Master Thesis

A Remotely-driven Hoverboard With Platform Learning Control

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TO DO: This is the abstract

Contents

Figure 1: Picture of a commercial *segway hover-board*

1 Objective

The objective of this project is to design, build and run reinforcement learning experiments on a dynamic robot. We wish to make this experiments easy and cheap to reproduce so we will try minimize its components and fabrication cost.

The chosen robot is inspired in a *segway hover-board*, similar to the one in Figure 1. The two wheels are controlled with classic control algorithms and the inclination of the central body is controlled with a reinforcement learning algorithm.

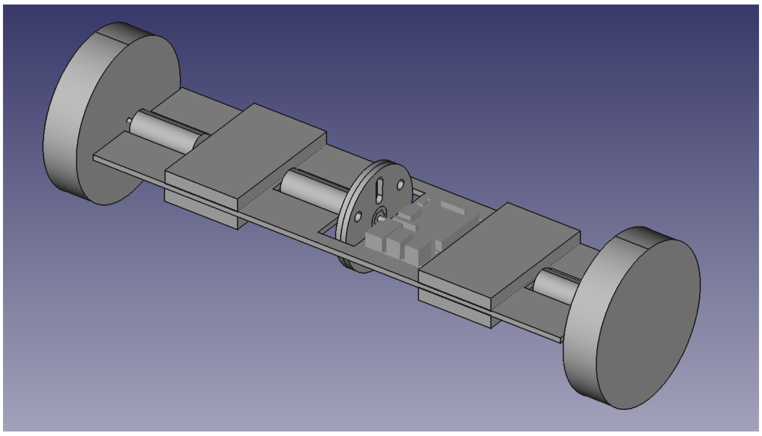


Figure 2: Isometric render view

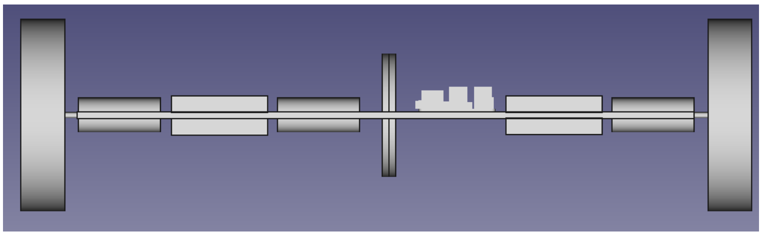


Figure 3: Front render view

2 Design

The design of the robot is done with the 3D design software Free-cad. All part files are uploaded to the GitHub repository `https://github.com/tarragoesteve/TFM` under the hardware folder.

You can see the main views on Figure 2, 3, 4 and 5.

We included three actuators in the robot because we want to control three degrees of freedom (inclination and speed of both wheels). Furthermore we introduced a flywheel/pendulum so we can control the inclination of the body.

We ensured symmetry along the axis formed by all motors in order to have an equilibrium in all possible inclinations without the need of external forces. We also took in consideration that the reinforcement learning algorithm starts being clumsy so none of the configurations should intersect with the ground. Figure 5 illustrates this restriction.

2.1 Flywheel design

To control the inclination of the body two strategies are taken in to account. Creating torque by a pendulum or accelerating the flywheel. In order to experiment with both of them we designed a part to allow both configuration by placing weights in different spots, see figure 6.

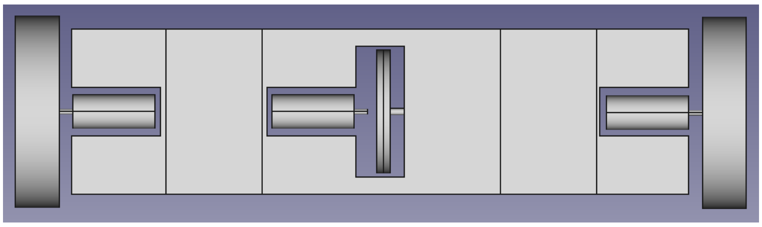


Figure 4: Top render view

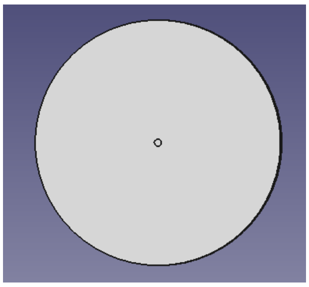


Figure 5: Side render view

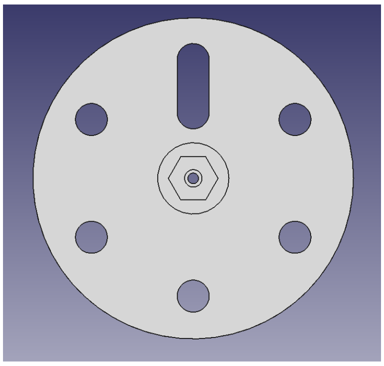


Figure 6: Fly wheel side render view

In order to create a configuration with maximum gravitational torque we have done the following computation. We denote the torque pendulum torque τ, consider the masses are cylinders of mass $m_{cylinder}$ with radius r_c and width w and the radius of the flywheel is r_f.

Each mass weights:

$$m_{cylinder} = \rho * w * \pi * r_c^2$$

We will neglect the mass of the flywheel structure versus the mass of the cylinders.

All the gravitational torque created by the masses will be compensated with the opposite weight except for the two masses with different radius.

One of the weight can be placed along a rail. The distance to the center will vary from $r_{min} = r_c + r_{motor-axis} \approx r_c$ to $r_{max} = r_f - r_c$.

The maximum torque takes place when these two masses are aligned horizontal with respect the ground and the movable weight is at distance r_{min} from the center.

$$\tau_{max}(r_c) = m_{cylinder} * g * r_{max} - m_{cylinder} * g * r_{min} = m_{cylinder} * g * (r_f - 2 * r_c)$$

In order to maximize τ it we first compute the derivative:

$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = g * (\frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) - m_{cylinder} * 2)$$

$$\frac{\partial m_{cylinder}}{\partial r_c} = 2 * \rho * w * \pi * r_c$$

An make it zero to find the maximum:

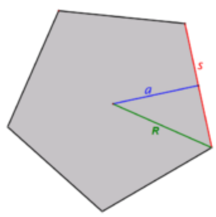
$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = 0$$

Substituting and simplifying we get:

$$\frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) = m_{cylinder} * 2 \Rightarrow 2 * \rho * w * \pi * r_c \ * (r_f - 2 * r_c) = \rho * w * \pi * r_c^2 * 2$$

$$\Rightarrow r_c \ * (r_f - 2 * r_c) = r_c^2 \Rightarrow (r_f - 2 * r_c) = r_c \Rightarrow \boxed{r_f = 3 * r_c}$$

The circumradius R from the center of a regular polygon to one of the vertices is related to the side length s by:



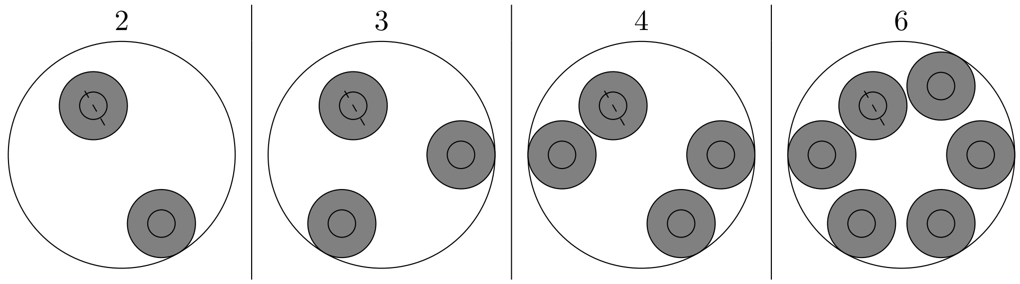
$$R = \frac{s}{2 * \sin \frac{\pi}{n}}$$

In our case:

$$R = r_f - r_c;$$

$$s = 2 * r_c$$

Substituting in the circumradius equation we get n = 6, so we will use up 6 masses in our flywheel. We will have a variable number of masses N that we will be able to add to the flywheel as shown in the following table.



3 Mechanical analysis

3.1 Inclination control

In order to keep the inclination of the platform at a certain angle ϕ we must be able to compensate all the torque being applied to the platform.

$$\ddot{\phi} \cdot I_{platform} = \tau_{platform}$$

Assuming that the platform is well balanced (the center of masses is located at the rotation axis) and neglecting the torque generated by the friction with air, the sum of all the torques in the motor axis applied to the platform is equal to the sum of the torque applied by the motors:

$$\tau_{platform} = \sum \tau_{motors}$$

The torque that the motors deliver to the wheels and to the flywheel create a reaction in the platform in the opposite direction.

$$\tau_{platform} = -\tau_{motor-right-wheel} - \tau_{motor-left-wheel} - \tau_{motor-flywheel}$$

If we want keep the inclination ϕ, we must be able to cancel $\tau_{platform}$. Observe that the angular acceleration $\ddot{\phi}$ of the platform is linearly dependent with the torque it receives.

$$\begin{gathered} 0 = -\tau_{motor-right-wheel} - \tau_{motor-left-wheel} - \tau_{motor-flywheel} \Rightarrow \\ \tau_{motor-right-wheel} + \tau_{motor-left-wheel} = -\tau_{motor-flywheel} \end{gathered} \tag{1}$$

In other words, we must compensate the torque of the wheels with the torque of the flywheel.

3.2 Wheels torque

The wheel torque we can induce is limited by the motor specifications. Note that the maximum torque of the motor is a function of velocity and in particular at max speed the torque is zero.

$$\tau_{motor-wheel}(\omega_{wheel})$$

We assume that the wheels just roll and do no slip. The robot is pushed by the wheels that make a force $F_{friction}$ against the ground in the contact point. See figure 8.

We can express the torque at the center of the of the wheel as:

$$\tau_{motor-wheel} + F_{friction} \cdot r_{wheel} = I_{wheel} \cdot \dot{w}_{wheel}$$

$$\tau_{motor-wheel} = I_{wheel} \cdot \dot{w}_{wheel} - F_{friction} \cdot r_{wheel} \tag{2}$$

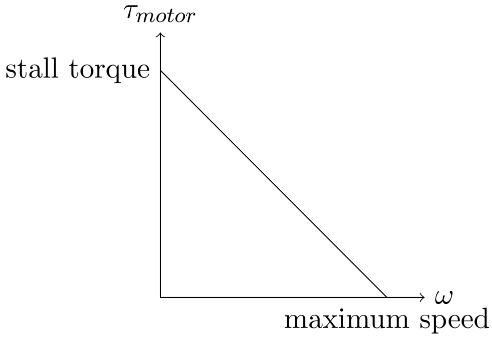


Figure 7: Motor torque

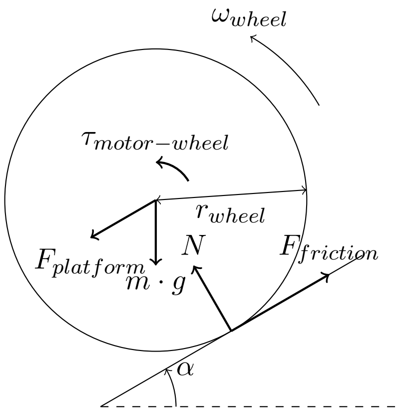


Figure 8: Wheel force diagram

3.3 Flywheel torque

The flywheel torque we can induce is also limited by the motor specifications.

Assuming a general configuration of the flywheel where the moving mass is at distance r and angle θ, see figure 9. we formulate its torque the following way:

$$\tau_{motor-flywheel} + m_{cylinder} \cdot g \cdot (r - r_{max}) \cdot \sin\theta = \ddot{\theta} \cdot I_{flywheel}(r)$$

$$\tau_{motor-flywheel} = \ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta \tag{3}$$

3.4 Maximum speed, acceleration and inclination

In this subsection we would like to study the maximum speed, acceleration and inclination the robot may surpass.

We will assume both wheels turn at the same speed, have the same $F_{friction}$ and the same τ_{wheel}:

$$\omega_{wheel-left} = \omega_{wheel-right} = \omega_{wheel}$$

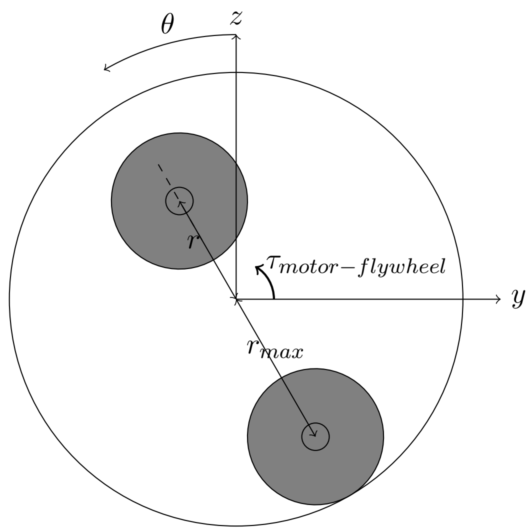


Figure 9: Flywheel diagram for $N = 2$

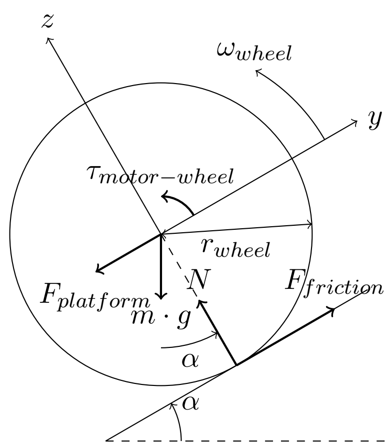


Figure 10: Wheel forward force diagram

Applying Newton's first law in the y axis of figure 10:

$$\ddot{y} \cdot m_{total} = 2 \cdot F_{friction} - m_{total} \cdot g \cdot sin(\alpha)$$

Substituting $F_{friction}$ taking in to account equation 2:

$$\ddot{y} \cdot m_{total} = 2 \cdot \frac{I_{wheel} \cdot \dot{w}_{wheel} - \tau_{motor-wheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha)$$

Using equation 1:

$$\Rightarrow \ddot{y} \cdot m_{total} = \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}} + \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha) \tag{4}$$

We will now study different cases to better understand this equation.

3.4.1 No terrain inclination ($\alpha = 0$)

The objective here is to obtain the maximum speed and acceleration we can get starting from rest in a plain surface.

The equation we get by substituting $\alpha = 0$ in equation 4:

$$\ddot{y} \cdot m_{total} = \frac{\tau_{motor-flywheel}}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

Substituting equation 3

$$\ddot{y} \cdot m_{total} = \frac{\ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}} \tag{5}$$

We will now split the study in two cases:

1. Flywheel case: r is fixed to $r = r_{max}$

   Then:

$$\ddot{y} \cdot m_{total} = -\frac{\ddot{\theta} \cdot I_{flywheel}(r_{max})}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

   Taking in two account the following relation:

$$-\omega_{wheel} \cdot r_{wheel} = \dot{y} \Rightarrow -\dot{w}_{wheel} \cdot r_{wheel} = \ddot{y}$$

   The moments of inertia are:

$$I_{wheel} \approx \frac{1}{2} \cdot m_{wheel} \cdot r_{wheel}^2$$

$$I_{flywheel} \approx N \cdot m_{cylinder} \cdot r_{max}^2$$

   Let's go for it:

$$-\dot{w}_{wheel} \cdot r_{wheel} \cdot m_{total} = \frac{\ddot{\theta} \cdot I_{flywheel}}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

$$-\dot{w}_{wheel} \cdot (r_{wheel} \cdot m_{total} + \frac{2 \cdot I_{wheel}}{r_{wheel}}) = \frac{\ddot{\theta} \cdot I_{flywheel}}{r_{wheel}}$$

We define R as the quotient between $\dot{w}_{wheel}$ and $-\ddot{\theta}$

$$R = \frac{\dot{w}_{wheel}}{-\ddot{\theta}} = \frac{I_{flywheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}} \approx \frac{N \cdot m_{cylinder} \cdot r_{max}^2}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}}$$

We can see that R will always be smaller than 1 because $N \cdot m_{cylinder} < m_{total}$ and $r_{max} < r_{wheel}$. This means that we will be limited by the acceleration of the flywheel.

The forward acceleration is:

$$\ddot{y} = -\dot{w}_{wheel} \cdot r_{wheel} = R \cdot \ddot{\theta} \cdot r_{wheel}$$

And using equation 3 we get that the maximum is:

$$\tau_{motor}(w) = \ddot{\theta} \cdot I_{flywheel}(r) \Rightarrow \ddot{\theta} = \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)}$$

$$\ddot{y}_{max} = R \cdot \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} \cdot r_{wheel}$$

$$\Rightarrow \ddot{y}_{max} = \frac{I_{flywheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}} \cdot \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} \cdot r_{wheel}$$

$$\boxed{\ddot{y}_{max} = \frac{\tau_{motor}(\dot{\theta}) \cdot r_{wheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}}} \tag{6}$$

In order to compute the maximum speed we will assume that the initial conditions are $\dot{\theta} = 0$ and $\omega_{wheel} = 0$

$$\omega_{wheel-max} = \int_{t=0}^{t=t_{max}} \dot{w}_{wheel} \cdot dt$$

Now we will proceed to do a change of variables in the integral.

$$\frac{\partial \dot{\theta}}{\partial t} = \ddot{\theta} \Rightarrow dt = \frac{d\dot{\theta}}{\ddot{\theta}}$$

$$\omega_{wheel-max} = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} \frac{\dot{w}_{wheel}}{\ddot{\theta}} \cdot d\dot{\theta} = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} -R \cdot d\dot{\theta} = -R \cdot \dot{\theta}_{max}$$

$$\boxed{\dot{y}_{max} = r_{wheel} \cdot R \cdot \dot{\theta}_{max}} \tag{7}$$

And $\dot{\theta}_{max}$ is a limitation imposed by the motor specifications. Note that this is the maximum speed we can get using the flywheel system starting from rest.

2. Pendulum: $\dot{\theta} = 0$, and r is fixed to $r = r_{min}$

   Using equation 5 and $\ddot{\theta} = 0$

$$\ddot{y} \cdot m_{total} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

   Multiplying by r_{wheel} both sides of the equation we get:

$$\ddot{y} \cdot m_{total} \cdot r_{wheel} = m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta + 2 \cdot I_{wheel} \cdot \dot{w}_{wheel}$$

   And using $\dot{w}_{wheel} = -\frac{\ddot{y}}{r_{wheel}}$

$$\ddot{y} \cdot m_{total} \cdot r_{wheel} + 2 \cdot I_{wheel} \cdot \frac{\ddot{y}}{r_{wheel}} = m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta$$

$$\ddot{y} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta}{m_{total} \cdot r_{wheel} + \frac{2 \cdot I_{wheel}}{r_{wheel}}}$$

   Which is maximum when $\sin\theta = 1$

$$\boxed{\ddot{y}_{max} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot}{m_{total} \cdot r_{wheel} + \frac{2 \cdot I_{wheel}}{r_{wheel}}}} \tag{8}$$

   We need to take into consideration air friction to see the speed limitation in the pendulum case.

$$F_{drag} = \frac{1}{2} \cdot \rho \cdot C_D \cdot A \cdot \dot{y}^2$$

   Adding this term to equation 4 we get:

$$\ddot{y} \cdot m_{total} = \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}} + \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha) - F_{drag}$$

   And making $\ddot{y} = 0$ and $\alpha = 0$.

$$F_{drag} = \frac{\tau_{motor-flywheel}}{r_{wheel}}$$

$$\frac{1}{2} \cdot \rho \cdot C_D \cdot A \cdot \dot{y}^2 = m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot sin(\theta)/r_{wheel}$$

The maximum $\dot{y}$ is then obtained when $\theta = \frac{\pi}{2}$:

$$\dot{y}_{max} = \sqrt{\frac{2 \cdot m_{cylinder} \cdot g \cdot (r_{max} - r_{min})}{\rho \cdot C_D \cdot A \cdot r_{wheel}}}$$

We will pick $C_D = 1$, $\rho = 1.2kg/m^3$ and $A = 0.01m^2$ for our computations.

Also note that the speed is also limited by maximum speed a motor can get $\dot{\theta}_{max}$.

$$\boxed{\dot{y}_{max} = min(\dot{\theta}_{max} \cdot r_{wheel}, \sqrt{\frac{2 \cdot m_{cylinder} \cdot g \cdot (r_{max} - r_{min})}{\rho \cdot C_D \cdot A \cdot r_{wheel}}})} \tag{9}$$

3.4.2 No acceleration, just inclination ($\alpha > 0$)

The goal of this subsubsection is to study which is the maximum inclination the robot can overpass.

Substituting $\ddot{y} = 0$ and $\dot{\omega}_{wheel} = 0$ in equation 4

$$0 = \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha)$$

$$m_{total} \cdot g \cdot sin(\alpha) = \frac{\tau_{motor-flywheel}}{r_{wheel}}$$

$$sin(\alpha) = min(1, \frac{\tau_{motor-flywheel}}{m_{total} \cdot g \cdot r_{wheel}})$$

Substituting equation 3

$$sin(\alpha) = min(1, \frac{\ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta}{m_{total} \cdot g \cdot r_{wheel}})$$

We are going to distinguish the same two cases as in the previous section:

1. Flywheel case: r is fixed to $r = r_{max}$ The maximum inclination at a certain moment:

$$\boxed{sin(\alpha_{max}) = min(1, \frac{\ddot{\theta} \cdot I_{flywheel}}{m_{total} \cdot g \cdot r_{wheel}}) = min(1, \frac{\tau_{motor-flywheel}(\dot{\theta})}{m_{total} \cdot g \cdot r_{wheel}})} \tag{10}$$

This angle doesn't give us a lot of information because it may not be fulfilled in a permanent state. That's why in addition we would like to compute the maximum height our robot can achieve start from rest position.

$$h = \int_{t=0}^{t=t_f} \dot{y} \cdot sin(\alpha)dt = \int_{t=0}^{t=t_f} sin(\alpha)(\int_{t=0}^{t=t} \ddot{y}dt)dt$$

$$\ddot{y}(\frac{2 \cdot I_{wheel} + m_{total} \cdot r_{wheel}^2}{r_{wheel}^2}) = \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha)$$

$$\ddot{y} = \frac{r_{wheel} \cdot \tau_{motor-flywheel}}{2 \cdot I_{wheel} + m_{total} \cdot r_{wheel}^2} - \frac{m_{total} \cdot g \cdot sin(\alpha) \cdot r_{wheel}^2}{2 \cdot I_{wheel} + m_{total} \cdot r_{wheel}^2}$$

$$\ddot{y} = A \cdot \tau_{motor-flywheel} + B$$

$$\int_{t=0}^{t=t} \ddot{y} dt = \int_{t=0}^{t=t} (A \cdot \tau_{motor-flywheel} + B) dt$$

$$= A \int_{t=0}^{t=t} \tau_{motor-flywheel} dt + Bt$$

From now on we will assume that we deliver the maximum torque to the flywheel.

$$\tau_{motor}(\dot{\theta}) \approx \tau_{max} - \dot{\theta} \cdot \frac{\tau_{max}}{\dot{\theta}_{max}}$$

$$\dot{\theta}(t) = \int \ddot{\theta}(t) dt = \frac{1}{I_f} \int \tau_{max} - \dot{\theta}(t) \cdot \frac{\tau_{max}}{\dot{\theta}_{max}} dt$$

$$\dot{\theta}(t) = \frac{\tau_{max} \cdot t}{I_f} - \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} \int \dot{\theta}(t) dt$$

$$\dot{\theta}(t) + \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} \cdot \theta(t) = \frac{\tau_{max} \cdot t}{I_f}$$

Which is a non-homogeneous first order differential equation with solution:

$$\mu(t) = e^{\int \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} dt} = e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1}$$

$$\theta(t) = \frac{\int (\mu(t) \cdot \frac{\tau_{max} \cdot t}{I_f}) dt}{\mu(t)}$$

Integrating by parts

$$\int (\mu(t) \cdot \frac{\tau_{max} \cdot t}{I_f}) dt = \frac{\tau_{max} \cdot t}{I_f} \cdot \int \mu(t) - \int (\int \mu(t) dt) \cdot \frac{\tau_{max}}{I_f} dt$$

$$\int \mu(t) dt = \frac{I_f \cdot \dot{\theta}_{max}}{\tau_{max}} \cdot (e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1} + C_2)$$

$$\dot{\theta}_{max} \cdot t \cdot (e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1} + C_2) - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}} \cdot (e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1} + C_2) + C_3$$

$$(\dot{\theta}_{max} \cdot t - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}}) \cdot (e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1} + C_2) + C_3$$

$$\theta(t) = \frac{(\dot{\theta}_{max} \cdot t - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}}) \cdot (e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1} + C_2) + C_3}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1}}$$

$$\theta(t) = \frac{(\dot{\theta}_{max} \cdot t - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}}) \cdot C_2}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1}} + (\dot{\theta}_{max} \cdot t - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}}) + \frac{C_3}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + C_1}}$$

Renaming the constants we get:

$$\theta(t) = \frac{(\dot{\theta}_{max} \cdot t - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}}) \cdot C_1}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}} + (\dot{\theta}_{max} \cdot t - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}}) + \frac{C_2}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}}$$

$$\theta(0) = -\frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}} \cdot C_1 - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}} + C_2 = 0$$

$$\dot{\theta(t)} = \frac{\dot{\theta}_{max} \cdot C_1}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}} - \frac{(\dot{\theta}_{max} \cdot t - \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}}) \cdot C_1 \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}} + \dot{\theta}_{max} - \frac{C_2 \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}}$$

$$\dot{\theta(0)} = \dot{\theta}_{max} \cdot C_1 + \frac{I_f \cdot \dot{\theta}_{max}^2}{\tau_{max}} \cdot C_1 \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} + \dot{\theta}_{max} - C_2 \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}$$

$$\int_{t=0}^{t=t} \tau_{motor-flywheel} dt = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} \tau_{motor-flywheel}(\dot{\theta}) \frac{d\dot{\theta}}{\ddot{\theta}}$$

$$\int_{\dot{\theta}=0}^{\dot{\theta}=t \cdot \ddot{\theta}} (\tau_{max} - \dot{\theta} \cdot \frac{\tau_{max}}{\dot{\theta}_{max}}) \frac{d\dot{\theta}}{\ddot{\theta}} = \frac{\tau_{max} \cdot t}{\ddot{\theta}} - \frac{1}{2} \cdot (t \cdot \ddot{\theta})^2 \frac{\tau_{max}}{\dot{\theta}_{max} \ddot{\theta}}$$

$$\frac{\tau_{max} \cdot t}{\ddot{\theta}} - \frac{1}{2} \cdot t^2 \cdot \frac{\ddot{\theta} \cdot \tau_{max}}{\dot{\theta}_{max}}$$

$$t_f = \frac{\dot{\theta}_{max}}{\ddot{\theta}}$$

$$h = sin(\alpha) \int_{t=0}^{t=t_f} (A \cdot (\frac{\tau_{max} \cdot t}{\ddot{\theta}} - \frac{1}{2} \cdot t^2 \cdot \frac{\ddot{\theta} \cdot \tau_{max}}{\dot{\theta}_{max}}) + Bt) dt$$

$$h = sin(\alpha)(\frac{1}{2}\frac{A \cdot \tau_{max}\dot{\theta}_{max}^2}{\ddot{\theta}^3} - \frac{1}{6}\frac{A\tau_{max}\dot{\theta}_{max}^2}{\ddot{\theta}^2} + \frac{1}{2}\frac{B\dot{\theta}_{max}^2}{\ddot{\theta}^2})$$

2n way

$$W = \int_{t_1}^{t_2} 2\tau_{motor-wheel}(t) \cdot \omega(t) \cdot dt$$

Using equation 1:

$$W = \int_{t_1}^{t_2} -\tau_{flywheel-wheel}(t) \cdot \omega(t) \cdot dt$$

From equation 4 we get:

$$\tau_{motor-flywheel} = \dot{\omega}_{wheel} \cdot (-r_w^2 \cdot m_{total} - 2 \cdot I_{wheel}) + m_{total} \cdot g \cdot sin(\alpha) \cdot r_w$$

So there is a linear relation between $\tau_{flywheel-motor}$ and $\dot{\omega}_{wheel}$:

$$\tau_f = A \cdot \dot{w} + B \tag{11}$$

$$\dot{w} = C \cdot \tau_f + D$$
$$C = 1/A$$
$$D = -B/A$$

Substituting equation 11:

$$W = \int_{t_1}^{t_2} -(A \cdot \dot{\omega}(t) + B) \cdot \omega(t) \cdot dt$$

Which we can split into kinetic energy and potential energy.

$$W = [\frac{-A}{2} \cdot \omega(t)^2)]_{t_1}^{t_2} - \int_{t_1}^{t_2} B \cdot \omega(t) \cdot dt$$

$$\int_{t_1}^{t_2} \omega(t) \cdot dt = \int_{t_1}^{t_2} \int_{t_1}^{t} \dot{\omega}(t) \cdot dt \cdot dt = \int_{t_1}^{t} C \cdot \tau_{flywheel-wheel}(t) + D \cdot dt$$

Using equation 3:

$$\int_{t_1}^{t} \tau_{flywheel-wheel}(t)dt = \int_{t_1}^{t} \ddot{\theta}(t) \cdot I_f dt = I_f \cdot \int_{t_1}^{t} \ddot{\theta}(t)dt =$$

$$\int_{t_1}^{t} \ddot{\theta}(t)dt = [\dot{\theta}(t)]_{t_1}^{t}$$

$$\int_{t_1}^{t_2} \omega(t) \cdot dt = \int_{t_1}^{t_2} [D]_{t_1}^{t} + C \cdot I_f \cdot [\dot{\theta}(t)]_{t_1}^{t} dt = \int_{t_1}^{t_2} D \cdot t - D \cdot t_1 + C \cdot I_f \cdot (\dot{\theta}(t) - \dot{\theta}(t_1)) dt$$

$$\int_{t_1}^{t_2} \dot{\theta}(t) dt = [\theta(t)]_{t_1}^{t_2}$$

$$[\frac{D \cdot t^2}{2}]_{t_1}^{t_2} - [D \cdot t_1 \cdot t]_{t_1}^{t_2} + C \cdot I_f \cdot ([\theta(t)]_{t_1}^{t_2} - [\dot{\theta}(t_1) \cdot t]_{t_1}^{t_2})$$

$$W = [\frac{-A}{2} \cdot \omega(t)^2)]_{t_1}^{t_2} - B \cdot ([\frac{D \cdot t^2}{2}]_{t_1}^{t_2} - [D \cdot t_1 \cdot t]_{t_1}^{t_2} + C \cdot I_f \cdot ([\theta(t)]_{t_1}^{t_2} - [\dot{\theta}(t_1) \cdot t]_{t_1}^{t_2})) \quad (12)$$

Replacing initial conditions:

- $\omega(t1) = 0$
- $\dot{\theta}(t1) = 0$
- $t1 = 0$

And forcing the final speed to be zero:

- $\omega(t2) = 0$
- $$\int_{t1}^{t2} \dot{w} = 0$$

$$\int_{t1}^{t2} \dot{w} = \int_{t1}^{t2} C \cdot I_f \cdot \theta(\ddot{t}) + D$$

$$W = -B \cdot (\frac{D \cdot t_2^2}{2} + C \cdot I_f \cdot [\theta(t)]_{t_1}^{t_2})$$

$$W = \frac{B^2 \cdot t_2^2}{A \cdot 2} - \frac{B \cdot I_f \cdot [\theta(t)]_{t_1}^{t_2}}{A}$$

...

$$\frac{\partial W}{\partial t_2} = \frac{B^2 \cdot t_2}{A} - \frac{B \cdot I_f \cdot \dot{\theta}(t_2)}{A} = 0$$

$$t_2 = \frac{I_f \cdot \dot{\theta}(t_2)}{B}$$

$$\dot{\theta}(t_2) = \frac{B \cdot t_2}{I_f}$$

$$\dot{\theta}(t_2) = \frac{B \cdot t_2}{I_f}$$

$$W = \frac{I_f^2 \cdot \dot{\theta(t_2)}^2}{A \cdot 2} - \frac{B \cdot I_f \cdot [\theta(t)]_{t_1}^{t_2}}{A}$$

2. Pendulum: $\dot{\theta} = 0$, and r is fixed to $r = r_{min}$

$$sin(\alpha) = \frac{min(\tau_{motor-flywheel}, m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta)}{m_{total} \cdot g \cdot r_{wheel}}$$

$$\boxed{sin(\alpha_{max}) = \frac{min(\tau_{motor-flywheel}, m_{cylinder} \cdot g \cdot (r_{max} - r_{min}))}{m_{total} \cdot g \cdot r_{wheel}}} \tag{13}$$

Note that the $sin(\alpha_{max})$ will not be more than 1.

4 Optimization

4.1 Restrictions

As a reminder: L is the length of the robot, and w the width of the flywheel cylinders.

1. Not crashing with the ground at any inclination can be translated as:

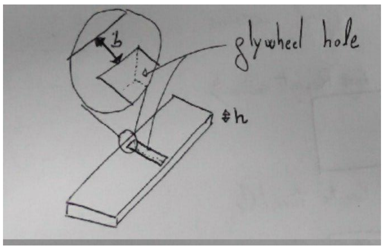


Figure 11: Flywheel hole diagram

$$r_{wheel} > \sqrt{(r_{flywheel} + b)^2 + (\frac{h}{2})^2}$$

2. We want to inserted the robot in to a external of diameter 0.5m so:

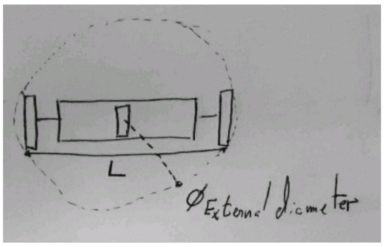


Figure 12: External diameter diagram

$$0.25m > \sqrt{r_{wheel}^2 + L^2/4}$$

3. We can place all the devices:
$$L > 0.3m + w$$

4. Maximum weight of the robot: 5kg

4.2 Requirements

Flywheel mode

1. $\dot{y}_{max}$ (equation 7) $> 0.1m/s$

2. $\ddot{y}_{max}$ (equation 6) $1m/s^2$

3. $sin(\alpha_{max})$ (equation 10) > 0.2

Pendulum mode

1. $\dot{y}_{max}$ (equation 9) $> 1m/s$

2. $\ddot{y}_{max}$ (equation 8) $> 0.1m/s^2$

3. $sin(\alpha_{max})$ (equation 13) > 0.02

4.3 Cost function

We will maximize the maximum sinus in the pendulum mode (equation 13) because it give the robot the capacity to deliver force in a permanent state.

We will also maximize the square of the max speed the robot can achieve in flywheel mode (equation 7) because it is proportional to the energy the robot can deliver at a certain moment.

$$cost(r_{flywheel}, r_{wheel}, w, N) = -sin(\alpha_{max})_{pendulum} - \dot{y}^2_{max-flywheel} \tag{14}$$

$$m_{cylinder} = \rho * w * \pi * (\frac{r_{flywheel}}{3})^2$$

$$sin(\alpha_{max})_{pendulum} = \frac{m_{cylinder} \cdot (r_{max} - r_{min})}{m_{total} \cdot r_{wheel}} = \frac{m_{cylinder} \cdot (\frac{r_{flywheel}}{3})}{(m_{rest} + N \cdot m_{cylinder}) \cdot r_{wheel}}$$

$$\dot{y}_{max} = r_{wheel} \cdot R \cdot \dot{\theta}_{max} = r_{wheel} \cdot \frac{N \cdot m_{cylinder} \cdot (\frac{2 \cdot r_{flywheel}}{3})^2}{r^2_{wheel} \cdot (m_{rest} + N \cdot m_{cylinder}) + 2 \cdot I_{wheel}} \cdot \dot{\theta}_{max}$$

4.4 Results

Our procedure has been making a grid with the four parameters of the robot design: w (width of the cylinders),N (number of cylinders), r_{wheel} and $r_{flywheel}$. We have fixed $r_{flywheel}$, iterated over the other two variables and keeped the best parameters for our cost function.

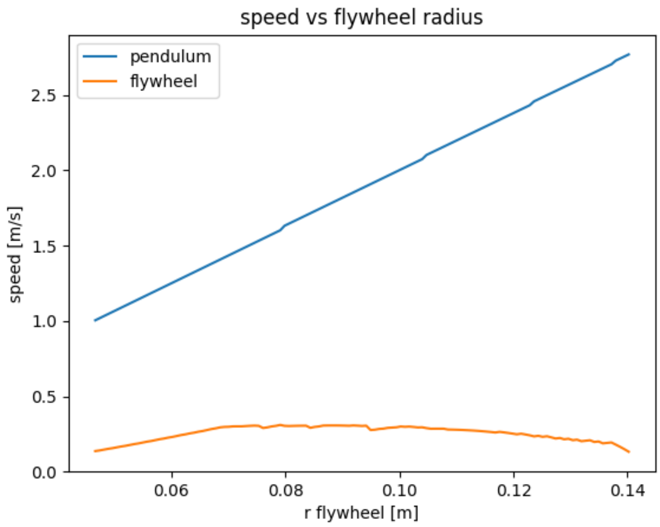


Figure 13: Plot of the equations 7 and 9 at the parameters that minimize the cost function and fullfil the requirements and restrictions

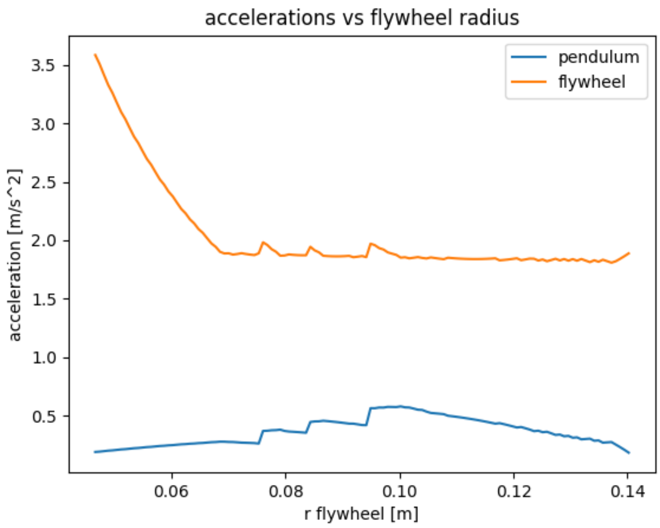


Figure 14: Plot of the equations 6 and 8 at the parameters that minimize the cost and fullfil the requirements and restrictions

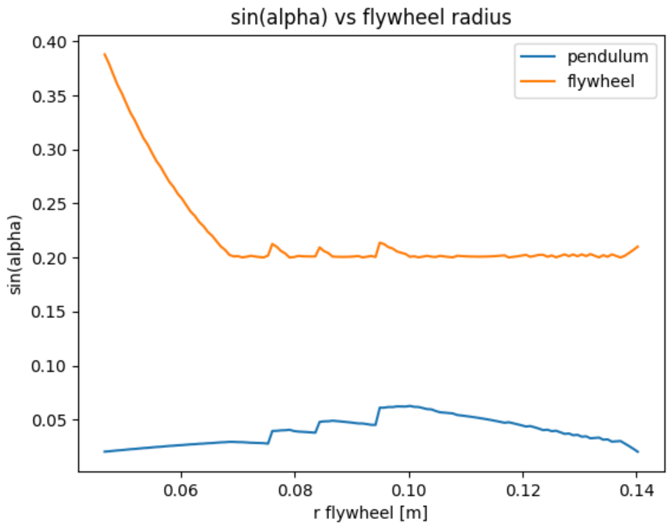


Figure 15: Plot of the equations 10 and 13 at the parameters that minimize the cost and fullfil the requirements and restrictions

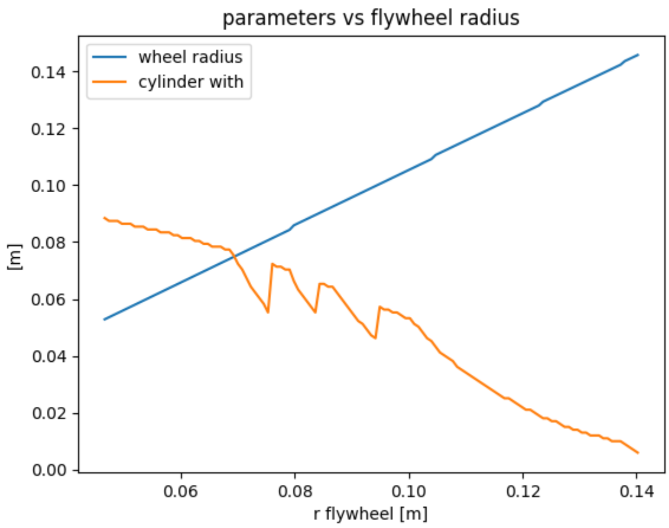


Figure 16: Plot of the parameters that minimize the cost function.

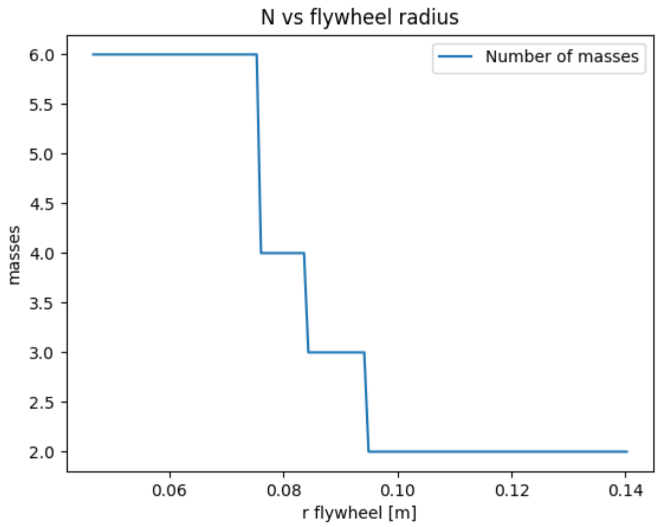


Figure 17: Plot of the N that minimize the cost function.

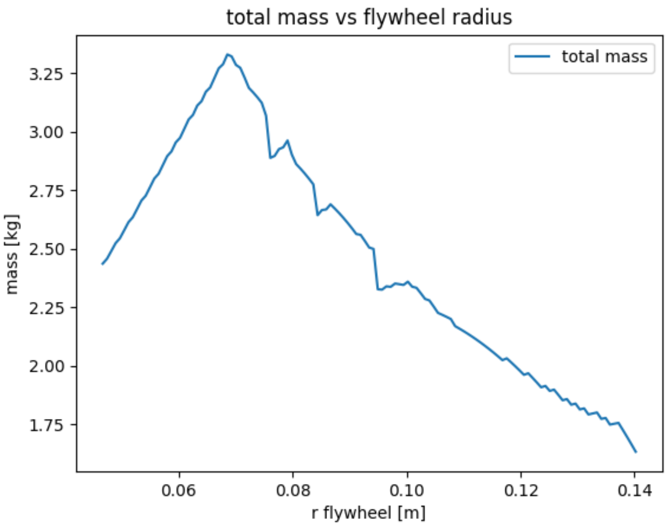


Figure 18: Plot of the mass for each configuration.

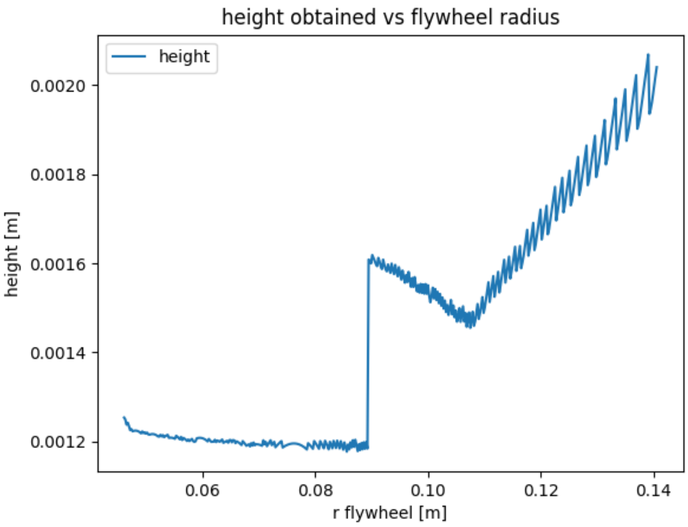


Figure 19: Plot of the equation **??** for each configuration.

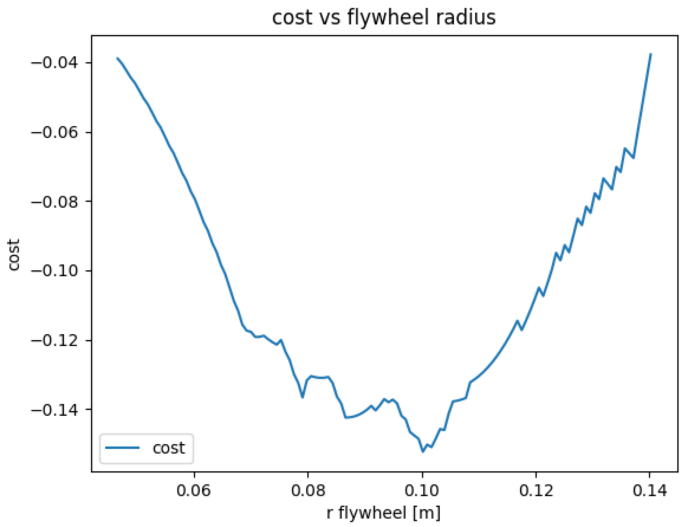


Figure 20: Plot of the equation 14 for each configuration.

Our selected parameters are:

$r_{flywheel}$	r_{wheel}	w
8cm	9cm	3cm

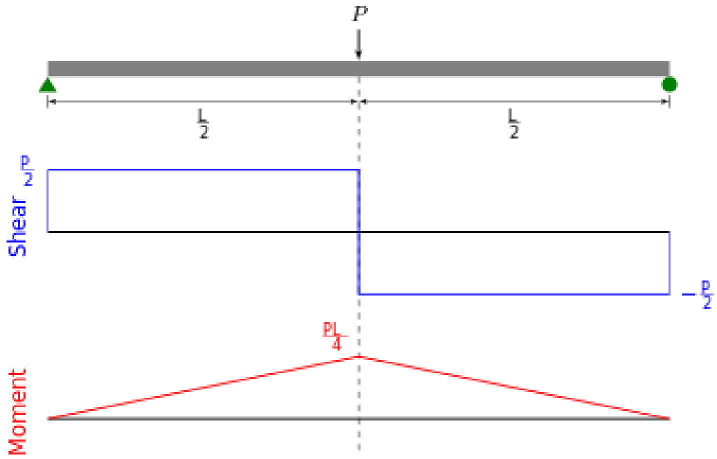


Figure 21: Bendig moment diagram

4.5 Minimum section in the section of the flywheel

The maximum bending moment is at the flywheel section:

$$M_y = \frac{P * L}{4}$$

Where P is the weight of the robot and L is the distance between the two wheels.

$$I_y = \frac{2 * b * h^3}{12} = \frac{b * h^3}{6}$$

$$\sigma = \frac{M_y}{I_y} * \frac{h}{2} = \frac{P * L}{48 * b * h^2}$$

We are planning to build our body structure with aluminum:

$$\sigma_{al} = 400MPa = 4E8Pa$$

We will impose the relation:

$$b * 5 = h$$

And set a target P of $2000N$ and a maximum length of $0.5m$

$$\sigma_{al} = 4E8Pa = \frac{P * L}{48 * b * h^2} = \frac{1000}{48 * 25 * b^3}$$

Therefore:

$$b = \sqrt[3]{\frac{1000}{48 * 25 * 4E8}} = 0,001277182m$$

We will use b=5mm and h=10mm from now on. Which is far more than what we need.

4.6 Motor specifications

Here we have the factory specifications of our motors:

- Operating voltage: between 3 V and 9 V
- Nominal voltage: 6 V
- Free-run speed at 6 V: 176 RPM
- Free-run current at 6 V: 80 mA
- Stall current at 6V: 900 mA
- Stall torque at 6V: 5 kgcm
- Gear ratio: 1:35
- Reductor size: 21 mm
- Weight: 85 g

5 Flywheel brake study

This section aims to study a possible way to break the flywheel without applying torque to the platform. The idea is the following: we will leave the moving weight free.

This way, when the weight is going upward will have a larger radius than when is going downward and will produce an average moment against the movement of the flywheel.

From an energetic point of view we are transforming the rotation energy of the flywheel in to translation of the free cylinder and then realising it trough collisions.

5.1 System of differential equations

As described in figure 9 we will use two variables to describe the flywheel position: r, θ

Using equation 3:

$$\tau_{motor-flywheel} = \ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r_{max} - r) * \sin\theta$$

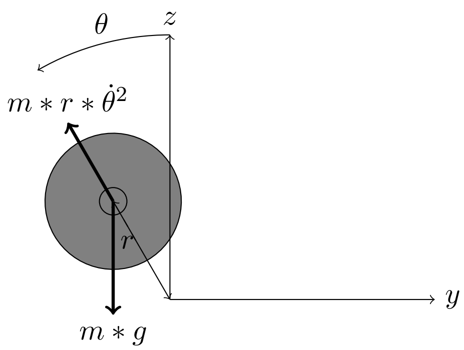


Figure 22: Cylinder force diagram

As seen in figure 22, we can deduce newtons equation for the distance from the cylinder to the center r. Note that we are adding the centrifugal force term due to the non-inertial frame.

$$\ddot{r} * m = -m * g * cos(\theta) + m * r * \dot{\theta}^2$$

$$\ddot{r} = -g * cos(\theta) + r * \dot{\theta}^2$$

The variables we will be using for our ODE system are: r,$\dot{r}$,θ,$\dot{\theta}$

Note that we impose $\tau_{flywheel} = 0$ so the motor is not applying any torque.

$$\begin{cases} \dot{r} = \dot{r} \\ \ddot{r} = -g * cos(\theta) + r * \dot{\theta}^2 \\ \dot{\theta} = \dot{\theta} \\ \ddot{\theta} = \frac{m_{cylinder} * g * (r - r_{max}) * \sin\theta}{I_{flywheel}(r)} \end{cases}$$

Our initial conditions will be the free cylinder mass lying on the bottom of the flywheel and the flywheel turning at a speed θ_0:

$$\begin{cases} r = r_{max} \\ \dot{r} = 0 \\ \theta = \pi \\ \dot{\theta} = \theta_0 \end{cases}$$

We will use a poincare map to simulate the bounce with the end of the guides at $r = r_{min}$ and $r = r_{max}$. At each bounce we will reduce its kinetic energy by a percentage $bounce\_percentage$.

5.2 Results

The parameters of the simulation where:

$$\begin{cases} r_{flywheel} = 8cm \\ r_{wheel} = 9cm \\ w = 7cm \\ \dot{\theta}_0 = 4.2\pi rad/s \\ bounce\_percentage = 0.0(totallyinelastic) \end{cases}$$

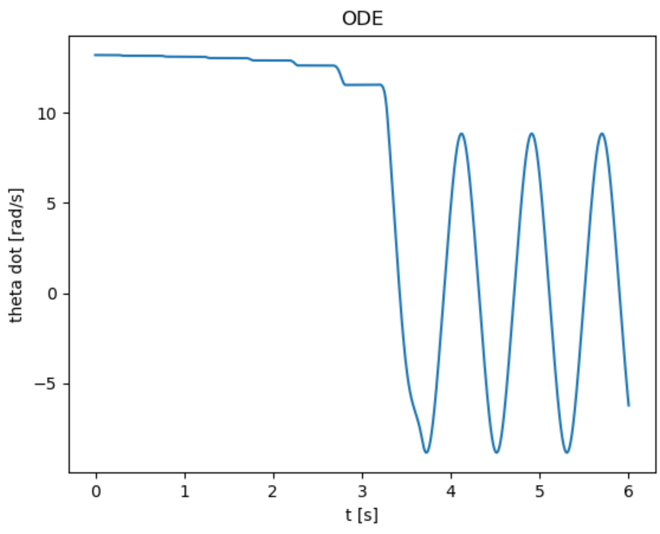


Figure 23: How the variable $\dot{\theta}$ evolve over time

As we can see in figure 23 the flywheel is braking until it becomes a pendulum and starts oscillating.

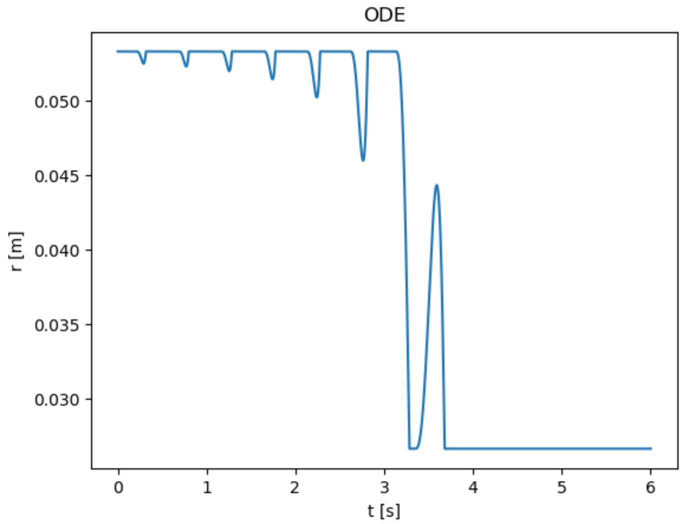


Figure 24: How the variable r evolves over time

In the first laps the cylinder is almost always at the maximum value of r, but as the speed decrease each lap the minimum r decrease until it hits the r_{min}.

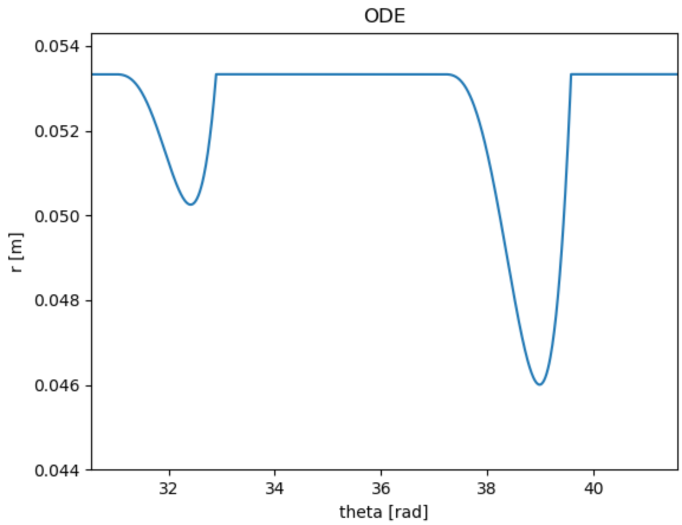


Figure 25: How the variable r evolve over θ zoomed

In image 25 we can appreciate that the r decrease slower that what it increase.

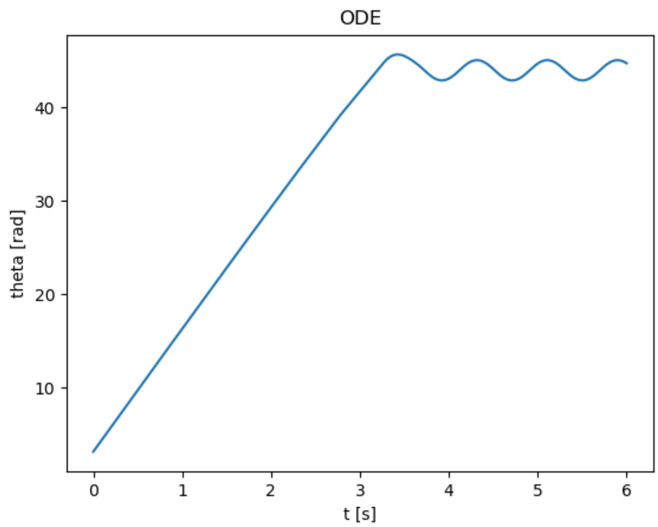


Figure 26: How the variable θ evolves over time

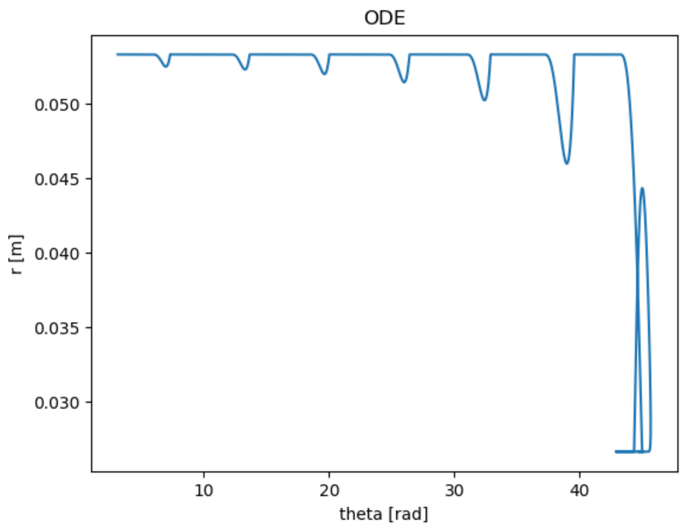


Figure 27: How the variable r evolves over θ