Master Thesis

A Remotely-driven Hoverboard With Platform Leaning Control

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TO DO: This is the abstract

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Figure 1: Picture of a commercial *segway hover-board*

1 Objective

The objective of this project is to design, build and run reinforcement learning experiments on a dynamic robot. We wish to make this experiments easy and cheap to reproduce so we will try minimize its components and fabrication cost.

The chosen robot is inspired in a *segway hover-board*, similar to the one in Figure 1. The two wheels are controlled with classic control algorithms and the inclination of the central body is controlled with a reinforcement learning algorithm.

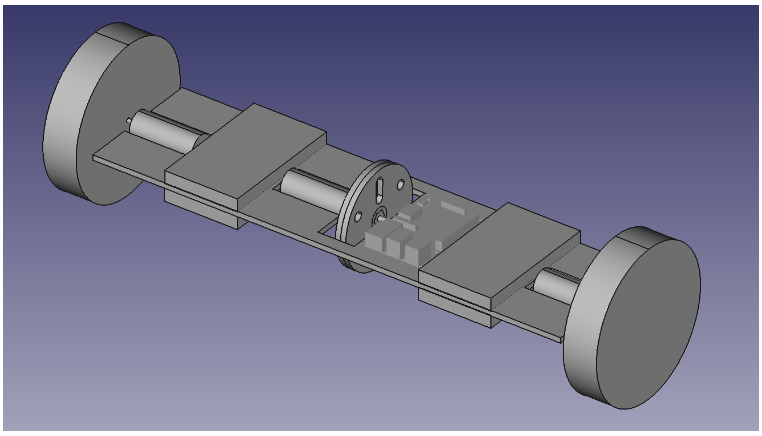


Figure 2: Isometric render view

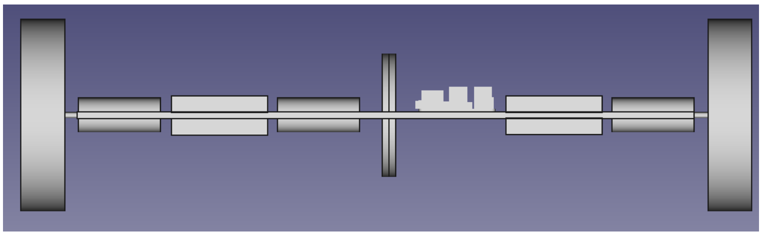


Figure 3: Front render view

2 Design

The design of the robot is done with the 3D design software Free-cad. All part files are uploaded to the GitHub repository `https://github.com/tarragoesteve/TFM` under the hardware folder.

You can see the main views on Figure 2, 3, 4 and 5.

We included three actuators in the robot because we want to control three degrees of freedom (inclination and speed of both wheels). Furthermore we introduced a flywheel/pendulum so we can control the inclination of the body.

We ensured symmetry along the axis formed by all motors in order to have an equilibrium in all possible inclinations without the need of external forces. We also took in consideration that the reinforcement learning algorithm starts being clumsy so none of the configurations should intersect with the ground. Figure 5 illustrates this restriction.

2.1 Flywheel design

To control the inclination of the body two strategies are taken in to account. Creating torque by a pendulum or accelerating the flywheel. In order to experiment with both of them we designed a part to allow both configuration by placing weights in different spots, see figure 6.

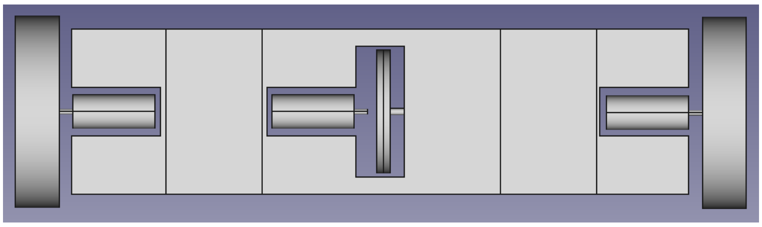


Figure 4: Top render view

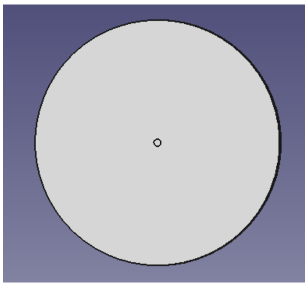


Figure 5: Side render view

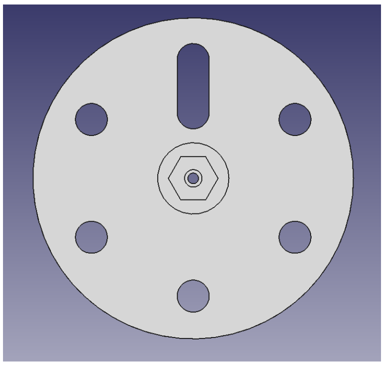


Figure 6: Fly wheel side render view

In order to create a configuration with maximum gravitational torque we have done the following computation. We denote the torque pendulum torque τ, consider the masses are cylinders of mass $m_{cylinder}$ with radius r_c and width w and the radius of the flywheel is r_f.

Each mass weights:

$$m_{cylinder} = \rho * w * \pi * r_c^2$$

All the gravitational torque created by the masses will be compensated with the opposite weight except for the two masses with different radius.

One of the weight can be placed along a rail. The distance to the center will vary from $r_{min} = r_c + r_{motor-axis} \approx r_c$ to $r_{max} = r_f - r_c$.

The maximum torque takes place when these two masses are aligned horizontal with respect the ground and the movable weight is at distance r_{min} from the center.

$$\tau_{max}(r_c) = m_{cylinder} * g * r_{max} - m_{cylinder} * g * r_{min} = m_{cylinder} * g * (r_f - 2 * r_c)$$

In order to maximize τ it we first compute the derivative:

$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = g * (\frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) - m_{cylinder} * 2)$$

$$\frac{\partial m_{cylinder}}{\partial r_c} = 2 * \rho * w * \pi * r_c$$

An make it zero to find the maximum:

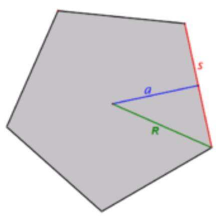
$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = 0$$

Substituting and simplifying we get:

$$\frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) = m_{cylinder} * 2 \Rightarrow 2 * \rho * w * \pi * r_c \ * (r_f - 2 * r_c) = \rho * w * \pi * r_c^2 * 2$$

$$\Rightarrow r_c \ * (r_f - 2 * r_c) = r_c^2 \Rightarrow (r_f - 2 * r_c) = r_c \Rightarrow \boxed{r_f = 3 * r_c}$$

The circumradius R from the center of a regular polygon to one of the vertices is related to the side length s by:



$$R = \frac{s}{2 * \sin \frac{\pi}{n}}$$

In our case:

$$R = r_f - r_c;$$

$$s = 2 * r_c$$

Substituting in the circumradius equation we get n = 6, so we will use 6 masses in our flywheel.

3 Mechanical analysis

3.1 Reference frames

In order to study the behavior of the robot we will use the following frames:

- Absolute frame: From a fix object in the room.
- Body frame: From the body of our robot.

TO DO: Diagrams of the frames in tikz.

3.2 Inclination control

In order to keep the inclination of the platform at a certain angle ϕ we must be able to compensate all the torque being to the body.

$$\ddot{\phi} * I_{body} = \tau_{body}$$

Assuming that the body is well balanced and neglecting the torque generated by the friction with air, the sum of all the torques in the motor axis applied to the body is equal to the sum of the torque applied by the motors:

$$\tau_{body} = \sum \tau_{motors}$$

The torque that the motors deliver to the wheels and to the flywheel create a reaction in the body in the opposite direction.

$$\tau_{body} = -\tau_{right-wheel} - \tau_{left-wheel} - \tau_{flywheel}$$

If we want to control the inclination ϕ, we must be able to control τ_{body} in a range $\tau_{body} \in (-\epsilon, \epsilon)$. Observe that the angular acceleration $\ddot{\phi}$ of the body is linearly dependent with the torque it receives. In order to simplify the calculations we will assume $\epsilon = 0$.

$$0 = -\tau_{right-wheel} - \tau_{left-wheel} - \tau_{flywheel} \Rightarrow \tau_{right-wheel} + \tau_{left-wheel} = -\tau_{flywheel} \quad (1)$$

In other words, we must compensate the torque of the wheels with the torque of the flywheel.

3.3 Wheels torque

The wheel torque we can induce is limited by the motor specifications. Note that the maximum torque of the motor is a function of velocity and in particular at max speed the torque is zero.

$$\tau_{motor}(w_{wheel})$$

We assume that the wheels just roll and do no slip. The robot is pushed by the wheels that make a force F_{drag} against the ground in the contact point. See figure 7.

We can express the torque at the center of the of the wheel as:

$$\tau_{wheel} - F_{drag} * r_{wheel} = I_{wheel} * \dot{w}_{wheel}$$

$$\tau_{wheel} = min(\tau_{motor}(w_{wheel}), I_{wheel} * \dot{w}_{wheel} + F_{drag} * r_{wheel}) \tag{2}$$

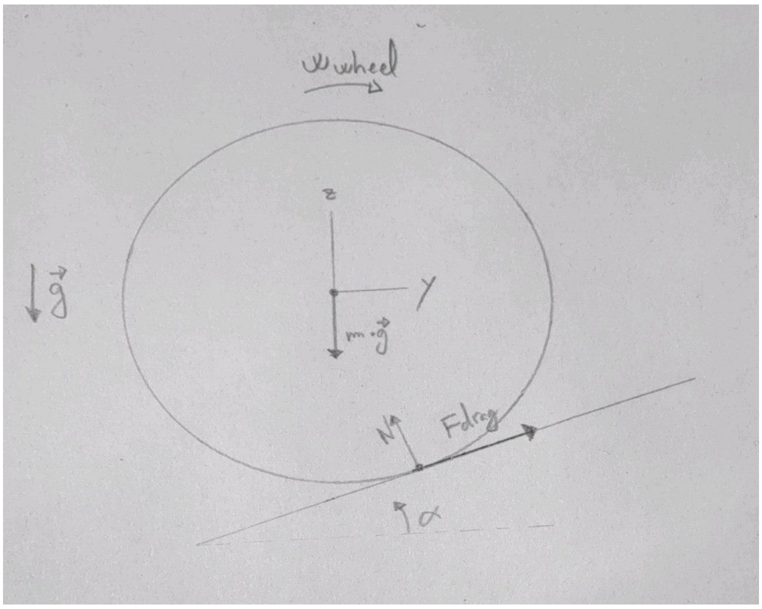


Figure 7: Wheel force diagram

3.4 Flywheel torque

The flywheel torque we can induce is also limited by the motor specifications.

Assuming a general configuration of the flywheel where the moving mass is at distance r and angle θ, see figure 8. we formulate its torque the following way:

$$\tau_{flywheel} = min(\tau_{motor}(w), \ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r - r_{max}) * \sin\theta) \tag{3}$$

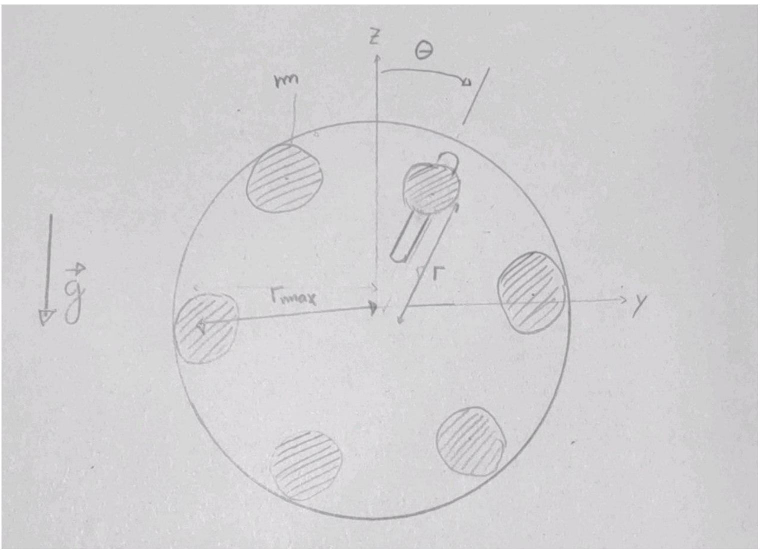


Figure 8: Flywheel force diagram

3.5 Maximum speed and acceleration

We will assume both wheels turn at the same speed, have the same F_{drag} and the same τ_{wheel}:

$$w_{wheel-left} = w_{wheel-right} = w$$

Applying Newton's first law in the y axis of figure 9

$$\ddot{y} * m_{total} = 2 * F_{drag} - m_{total} * g * sin(\alpha)$$

Substituting F_{drag} taking in to account equation 2:

$$\ddot{y} * m_{total} = 2 * \frac{\tau_{wheel} - I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} - m_{total} * g * sin(\alpha)$$

Using equation 1:

$$\Rightarrow \ddot{y} * m_{total} = -\frac{\tau_{flywheel}}{r_{wheel}} - \frac{I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} - m_{total} * g * sin(\alpha) \tag{4}$$

We will now study different cases to better understand this equation.

3.5.1 No terrain inclination ($\alpha = 0$)

The equation we get by substituting $\alpha = 0$ in equation 4:

$$\ddot{y} * m_{total} = -\frac{\tau_{flywheel}}{r_{wheel}} - \frac{I_{wheel} * \dot{w}_{wheel}}{r_{wheel}}$$

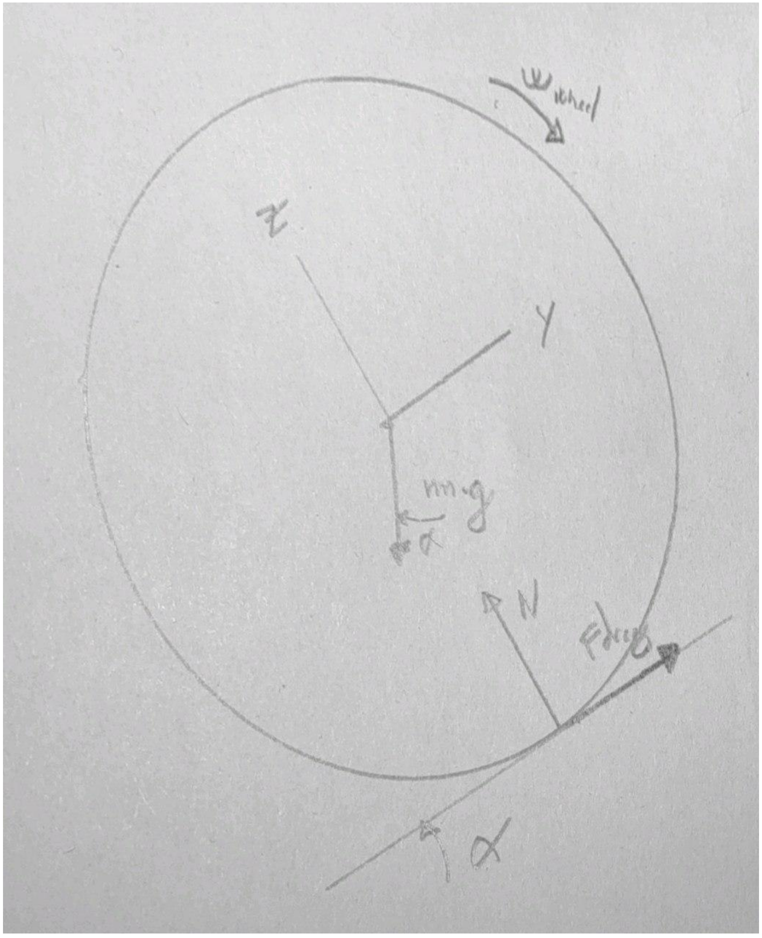


Figure 9: Wheel force diagram

Substituting equation 3

$$\ddot{y} * m_{total} = -\frac{\ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r - r_{max}) * \sin\theta}{r_{wheel}} - \frac{I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} \tag{5}$$

We will now split the study in three cases:

1. Flywheel case: r is fixed to $r = r_{max}$

   Then:

   $$\ddot{y} * m_{total} = -\frac{\ddot{\theta} * I_{flywheel}(r_{max})}{r_{wheel}} - \frac{I_{wheel} * \dot{w}_{wheel}}{r_{wheel}}$$

   Taking in two account the following relation:

   $$w_{wheel} * r_{wheel} = \dot{y} \Rightarrow \dot{w}_{wheel} * r_{wheel} = \ddot{y}$$

   The moment of inertia are:

   $$I_{wheel} = \frac{1}{2} * m_{wheel} * r_{wheel}^2$$

   $$I_{flywheel} \approx 6 * m_{cylinder} * r_{max}^2$$

   Let's go for it:

   $$\dot{w}_{wheel} * r_{wheel} * m_{total} = -\frac{\ddot{\theta} * 6 * m_{cylinder} * r_{max}^2}{r_{wheel}} - \frac{1}{2} * m_{wheel} * r_{wheel} * \dot{w}_{wheel}$$

   $$\dot{w}_{wheel} * r_{wheel} * (m_{total} + \frac{1}{2} * m_{wheel}) = -\frac{\ddot{\theta} * 6 * m_{cylinder} * r_{max}^2}{r_{wheel}}$$

   We define R as the quotient between $\dot{w}_{wheel}$ and $-\ddot{\theta}$

   $$R = \frac{\dot{w}_{wheel}}{-\ddot{\theta}} = \frac{6 * m_{cylinder} * r_{max}^2}{(m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}^2}$$

   We can see that R will always be smaller than 1 because $6 * m_{cylinder} < m_{total}$ and $r_{max} < r_{wheel}$. This means that we will be limited by the acceleration of the flywheel.

   The forward acceleration is:

   $$\ddot{y} = \dot{w}_{wheel} * r_{wheel} = -R * \ddot{\theta} * r_{wheel}$$

   And using equation 3 we get that the maximum is:

   $$\tau_{motor}(w) = \ddot{\theta} * I_{flywheel}(r) \Rightarrow \ddot{\theta} = \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)}$$

$$\ddot{y}_{max} = -R * \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} * r_{wheel}$$

$$\Rightarrow \ddot{y}_{max} = -\frac{6 * m_{cylinder} * r_{max}^2}{(m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}^2} * \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} * r_{wheel}$$

$$\boxed{\Rightarrow \ddot{y}_{max} = -\frac{\tau_{motor}(\dot{\theta})}{(m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}}}$$

In order to compute the maximum speed we will assume that the initial conditions are $\dot{\theta} = 0$ and $w_{wheel-max} = 0$

$$w_{wheel-max} = \int_{w_{wheel-0}}^{w_{wheel-max}} \dot{w}_{wheel} dt$$

Now we will proceed to do a change of variables in the integral.

$$\ddot{\theta} = -\frac{\dot{w}_{wheel}}{R}$$

$$\Rightarrow \dot{w}_{wheel} = -\ddot{\theta} * R$$

$$w_{wheel-max} = -R * \int_{\dot{\theta}_0}^{\dot{\theta}_{max}} \ddot{\theta} * dt = -R * \dot{\theta}_{max}$$

$$\boxed{\dot{y}_{max} = -r_{wheel} * R * \dot{\theta}_{max}}$$

And $\dot{\theta}_{max}$ is a limitation imposed by the motor specifications.

2. Pendulum: $\dot{\theta} = 0$, and r is fixed to $r = r_{min}$

   Using equation 5 and $\ddot{\theta} = 0$

$$\ddot{y} * m_{total} = -\frac{m_{cylinder} * g * (r - r_{max}) * \sin\theta}{r_{wheel}} - \frac{I_{wheel} * \dot{w}_{wheel}}{r_{wheel}}$$

   Multiplying by r_{wheel} both sides of the equation we get:

$$\ddot{y} * m_{total} * r_{wheel} = -m_{cylinder} * g * (r - r_{max}) * \sin\theta - I_{wheel} * \dot{w}_{wheel}$$

   And using $\dot{w}_{wheel} = \frac{\ddot{y}}{r_{wheel}}$

$$\ddot{y} * m_{total} * r_{wheel} + I_{wheel} * \frac{\ddot{y}}{r_{wheel}} = -m_{cylinder} * g * (r - r_{max}) * \sin\theta$$

$$\ddot{y} = -\frac{m_{cylinder} * g * (r - r_{max}) * \sin\theta}{m_{total} * r_{wheel} + \frac{I_{wheel}}{r_{wheel}}}$$

Replacing I_{wheel} and $r = r_{min}$

$$\ddot{y} = -\frac{m_{cylinder} * g * (r_{min} - r_{max}) * \sin\theta}{r_{wheel} * (m_{total} + \frac{1}{2} * m_{wheel})}$$

Which is maximum when $\sin\theta = 1$

$$\boxed{\ddot{y}_{max} = \frac{m_{cylinder} * g * (r_{max} - r_{min})}{r_{wheel} * (m_{total} + \frac{1}{2} * m_{wheel})}}$$

And there is no limitation on the maximum speed.

3. We leave the weight free:

   TO DO: ODE

3.5.2 No acceleration, just inclination ($\alpha > 0$)

Substituting $\ddot{y} = 0$ in equation 4

$$m_{total} * g * sin(\alpha) = -\frac{\tau_{flywheel}}{r_{wheel}}$$

$$sin(\alpha) = -\frac{\tau_{flywheel}}{m_{total} * g * r_{wheel}}$$

We are going to distinguish the same three cases as in the previous section:

1. Flywheel case: r is fixed to $r = r_{max}$

   Substituting equation 3

$$sin(\alpha) = -\frac{min(\tau_{motor}(w), \ddot{\theta} * I_{flywheel}(r))}{m_{total} * g * r_{wheel}}$$

   The maximum inclination at a certain moment:

$$sin(\alpha_{max}) = -\frac{\tau_{motor}(w)}{m_{total} * g * r_{wheel}}$$

   Maximum height can that the robot can achieve under the initial conditions $\dot{y} = \dot{y}_0$ and $\dot{\theta} = \dot{\theta}_0$:

$$h_{max} = \int_{\dot{\theta}_0}^{\dot{\theta}_{max}} \dot{y}_0 * sin(\alpha) * dt = \dot{y}_0 * \int_{\dot{\theta}_0}^{\dot{\theta}_{max}} -\frac{\ddot{\theta} * I_{flywheel}(r)}{m_{total} * g * r_{wheel}} * dt$$

$$\boxed{\Rightarrow h_{max} = \dot{y}_0 * (\dot{\theta}_0 - \dot{\theta}_{max}) * \frac{I_{flywheel}(r)}{m_{total} * g * r_{wheel}}}$$

2. Pendulum: $\dot{\theta} = 0$, and r is fixed to $r = r_{min}$

$$\tau_{flywheel} = m_{cylinder} * g * (r - r_{max}) * \sin\theta$$

$$sin(\alpha) = -\frac{m_{cylinder} * g * (r - r_{max}) * \sin\theta}{m_{total} * g * r_{wheel}} = \frac{m_{cylinder} * (r_{max} - r_{min}) * \sin\theta}{m_{total} * r_{wheel}}$$

$$sin(\alpha_{max}) = -\frac{m_{cylinder} * g * (r - r_{max}) * \sin\theta}{m_{total} * g * r_{wheel}}$$

$$\boxed{sin(\alpha_{max}) = \frac{m_{cylinder} * (r_{max} - r_{min}) * \sin\theta}{m_{total} * r_{wheel}}}$$

3. We leave the weight free: ODE

4 Optimizing

4.1 Restrictions

As a reminder: L is the length of the robot, and w the width of the flywheel cylinders.

1. Not crashing with the ground at any inclination can be translated as:

$$r_{wheel} > \sqrt{(r_{flywheel} + b)^2 + \frac{h}{2}^2} + 2 * \epsilon$$

2. We want to inserted the robot in to a bicycle wheel of diameter 0.5m so:

$$0.25^2 m > \sqrt{r_{wheel}^2 + L/2}$$

3. We can place all the devices:

$$L > 0.3m + w$$

4.2 Motor specifications

Here we have the factory specifications of our motors:

- Operating voltage: between 3 V and 9 V
- Nominal voltage: 6 V
- Free-run speed at 6 V: 176 RPM
- Free-run current at 6 V: 80 mA
- Stall current at 6V: 900 mA
- Stall torque at 6V: 5 kgcm
- Gear ratio: 1:35
- Reductor size: 21 mm
- Weight: 85 g

4.3 Optimizing the design

The function we want to maximize is the maxim height we can reach because it has into account the maximum speed and the maximum resistance we can face. Our hypothesis is that we start our resistance at maximum speed $\dot{y}_m ax$.

supposing we where travelling at the maximum velocity in plane:

$$h_{max} = -r_{wheel} * R * \dot{\theta}_{max} * (\dot{\theta}_0 - \dot{\theta}_{max}) * \frac{I_{flywheel}(r)}{m_{total} * g * r_{wheel}}$$

$\dot{\theta}_{max}$ will be given by the motor specifications and we won't optimize this parameter so we can remove it. The mass will be a constant + the mass of the flywheel which will depend linearly on w.

$$g = r_{wheel} * \frac{I_{flywheel}(r)^2}{m_{total} * g * r_{wheel} * (m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}^2}$$

$$g = \frac{I_{flywheel}(r)^2}{m_{total} * g * (m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}^2}$$

$$g = \frac{I_{flywheel}(r)^2}{m_{total} * g * (m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}^2}$$

$$g = \frac{(6 * m_{cylinder} * r_{max}^2)^2}{m_{total} * g * (m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}^2}$$

$$g = \frac{(6 * \rho * w * \pi * r_c^2 * r_{max}^2)^2}{m_{total} * g * (m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}^2}$$

$$g = \frac{(6 * \rho * w * \pi * (\frac{r_{flywheel}}{3})^2 * (\frac{2 * r_{flywheel}}{3})^2)^2}{m_{total} * g * (m_{total} + \frac{1}{2} * m_{wheel}) * r_{wheel}^2}$$

$$g = \frac{(\frac{8}{3} * \rho * w * \pi * r_{flywheel}^4)^2}{m_{total} * g * (m_{total} + \frac{1}{2} * m_{wheel}) * ((r_{flywheel} + b)^2 + \frac{h}{2}^2)} + 2 * \epsilon$$

$$m_{total} = 6 * m_{cylinder} + m_{rest} = 6 * \rho * w * \pi * r_c^2 + m_{rest} = 6 * \rho * w * \pi * (\frac{r_{flywheel}}{3})^2 + m_{rest}$$

$$m_{total} = \frac{1}{3} * \rho * w * \pi * r_{flywheel}^2 + m_{rest}$$

$$0.25 = r_{wheel}^2 + L/2$$

$$0.25 - r_{wheel}^2 = +L/2 \Rightarrow L = 2 * (0.25 - r_{wheel}^2) = .5 - r_{wheel}^2$$

$$L = .5 - (r_{flywheel} + b)^2 + \frac{h}{2}^2$$

$$w = L - 0.3m = .5 - (r_{flywheel} + b)^2 + \frac{h}{2}^2 - 0.3 = .2 - (r_{flywheel} + b)^2 + \frac{h}{2}^2$$

This plot shows how g varies when we change $R_{flywheel}$ between 0 and .5:
The maximum will be find on the border of all the restrictions?

4.4 Hypothesis

Assuming that the body is well balanced