

# **Master Thesis**

## **A Remotely-driven Hoverboard With Platform Leaning Control**

Author: Esteve Tarragó

Advisors: Lluís Ros Enric Celaya

Tutor: Mercè Ollé

MSc in Advanced Mathematics and Mathematical Engineering

Institut de Robtica i Informtica Industrial

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TO DO: This is the abstract

# Contents

<b>1</b>	<b>Objective</b>	<b>4</b>
<b>2</b>	<b>Design</b>	<b>5</b>
2.1	Flywheel design . . . . .	5
<b>3</b>	<b>Mechanical analysis</b>	<b>9</b>
3.1	Reference frames . . . . .	9
3.2	Inclination control . . . . .	9
3.3	Wheels torque . . . . .	10
3.4	Flywheel torque . . . . .	11
3.5	Maximum speed and acceleration . . . . .	11
3.5.1	No terrain inclination ( $\alpha = 0$ ) . . . . .	12
3.5.2	No acceleration, just inclination ( $\alpha > 0$ ) . . . . .	14
3.5.3	No acceleration, no inclination, just keeping speed $v$ . . . . .	15
<b>4</b>	<b>Optimization</b>	<b>17</b>
4.1	Restrictions . . . . .	17
4.2	Cost function . . . . .	17
4.3	Results . . . . .	18
4.4	Minimum section in the section of the flywheel . . . . .	20
4.5	Motor specifications . . . . .	21
<b>5</b>	<b>Flywheel brake study</b>	<b>22</b>
5.1	System of differential equations . . . . .	22
5.2	Results . . . . .	23



Figure 1: Picture of a commercial *segway hover-board*

## 1 Objective

The objective of this project is to design, build and run reinforcement learning experiments on a dynamic robot. We wish to make this experiments easy and cheap to reproduce so we will try minimize its components and fabrication cost.

The chosen robot is inspired in a *segway hover-board*, similar to the one in Figure 1. The two wheels are controlled with classic control algorithms and the inclination of the central body is controlled with a reinforcement learning algorithm.

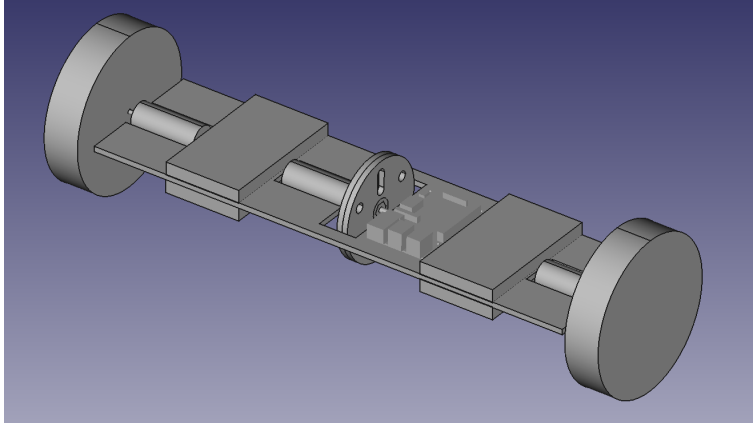


Figure 2: Isometric render view

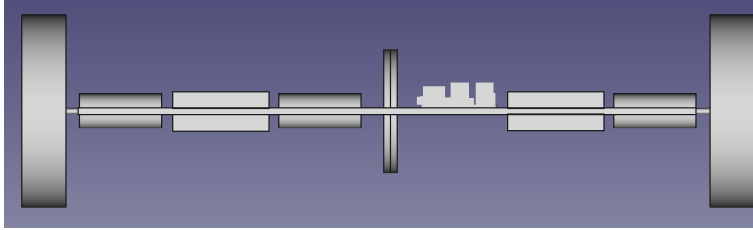


Figure 3: Front render view

## 2 Design

The design of the robot is done with the 3D design software Free-cad. All part files are uploaded to the GitHub repository <https://github.com/tarragoesteve/TFM> under the hardware folder.

You can see the main views on Figure 2, 3, 4 and 5.

We included three actuators in the robot because we want to control three degrees of freedom (inclination and speed of both wheels). Furthermore we introduced a fly-wheel/pendulum so we can control the inclination of the body.

We ensured symmetry along the axis formed by all motors in order to have an equilibrium in all possible inclinations without the need of external forces. We also took in consideration that the reinforcement learning algorithm starts being clumsy so none of the configurations should intersect with the ground. Figure 5 illustrates this restriction.

### 2.1 Flywheel design

To control the inclination of the body two strategies are taken in to account. Creating torque by a pendulum or accelerating the flywheel. In order to experiment with both of them we designed a part to allow both configuration by placing weights in different spots, see figure 6.

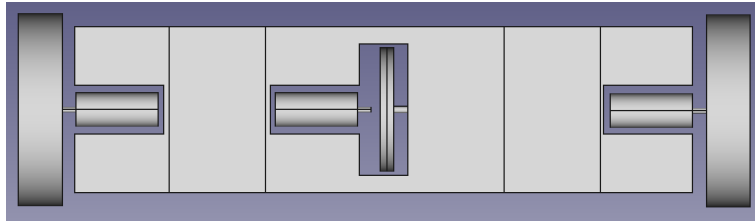


Figure 4: Top render view

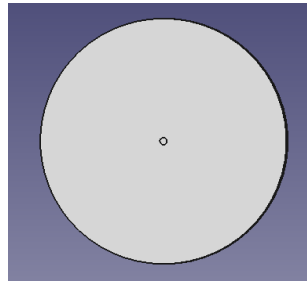


Figure 5: Side render view

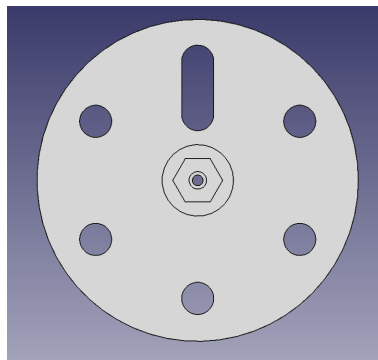


Figure 6: Fly wheel side render view

In order to create a configuration with maximum gravitational torque we have done the following computation. We denote the torque pendulum torque  $\tau$ , consider the masses are cylinders of mass  $m_{cylinder}$  with radius  $r_c$  and width  $w$  and the radius of the flywheel is  $r_f$ .

Each mass weights:

$$m_{cylinder} = \rho * w * \pi * r_c^2$$

All the gravitational torque created by the masses will be compensated with the opposite weight except for the two masses with different radius.

One of the weight can be placed along a rail. The distance to the center will vary from  $r_{min} = r_c + r_{motor-axis} \approx r_c$  to  $r_{max} = r_f - r_c$ .

The maximum torque takes place when these two masses are aligned horizontal with respect the ground and the movable weight is at distance  $r_{min}$  from the center.

$$\tau_{max}(r_c) = m_{cylinder} * g * r_{max} - m_{cylinder} * g * r_{min} = m_{cylinder} * g * (r_f - 2 * r_c)$$

In order to maximize  $\tau$  it we first compute the derivative:

$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = g * \left( \frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) - m_{cylinder} * 2 \right)$$

$$\frac{\partial m_{cylinder}}{\partial r_c} = 2 * \rho * w * \pi * r_c$$

An make it zero to find the maximum:

$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = 0$$

Substituting and simplifying we get:

$$\frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) = m_{cylinder} * 2 \Rightarrow 2 * \rho * w * \pi * r_c * (r_f - 2 * r_c) = \rho * w * \pi * r_c^2 * 2$$

$$\Rightarrow r_c * (r_f - 2 * r_c) = r_c^2 \Rightarrow (r_f - 2 * r_c) = r_c \Rightarrow \boxed{r_f = 3 * r_c}$$

The circumradius  $R$  from the center of a regular polygon to one of the vertices is related to the side length  $s$  by:



$$R = \frac{s}{2 * \sin \frac{\pi}{n}}$$

In our case:

$$R = r_f - r_c;$$

$$s = 2 * r_c$$

Substituting in the circumradius equation we get  $n = 6$ , so we will use 6 masses in our flywheel.



### 3 Mechanical analysis

#### 3.1 Reference frames

In order to study the behavior of the robot we will use the following frames:

- Absolute frame.
- Inclination frame.

Both frames are orthonormal frames that have the  $x$  axis in the same direction that the axis of the motors.

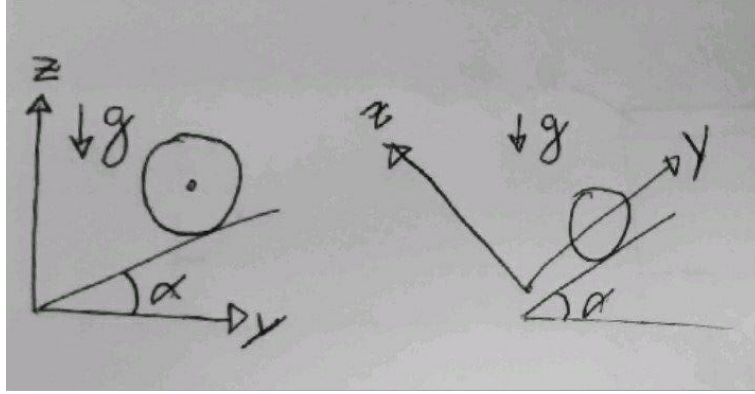


Figure 7: Reference frames diagram

#### 3.2 Inclination control

In order to keep the inclination of the platform at a certain angle  $\phi$  we must be able to compensate all the torque being to the body.

$$\ddot{\phi} * I_{body} = \tau_{body}$$

Assuming that the body is well balanced and neglecting the torque generated by the friction with air, the sum of all the torques in the motor axis applied to the body is equal to the sum of the torque applied by the motors:

$$\tau_{body} = \sum \tau_{motors}$$

The torque that the motors deliver to the wheels and to the flywheel create a reaction in the body in the opposite direction.

$$\tau_{body} = -\tau_{motor-right-wheel} - \tau_{motor-left-wheel} - \tau_{motor-flywheel}$$

If we want to control the inclination  $\phi$ , we must be able to control  $\tau_{body}$  in a range  $\tau_{body} \in (-\epsilon, \epsilon)$ . Observe that the angular acceleration  $\ddot{\phi}$  of the body is linearly dependent with the torque it receives. In order to simplify the calculations we will assume  $\epsilon = 0$ .

$$0 = -\tau_{motor-right-wheel} - \tau_{motor-left-wheel} - \tau_{motor-flywheel} \Rightarrow \tau_{motor-right-wheel} + \tau_{motor-left-wheel} = -\tau_{motor-flywheel} \quad (1)$$

In other words, we must compensate the torque of the wheels with the torque of the flywheel.

### 3.3 Wheels torque

The wheel torque we can induce is limited by the motor specifications. Note that the maximum torque of the motor is a function of velocity and in particular at max speed the torque is zero.

$$\tau_{motor-wheel}(w_{wheel})$$

We assume that the wheels just roll and do no slip. The robot is pushed by the wheels that make a force  $F_{friction}$  against the ground in the contact point. See figure 8.

We can express the torque at the center of the wheel as:

$$\begin{aligned} \tau_{motor-wheel} + F_{friction} * r_{wheel} &= I_{wheel} * \dot{w}_{wheel} \\ \tau_{motor-wheel} &= I_{wheel} * \dot{w}_{wheel} - F_{friction} * r_{wheel} \end{aligned} \quad (2)$$

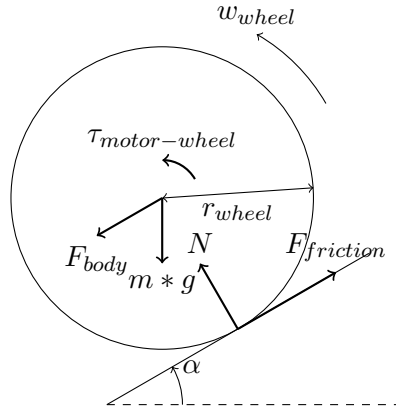


Figure 8: Wheel force diagram

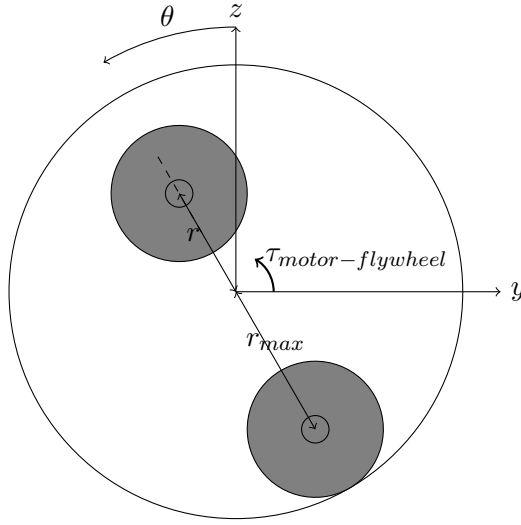


Figure 9: Flywheel diagram

### 3.4 Flywheel torque

The flywheel torque we can induce is also limited by the motor specifications.

Assuming a general configuration of the flywheel where the moving mass is at distance  $r$  and angle  $\theta$ , see figure 9. we formulate its torque the following way:

$$\begin{aligned}\tau_{motor-flywheel} + m_{cylinder} * g * (r - r_{max}) * \sin \theta &= \ddot{\theta} * I_{flywheel}(r) \\ \tau_{motor-flywheel} &= \ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r_{max} - r) * \sin \theta\end{aligned}\quad (3)$$

### 3.5 Maximum speed and acceleration

We will assume both wheels turn at the same speed, have the same  $F_{friction}$  and the same  $\tau_{wheel}$ :

$$w_{wheel-left} = w_{wheel-right} = w_{wheel}$$

Applying Newton's first law in the y axis of figure 10:

$$\ddot{y} * m_{total} = 2 * F_{friction} - m_{total} * g * \sin(\alpha)$$

Substituting  $F_{friction}$  taking in to account equation 2:

$$\ddot{y} * m_{total} = 2 * \frac{I_{wheel} * \dot{w}_{wheel} - \tau_{motor-wheel}}{r_{wheel}} - m_{total} * g * \sin(\alpha)$$

Using equation 1:

$$\Rightarrow \ddot{y} * m_{total} = \frac{2 * I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} + \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} * g * \sin(\alpha) \quad (4)$$

We will now study different cases to better understand this equation.

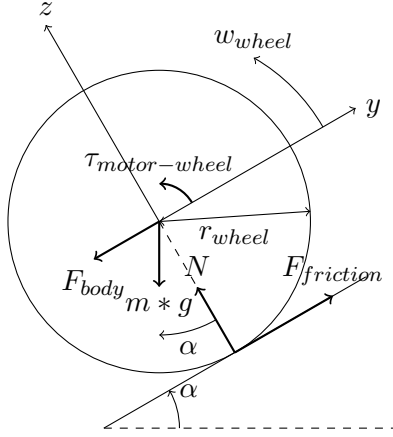


Figure 10: Wheel forward force diagram

### 3.5.1 No terrain inclination ( $\alpha = 0$ )

The equation we get by substituting  $\alpha = 0$  in equation 4:

$$\ddot{y} * m_{total} = \frac{\tau_{motor-flywheel}}{r_{wheel}} + \frac{2 * I_{wheel} * \dot{w}_{wheel}}{r_{wheel}}$$

Substituting equation 3

$$\ddot{y} * m_{total} = \frac{\ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r_{max} - r) * \sin \theta}{r_{wheel}} + \frac{2 * I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} \quad (5)$$

We will now split the study in two cases:

1. Flywheel case:  $r$  is fixed to  $r = r_{max}$

Then:

$$\ddot{y} * m_{total} = -\frac{\ddot{\theta} * I_{flywheel}(r_{max})}{r_{wheel}} + \frac{2 * I_{wheel} * \dot{w}_{wheel}}{r_{wheel}}$$

Taking in two account the following relation:

$$-w_{wheel} * r_{wheel} = \dot{y} \Rightarrow -\dot{w}_{wheel} * r_{wheel} = \ddot{y}$$

The moments of inertia are:

$$I_{wheel} = \frac{1}{2} * m_{wheel} * r_{wheel}^2$$

$$I_{flywheel} \approx 6 * m_{cylinder} * r_{max}^2$$

Let's go for it:

$$-\dot{w}_{wheel} * r_{wheel} * m_{total} = \frac{\ddot{\theta} * 6 * m_{cylinder} * r_{max}^2}{r_{wheel}} + m_{wheel} * r_{wheel} * \dot{w}_{wheel}$$

$$-\dot{w}_{wheel} * r_{wheel} * (m_{total} + m_{wheel}) = \frac{\ddot{\theta} * 6 * m_{cylinder} * r_{max}^2}{r_{wheel}}$$

We define R as the quotient between  $\dot{w}_{wheel}$  and  $-\ddot{\theta}$

$$R = \frac{\dot{w}_{wheel}}{-\ddot{\theta}} = \frac{6 * m_{cylinder} * r_{max}^2}{(m_{total} + m_{wheel}) * r_{wheel}^2}$$

We can see that R will always be smaller than 1 because  $6 * m_{cylinder} < m_{total}$  and  $r_{max} < r_{wheel}$ . This means that we will be limited by the acceleration of the flywheel.

The forward acceleration is:

$$\ddot{y} = \dot{w}_{wheel} * r_{wheel} = -R * \ddot{\theta} * r_{wheel}$$

And using equation 3 we get that the maximum is:

$$\tau_{motor}(w) = \ddot{\theta} * I_{flywheel}(r) \Rightarrow \ddot{\theta} = \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)}$$

$$\ddot{y}_{max} = -R * \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} * r_{wheel}$$

$$\Rightarrow \ddot{y}_{max} = -\frac{6 * m_{cylinder} * r_{max}^2}{(m_{total} + m_{wheel}) * r_{wheel}^2} * \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} * r_{wheel}$$

$$\Rightarrow \ddot{y}_{max} = -\frac{\tau_{motor}(\dot{\theta})}{(m_{total} + m_{wheel}) * r_{wheel}}$$

In order to compute the maximum speed we will assume that the initial conditions are  $\dot{\theta} = 0$  and  $w_{wheel-0} = 0$

$$w_{wheel-max} = \int_{t=0}^{t=t_{max}} \dot{w}_{wheel} * dt$$

Now we will proceed to do a change of variables in the integral.

$$\frac{\partial \dot{\theta}}{\partial t} = \ddot{\theta} \Rightarrow dt = \frac{d\dot{\theta}}{\ddot{\theta}}$$

$$w_{wheel-max} = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} \frac{\dot{w}_{wheel}}{\ddot{\theta}} * d\dot{\theta} = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} -R * d\dot{\theta} = -R * \dot{\theta}_{max}$$

$$\dot{y}_{max} = -r_{wheel} * R * \dot{\theta}_{max}$$

And  $\dot{\theta}_{max}$  is a limitation imposed by the motor specifications.

2. Pendulum:  $\dot{\theta} = 0$ , and  $r$  is fixed to  $r = r_{min}$

Using equation 5 and  $\ddot{\theta} = 0$

$$\ddot{y} * m_{total} = \frac{m_{cylinder} * g * (r_{max} - r) * \sin \theta}{r_{wheel}} + \frac{2 * I_{wheel} * \dot{\omega}_{wheel}}{r_{wheel}}$$

Multiplying by  $r_{wheel}$  both sides of the equation we get:

$$\ddot{y} * m_{total} * r_{wheel} = m_{cylinder} * g * (r_{max} - r) * \sin \theta + 2 * I_{wheel} * \dot{\omega}_{wheel}$$

And using  $\dot{\omega}_{wheel} = -\frac{\ddot{y}}{r_{wheel}}$

$$\ddot{y} * m_{total} * r_{wheel} + 2 * I_{wheel} * \frac{\ddot{y}}{r_{wheel}} = m_{cylinder} * g * (r_{max} - r) * \sin \theta$$

$$\ddot{y} = \frac{m_{cylinder} * g * (r_{max} - r) * \sin \theta}{m_{total} * r_{wheel} + \frac{2 * I_{wheel}}{r_{wheel}}}$$

Replacing  $I_{wheel}$  and  $r = r_{min}$

$$\ddot{y} = \frac{m_{cylinder} * g * (r_{max} - r_{min}) * \sin \theta}{r_{wheel} * (m_{total} + m_{wheel})}$$

Which is maximum when  $\sin \theta = 1$

$$\ddot{y}_{max} = \frac{m_{cylinder} * g * (r_{max} - r_{min})}{r_{wheel} * (m_{total} + m_{wheel})}$$

And there is no limitation on the maximum speed without considering air friction.

### 3.5.2 No acceleration, just inclination ( $\alpha > 0$ )

Substituting  $\ddot{y} = 0$  in equation 4

$$0 = \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} * g * \sin(\alpha)$$

$$m_{total} * g * \sin(\alpha) = \frac{\tau_{motor-flywheel}}{r_{wheel}}$$

$$\sin(\alpha) = \min(1, \frac{\tau_{motor-flywheel}}{m_{total} * g * r_{wheel}})$$

Substituting equation 3

$$\sin(\alpha) = \min(1, \frac{\ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r_{max} - r) * \sin \theta}{m_{total} * g * r_{wheel}})$$

We are going to distinguish the same two cases as in the previous section:

1. Flywheel case:  $r$  is fixed to  $r = r_{max}$  The maximum inclination at a certain moment:

$$\sin(\alpha_{max}) = \min\left(1, \frac{\ddot{\theta} * I_{flywheel}(r_{max})}{m_{total} * g * r_{wheel}}\right) = \min\left(1, \frac{\tau_{motor-flywheel}(w_{wheel})}{m_{total} * g * r_{wheel}}\right) \quad (6)$$

Maximum height can that the robot can achieve keeping its speed constant under the initial conditions  $\dot{y} = \dot{y}_0$  and  $\dot{\theta} = \dot{\theta}_0$ :

$$h_{max} = \int_{t=0}^{t=t_{max}} \dot{y}_0 * \sin(\alpha) * dt = \dot{y}_0 * \int_{t=0}^{t=t_{max}} \sin(\alpha) * dt$$

Using the same change of variables as in previous section:

$$\frac{\partial \dot{\theta}}{\partial t} = \ddot{\theta} \Rightarrow dt = \frac{d\dot{\theta}}{\ddot{\theta}}$$

$$h_{max} = \dot{y}_0 * \int_{\dot{\theta}=\dot{\theta}_0}^{\dot{\theta}=\dot{\theta}_{max}} \frac{\sin(\alpha)}{\ddot{\theta}} * d\dot{\theta} = \dot{y}_0 * \int_{\dot{\theta}=\dot{\theta}_0}^{\dot{\theta}=\dot{\theta}_{max}} \frac{I_{flywheel}(r_{max})}{m_{total} * g * r_{wheel}} * d\dot{\theta}$$

$$\Rightarrow h_{max} = \dot{y}_0 * (\dot{\theta}_{max} - \dot{\theta}_0) * \frac{I_{flywheel}(r_{max})}{m_{total} * g * r_{wheel}}$$

2. Pendulum:  $\dot{\theta} = 0$ , and  $r$  is fixed to  $r = r_{min}$

$$\sin(\alpha) = \min\left(1, \frac{\min(\tau_{motor-flywheel}, m_{cylinder} * g * (r_{max} - r_{min}) * \sin \theta)}{m_{total} * g * r_{wheel}}\right)$$

$$\sin(\alpha_{max}) = \min\left(1, \frac{\min(\tau_{motor-flywheel}, m_{cylinder} * g * (r_{max} - r_{min}))}{m_{total} * g * r_{wheel}}\right) \quad (7)$$

### 3.5.3 No acceleration, no inclination, just keeping speed $v$

We need to take into consideration air friction to see the speed limitation in the pendulum case.

$$F_{drag} = \frac{1}{2} * \rho * C_D * A * v^2$$

Adding this term to equation 4 we get:

$$\ddot{y} * m_{total} = \frac{2 * I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} + \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} * g * \sin(\alpha) - F_{drag}$$

And making  $\ddot{y} = 0$  and  $\alpha = 0$ .

$$F_{drag} = \frac{\tau_{motor-flywheel}}{r_{wheel}}$$

$$\frac{1}{2} * \rho * C_D * A * v^2 = m_{cylinder} * g * (r_{max} - r_{min}) * \sin(\theta) / r_{wheel}$$

The maximum v is then:

$$v_{max} = \sqrt{\frac{2 * m_{cylinder} * g * (r_{max} - r_{min})}{\rho * C_D * A * r_{wheel}}}$$

We will pick  $C_D = 1$ ,  $\rho = 1.2kg/m^3$  and  $A = 0.01m^2$  for our computations.



## 4 Optimization

### 4.1 Restrictions

As a reminder:  $L$  is the length of the robot, and  $w$  the width of the flywheel cylinders.

1. Not crashing with the ground at any inclination can be translated as:

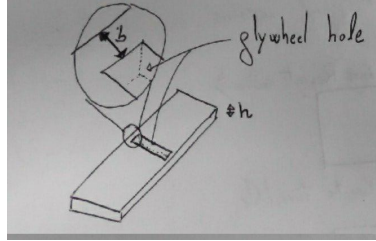


Figure 11: Flywheel hole diagram

$$r_{wheel} > \sqrt{(r_{flywheel} + b)^2 + \frac{h^2}{2}}$$

2. We want to insert the robot into an external of diameter 0.5m so:

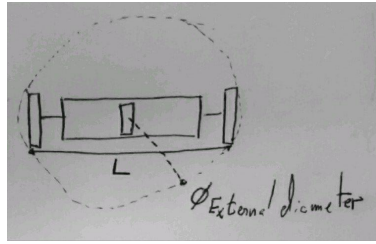


Figure 12: External diameter diagram

$$0.25m > \sqrt{r_{wheel}^2 + L^2/4}$$

3. We can place all the devices:

$$L > 0.3m + w$$

4. Maximum weight of the robot: 5kg

### 4.2 Cost function

The function we would like to maximize is the maximum angle we can keep our speed using the pendulum system. It's described in equation 7:

$$\sin(\alpha_{max}) = \frac{m_{cylinder} * (r_{max} - r_{min})}{m_{total} * r_{wheel}}$$

Also we have imposed a minimum  $\sin(\alpha_{max})$  for the flywheel system which was set to 0.3.

### 4.3 Results

Our procedure has been making a grid of the three parameters of the robot design:  $w$  (width of the cylinders),  $r_{wheel}$  and  $r_{flywheel}$ . We have fixed  $r_{flywheel}$ , iterated over the other two variables and kepted the best parameters for our cost function.

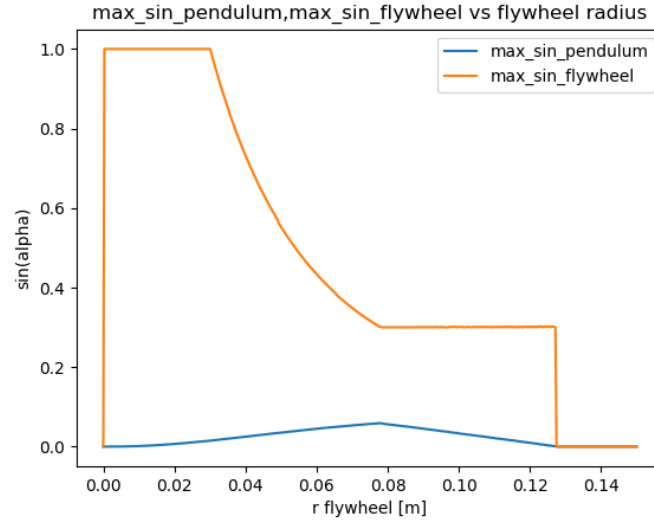


Figure 13: Plot of the sinus of the maximum inclination in which the robot can keep his speed.

In figure 13 we can see how the sinus of the flywheel is saturated to 1 and as long as it's possible it does not go below 0.3.

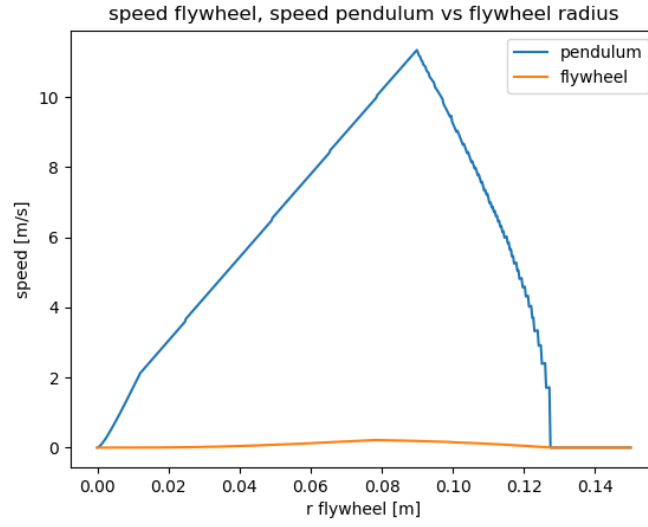


Figure 14: Plot of the maximum speed that the robot can achieve.

In figure 14 we can observe that the pendulum systems allows as to get a much higher speed than the flywheel.

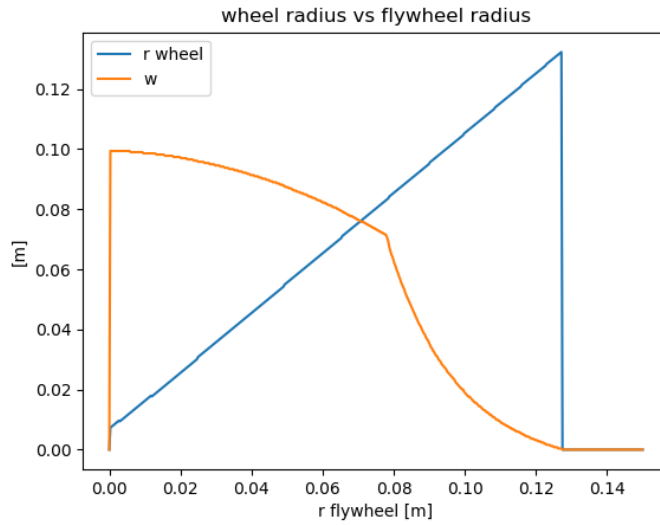


Figure 15: Plot of the parameters that maximize the cost function.

Just for reference this plot in figure 15 shows which parameters maximize the cost function so we can then design the robot accordingly.

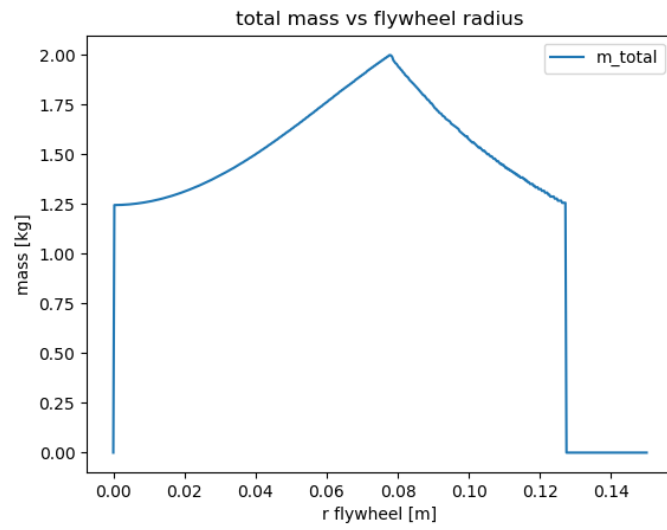


Figure 16: Plot of the mass for each configuration.

This plot 16 shows us which is the total mass of the flywheel in each of the configurations.

Our selected parameters are:

$r_{flywheel}$	$r_{wheel}$	$w$
8cm	9cm	7cm

#### 4.4 Minimum section in the section of the flywheel

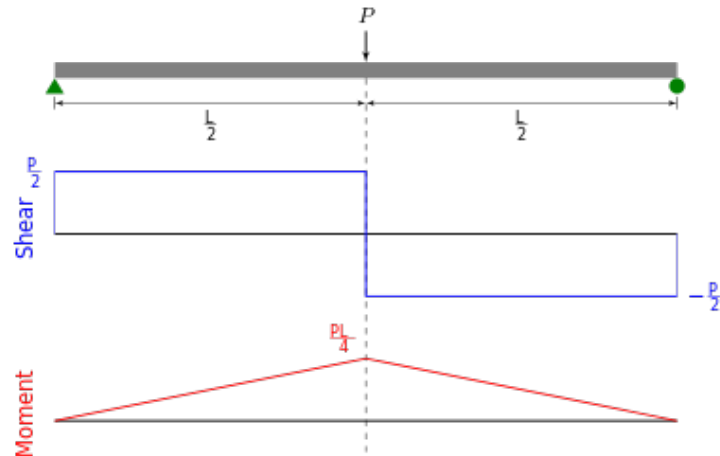


Figure 17: Bending moment diagram

The maximum bending moment is at the flywheel section:

$$M_y = \frac{P * L}{4}$$

Where P is the weight of the robot and L is the distance between the two wheels.

$$I_y = \frac{2 * b * h^3}{12} = \frac{b * h^3}{6}$$

$$\sigma = \frac{M_y}{I_y} * \frac{h}{2} = \frac{P * L}{48 * b * h^2}$$

We are planning to build our body structure with aluminum:

$$\sigma_{al} = 400MPa = 4E8Pa$$

We will impose the relation:

$$b * 5 = h$$

And set a target P of 2000N and a maximum length of 0.5m

$$\sigma_{al} = 4E8Pa = \frac{P * L}{48 * b * h^2} = \frac{1000}{48 * 25 * b^3}$$

Therefore:

$$b = \sqrt[3]{\frac{1000}{48 * 25 * 4E8}} = 0,001277182m$$

We will use b=5mm and h=10mm from now on. Which is far more than what we need.

## 4.5 Motor specifications

Here we have the factory specifications of our motors:

- Operating voltage: between 3 V and 9 V
- Nominal voltage: 6 V
- Free-run speed at 6 V: 176 RPM
- Free-run current at 6 V: 80 mA
- Stall current at 6V: 900 mA
- Stall torque at 6V: 5 kgcm
- Gear ratio: 1:35
- Reductor size: 21 mm
- Weight: 85 g

## 5 Flywheel brake study

This section aims to study a possible way to break the flywheel without applying torque to the platform. The idea is the following: we will leave the moving weight free.

This way, when the weight is going upward will have a larger radius than when is going downward and will produce an average moment against the movement of the flywheel.

From an energetic point of view we are transforming the rotation energy of the flywheel in to translation of the free cylinder and then realising it trough collisions.

### 5.1 System of differential equations

As described in figure 9 we will use two variables to describe the flywheel position:  $r, \theta$

Using equation 3:

$$\tau_{motor-flywheel} = \ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r_{max} - r) * \sin \theta$$

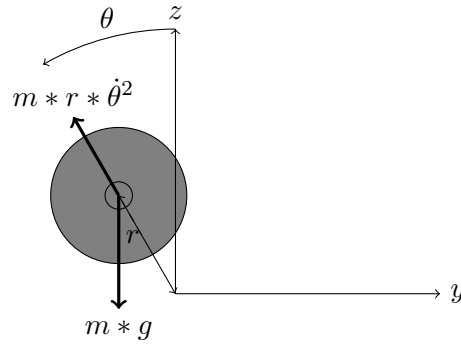


Figure 18: Cylinder force diagram

As seen in figure 18, we can deduce newtons equation for the distance from the cylinder to the center  $r$ . Note that we are adding the centrifugal force term due to the non-inertial frame.

$$\begin{aligned} \ddot{r} * m &= -m * g * \cos(\theta) + m * r * \dot{\theta}^2 \\ \ddot{r} &= -g * \cos(\theta) + r * \dot{\theta}^2 \end{aligned}$$

The variables we will be using for our ODE system are:  $r, \dot{r}, \theta, \dot{\theta}$

Note that we impose  $\tau_{flywheel} = 0$  so the motor is not applying any torque.

$$\begin{cases} \dot{r} = \dot{r} \\ \ddot{r} = -g * \cos(\theta) + r * \dot{\theta}^2 \\ \dot{\theta} = \dot{\theta} \\ \ddot{\theta} = \frac{m_{cylinder} * g * (r - r_{max}) * \sin \theta}{I_{flywheel}(r)} \end{cases}$$

Our initial conditions will be the free cylinder mass lying on the bottom of the flywheel and the flywheel turning at a speed  $\theta_0$ :

$$\begin{cases} r = r_{max} \\ \dot{r} = 0 \\ \theta = \pi \\ \dot{\theta} = \theta_0 \end{cases}$$

We will use a poincare map to simulate the bounce with the end of the guides at  $r = r_{min}$  and  $r = r_{max}$ . At each bounce we will reduce its kinetic energy by a percentage *bounce\_percentage* .

## 5.2 Results

The parameters of the simulation where:

$$\begin{cases} r_{flywheel} = 8cm \\ r_{wheel} = 9cm \\ w = 7cm \\ \dot{\theta}_0 = 4.2\pi rad/s \\ bounce\_percentage = 0.0(totallyinelastic) \end{cases}$$

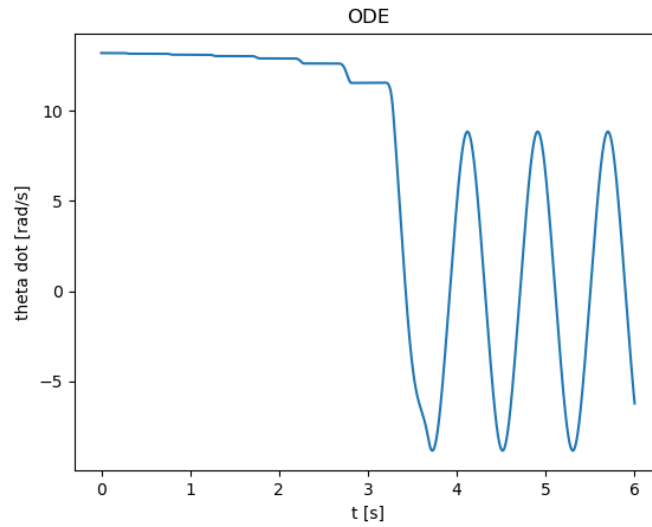


Figure 19: How the variable  $\dot{\theta}$  evolve over time

As we can see in figure 19 the flywheel is braking until it becomes a pendulum and starts oscillating.

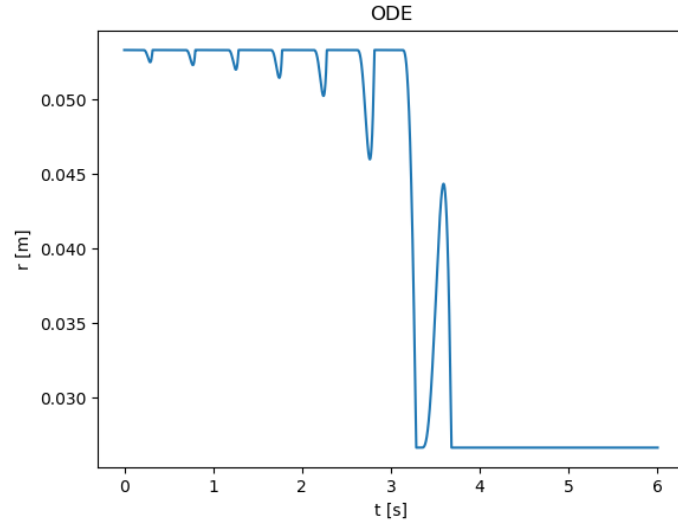


Figure 20: How the variable  $r$  evolves over time

In the first laps the cylinder is almost always at the maximum value of  $r$ , but as the speed decrease each lap the minimum  $r$  decrease until it hits the  $r_{min}$ .

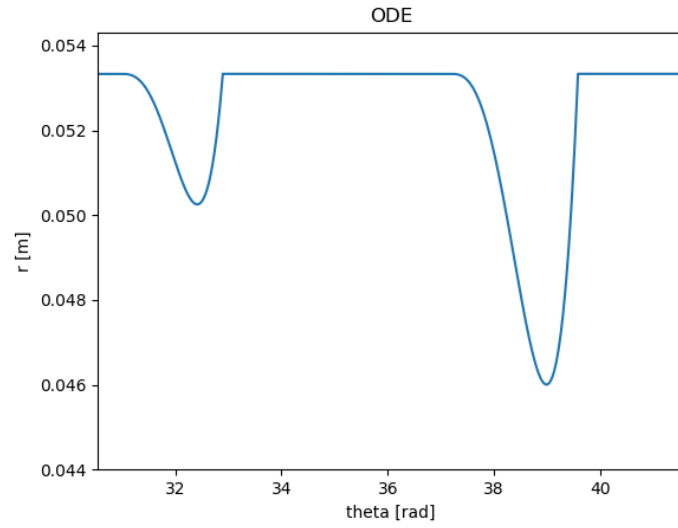


Figure 21: How the variable  $r$  evolve over  $\theta$  zoomed

In image 21 we can appreciate that the  $r$  decrease slower than what it increase.



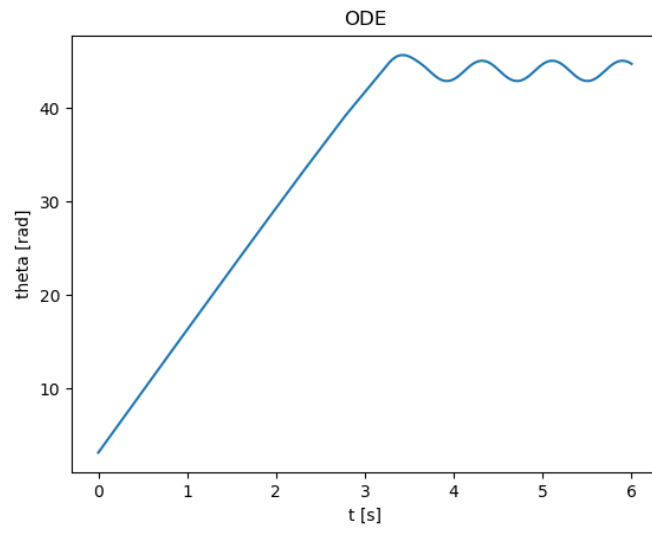


Figure 22: How the variable  $\theta$  evolves over time

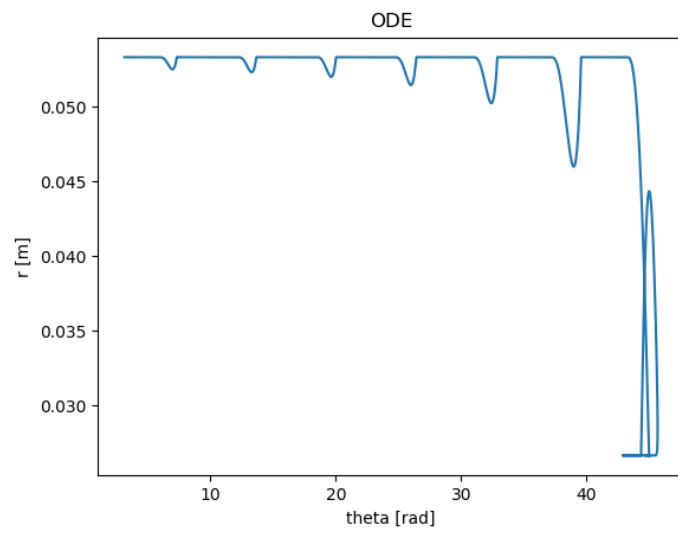


Figure 23: How the variable  $r$  evolves over  $\theta$