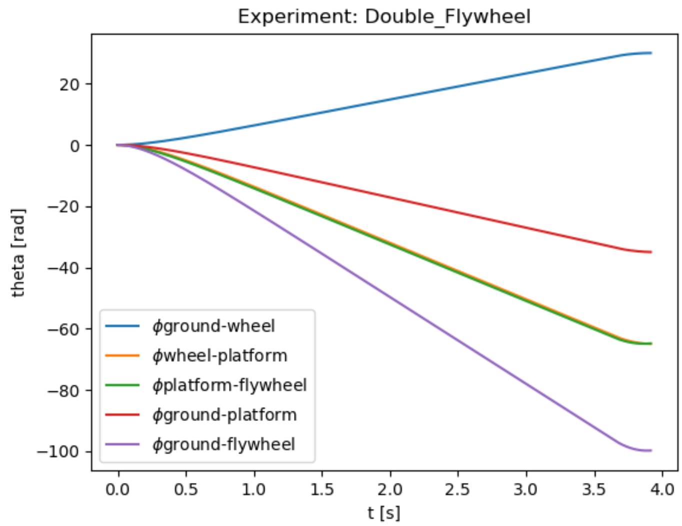


Figure 27: Plot of the angles for the double flywheel experiment.

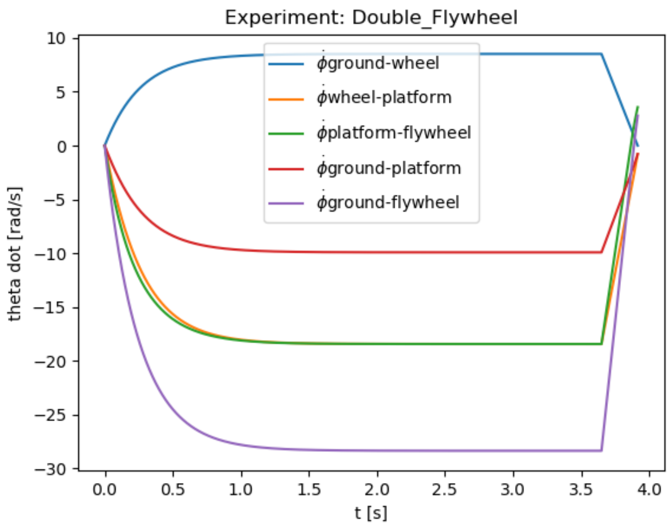


Figure 28: Plot of the angular velocities for the double flywheel experiment.

2. Compose

The movable weight is fixed at r_{min}. In this experiment we add the pendulum effect to the flywheel effect produced by the flywheel. We do so with a PID controller for the angle ground-flywheel and making the maximum possible torque with the platform-wheel motors.

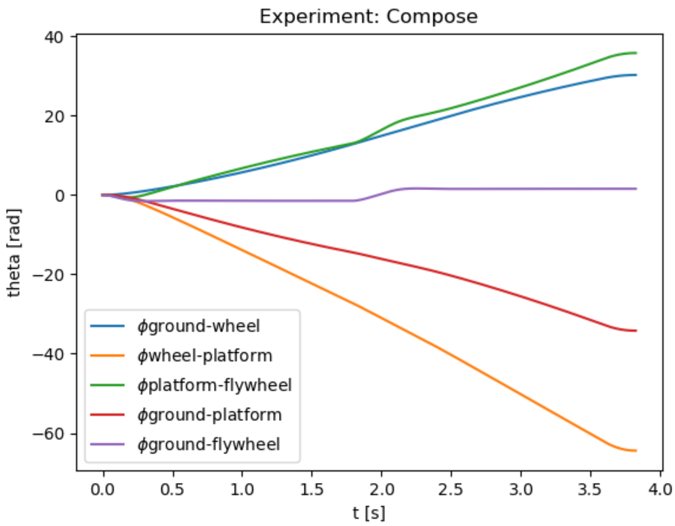


Figure 29: Plot of the angles for the compose experiment.

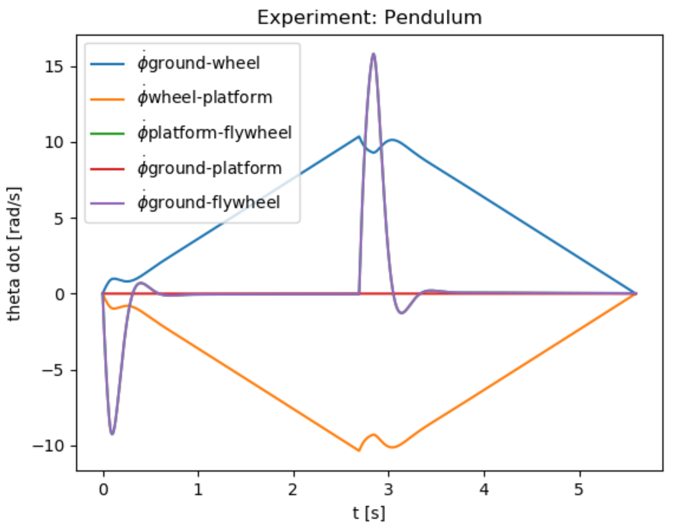


Figure 30: Plot of the angular velocities for the pendulum experiment.

7. Flywheel brake study

This section aims to study a possible way to break the flywheel without applying torque to the platform. The idea is combining both methods: we will leave the moving weight free.

This way, when the weight is going upward will have a larger radius than when is going downward and will produce an average moment against the movement of the flywheel.

From an energetic point of view we are transforming the rotation energy of the flywheel in to translation of the free cylinder and then realizing it trough collisions.

7.1 System of differential equations

As described in figure 12 we will use two variables to describe the flywheel position: r and θ.

Using equation 4:

$$\tau_{motor-flywheel} = \ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r_{max} - r) * \sin(\theta)$$

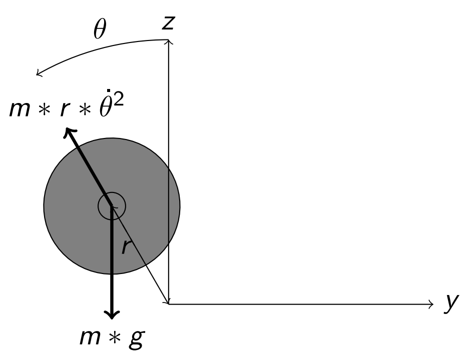


Figure 31: Cylinder force diagram.

As seen in figure 31, we can deduce newtons equation for the distance from the cylinder to the center r. Note that we are adding the centrifugal force term due to the non-inertial frame.

$$\ddot{r} * m = -m * g * cos(\theta) + m * r * \dot{\theta}^2$$

$$\ddot{r} = -g * cos(\theta) + r * \dot{\theta}^2$$

The variables we will be using for our ODE system are: r,$\dot{r}$,θ,$\dot{\theta}$

Note that we impose $\tau_{flywheel} = 0$ so the motor is not applying any torque.

$$\begin{cases} \dot{r} = \dot{r} \\ \ddot{r} = -g * cos(\theta) + r * \dot{\theta}^2 \\ \dot{\theta} = \dot{\theta} \\ \ddot{\theta} = \frac{m_{cylinder} * g * (r - r_{max}) * \sin\theta}{I_{flywheel}(r)} \end{cases}$$

Our initial conditions will be the free cylinder mass lying on the bottom of the flywheel and the flywheel turning at a speed θ_0:

$$\begin{cases} r = r_{max} \\ \dot{r} = 0 \\ \theta = \pi \\ \dot{\theta} = \theta_0 \end{cases}$$

We will use a Poincar map to simulate the bounce with the end of the guides at $r = r_{min}$ and $r = r_{max}$. At each bounce we will reduce its kinetic energy by a percentage *bounce_percentage*.

7.2 Results

The parameters of the simulation where:

$$\begin{cases} r_{flywheel} = 8cm \\ r_{wheel} = 9cm \\ w = 7cm \\ \dot{\theta}_0 = 4.2\pi rad/s \\ bounce\_percentage = 0.0(totallyinelastic) \end{cases}$$

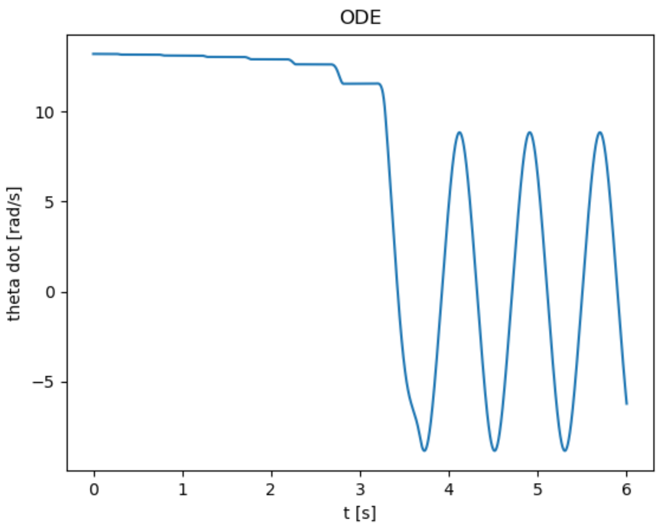


Figure 32: How the variable $\dot{\theta}$ evolve over time

As we can see in figure 32 the flywheel is braking until it becomes a pendulum and starts oscillating.

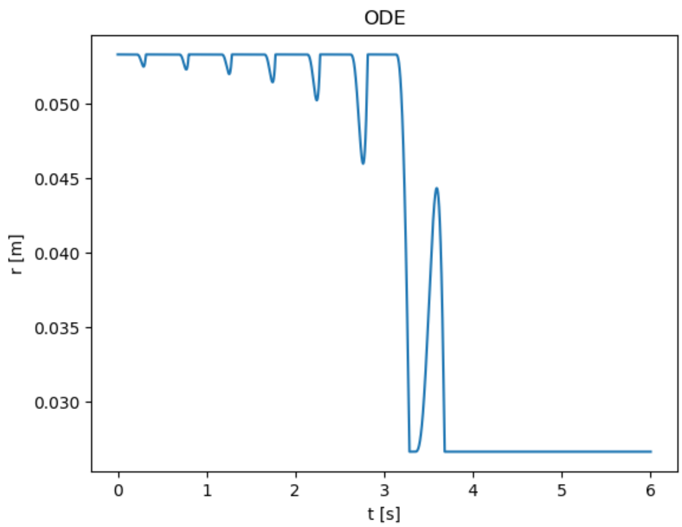


Figure 33: How the variable r evolves over time

In the first laps the cylinder is almost always at the maximum value of r, but as the speed decrease each lap the value of r decrease until it hits the r_{min}. In other words, at the begining is operating as a flywheel but then thanks to the collisions it becomes a pendulum.

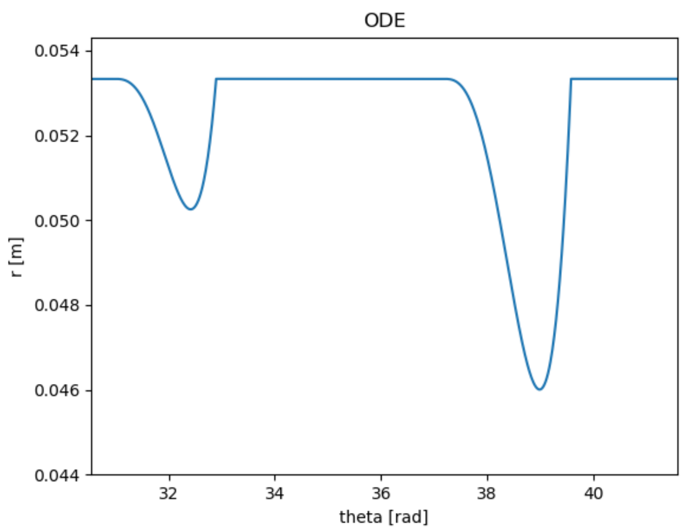


Figure 34: How the variable r evolve over θ zoomed

In image 34 we can appreciate that the r decrease slower that what it increase.

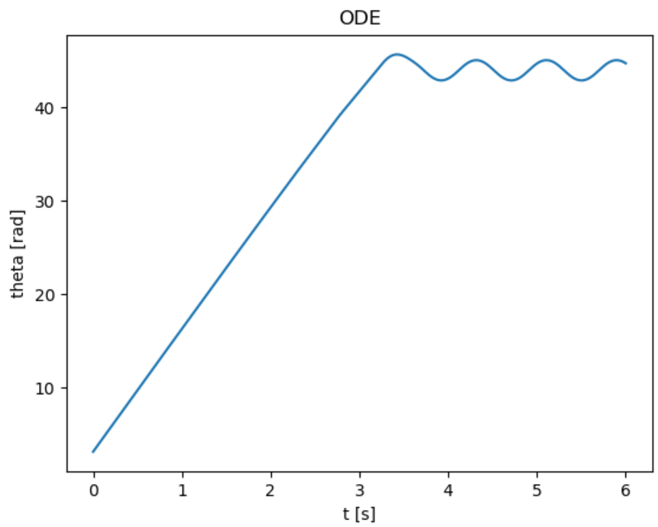


Figure 35: How the variable θ evolves over time

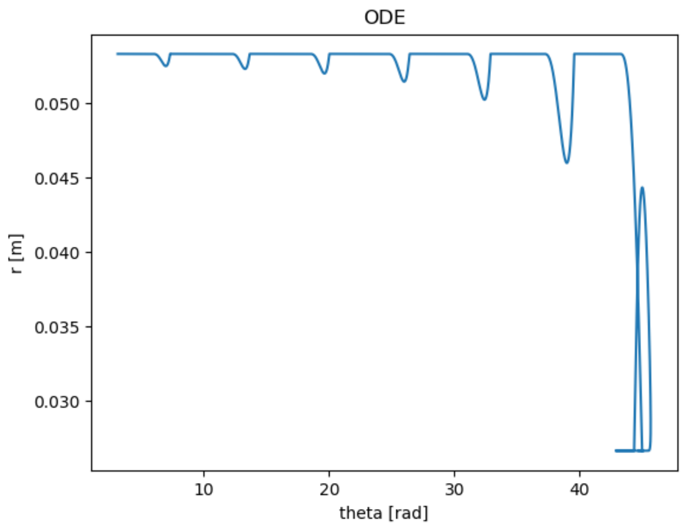


Figure 36: How the variable r evolves over θ

8. Rectilinear movement with $r_{flywheel}$ free

In this section we have developed the Lagrange Mechanics for the system with one mass of the flywheel being free (i.e. Movable along a rail).

The generalized coordinates (q) will be:

1. $\phi_{ground-wheel}$: rotation of the wheel respect the ground.
2. $\phi_{wheel-platform}$: rotation of the platform respect the wheel.
3. $\phi_{platform-flywheel}$: rotation of the flywheel respect the platform.
4. r: distance from the center of the flywheel to the free cylinder.

We will use two auxiliary variables:

1. $\phi_{ground-platform} = \phi_{ground-wheel} + \phi_{wheel-platform}$: rotation of the platform respect the ground.
2. $\phi_{ground-flywheel} = \phi_{ground-platform} + \phi_{platform-flywheel}$: rotation of the flywheel respect the ground.

The total potential energy:

$$V = m_{cylinder} \cdot (r - r_{flywheel-max}) \cdot cos(\phi_{ground-flywheel}) \cdot g \tag{29}$$

The total kinetic energy:

$$T = \frac{1}{2} \cdot [\dot{\phi}^2_{ground-wheel} \cdot I_{wheel} + \dot{\phi}^2_{ground-platform} \cdot I_{platform} + \dot{\phi}^2_{ground-flywheel} \cdot I_{flywheel}(r) + \dot{\phi}^2_{ground-wheel} \cdot r^2_{wheel} \cdot m_{total} + \dot{r}^2 \cdot m_{cylinder}] \tag{30}$$

The Lagrangian is defined as:

$$L = T - V \tag{31}$$

Lagrange's equation is:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_j}) = \frac{\partial L}{\partial q_j} + F_j \tag{32}$$

So in our case:

$$\frac{\partial L}{\partial \dot{\phi}_{ground-wheel}} = \dot{\phi}_{ground-wheel} \cdot I_{wheel} + \dot{\phi}_{ground-platform} \cdot I_{platform} + \dot{\phi}_{ground-flywheel} \cdot I_{flywheel}(r) + \dot{\phi}_{ground-wheel} \cdot r^2_{wheel} \cdot m_{total} \tag{33}$$

$$\frac{\partial L}{\partial \dot{\phi}_{wheel-platform}} = \dot{\phi}_{ground-platform} \cdot I_{platform} + \dot{\phi}_{ground-flywheel} \cdot I_{flywheel}(r) \tag{34}$$

$$\frac{\partial L}{\partial \dot{\phi}_{platform-flywheel}} = \dot{\phi}_{ground-flywheel} \cdot I_{flywheel}(r) \tag{35}$$

$$\frac{\partial L}{\partial \dot{r}} = \dot{r} \cdot m_{cylinder} \tag{36}$$

We define M as the following matrix:

$$\begin{pmatrix} I_{wheel} + I_{platform} + I_{flywheel}(r) + r_{wheel}^2 \cdot m_{total} & I_{platform} + I_{flywheel}(r) & I_{flywheel}(r) & 0 \\ I_{platform} + I_{flywheel}(r) & I_{platform} + I_{flywheel}(r) & I_{flywheel}(r) & 0 \\ I_{flywheel}(r) & I_{flywheel}(r) & I_{flywheel}(r) & 0 \\ 0 & 0 & 0 & m_{cylinder} \end{pmatrix} \tag{37}$$

In matrix form and with our generalized coordinates:

$$\frac{\partial L}{\partial \dot{q}} = M \cdot \dot{q} \tag{38}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = M \cdot \ddot{q} + \dot{M} \cdot \dot{q} \tag{39}$$

Let's recall the definition of $I_{flywheel}(r)$

$$I_{flywheel}(r) = m_{cylinder} \cdot r^2 + C \tag{40}$$

Compute the derivative

$$\dot{I}_{flywheel}(r) = 2 \cdot m_{cylinder} \cdot r \cdot \dot{r} \tag{41}$$

We compute $\dot{M}$ as the following matrix:

$$\dot{M} = \dot{I}_{flywheel}(r) \cdot \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{42}$$

Let a be::

$$\frac{\partial L}{\partial q_{1..3}} = a = m_{cylinder} \cdot (r - r_{flywheel-max}) \cdot g \cdot sin(\phi_{ground-flywheel}) \tag{43}$$

Let b be:

$$\frac{\partial L}{\partial r} = b = \cdot m_{cylinder} \cdot r \cdot \dot{\phi}_{ground-flywheel}^2 - m_{cylinder} \cdot g \cdot cos(\phi_{ground-flywheel}) \tag{44}$$

$$\frac{\partial L}{\partial q} = \begin{pmatrix} a \\ a \\ a \\ b \end{pmatrix} \tag{45}$$

So using Lagrange's equation we get:

$$\boxed{M \cdot \ddot{q} + \dot{M} \cdot \dot{q} = \begin{pmatrix} a \\ a \\ a \\ b \end{pmatrix} + F} \tag{46}$$

9. Components

In this section we will explain all the components used during the building of the robot and how to get them.

9.1 Mechanical components

These components do not have any electronics and the important thing about them is their mechanical function.

9.1.1 Central Body

Used to host the flywheel, a motor and a bearing. It also has lateral tabs so it can easily be joined with the lateral body part. It's 3D printed and in the same orientation you see in figure 37.

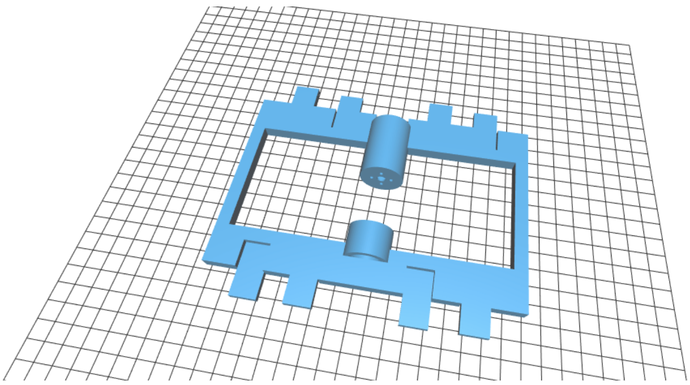


Figure 37: 3D Model of the central body.

9.1.2 Lateral Body

Used to host the wheel motor and the rest of the electronic components. It also has lateral tabs so it can easily be joined with the central body part. It's 3D printed and in the same orientation you see in figure 38.

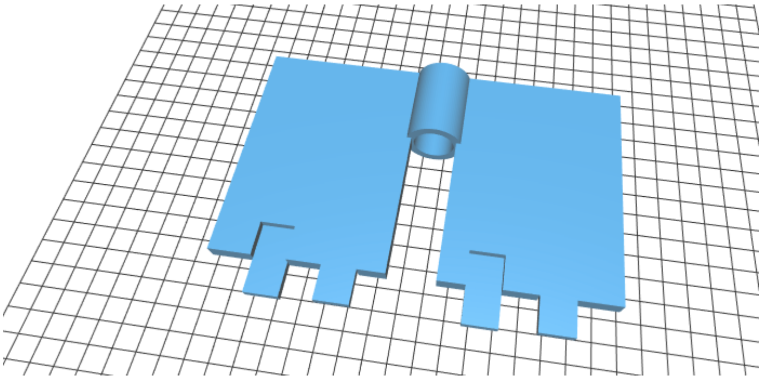


Figure 38: 3D Model of lateral body.

9.1.3 Wheels

The wheels are 3D printed. The orientation we used to print them is putting the motor axis vertical. Each wheel is produced by two symmetric parts that are glued together around the motor.

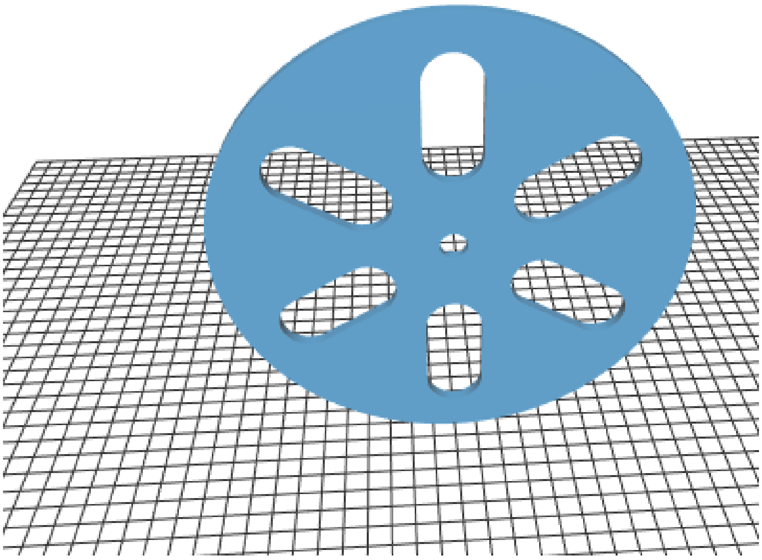


Figure 39: 3D Model of a wheel.

9.1.4 Flywheel

The flywheel is composed by 3 parts:

1. 3D printed wheels very similar to the one in figure 39
2. Steel bolts, washers and nuts.
3. Metallic axis.

9.1.5 Bearings

We are using angular balls bearing as the one seen in the figure 40 to surround the flywheel axis. This bearing is tight fit in to the 3D printed central body. The main purpose of the bearing is to support the flywheel metal axis and help the motor with the flywheel weight stress.

Figure 40: Photo of a bearing.

9.1.6 Reinforcements

After we built the first prototype, we saw that the robot was suffering from bending. So we decided to reinforce the structure with additional 3D printed tabs and two aluminum 'U' profiles to solve the problem.

9.2 Electronic components

9.2.1 Raspberry Pi

The Raspberry Pi is a small single-board computer. We are using Raspberry Pi 3 Model B. It has some features that we will use:

- GPIO pins: General Purpose Input/ Output pins are used to communicate with the motors and other peripherals.
- Ethernet connection: We used to be able to set up the Wi-Fi's connection.
- Wi-Fi's connection: We used to be able to control the robot without cables.
- 5V and 3V lines: It's useful to get the 3V to power the encoders.
- DC's hardware power: So we can power the Raspberry the same way as the other
- I2C: I2C is a serial protocol for two-wire interface to connect low-speed devices like micro-controllers. We use it to communicate the Raspberry Pi

Figure 41: Raspberry Pi picture

Weight	42 g
Price per unit	35 euros
Number of units	1

9.2.2 Batteries

We are using four identical batteries to power our system. They are commercial external power supplies for smartphones. They can be recharged via a micro-USB. We use USB cables to transmit the power to an outlet that connects two batteries in serial that power the DC Bridges. These bridges are powering the motors and the Raspberry.

These batteries do not always work as expected because they have integrated security measures that disconnect them when they detect that not power is being used.

In the other hand they incorporate many features as battery level indicator, incorporated recharge mechanisms and on/off buttons.

Weight	200 g
Price per unit	10 euros
Number of units	4

9.2.3 Printed Circuit Board

In this project we tested the basic electronic with a breadboard, once we knew the electrical components where working we decided to transfer our circuit to a Printed Circuit Board (PCB). In particular, we used a Double Sided PCB that allows routing traces in both sides.

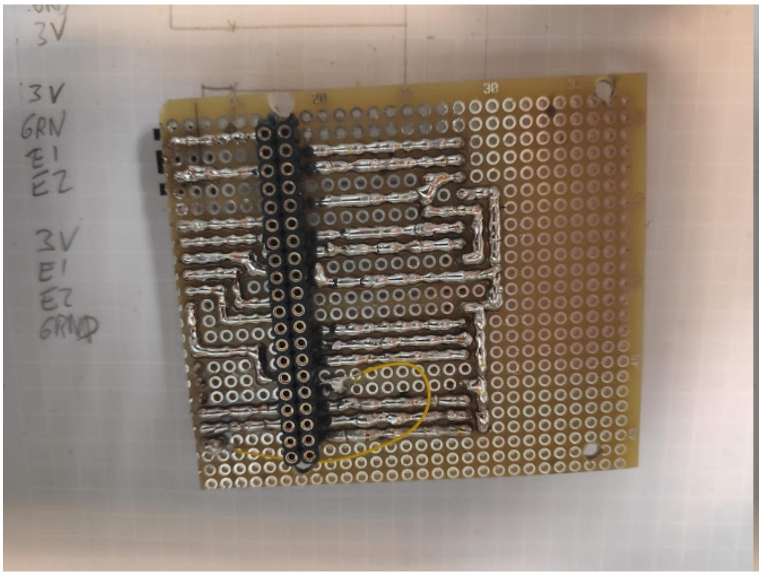


Figure 42: Printed circuit board

9.2.4 DC Motor

A DC Motor is a rotary electric machine that transforms electrical energy (in the form of direct current) into mechanical energy through electromagnetic interactions.

Here are the specifications of our three motors:

Operating voltage	between 3 V and 9 V
Free-run speed at 6 V	176 RPM
Free-run current at 6 V	80 mA
Stall current at 6V	900 mA
Stall current at 6V	5 kgcm
Gear ratio	1:35
Reductor size	21 mm
Weight	85 g
Price per unit	10 euros
Number of units	3

Figure 43: DC Motor

9.2.5 H Bridge

An H bridge is an electronic circuit that switches the polarity of a voltage applied to a load (in our case DC motors). This allows the motor to go forwards and reverse.

Also, it has the possibility to modulate the output power through a PWM signal.

Our particular H Bridge (L298N) is capable of supporting two motors at the same time. This means that it has 6 input signals, 2 PWM (one for each motor) and 4 enable signals (2 for each motor).

The input power for the H bridge is 10V but it has lost and it arrives to the motor at around 8,5 V. Also it has a 5V power output that we used to feed the Raspberry Pi.

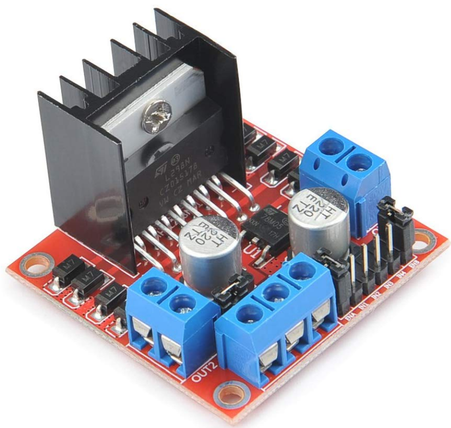


Figure 44: H Bridge

9.2.6 Rotatory Encoder

A rotary encoder, also called a shaft encoder, is an electro-mechanical device that converts the angular position or motion of a shaft or axle to analog or digital output signals.

Our encoder has two signals, channel A and channel B offset by 90 degrees (in quadrature). The direction of rotation can be determined by which channel is leading. If channel A is leading, the direction is taken to be clockwise, and if channel B is leading, the direction is counterclockwise.

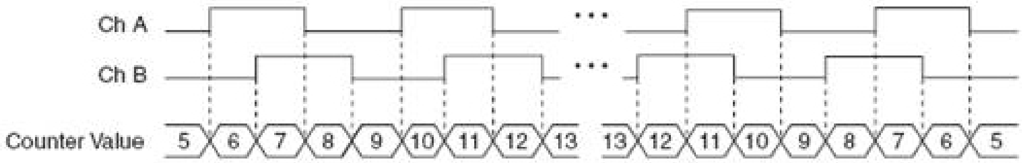


Figure 45: Encoder Signals. Image credit: National Instruments Corporation.

We programmed an interruption that fires every time a channel changes the state. It increments (or decrements) a counter, so we can then compute the position and the speed of each motor.

9.2.7 Accelerometer

An accelerometer is a device that measures proper acceleration. The one we are using is MPU-6050. We use the acceleration measured by the accelerometer to know in which inclination is the platform. The accelerometer is connected to the Raspberry Pi via I2C.

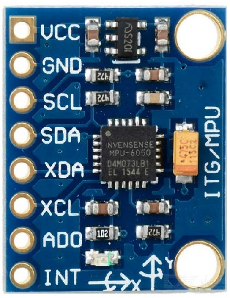


Figure 46: Pictures of a MPU-6050

10. Control

So far we have analyzed our system as a continuous system. However, we are using the Raspberry Pi to control the robot, so we will be using digital control. It's a branch of control theory that uses digital computers to act as system controllers.

In order to sample the position and the speed we use the rotatory encoders. They count the number of flags so there is a constant to convert them in to radiants.

All our digital control was made in periodical loops. To get the speed we divided the incremental position by the period of the loop. All the controllers are PID controllers each of time with different inputs, constants and outputs.

A PID controller continuously calculates an error value $e(t)$ as the difference between a desired set point and a measured process variable and applies a correction based on proportional, integral, and derivative terms (denoted P, I, and D respectively).

To adjust the controller constants in all cases we did the following way:

1. Set all gains to zero.
2. Increase the P gain until the response to a disturbance is steady oscillation.
3. Increase the D gain until the oscillations go away (i.e. it's critically damped).
4. Repeat steps 2 and 3 until increasing the D gain does not stop the oscillations.
5. Set P and D to the last stable values.
6. Increase the I gain until it brings you to the set point with no oscillations desired.

10.1 Position control

The first steep was to control the position of a peg attached to the motor. We started by making sure that the encoder was reading the correct position.

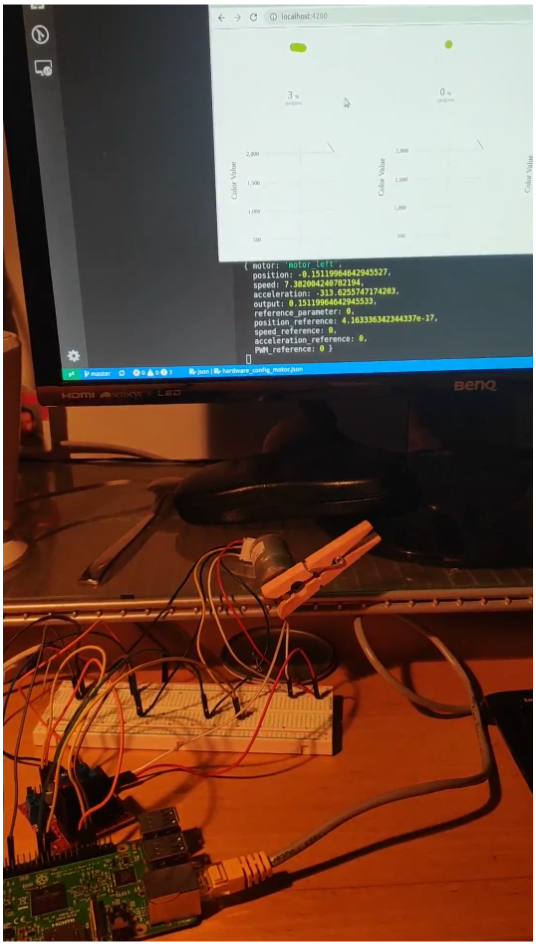


Figure 47: Picture of the set up to control the peg.

10.2 Speed control

10.3 Inclination control

11. Software Design

11.1 User Interface

11.2 Planner

11.3 Controller

12. Conclusion