Universitat Politècnica de Catalunya
Facultat de Matemàtiques i Estadística

Master in Advanced Mathematics and Mathematical Engineering
Master's thesis

# A Remotely-driven Hoverboard With Platform Leaning Control

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June, 2019

Thanks to Patrick Grosch, Lluís Ros, Enric Celaya, Iñigo Moreno

## Abstract

This thesis is centered arround the design, construction and control of a remote hoverboard. The main challenge of this project is the study of the leaning control of the hoverboard platform.

Nowadays segways robots have they inclination determined by their speed and they do not have it as a degree of freedom. Unblocking this degree of freedom may help two-wheel robots perform new task as mesurments, taking images or samples from other inclinations, avoiding obstacles, etc.

This project also aims to provide a benchmark and an easy to build robot that help experiment with non-linear control methods.



### Keywords

Dynamic, System, Control, Design

# Contents

Figure 1: Picture of a commercial *segway hover-board*

# 1. Objective

The objective of this project is to design, build and run reinforcement learning experiments on a dynamic robot. We wish to make this experiments easy and cheap to reproduce so we will try minimize its components and fabrication cost.

The chosen robot is inspired in a *segway hover-board*, similar to the one in Figure 1. The two wheels are controlled with classic control algorithms and the inclination of the central body is controlled with a reinforcement learning algorithm.

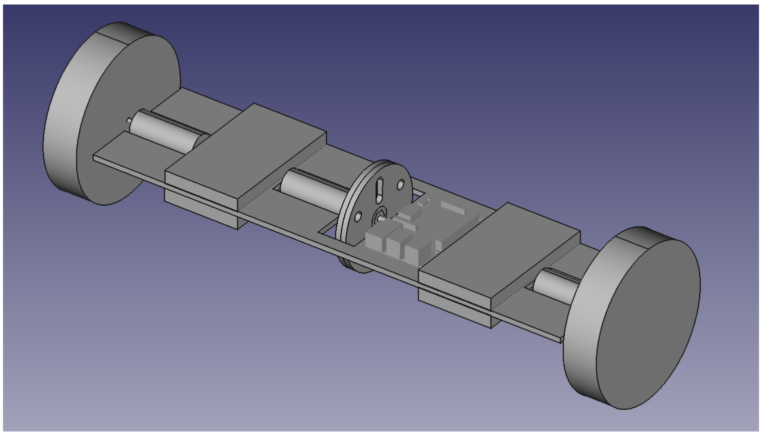


Figure 2: Isometric render view

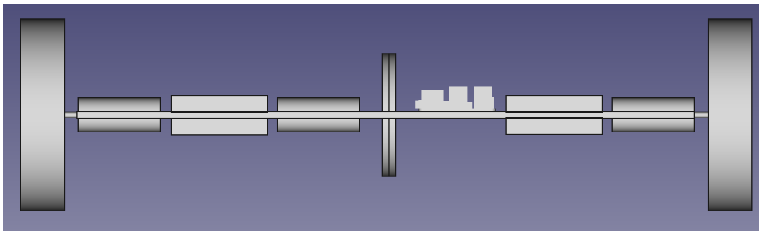


Figure 3: Front render view

# 2. Initial design considerations

The first thing we decided is the number of actuators. Most of segway robot include two motors for the motion control but we added a third on in order to control the inclination. We included three motors in total because we want to control three degrees of freedom (inclination and speed of both wheels).

In order to control the inclination of the platform we needed to add an external torque to the platform. We considered three methods: accelerating a flywheel, holding a pendulum in a non-vertical position and air friction with a fan. We discarded the last one due to the high speeds we needed to obtain a reasonable torque on the platform.

Both methods have strengths in different situations so we decided to build a mixed method that could combine both.

We took two restriction in our design. The first one symmetry along the inclination axis in order to have an equilibrium in all possible inclinations without the need of external forces. We also took in consideration that the reinforcement learning algorithms starts being clumsy so none of the configurations should touch the ground. Figure 5 illustrates this restriction.

The design of the robot is done with the 3D design software Free-cad. All part files are uploaded to the GitHub repository `https://github.com/tarragoesteve/TFM` under the hardware folder.

You can see the main views on Figure 2, 3, 4 and 5.

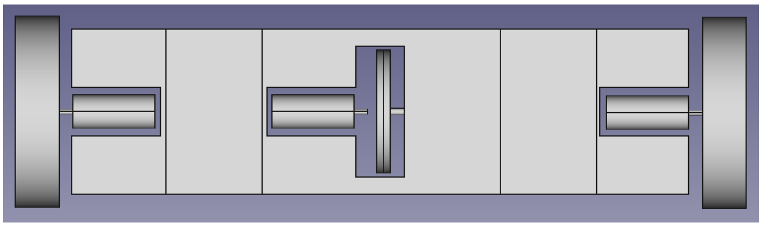


Figure 4: Top render view

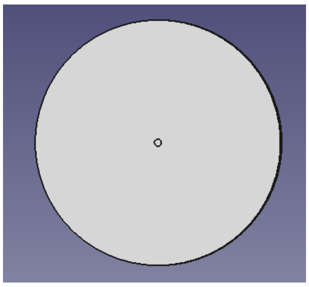


Figure 5: Side render view

## 2.1 Flywheel design

To control the inclination of the body two strategies are taken in to account. Creating torque by a pendulum or accelerating the flywheel. In order to experiment with both of them we designed a part to allow both configuration by placing weights in different spots, see figure 6.

In order to create a configuration with maximum gravitational torque we have done the following computation. We denote the torque pendulum torque $\tau$, consider the masses are cylinders of mass $m_{cylinder}$ with radius $r_c$ and width $w$ and the radius of the flywheel is $r_f$.

Each mass weights:

$$m_{cylinder} = \rho * w * \pi * r_c^2$$

We neglect the mass of the flywheel structure versus the mass of the cylinders so all the gravitational torque created by the masses will be compensated with the opposite weight except for the two masses with

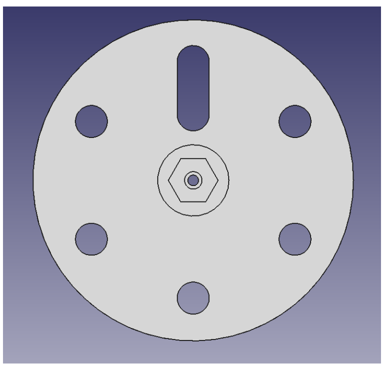


Figure 6: Fly wheel side render view

different radius.

One of the weight can be placed along a rail. The distance to the center will vary from $r_{min} = r_c + r_{motor-axis} \approx r_c$ to $r_{max} = r_f - r_c$.

The maximum torque takes place when these two masses are aligned horizontal with respect the ground and the movable weight is at distance $r_{min}$ from the center.

$$\tau_{max}(r_c) = m_{cylinder} * g * r_{max} - m_{cylinder} * g * r_{min} = m_{cylinder} * g * (r_f - 2 * r_c)$$

In order to maximize $\tau$ it we first compute the derivative:

$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = g * (\frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) - m_{cylinder} * 2)$$

$$\frac{\partial m_{cylinder}}{\partial r_c} = 2 * \rho * w * \pi * r_c$$

An make it zero to find the maximum:

$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = 0$$

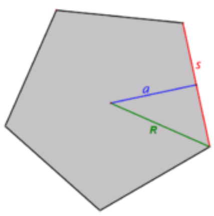
Substituting and simplifying we get:

$$\frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) = m_{cylinder} * 2 \Rightarrow 2 * \rho * w * \pi * r_c \ * (r_f - 2 * r_c) = \rho * w * \pi * r_c^2 * 2$$

$$\Rightarrow r_c \ * (r_f - 2 * r_c) = r_c^2 \Rightarrow (r_f - 2 * r_c) = r_c \Rightarrow \boxed{r_f = 3 * r_c}$$

The circumradius $R$ from the center of a regular polygon to one of the vertices is related to the side length $s$ by:

$$R = \frac{s}{2 * \sin \frac{\pi}{n}}$$

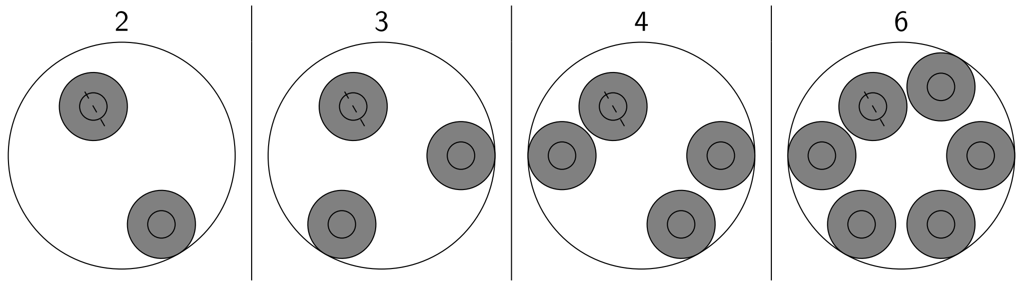



In our case:

$$R = r_f - r_c;$$

$$s = 2 * r_c$$

Substituting in the circumradius equation we get n = 6, so we will use up 6 masses in our flywheel. We will have a variable number of masses $N$ that we will be able to add to the flywheel as shown in the following table.


2
3
4
6

# 3. Mechanical analysis

In this section we will analyze and understand the dynamics of our robot so we can choose the design parameters based on performance indicators. As parameters we have the with of the flywheel masses $w$, the number of masses $N$, the radius of the flywheel $r_{flywheel}$, and the radius of the wheels $r_{wheel}$

## 3.1 Inclination control

In order to keep the inclination of the platform at a certain angle $\phi$ we must be able to compensate all the torque being applied to the platform.

$$\ddot{\phi} \cdot I_{platform} = \tau_{platform}$$

Assuming that the platform is well balanced (the center of masses is located at the rotation axis by our design restriction) and neglecting the torque generated by the friction with air, the sum of all the torques in the motor axis applied to the platform is equal to the sum of the torque applied by the motors:

$$\tau_{platform} = \sum \tau_{motors}$$

The torque that the motors deliver to the wheels and to the flywheel create a reaction in the platform in the opposite direction.

$$\tau_{platform} = -\tau_{motor-right-wheel} - \tau_{motor-left-wheel} - \tau_{motor-flywheel}$$

If we want keep the inclination $\phi$, we must be able to cancel $\tau_{platform}$. Observe that the angular acceleration $\ddot{\phi}$ of the platform is linearly dependent with the torque it receives.

$$\begin{gathered} 0 = -\tau_{motor-right-wheel} - \tau_{motor-left-wheel} - \tau_{motor-flywheel} \Rightarrow \\ \tau_{motor-right-wheel} + \tau_{motor-left-wheel} = -\tau_{motor-flywheel} \end{gathered} \tag{1}$$

In other words, we must overpass the torque of the wheels with the torque of the flywheel if we want to control the inclination.

## 3.2 Wheels torque

The wheel torque we can induce is limited by the motor specifications. Note that the maximum torque of the motor is a function of velocity and in particular at max speed the torque is zero.

$$\tau_{motor-wheel}(\omega_{wheel})$$

We assume that the wheels just roll and do no slip. The robot is pushed by the wheels that make a force $F_{friction}$ against the ground in the contact point. See figure 8.

We can express the torque at the center of the of the wheel as:

$$\tau_{motor-wheel} + F_{friction} \cdot r_{wheel} = I_{wheel} \cdot \dot{w}_{wheel}$$

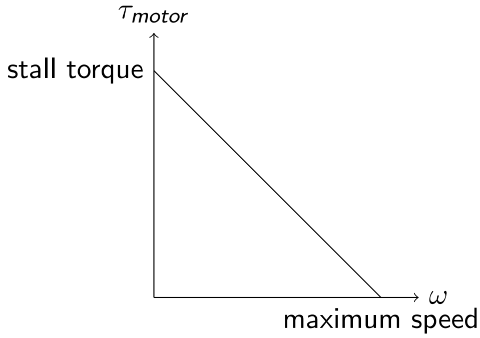



Figure 7: Motor torque

$$\tau_{motor-wheel} = I_{wheel} \cdot \dot{w}_{wheel} - F_{friction} \cdot r_{wheel} \tag{2}$$

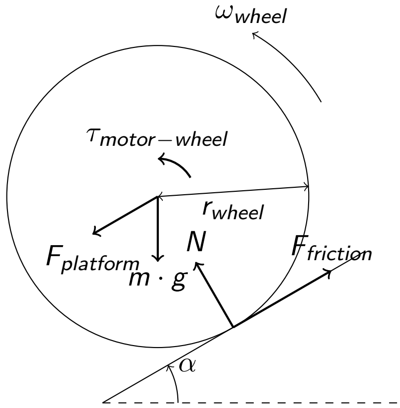



Figure 8: Wheel force diagram

## 3.3 Flywheel torque

The flywheel torque we can induce is also limited by the motor specifications.

Assuming a general configuration of the flywheel where the moving mass is at distance $r$ from the axis and angle $\theta$, see figure 9. We formulate its torque the following way:

$$\tau_{motor-flywheel} + m_{cylinder} \cdot g \cdot (r - r_{max}) \cdot \sin\theta = \ddot{\theta} \cdot I_{flywheel}(r)$$

$$\tau_{motor-flywheel} = \ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta \tag{3}$$

Note that in equation 3 the two terms correspond to the two methods: accelerating and non vertical position.

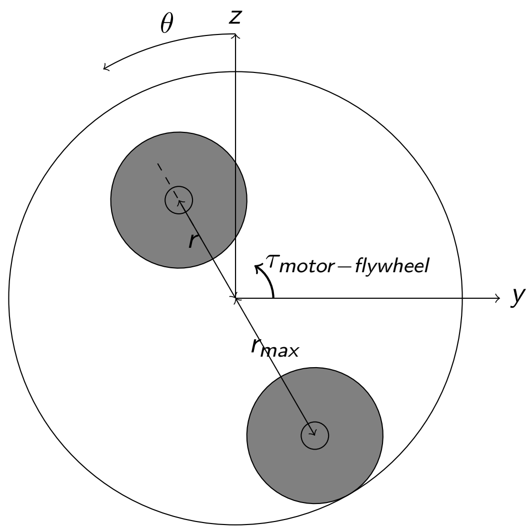



Figure 9: Flywheel diagram for $N = 2$

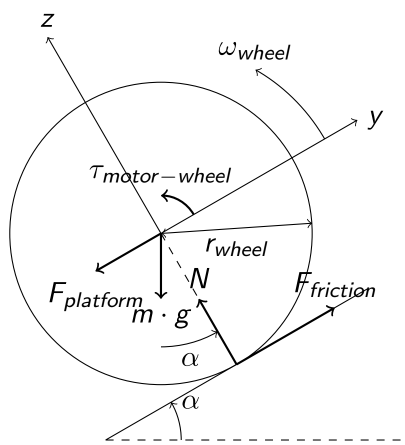



Figure 10: Wheel forward force diagram

## 3.4 Maximum speed, acceleration and inclination

In this subsection we would like to study the maximum speed and acceleration the robot may obtain in straight direction facing and inclination $\alpha$.

We will assume both wheels turn at the same speed, have the same $F_{friction}$ and the same $\tau_{wheel}$:

$$\omega_{wheel-left} = \omega_{wheel-right} = \omega_{wheel}$$

Applying Newton's first law in the y axis of figure 10:

$$\ddot{y} \cdot m_{total} = 2 \cdot F_{friction} - m_{total} \cdot g \cdot sin(\alpha)$$

Substituting $F_{friction}$ taking in to account equation 2:

$$\ddot{y} \cdot m_{total} = 2 \cdot \frac{I_{wheel} \cdot \dot{w}_{wheel} - \tau_{motor-wheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha)$$

Using equation 1:

$$\Rightarrow \ddot{y} \cdot m_{total} = \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}} + \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha) \tag{4}$$

We will now study different cases to better understand this equation.

### 3.4.1 No terrain inclination ($\alpha = 0$)

The objective here is to obtain the maximum speed and acceleration we can get starting from rest in a plain surface.

The equation we get by substituting $\alpha = 0$ in equation 4:

$$\ddot{y} \cdot m_{total} = \frac{\tau_{motor-flywheel}}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

Substituting equation 3:

$$\ddot{y} \cdot m_{total} = \frac{\ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}} \tag{5}$$

We will now split the study in two cases:

1. Flywheel case: $r$ is fixed to $r = r_{max}$

   Then:

   $$\ddot{y} \cdot m_{total} = -\frac{\ddot{\theta} \cdot I_{flywheel}(r_{max})}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

   Taking in to account the following relation:

   $$-\omega_{wheel} \cdot r_{wheel} = \dot{y} \Rightarrow -\dot{w}_{wheel} \cdot r_{wheel} = \ddot{y} \tag{6}$$

   Substituting in equation 5:

   $$-\dot{w}_{wheel} \cdot r_{wheel} \cdot m_{total} = \frac{\ddot{\theta} \cdot I_{flywheel}}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

   $$-\dot{w}_{wheel} \cdot (r_{wheel} \cdot m_{total} + \frac{2 \cdot I_{wheel}}{r_{wheel}}) = \frac{\ddot{\theta} \cdot I_{flywheel}}{r_{wheel}}$$

   We now define R as a non dimensional constant beeing the the quotient between $\dot{w}_{wheel}$ and $-\ddot{\theta}$.

   $$R = \frac{\dot{w}_{wheel}}{-\ddot{\theta}} = \frac{I_{flywheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}} \tag{7}$$

   The moments of inertia are:

   $$I_{wheel} \approx \frac{1}{2} \cdot m_{wheel} \cdot r_{wheel}^2$$

   $$I_{flywheel} \approx N \cdot m_{cylinder} \cdot r_{max}^2$$

Substituting those in equation 7 we get:

$$R \approx \frac{N \cdot m_{cylinder} \cdot r_{max}^2}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}} < 1$$

We can see that R will always be smaller than 1 because $N \cdot m_{cylinder} < m_{total}$ and $r_{max} < r_{wheel}$.

This means that we will the maximum acceleration of the wheels will be limited by the acceleration of the flywheel. The same is true for the speed.

In order to get the forward acceleration we can use equation 6

$$\ddot{y} = -\dot{w}_{wheel} \cdot r_{wheel} = R \cdot \ddot{\theta} \cdot r_{wheel}$$

And using equation 3 we get that the maximum is:

$$\tau_{motor}(w) = \ddot{\theta} \cdot I_{flywheel}(r) \Rightarrow \ddot{\theta} = \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)}$$

$$\ddot{y}_{max} = R \cdot \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} \cdot r_{wheel}$$

$$\Rightarrow \ddot{y}_{max} = \frac{I_{flywheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}} \cdot \frac{\tau_{motor}(\dot{\theta})}{I_{flywheel}(r)} \cdot r_{wheel}$$

$$\boxed{\ddot{y}_{max} = \frac{\tau_{motor}(\dot{\theta}) \cdot r_{wheel}}{r_{wheel}^2 \cdot m_{total} + 2 \cdot I_{wheel}}} \tag{8}$$

In order to compute the maximum speed we assume that the initial conditions are $\dot{\theta} = 0$ and $\omega_{wheel} = 0$

$$\omega_{wheel-max} = \int_{t=0}^{t=t_{max}} \dot{w}_{wheel} \cdot dt$$

Now we will proceed to do a change of variables in the integral.

$$\frac{\partial \dot{\theta}}{\partial t} = \ddot{\theta} \Rightarrow dt = \frac{d\dot{\theta}}{\ddot{\theta}}$$

$$\omega_{wheel-max} = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} \frac{\dot{w}_{wheel}}{\ddot{\theta}} \cdot d\dot{\theta} = \int_{\dot{\theta}=0}^{\dot{\theta}=\dot{\theta}_{max}} -R \cdot d\dot{\theta} = -R \cdot \dot{\theta}_{max}$$

$$\boxed{\dot{y}_{max} = r_{wheel} \cdot R \cdot \dot{\theta}_{max}} \tag{9}$$

And $\dot{\theta}_{max}$ is a limitation imposed by the motor specifications. Note that this is the maximum speed we can get using the flywheel system starting from rest.

2. Pendulum: $\dot{\theta} = 0$, and $r$ is fixed to $r = r_{min}$
   Using equation 5 and $\ddot{\theta} = 0$

$$\ddot{y} \cdot m_{total} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta}{r_{wheel}} + \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}}$$

   Multiplying by $r_{wheel}$ both sides of the equation we get:

$$\ddot{y} \cdot m_{total} \cdot r_{wheel} = m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta + 2 \cdot I_{wheel} \cdot \dot{w}_{wheel}$$

   And using $\dot{w}_{wheel} = -\frac{\ddot{y}}{r_{wheel}}$

$$\ddot{y} \cdot m_{total} \cdot r_{wheel} + 2 \cdot I_{wheel} \cdot \frac{\ddot{y}}{r_{wheel}} = m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta$$

   With some manipulation:

$$\ddot{y} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta}{m_{total} \cdot r_{wheel} + \frac{2 \cdot I_{wheel}}{r_{wheel}}}$$

   Which is maximum when $\sin\theta = 1$

$$\boxed{\ddot{y}_{max} = \frac{m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot}{m_{total} \cdot r_{wheel} + \frac{2 \cdot I_{wheel}}{r_{wheel}}}} \tag{10}$$

   We need to take into consideration air friction to see the speed limitation in the pendulum case.

$$F_{drag} = \frac{1}{2} \cdot \rho \cdot C_D \cdot A \cdot \dot{y}^2$$

   Adding this term to equation 4 we get:

$$\ddot{y} \cdot m_{total} = \frac{2 \cdot I_{wheel} \cdot \dot{w}_{wheel}}{r_{wheel}} + \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha) - F_{drag}$$

   And making $\ddot{y} = 0$ and $\alpha = 0$.

$$F_{drag} = \frac{\tau_{motor-flywheel}}{r_{wheel}}$$

$$\frac{1}{2} \cdot \rho \cdot C_D \cdot A \cdot \dot{y}^2 = m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot sin(\theta)/r_{wheel}$$

   The maximum $\dot{y}$ is then obtained when $\theta = \frac{\pi}{2}$:

$$\dot{y}_{max} = \sqrt{\frac{2 \cdot m_{cylinder} \cdot g \cdot (r_{max} - r_{min})}{\rho \cdot C_D \cdot A \cdot r_{wheel}}}$$

   We will pick $C_D = 1$, $\rho = 1.2kg/m^3$ and $A = 0.01m^2$ for our computations.
   Also note that the speed is also limited by maximum speed a motor can get $\dot{\theta}_{max}$.

$$\boxed{\dot{y}_{max} = min(\dot{\theta}_{max} \cdot r_{wheel}, \sqrt{\frac{2 \cdot m_{cylinder} \cdot g \cdot (r_{max} - r_{min})}{\rho \cdot C_D \cdot A \cdot r_{wheel}}})} \tag{11}$$

### 3.4.2 No acceleration, just inclination ($\alpha > 0$)

The goal of this subsubsection is to study which is the maximum inclination $\alpha_{max}$ the robot can overpass.

Substituting $\ddot{y} = 0$ and $\dot{\omega}_{wheel} = 0$ in equation 4

$$0 = \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha)$$

$$m_{total} \cdot g \cdot sin(\alpha) = \frac{\tau_{motor-flywheel}}{r_{wheel}}$$

$$sin(\alpha) = min(1, \frac{\tau_{motor-flywheel}}{m_{total} \cdot g \cdot r_{wheel}})$$

Substituting equation 3

$$sin(\alpha) = min(1, \frac{\ddot{\theta} \cdot I_{flywheel}(r) + m_{cylinder} \cdot g \cdot (r_{max} - r) \cdot \sin\theta}{m_{total} \cdot g \cdot r_{wheel}})$$

We are going to distinguish the same two cases as in the previous section:

1. Flywheel case: $r$ is fixed to $r = r_{max}$ The maximum inclination at a certain moment:

$$\boxed{sin(\alpha_{max}) = min(1, \frac{\ddot{\theta} \cdot I_{flywheel}}{m_{total} \cdot g \cdot r_{wheel}}) = min(1, \frac{\tau_{motor-flywheel}(\dot{\theta})}{m_{total} \cdot g \cdot r_{wheel}})} \quad (12)$$

This angle doesn't give us a lot of information because it may not be fulfilled in a permanent state. That's why in addition we would like to compute the maximum height our robot can achieve start from rest position.

2. Pendulum: $\dot{\theta} = 0$, and $r$ is fixed to $r = r_{min}$

$$sin(\alpha) = \frac{min(\tau_{motor-flywheel}, m_{cylinder} \cdot g \cdot (r_{max} - r_{min}) \cdot \sin\theta)}{m_{total} \cdot g \cdot r_{wheel}}$$

Which is maximum when $\sin\theta = 1$

$$\boxed{sin(\alpha_{max}) = \frac{min(\tau_{motor-flywheel}, m_{cylinder} \cdot g \cdot (r_{max} - r_{min}))}{m_{total} \cdot g \cdot r_{wheel}}} \quad (13)$$

Note that the $sin(\alpha_{max})$ will not be more than 1.

### 3.4.3 Flywheel maximum height

**Computing $\theta(t)$:** Before we compute the height the robot can obtain using the flywheel system we need to define how the flywheel will be turning. We will assume that $\tau_{motor-flywheel}$ is always equal to the maximum torque the motor may apply.

$$\tau_{motor}(\dot{\theta}) \approx \tau_{max} - \dot{\theta} \cdot \frac{\tau_{max}}{\dot{\theta}_{max}}$$

Using equation 3:

$$\dot{\theta}(t) = \int \ddot{\theta}(t)dt = \frac{1}{I_f}\int \tau_{max} - \dot{\theta}(t)\cdot\frac{\tau_{max}}{\dot{\theta}_{max}}dt$$

$$\dot{\theta}(t) = \frac{\tau_{max}\cdot t}{I_f} - \frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}\int \dot{\theta}(t)dt$$

$$\dot{\theta}(t) + \frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}\cdot\theta(t) = \frac{\tau_{max}\cdot t}{I_f}$$

Which is a non-homogeneous first order differential equation with solution:

$$\theta(t) = \frac{\int(\mu(t)\cdot\frac{\tau_{max}\cdot t}{I_f})dt}{\mu(t)} \tag{14}$$

Being $\mu(t)$:

$$\mu(t) = e^{\int\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}dt} = e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1}$$

Integrating by parts to get the numerator:

$$\int(\mu(t)\cdot\frac{\tau_{max}\cdot t}{I_f})dt = \frac{\tau_{max}\cdot t}{I_f}\cdot\int\mu(t) - \int(\int\mu(t)dt)\cdot\frac{\tau_{max}}{I_f}dt$$

$$\int\mu(t)dt = \frac{I_f\cdot\dot{\theta}_{max}}{\tau_{max}}\cdot(e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1} + C_2)$$

$$\dot{\theta}_{max}\cdot t\cdot(e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1} + C_2) - \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}}\cdot(e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1} + C_2) + C_3$$

$$(\dot{\theta}_{max}\cdot t - \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}})\cdot(e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1} + C_2) + C_3$$

Substituting on equation 14:

$$\theta(t) = \frac{(\dot{\theta}_{max}\cdot t - \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}})\cdot(e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1} + C_2) + C_3}{e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1}}$$

$$\theta(t) = \frac{(\dot{\theta}_{max}\cdot t - \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}})\cdot C_2}{e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1}} + (\dot{\theta}_{max}\cdot t - \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}}) + \frac{C_3}{e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}+C_1}}$$

Renaming the constants we get:

$$\theta(t) = \frac{(\dot{\theta}_{max}\cdot t - \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}})\cdot C_1}{e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}}} + (\dot{\theta}_{max}\cdot t - \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}}) + \frac{C_2}{e^{t\cdot\frac{\tau_{max}}{I_f\cdot\dot{\theta}_{max}}}}$$

Applying the initial position condition:

$$\theta(0) = -\frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}}\cdot C_1 - \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}} + C_2 = 0$$

$$\rightarrow C_2 = \frac{I_f\cdot\dot{\theta}^2_{max}}{\tau_{max}}\cdot(C_1+1)$$

Applying the initial speed condition:

$$\dot{\theta}(t) = \frac{\dot{\theta}_{max} \cdot C_1}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}} - \frac{(\dot{\theta}_{max} \cdot t - \frac{I_f \cdot \dot{\theta}^2_{max}}{\tau_{max}}) \cdot C_1 \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}} + \dot{\theta}_{max} - \frac{C_2 \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}}$$

$$\dot{\theta}(0) = 2 \cdot \dot{\theta}_{max} \cdot C_1 + \dot{\theta}_{max} - C_2 \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}} = 0$$

$$\rightarrow C_2 = \frac{\dot{\theta}^2_{max} \cdot I_f}{\tau_{max}} \cdot (2 \cdot C_1 + 1)$$

Solving the system of equations we get:

$$C_1 = 0$$

$$C_2 = \frac{\dot{\theta}^2_{max} \cdot I_f}{\tau_{max}}$$

So our final speed and position functions will be:

$$\theta(t) = \dot{\theta}_{max} \cdot t + \frac{\dot{\theta}^2_{max} \cdot I_f}{\tau_{max}} \cdot (\frac{1}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}} - 1) \tag{15}$$

$$\dot{\theta}(t) = \dot{\theta}_{max} - \frac{\dot{\theta}_{max}}{e^{t \cdot \frac{\tau_{max}}{I_f \cdot \dot{\theta}_{max}}}} \tag{16}$$

**Height integral**

$$h = \int_{t=0}^{t=t_f} \dot{y} \cdot sin(\alpha) dt = \int_{t=0}^{t=t_f} sin(\alpha) (\int_{t=0}^{t=t} \ddot{y} dt) dt$$

Where $t_f$ is the time when the robot gets to the maximum height.
From equation 4 we get:

$$\ddot{y}(\frac{2 \cdot I_{wheel} + m_{total} \cdot r^2_{wheel}}{r^2_{wheel}}) = \frac{\tau_{motor-flywheel}}{r_{wheel}} - m_{total} \cdot g \cdot sin(\alpha)$$

$$\ddot{y} = \frac{r_{wheel} \cdot \tau_{motor-flywheel}}{2 \cdot I_{wheel} + m_{total} \cdot r^2_{wheel}} - \frac{m_{total} \cdot g \cdot sin(\alpha) \cdot r^2_{wheel}}{2 \cdot I_{wheel} + m_{total} \cdot r^2_{wheel}}$$

$$\ddot{y} = A \cdot \tau_{motor-flywheel} + B$$

$$\dot{y} = \int_{t=0}^{t=t} \ddot{y} dt = \int_{t=0}^{t=t} (A \cdot \tau_{motor-flywheel} + B) dt$$

$$= A \int_{t=0}^{t=t} \tau_{motor-flywheel} dt + Bt$$

Lets check now $\tau$ using equation 3:

$$\int_{t=0}^{t=t} \tau_{motor-flywheel}(t) dt = I_f \int_{t=0}^{t=t} \ddot{\theta}(t) dt = I_f \cdot \dot{\theta}(t)$$

$$\dot{y} = A \cdot I_f \cdot \dot{\theta}(t) + B \cdot t$$

$$h = sin(\alpha) \cdot \int_{t=0}^{t=t_f} \dot{y} dt = sin(\alpha) \cdot (A \cdot I_f \cdot \theta(t) + \frac{B}{2} \cdot t^2)$$

# 4. Flywheel brake study

This section aims to study a possible way to break the flywheel without applying torque to the platform. The idea is the following: we will leave the moving weight free.

This way, when the weight is going upward will have a larger radius than when is going downward and will produce an average moment against the movement of the flywheel.

From an energetic point of view we are transforming the rotation energy of the flywheel in to translation of the free cylinder and then realising it trough collisions.

## 4.1 System of differential equations

As described in figure 9 we will use two variables to describe the flywheel position: $r$, $\theta$

Using equation 3:

$$\tau_{motor-flywheel} = \ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r_{max} - r) * \sin\theta$$

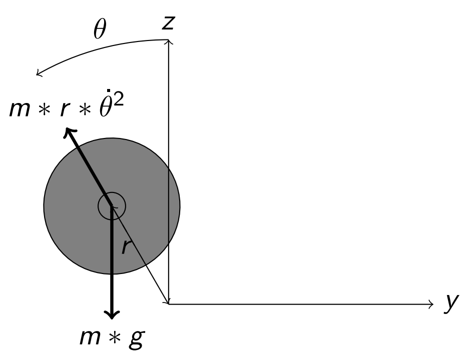



Figure 11: Cylinder force diagram

As seen in figure 11, we can deduce newtons equation for the distance from the cylinder to the center $r$. Note that we are adding the centrifugal force term due to the non-inertial frame.

$$\ddot{r} * m = -m * g * cos(\theta) + m * r * \dot{\theta}^2$$

$$\ddot{r} = -g * cos(\theta) + r * \dot{\theta}^2$$

The variables we will be using for our ODE system are: $r$,$\dot{r}$,$\theta$,$\dot{\theta}$

Note that we impose $\tau_{flywheel} = 0$ so the motor is not applying any torque.

$$\begin{cases} \dot{r} = \dot{r} \\ \ddot{r} = -g * cos(\theta) + r * \dot{\theta}^2 \\ \dot{\theta} = \dot{\theta} \\ \ddot{\theta} = \frac{m_{cylinder} * g * (r - r_{max}) * \sin\theta}{I_{flywheel}(r)} \end{cases}$$

Our initial conditions will be the free cylinder mass lying on the bottom of the flywheel and the flywheel turning at a speed $\theta_0$:

$$\begin{cases} r = r_{max} \\ \dot{r} = 0 \\ \theta = \pi \\ \dot{\theta} = \theta_0 \end{cases}$$

We will use a poincare map to simulate the bounce with the end of the guides at $r = r_{min}$ and $r = r_{max}$. At each bounce we will reduce its kinetic energy by a percentage *bounce_percentage* .

## 4.2 Results

The parameters of the simulation where:

$$\begin{cases} r_{flywheel} = 8cm \\ r_{wheel} = 9cm \\ w = 7cm \\ \dot{\theta}_0 = 4.2\pi rad/s \\ bounce\_percentage = 0.0(totallyinelastic) \end{cases}$$

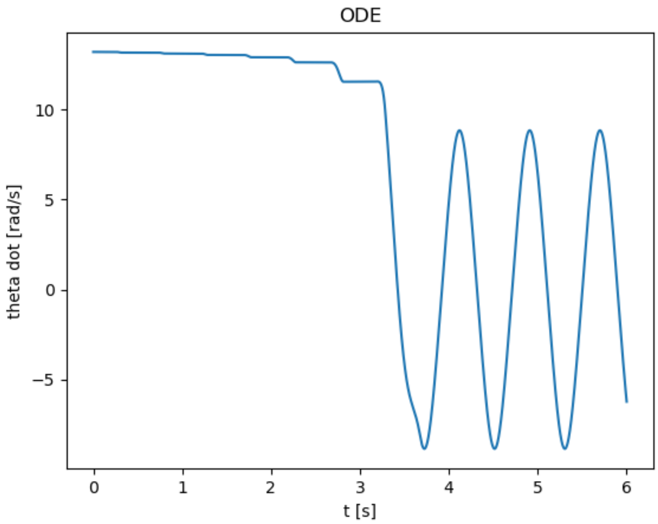



Figure 12: How the variable $\dot{\theta}$ evolve over time

As we can see in figure 12 the flywheel is braking until it becomes a pendulum and starts oscillating.

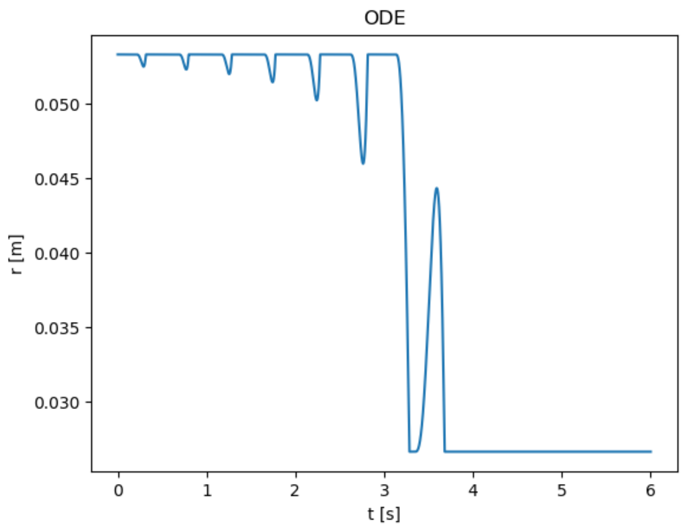



Figure 13: How the variable $r$ evolves over time

In the first laps the cylinder is almost always at the maximum value of r, but as the speed decrease each lap the minimum $r$ decrease until it hits the $r_{min}$.

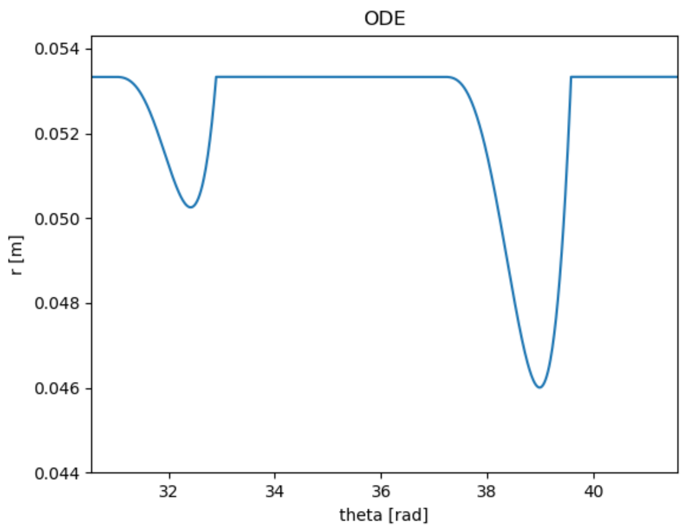



Figure 14: How the variable $r$ evolve over $\theta$ zoomed

In image 14 we can appreciate that the r decrease slower that what it increase.

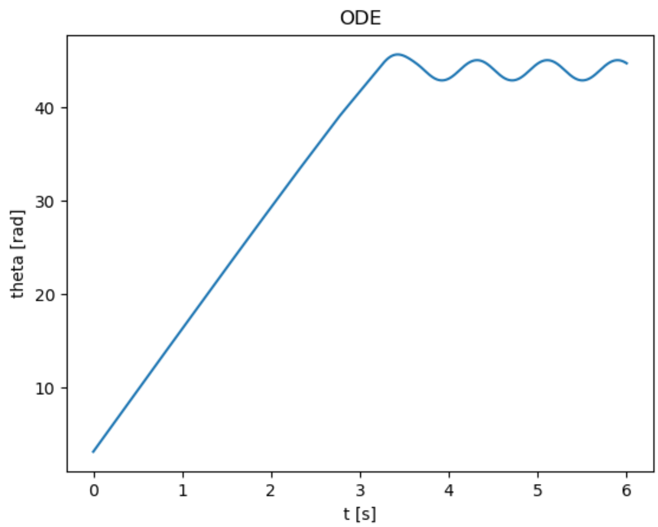



Figure 15: How the variable $\theta$ evolves over time

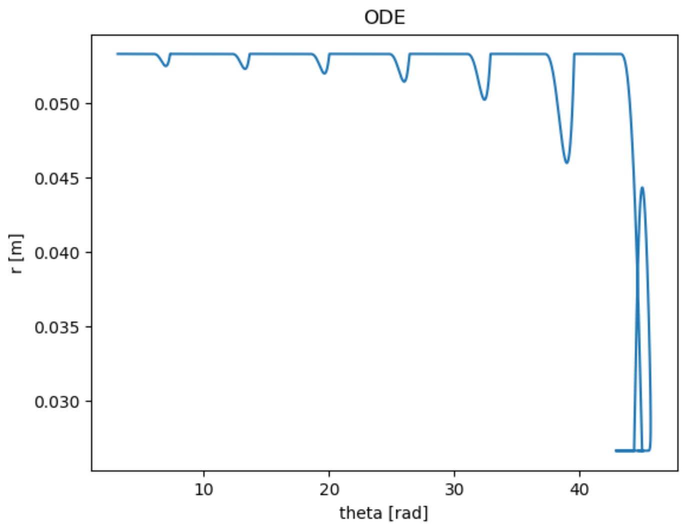



Figure 16: How the variable $r$ evolves over $\theta$

# 5. Dynamics

In order to study the dynamics of the robot we will use Lagrange mechanics.

## 5.1 Rectilinear movement with $r_{flywheel}$ fixed to $r_{flywheel-min}$

To reduce the number of variables we wil study the case of rectilinear movement by imposing that both wheels turn at the same speed. We will also set the radius of the free weight to $r_{flywheel-min}$.

The generalized coordinates ($q$) will be:

1. $\phi_{ground-wheel}$: rotation of the wheel respect the ground.
2. $\phi_{wheel-platform}$: rotation of the platform respect the wheel.
3. $\phi_{platform-flywheel}$: rotation of the flywheel respect the platform.

We will use two auxiliary variables:

1. $\phi_{ground-platform} = \phi_{ground-wheel} + \phi_{wheel-platform}$: rotation of the platform respect the ground.
2. $\phi_{ground-flywheel} = \phi_{ground-platform} + \phi_{platform-flywheel}$: rotation of the flywheel respect the ground.

The total potential energy:

$$V = m_{cylinder} \cdot (r_{flywheel-min} - r_{flywheel-max}) \cdot cos(\phi_{ground-flywheel}) \cdot g \tag{17}$$

The total kinetic energy:

$$T = \frac{1}{2} \cdot [\dot{\phi}^2_{ground-wheel} \cdot I_{wheel} + \dot{\phi}^2_{ground-platform} \cdot I_{platform} + \dot{\phi}^2_{ground-flywheel} \cdot I_{flywheel} + \dot{\phi}^2_{ground-wheel} \cdot r^2_{wheel} \cdot m_{total}] \tag{18}$$

The Lagrangian is defined as:

$$L = T - V \tag{19}$$

Lagrange's equation is:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_j}) = \frac{\partial L}{\partial q_j} + F_j \tag{20}$$

So in our case:

$$\frac{\partial L}{\partial \dot{\phi}_{ground-wheel}} = \dot{\phi}_{ground-wheel} \cdot I_{wheel} + \dot{\phi}_{ground-platform} \cdot I_{platform} + \dot{\phi}_{ground-flywheel} \cdot I_{flywheel} + \dot{\phi}_{ground-wheel} \cdot r^2_{wheel} \cdot m_{total} \tag{21}$$

$$\frac{\partial L}{\partial \dot{\phi}_{wheel-platform}} = \dot{\phi}_{ground-platform} \cdot I_{platform} + \dot{\phi}_{ground-flywheel} \cdot I_{flywheel} \tag{22}$$

$$\frac{\partial L}{\partial \dot{\phi}_{platform-flywheel}} = \dot{\phi}_{ground-flywheel} \cdot I_{flywheel} \tag{23}$$

We define M as the following matrix:

$$\begin{pmatrix} I_{wheel} + I_{platform} + I_{flywheel} + r_{wheel}^2 \cdot m_{total} & I_{platform} + I_{flywheel} & I_{flywheel} \\ I_{platform} + I_{flywheel} & I_{platform} + I_{flywheel} & I_{flywheel} \\ I_{flywheel} & I_{flywheel} & I_{flywheel} \end{pmatrix} \tag{24}$$

In matrix form and with our generalized coordinates:

$$\frac{\partial L}{\partial \dot{q}} = M \cdot \dot{q} \tag{25}$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}) = M \cdot \ddot{q} \tag{26}$$

Let a be a constant:

$$a = m_{cylinder} \cdot (r_{flywheel-min} - r_{flywheel-max}) \cdot g \tag{27}$$

$$\frac{\partial L}{\partial q} = a \cdot sin(\phi_{ground-flywheel}) \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{28}$$

So using Lagrange's equation we get:

$$\boxed{M \cdot \ddot{q} = a \cdot sin(\phi_{ground-flywheel}) \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + F} \tag{29}$$

## 5.2 Rectilinear movement with $r_{flywheel}$ free

The generalized coordinates ($q$) will be:

1. $\phi_{ground-wheel}$: rotation of the wheel respect the ground.
2. $\phi_{wheel-platform}$: rotation of the platform respect the wheel.
3. $\phi_{platform-flywheel}$: rotation of the flywheel respect the platform.
4. $r$: distance from the center of the flywheel of the free cylinder.

We will use two auxiliary variables:

1. $\phi_{ground-platform} = \phi_{ground-wheel} + \phi_{wheel-platform}$: rotation of the platform respect the ground.
2. $\phi_{ground-flywheel} = \phi_{ground-platform} + \phi_{platform-flywheel}$: rotation of the flywheel respect the ground.

The total potential energy:

$$V = m_{cylinder} \cdot (r - r_{flywheel-max}) \cdot cos(\phi_{ground-flywheel}) \cdot g \tag{30}$$

The total kinetic energy:

$$T = \frac{1}{2} \cdot [\dot{\phi}^2_{ground-wheel} \cdot I_{wheel} + \dot{\phi}^2_{ground-platform} \cdot I_{platform} + \dot{\phi}^2_{ground-flywheel} \cdot I_{flywheel}(r) \\ + \dot{\phi}^2_{ground-wheel} \cdot r^2_{wheel} \cdot m_{total} + \dot{r}^2 \cdot m_{cylinder}] \quad (31)$$

The Lagrangian is defined as:

$$L = T - V \quad (32)$$

Lagrange's equation is:

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_j}) = \frac{\partial L}{\partial q_j} + F_j \quad (33)$$

So in our case:

$$\frac{\partial L}{\partial \dot{\phi}_{ground-wheel}} = \dot{\phi}_{ground-wheel} \cdot I_{wheel} + \dot{\phi}_{ground-platform} \cdot I_{platform} \\ + \dot{\phi}_{ground-flywheel} \cdot I_{flywheel}(r) + \dot{\phi}_{ground-wheel} \cdot r^2_{wheel} \cdot m_{total} \quad (34)$$

$$\frac{\partial L}{\partial \dot{\phi}_{wheel-platform}} = \dot{\phi}_{ground-platform} \cdot I_{platform} + \dot{\phi}_{ground-flywheel} \cdot I_{flywheel}(r) \quad (35)$$

$$\frac{\partial L}{\partial \dot{\phi}_{platform-flywheel}} = \dot{\phi}_{ground-flywheel} \cdot I_{flywheel}(r) \quad (36)$$

$$\frac{\partial L}{\partial \dot{r}} = \dot{r} \cdot m_{cylinder} \quad (37)$$

We define M as the following matrix:

$$\begin{pmatrix} I_{wheel} + I_{platform} + I_{flywheel}(r) + r^2_{wheel} \cdot m_{total} & I_{platform} + I_{flywheel}(r) & I_{flywheel}(r) & 0 \\ I_{platform} + I_{flywheel}(r) & I_{platform} + I_{flywheel}(r) & I_{flywheel}(r) & 0 \\ I_{flywheel}(r) & I_{flywheel}(r) & I_{flywheel}(r) & 0 \\ 0 & 0 & 0 & m_{cylinder} \end{pmatrix} \quad (38)$$

In matrix form and with our generalized coordinates:

$$\frac{\partial L}{\partial \dot{q}} = M \cdot \dot{q} \quad (39)$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}) = M \cdot \ddot{q} + \dot{M} \cdot \dot{q} \quad (40)$$

Let's recall the definition of $I_{flywheel}(r)$

$$I_{flywheel}(r) = m_{cylinder} \cdot r^2 + C \quad (41)$$

Compute the derivative

$$\dot{I}_{flywheel}(r) = 2 \cdot m_{cylinder} \cdot r \cdot \dot{r} \quad (42)$$

We compute $\dot{M}$ as the following matrix:

$$\dot{M} = \dot{I}_{flywheel}(r) \cdot \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{43}$$

Let $a$ be::

$$\frac{\partial L}{\partial q_{1..3}} = a = m_{cylinder} \cdot (r - r_{flywheel-max}) \cdot g \cdot sin(\phi_{ground-flywheel}) \tag{44}$$

Let $b$ be:

$$\frac{\partial L}{\partial r} = b = 2 \cdot m_{cylinder} \cdot r \cdot \dot{\phi}^2_{ground-flywheel} - m_{cylinder} \cdot g \cdot cos(\phi_{ground-flywheel}) \tag{45}$$

$$\frac{\partial L}{\partial q} = \begin{pmatrix} a \\ a \\ a \\ b \end{pmatrix} \tag{46}$$

So using Lagrange's equation we get:

$$\boxed{M \cdot \ddot{q} + \dot{M} \cdot \dot{q} = \begin{pmatrix} a \\ a \\ a \\ b \end{pmatrix} + F} \tag{47}$$

# 6. Optimization

In this section we select which parameters are going to be used when building our robot.

## 6.1 Minimum section in the section of the flywheel

We wil place our flywheel in a hole on our robot. We will now compute the minimum section in hole so it doesn't bend. See figure 18.

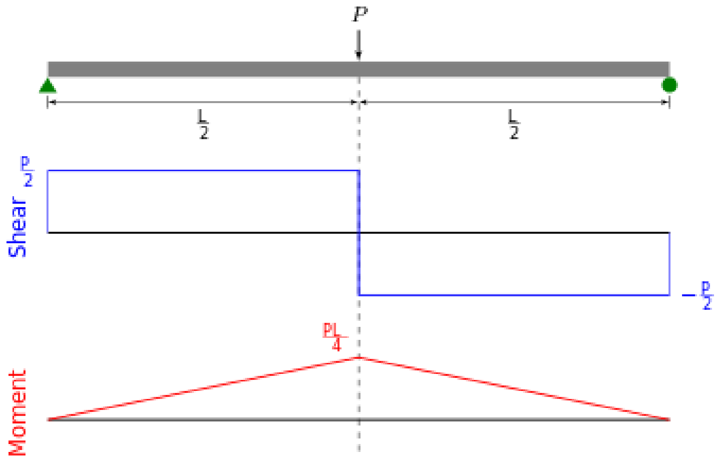



Figure 17: Bendig moment diagram

The maximum bending moment is at the flywheel section:

$$M_y = \frac{P * L}{4}$$

Where P is the weight of the robot and L is the distance between the two wheels.

$$I_y = \frac{2 * b * h^3}{12} = \frac{b * h^3}{6}$$

$$\sigma = \frac{M_y}{I_y} * \frac{h}{2} = \frac{P * L}{48 * b * h^2}$$

We are planning to build our body structure with plastic:

$$\sigma_{plastic} = 4MPa = 4E6Pa$$

We will impose the relation:

$$b = h$$

And set a target P of $2000N$ and a maximum length of $0.5m$

$$\sigma_{plastic} = 4E6Pa = \frac{P * L}{48 * b * h^2} = \frac{1000}{48 * b^3}$$

Therefore:

$$b = \sqrt[3]{\frac{1000}{48 * 4E6}} = 1,277182m$$

We will use b=10mm and h=10mm from now on. Which is far more than what we need.

## 6.2 Restrictions

The first restrictions is due to the initial design requirements. The other three are somehow arbitrary but will help us to reduce the size of the robot.

1. We wil place our flywheel in a hole on our robot. We don't want to touch the ground in any configuration so:

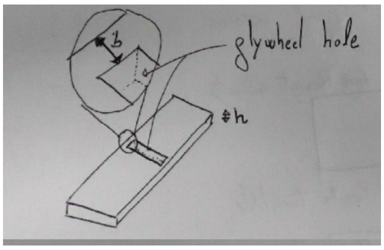



Figure 18: Flywheel hole diagram

$$r_{wheel} > \sqrt{(r_{flywheel} + b)^2 + (\frac{h}{2})^2}$$

2. Being able to insert the robot in to a wheel of diameter 0.5m so:

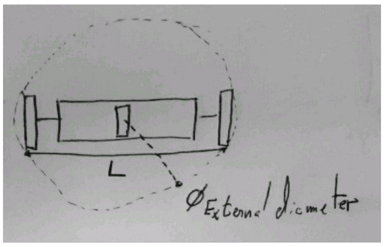



Figure 19: External diameter diagram

$$0.25m > \sqrt{r_{wheel}^2 + L^2/4}$$

3. We can place all electronic the devices:

$$L > 0.3m + w$$

4. Maximum weight of the robot: 5kg

## 6.3 Requirements

Based on the mechanical analysis we have set some requirements that we would like our robot to fullfil:
**Flywheel mode**

1. $\dot{y}_{max}$ (equation 9) $> 0.1 m/s$

2. $\ddot{y}_{max}$ (equation 8)¿ $1 m/s^2$

3. $sin(\alpha_{max})$ (equation 12) $> 0.2$

**Pendulum mode**

1. $\dot{y}_{max}$ (equation 11) $> 1 m/s$

2. $\ddot{y}_{max}$ (equation 10) $> 0.1 m/s^2$

3. $sin(\alpha_{max})$ (equation 13) $> 0.02$

## 6.4 Cost function

Apparat from fulfilling the requirements we will try to minimize a cost function.

We will maximize the maximum sinus in the pendulum mode (equation 13) because it give the robot the capacity to deliver force in a permanent state.

We will also maximize the square of the max speed the robot can achieve in flywheel mode (equation 9) because it is proportional to the energy the robot can deliver using the flywheel at a certain moment.

$$cost(r_{flywheel}, r_{wheel}, w, N) = -sin(\alpha_{max})_{pendulum} - \dot{y}^2_{max-flywheel} \tag{48}$$

$$m_{cylinder} = \rho * w * \pi * (\frac{r_{flywheel}}{3})^2$$

$$sin(\alpha_{max})_{pendulum} = \frac{m_{cylinder} \cdot (r_{max} - r_{min})}{m_{total} \cdot r_{wheel}} = \frac{m_{cylinder} \cdot (\frac{r_{flywheel}}{3})}{(m_{rest} + N \cdot m_{cylinder}) \cdot r_{wheel}}$$

$$\dot{y}_{max} = r_{wheel} \cdot R \cdot \dot{\theta}_{max} = r_{wheel} \cdot \frac{N \cdot m_{cylinder} \cdot (\frac{2 \cdot r_{flywheel}}{3})^2}{r^2_{wheel} \cdot (m_{rest} + N \cdot m_{cylinder}) + 2 \cdot I_{wheel}} \cdot \dot{\theta}_{max}$$

## 6.5 Results

Our procedure has been making a grid with the four parameters of the robot design: $w$ (width of the cylinders), $N$(number of cylinders), $r_{wheel}$ and $r_{flywheel}$. We have fixed $r_{flywheel}$, iterated over the other three variables and keeped the best parameters to reduce our cost function.

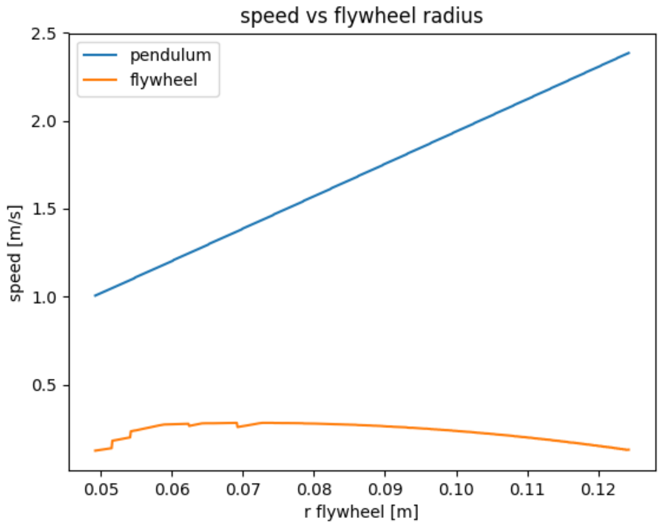



Figure 20: Plot of the equations 9 and 11 at the parameters that minimize the cost function and fullfil the requirements and restrictions

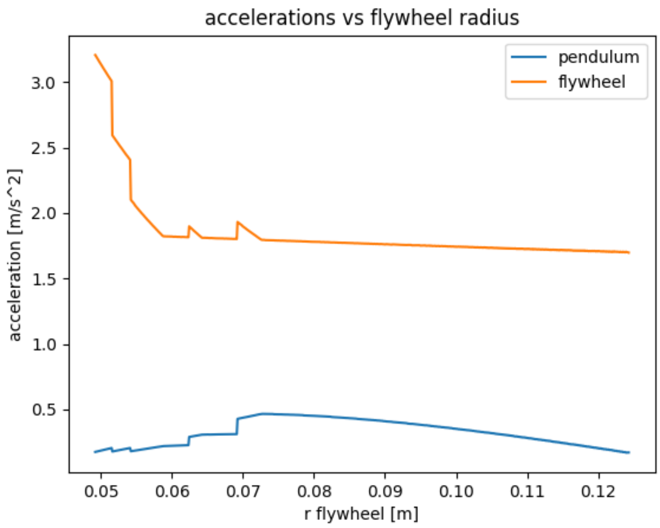



Figure 21: Plot of the equations 8 and 10 at the parameters that minimize the cost and fullfil the requirements and restrictions

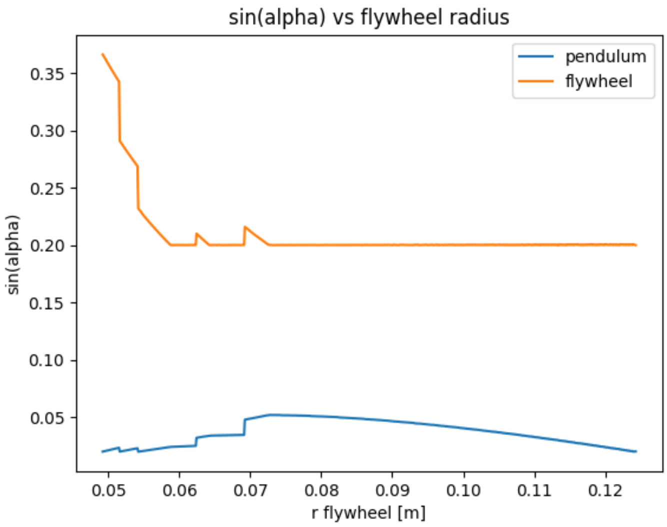



Figure 22: Plot of the equations 12 and 13 at the parameters that minimize the cost and fullfil the requirements and restrictions

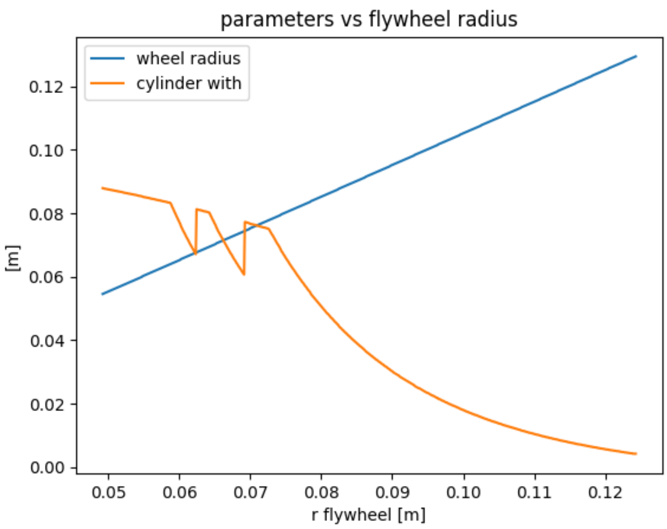



Figure 23: Plot of the parameters that minimize the cost function.

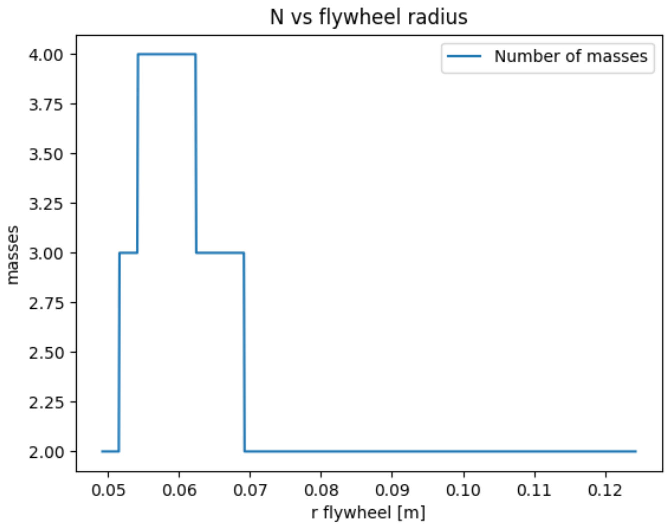



Figure 24: Plot of the N that minimize the cost function.

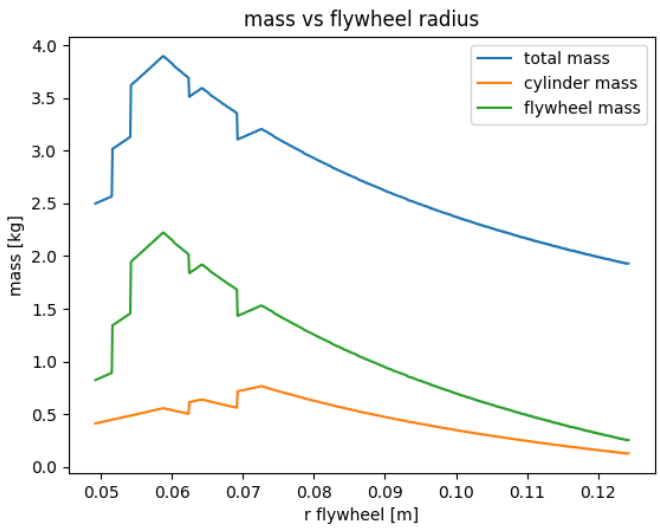



Figure 25: Plot of the mass for each configuration.

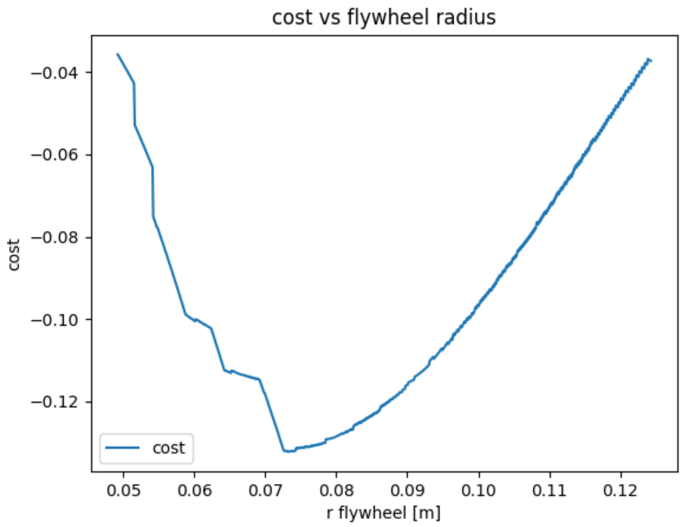



Figure 26: Plot of the equation 48 for each configuration.

Our selected parameters are:

| $r_{flywheel}$ | $r_{wheel}$ | $w$ | $N$ |
|---|---|---|---|
| 8cm | 10cm | 5cm | 2 |

With this parameters we get the following specifications:

| | |
|---|---|
| Total mass | 2,91kg |
| **Pendulum** | |
| Maximum sinus | 0,042 |
| Maximum speed horizontal | 1,84 $m/s$ |
| Maximum acceleration horizontal | 0,38 $m/s^2$ |
| **Flywheel** | |
| Maximum sinus | 0,171 |
| Maximum speed horizontal | 0,24 $m/s$ |
| Maximum acceleration horizontal | 1.52 $m/s^2$ |

# 7. Components

## 7.1 Mechanical components

### 7.1.1 Central Body

Used to host the flywheel, a motor and a bearing.

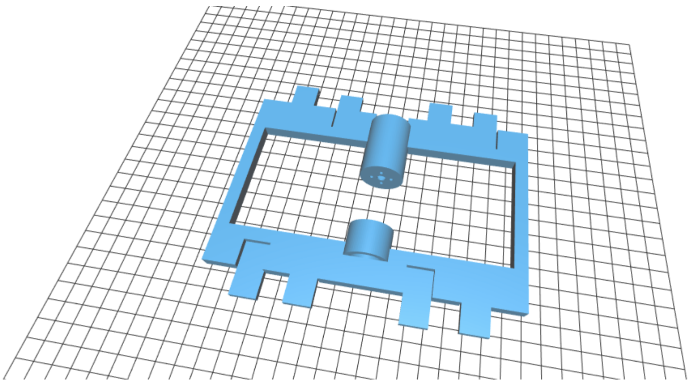


Figure 27: 3D Model of the central body.

### 7.1.2 Lateral Body

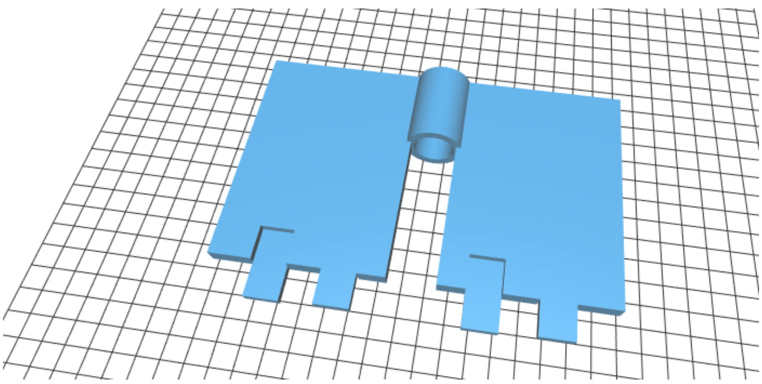


Figure 28: 3D Model of lateral body.

#### 7.1.3 Wheels

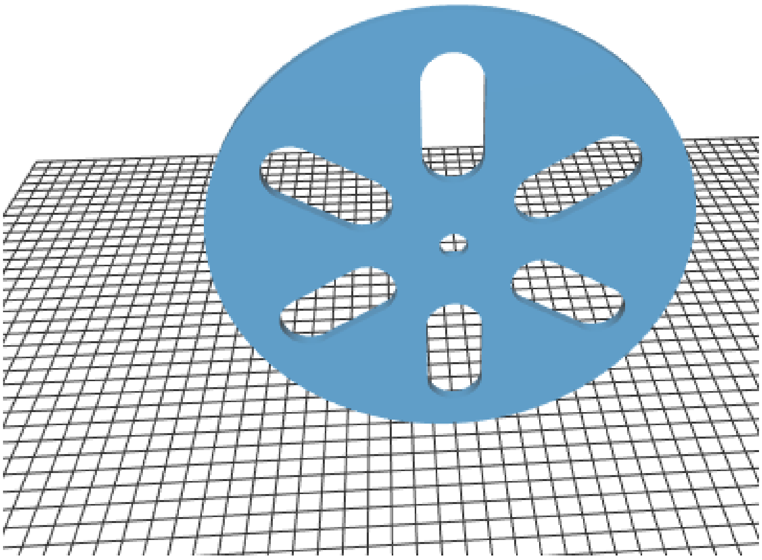


Figure 29: 3D Model of a wheel.

#### 7.1.4 Flywheel

#### 7.1.5 Bearings

### 7.2 Electronic components

#### 7.2.1 Raspberry Pi

The Raspberry Pi is a small single-board computer. We are using Raspberry Pi 3 Model B. It has GPIO pins.

Figure 30: Raspberry Pi picture

| | |
|---|---|
| Weight | 42 g |
| Price per unit | 35 euros |
| Number of units | 1 |

#### 7.2.2 Batteries

Provide power to our motors and to the Raspberry Pi.

#### 7.2.3 Breadboard

A breadboard is a construction base for prototyping of electronics.

#### 7.2.4 DC Motor

A DC motor is a class of rotary electrical machines that converts direct current electrical energy into mechanical energy. The most common types rely on the forces produced by magnetic fields.

| | |
|---|---|
| Operating voltage | between 3 V and 9 V |
| Free-run speed at 6 V | 176 RPM |
| Free-run current at 6 V | 80 mA |
| Stall current at 6V | 900 mA |
| Stall current at 6V | 5 kgcm |
| Gear ratio | 1:35 |
| Reductor size | 21 mm |
| Weight | 85 g |
| Price per unit | 10 euros |
| Number of units | 3 |

### 7.2.5 H Bridge

An H bridge is an electronic circuit that switches the polarity of a voltage applied to a load. These circuits are often used in robotics and other applications to allow DC motors to run forwards or backwards.

### 7.2.6 Rotatory Encoder

A rotary encoder, also called a shaft encoder, is an electro-mechanical device that converts the angular position or motion of a shaft or axle to analog or digital output signals.

### 7.2.7 Accelerometer

An accelerometer is a device that measures proper acceleration. Proper acceleration, being the acceleration (or rate of change of velocity) of a body in its own instantaneous rest frame, is not the same as coordinate acceleration, being the acceleration in a fixed coordinate system. For example, an accelerometer at rest on the surface of the Earth will measure an acceleration due to Earth's gravity, straight upwards (by definition) of g 9.81 m/s2.

# 8. Control

## 8.1 Position control

## 8.2 Speed control

## 8.3 Inclination control

# 9. Software Design

## 9.1 User Interface

## 9.2 Planner

## 9.3 Controller

# 10. Conclusion