Master Thesis

Design and implementation of a remote controlled segway

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This is the abstract

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Figure 1: Picture of a commercial *segway hover-board*

1 Objective

The objective of this project is to build a dynamic robot in order to experiment with reinforcement learning algorithm with real data. We wish to make this experiments easy and cheap to reproduce so we will try minimize its components and fabrication cost.

The chosen robot is inspired in a *segway hover-board*, similar to the one in Figure 1. The two wheels are controlled with classic control algorithms and the inclination of the central body is controlled with a reinforcement learning algorithm.

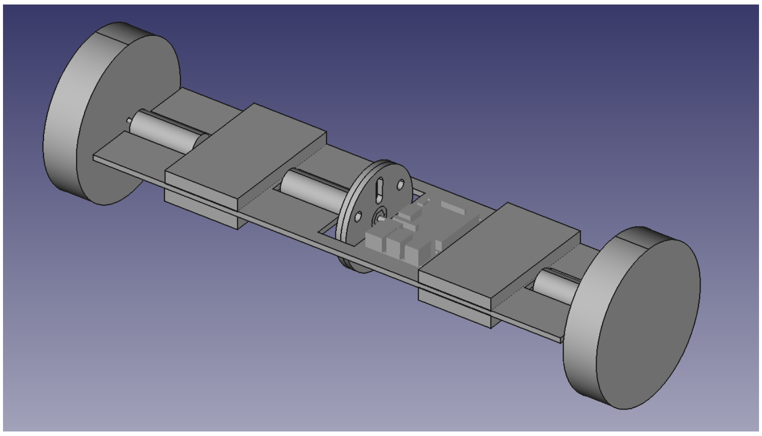


Figure 2: Isometric render view

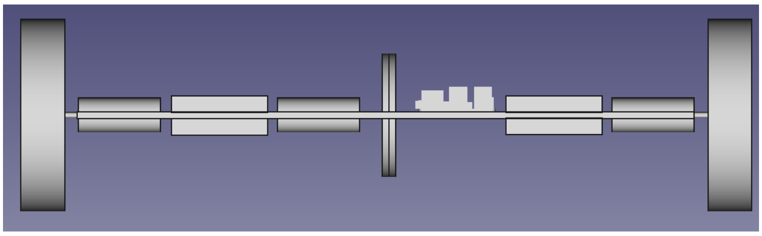


Figure 3: Front render view

2 Design

The design of the body is done with a 3D design software Free-cad. All part files are uploaded to the GitHub repository `https://github.com/tarragoesteve/TFM` under the hardware folder. You can see the main views on Figure 2, 3, 4 and 5

We included three actuators in the robot because we want to control 3 degrees of freedom (inclination and speed of both wheels). Furthermore we introduced a fly-wheel to control the inclination of the body.

We ensured symmetry along the axis formed by all motors in order to have an equilibrium in all possible inclinations without the need of external forces. We also took in consideration that the reinforcement learning algorithm starts being clumsy so none of the configurations should intersect with the ground. Figure 5 illustrates this restriction.

2.1 Flywheel design

To control the inclination of the body two strategies where taken in to account. Creating torque by a pendulum or by a the acceleration of flywheel. In order to experiment with both of them we designed a part to allow both configuration by placing weights in different spots, figure 6.

One of the weight can be placed along a rail. The distance to the center will vary from

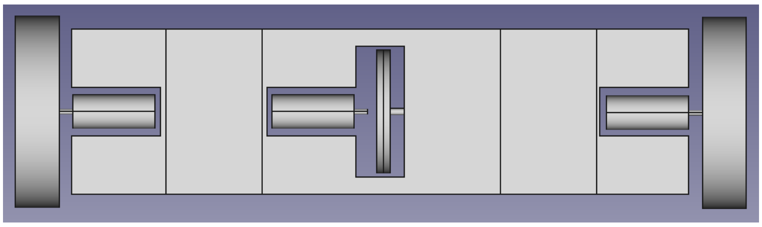


Figure 4: Top render view

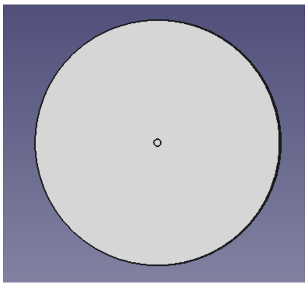


Figure 5: Side render view

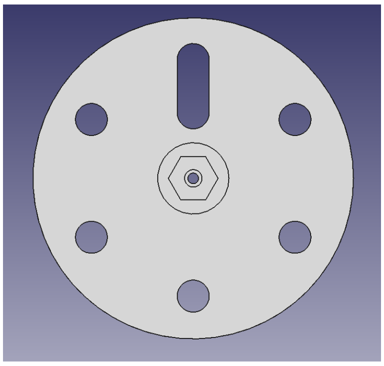


Figure 6: Fly wheel side render view

$r_{min} = r_c + r_{motor-axis} \approx r_c$ to $r_{max} = r_f - r_c$. The pendulum torque τ, considering the masses are cylinders of width w and radius r_c, and the flywheel has a radius r_f is the following:

Each mass weights:

$$m = \rho * w * \pi * r_c^2$$

All mass will be compensate the opposed one except the two with different radius. The maximum torque takes place when these two masses are aligned horizontal with respect the ground and the movable weight is at distance r_{min} from the center.

$$\tau_{max}(r_c) = m * r_{max} - m * r_{min} = m * (r_f - 2 * r_c)$$

In order to maximize it we first compute the derivative:

$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = \frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) - m * 2$$

$$\frac{\partial m}{\partial r_c} = 2 * \rho * w * \pi * r_c$$

An make it zero to find the maximum:

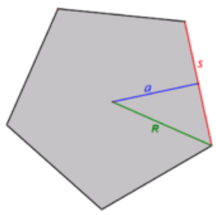
$$\frac{\partial \tau_{max}(r_c)}{\partial r_c} = 0$$

Substituting and simplifying we get:

$$\frac{\partial m}{\partial r_c} * (r_f - 2 * r_c) = m * 2 \Rightarrow 2 * \rho * w * \pi * r_c \ * (r_f - 2 * r_c) = \rho * w * \pi * r_c^2 * 2$$

$$\Rightarrow r_c \ * (r_f - 2 * r_c) = r_c^2 \Rightarrow (r_f - 2 * r_c) = r_c \Rightarrow r_f = 3 * r_c$$

The circumradius R from the center of a regular polygon to one of the vertices is related to the side length s by:



$$R = \frac{s}{2 * \sin \frac{\pi}{n}}$$

In our case:

$$R = r_f - r_c;$$

$$s = 2 * r_c$$

Substituting in the circumradius equation we get n = 6, so we will use 6 masses in our flywheel.

3 Mechanical analysis

3.1 Reference frames

In order to study the behavior of the robot we will use the following frames:

- Absolute frame: From a fix object in the room.
- Body frame: From the body of our robot.

3.2 Inclination control

In order to keep the inclination at a certain angle we must be able to compensate all the torque being applied to the body.

Assuming that the body is well balanced and neglecting the torque generated by the friction with air, the sum of all the torques in the motor axis applied to the body is equal to the sum of the torque applied by the motors:

$$\tau_{body} = \sum \tau_{motors}$$

The torque of the motors produce a reaction in the body opposite to the torque that the motors deliver to the wheels and the flywheel.

$$\tau_{body} = -\tau_{right-wheel} - \tau_{left-wheel} - \tau_{flywheel}$$

If we want to control the inclination θ, we must be able to control τ_{body} in a range $\tau_{body} \in (-\epsilon, \epsilon)$. Observe that the angular acceleration of the body is linearly dependent with the torque it receives. In the limit case $\epsilon = 0$. In order to simplify the calculations we will assume $\epsilon = 0$.

$$0 = -\tau_{right-wheel} - \tau_{left-wheel} - \tau_{flywheel} \Rightarrow \tau_{right-wheel} + \tau_{left-wheel} = -\tau_{flywheel} \quad (1)$$

In other words, we must compensate the torque of the wheels with the torque of the flywheel.

3.3 Wheels torque

The wheel torque we can induce is limited by the motor specifications. Note that the maximum torque of the motor is a function of velocity and in particular at max speed the torque is zero.

$$\tau_{motor}(w_{wheel})$$

We assume that the wheels just roll and do no slip. The robot is pushed by the wheels that make a force F_{drag} against the ground in the contact point. See figure 7.

We can express the torque of the wheel as:

$$\tau_{wheel} - F_{drag} * r_{wheel} = I_{wheel} * \dot{w}_{wheel}$$

$$\tau_{wheel} = min(\tau_{motor}(w_{wheel}), I_{wheel} * \dot{w}_{wheel} + F_{drag} * r_{wheel}) \quad (2)$$

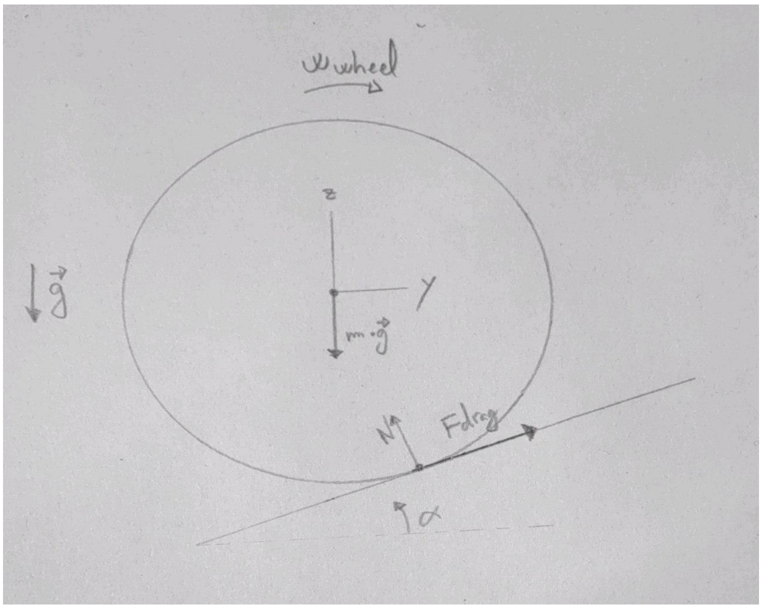


Figure 7: Wheel force diagram

3.4 Flywheel torque

The flywheel torque we can induce is also limited by the motor specifications.

Assuming that a general configuration of the flywheel, see figure 8. we formulate its torque the following way:

$$\tau_{flywheel} = min(\tau_{motor}(w), \ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r - r_{max}) * \sin\theta) \quad (3)$$

3.5 Maximum speed and acceleration

3.5.1 Forward movement

Both wheels turn at the same speed:

$$w_{wheel-left} = w_{wheel-right} = w$$

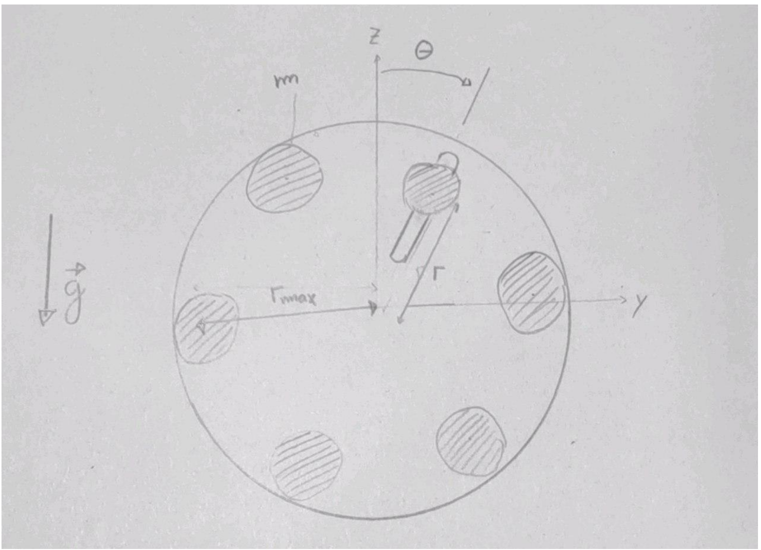


Figure 8: Flywheel force diagram

Applying Newton's first law in the y axis, see figure 9

$$\ddot{y} * m_{total} = 2 * F_{drag} - m_{total} * g * sin(\alpha)$$

Substituting F_{drag} taking in to account equation 2:

$$\ddot{y} * m_{total} = 2 * \frac{\tau_{wheel} - I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} - m_{total} * g * sin(\alpha)$$

Using equation 1:

$$\Rightarrow \ddot{y} * m_{total} = -\frac{\tau_{flywheel}}{r_{wheel}} - \frac{I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} - m_{total} * g * sin(\alpha)$$

Using equation 3:

$$\ddot{y} * m_{total} = -\frac{\ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r - r_{max})}{r_{wheel}} - \frac{I_{wheel} * \dot{w}_{wheel}}{r_{wheel}} - m_{total} * g * sin(\alpha) \tag{4}$$

Taking in two account the following relation:

$$w_{wheel} * r_{wheel} = \dot{y} \Rightarrow \dot{w}_{wheel} * r_{wheel} = \ddot{y}$$

$$\ddot{y} * m_{total} = -\frac{\ddot{\theta} * I_{flywheel}(r) + m_{cylinder} * g * (r - r_{max})}{r_{wheel}} - \frac{I_{wheel} * \ddot{y}}{r_{wheel}^2} - m_{total} * g * sin(\alpha) \tag{5}$$

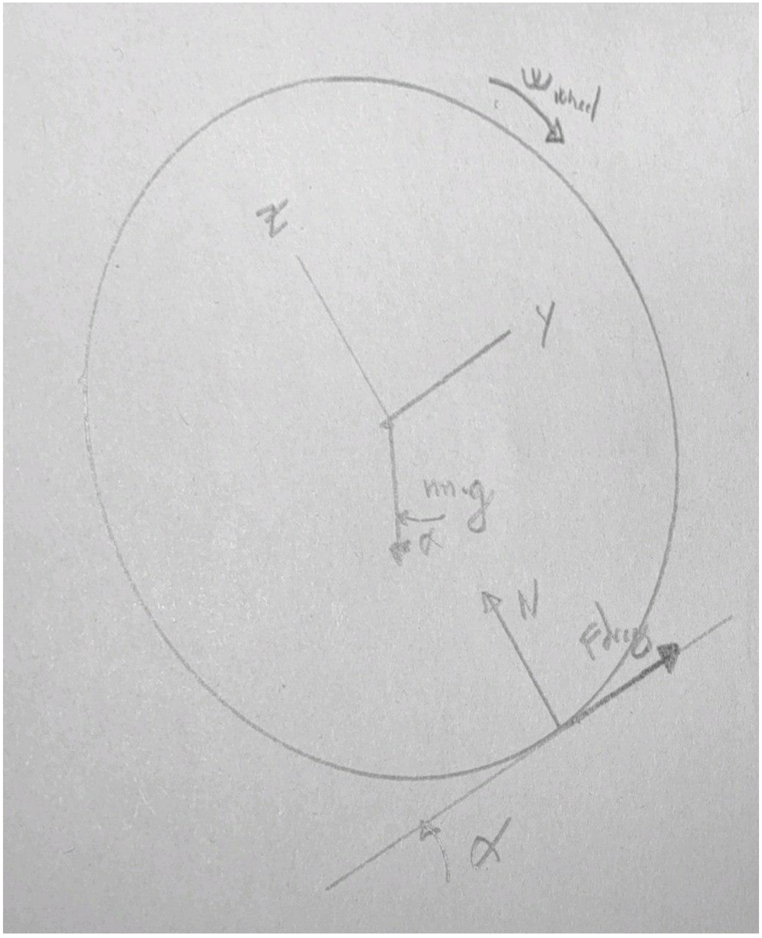


Figure 9: Wheel force diagram

The moment of inertia are:

$$I_{wheel} = \frac{1}{2} * m_{wheel} * r_{wheel}^2$$

$$I_{flywheel} \approx 5 * m_{cylinder} * r_{max}^2 + m_{cylinder} * r^2 = m_{cylinder} * (5 * r_{max}^2 + r^2)$$

[1]

And substituting them on equation 5

$$\ddot{y}*m_{total} = -\frac{m_{cylinder} * (\ddot{\theta} * (5 * r_{max}^2 + r^2) + g * (r - r_{max}))}{r_{wheel}} - \frac{\ddot{y} * m_{wheel}}{2} - m_{total}*g*sin(\alpha) \quad (6)$$

$$\ddot{y}*m_{total} = -\frac{m_{cylinder} * (\ddot{\theta} * (5 * r_{max}^2 + r^2) + g * (r - r_{max}))}{r_{wheel}} - \frac{\ddot{y} * m_{wheel}}{2} - m_{total}*g*sin(\alpha) \quad (7)$$

3.6 Motor specifications

Here we have the factory specifications of our motors:

- Operating voltage: between 3 V and 9 V
- Nominal voltage: 6 V
- Free-run speed at 6 V: 176 RPM
- Free-run current at 6 V: 80 mA
- Stall current at 6V: 900 mA
- Stall torque at 6V: 5 kgcm
- Gear ratio: 1:35
- Reductor size: 21 mm
- Weight: 85 g

3.7 Hypothesis

Assuming that the body is well balanced

[1] We are neglecting the plastic part