习题 5.1

1.C

解析：$\mathrm{F}(x)$仅为I区间内$f(x)$的原函数，非整个区间$f(x)$的原函数，故 C 错误。

2.

（1）$\int f(x)\,dx = C \ \Rightarrow C' = (\int f(x)\,dx)' = 0 = f(x)$

（2）（3）区间I需连续，并非整个定义域内　　例$f(x) = \frac{1}{\mathrm{x}}$

（3）定义 5.1.1：设函数$f(x)$在某区间I上有定义，如果存在可导函数$\mathrm{F}(x)$，使得对I内每一点x，都有$\mathrm{F}'(x) = f(x)$或$d\mathrm{F}(x) = f(x)\,dx$，则称$\mathrm{F}(x)$为$f(x)$在区间I上的一个原函数。

3.

（1）$\int \left(3x^3 - 5x^2 + \frac{3}{x^2}\right) dx = \int (3x^3)\,dx - \int (5x^2)\,dx + \int \left(\frac{3}{x^2}\right) dx = \frac{3}{4}x^4 - \frac{5}{3}x^3 - \frac{3}{x} + C$

（2）$\int \sqrt{x\sqrt{x\sqrt{x}}}\,dx = \int \sqrt{x\sqrt{x \cdot x^{\frac{1}{2}}}}\,dx = \int \sqrt{x\sqrt{x^{\frac{3}{2}}}}\,dx = \int \sqrt{x \cdot x^{\frac{3}{4}}}\,dx = \int x^{\frac{7}{8}}\,dx = \frac{8}{15}x^{\frac{15}{8}} + C$

（3）$\int (2\tan x + 3\cot x)^2\,dx = \int (4\tan^2 x + 12\tan x \cdot \cot x + 9\cot^2 x)^2\,dx$

$= \int \left[4\left(\frac{1-\cos^2 x}{\cos^2 x}\right) + 12 + 9\left(\frac{1-\sin^2 x}{\sin^2 x}\right)\right] dx = \int \left(4\frac{1}{\cos^2 x} + 9\frac{1}{\sin^2 x} - 1\right) dx$

$= 4\tan x - 9\cot x - x + C$

（4）$\int e^{3x}(3^x - e^{-2x})\,dx = \int e^{3x} \cdot e^{x ln3}\,dx - \int e^x\,dx$

$= \int e^{(ln3+3)x}\,dx - \int e^x\,dx = \frac{\mathrm{e}^{(ln3+3)x}}{ln3+3} - e^x + C$

$= \frac{e^x \cdot 3^x}{3+ln3} - e^x + C$

（5）$\int \left(\frac{1}{x} - \frac{3}{\sqrt{1-x^2}}\right) dx = ln|x| - 3\,arc\sin x + C$

（6）$\int \frac{\sqrt{x} - 2\sqrt[3]{x^2} + 1}{4\sqrt{x}}\,dx = \int \frac{\sqrt{x} - 2\sqrt[3]{x^2} + 1}{2}\,d\sqrt{x}$

令$t = \sqrt{x}$

原式$= \int \frac{t}{2}\,dt - \int t^{\frac{4}{3}}\,dt + \frac{1}{2}\int dt$

$= \frac{x}{4} - \frac{3}{7}x^{\frac{7}{6}} + \frac{\sqrt{x}}{2} + C$

（7）$\int \frac{2^{x-1} - 5^{x-1}}{10^x}\,dx = \int \frac{1}{2}\left(\frac{1}{5}\right)^x dx - \int \frac{1}{5}\left(\frac{1}{2}\right)^x dx = \frac{1}{5 \cdot 2^x ln2} - \frac{1}{2 \cdot 5^x ln5} + C$

（8）$\int \frac{(1-x)^2}{x(1+x^2)}\,dx = \int \frac{x^2+1-2x}{x(1+x^2)}\,dx = \int \frac{1}{x}\,dx - \int \frac{2}{x^2+1}\,dx = ln|x| - 2arc\tan x + C$

（9）$\int \frac{x^2}{1+x^2}\,dx = \int \frac{x^2+1-1}{1+x^2}\,dx = \int dx - \int \frac{1}{1+x^2}\,dx = x - arc\tan x + C$

（10）$\int \frac{1+\cos^2 x}{1+\cos 2x}\,dx = \int \frac{2-\sin^2 x}{2\sin^2 x}\,dx = -\frac{1}{2}\int dx + \int \frac{1}{\sin^2 x}\,dx = \cot x - \frac{1}{2}x + C$

4. 解：由题意得 $f'(x) = \frac{2}{\sqrt{1-x^2}}$

$f(x) = \int f'(x)\, dx = 2arc\sin x + C$

因为$f\left(\frac{1}{2}\right) = 0$ 得C=$-\frac{\pi}{3}$

所以$f(x) = 2arc\sin x - \frac{\pi}{3}$

5. 解：由题意得 $x = \int v\, dt = t^3 - t + C\ m$

因为在$x(t)$中，$x(1) = 10m$

所以$C = 10$

所以当$t = 3$时 $x(3) = 34m$

6. 证明：因为$\int f(x)\, dx = \mathrm{F}(x) + C$

所以$\mathrm{F}'(x) = f(x)$ $\mathrm{F}'(ax+b) = af(ax+b)$

对两边积分得 $\mathrm{F}(ax+b) = a\int f(ax+b)\, dx + C$

因为$C \in R$

所以$\frac{1}{a}\mathrm{F}(ax+b) + C = af(ax+b)\, dx$

习题 5.2

1、计算下列不定积分

(1)、$\int \frac{dx}{(3-2x)^2}$

$=-\frac{1}{2}\int \frac{1}{(3-2x)^2} d(3-2x)$

$=\frac{1}{2}\int \left(\frac{1}{3-2x}\right)^1 d(3-2x)$

$=\frac{1}{2}(3-2x)^{-1}+C$

(2)、$\int \tan(5x-3)\,dx$

$=\frac{1}{5}\int \frac{\sin(5x-3)}{\cos(5x-3)} d(5x-3)$

$=-\frac{1}{5}\int \frac{1}{\cos(5x-3)} d[\cos(5x-3)]$

$=-\frac{1}{5}\int \ln|\cos(5x-3)|+C$

(3)、$\int x^3\, e^{-x^4} dx$

$=-\frac{1}{4}\int e^{-x^4}\, d(-x^4)$

$=-\frac{1}{4}e^{-x^4}+C$

(4)、$\int \frac{dx}{x\ln x}$

$=\frac{1}{\ln x} d\ln x$

$=\ln|\ln x|+C$

(5)、$\int \frac{\cos x-\sin x}{\sin x+\cos x} dx$

$=\int \frac{1}{\sin x+\cos x} d(\sin x+\cos x)$

$=\ln|\sin x+\cos x|+C$

(6)、$\int \frac{1}{x^2} a^{\frac{1}{x}} dx$

$=-\int -\frac{1}{x^2}a^{\frac{1}{x}}dx$

$=-\int a^{\frac{1}{x}}d\frac{1}{x}$

$=-\frac{1}{\ln a}\int \ln a\, a^{\frac{1}{x}}d\frac{1}{x}$

$=-\frac{1}{\ln a}a^{\frac{1}{x}}+C$

(7)、$\int \frac{x^3}{\sqrt[3]{x^4+1}}dx$

$=\frac{1}{4}\int \frac{4x^3}{\sqrt[3]{x^4+1}}dx$

$=\frac{1}{4}\int \frac{1}{(x^4+1)^{\frac{1}{3}}}d(x^4+1)$

$=\frac{3}{8}(x^4+1)^{\frac{2}{3}}+C$

(8)、$\int \frac{f\prime(x)}{\sqrt{f(x)}}dx$

$=\int [f(x)]^{\frac{-1}{2}}df(x)$

$=2\sqrt{f(x)}+C$

(9)、$\int \frac{1}{\sqrt{\tan x}\cdot cos^2x}dx$

$=\int \frac{1}{\sqrt{\tan x}}d\tan x$

$=\int (\tan x)^{\frac{-1}{2}}d\tan x$

$=2\int \frac{1}{2}(\tan x)^{\frac{-1}{2}}d\tan x$

$=2\sqrt{\tan x}+C$

(10)、$\int \frac{1}{\sqrt{1-x^2}(arcsinx)^2}dx$

$=\int (arcsinx)^{-2}\, d\ arcsinx$

$=-\frac{1}{arcsinx}+C$

(11)、$\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$

$=2\int \frac{1}{2}\frac{\cos\sqrt{x}}{\sqrt{x}}dx$

$=2\int \cos\sqrt{x}\, d\sqrt{x}$

$=2\sin\sqrt{x}+C$

(12)、$\int \frac{x}{4+x^4}dx$

$=-\frac{1}{4}\int \frac{-4x}{4+x^4}dx$

$=-\frac{1}{4}\int \frac{1}{x^2+2x+2}-\frac{1}{x^2-2x+2}dx$

$=-\frac{1}{4}\int \frac{1}{x^2+2x+2}dx+\frac{1}{4}\int \frac{1}{x^2-2x+2}dx$

$=-\frac{1}{4}\int \frac{1}{(x+1)^2+1}d(x+1)+\frac{1}{4}\int \frac{1}{(x-1)^2+1}d(x-1)$

$=-\frac{1}{4}arc\tan(x+1)+\frac{1}{4}arc\tan(x-1)+C$

(13)、$\int sin^3x\,dx$

$=-\int sin^2x\,d\cos x$

$=-\int(1-cos^2x)\,d\cos x$

$=-\left(\cos x-\frac{1}{3}cos^3x\right)+C$

$=\frac{1}{3}cos^3x-\cos x+C$

(14)、$\int(x^2-3x+1)^{10}(2x-3)dx$

$=\int(x^2-3x+1)^{10}d(x^2-3x+1)$

$=\frac{1}{11}(x^2-3x+1)^{11}+C$

(15)、$\int \frac{1}{x^2}\cot\frac{1}{x}dx$

$=-\int \frac{\cos\frac{1}{x}}{\sin\frac{1}{x}}d\frac{1}{x}$

$=-\int \frac{1}{\sin\frac{1}{x}}d\frac{1}{\sin\frac{1}{x}}$

$=\ln\left|\sin\frac{1}{x}\right|+C$

(16)、$\int sin^5\,xcos^3xdx$

$=\frac{1}{2}\int 2\sin x\cos x\cdot sin^4xcos^2xdx$

$=\frac{1}{2}\int \sin 2x\left(\frac{1-\cos 2x}{2}\right)^2\frac{1+\cos 2x}{2}dx$

$=-\frac{1}{4}\int -2\sin 2x\left(\frac{1-\cos 2x}{2}\right)^2\frac{1+\cos 2x}{2}dx$

$=-\frac{1}{4}\int \frac{1}{8}(1-\cos 2x)^2(1+\cos 2x)d\cos 2x$

$=-\frac{1}{32}\int(1-\cos 2x-cos^2 2x+cos^3 2x)\,d\cos 2x$

$=-\frac{1}{32}cos2x+\frac{1}{64}cos^2 2x+\frac{1}{96}cos^3 2x-\frac{1}{128}cos^4 2x+C$

(17)、$\int\frac{1}{x(x^8+1)}dx$

$=\int(\frac{1}{x}-\frac{x^7}{x^8+1})dx$

$=\int\frac{1}{x}dx-\int\frac{x^7}{x^8+1}dx$

$=\ln|x|-\frac{1}{8}\int\frac{8x^7}{x^8+1}dx$

$=\ln|x|-\frac{1}{8}\int\frac{1}{x^8+1}d(x^8+1)$

$=\ln|x|-\frac{1}{8}\ln(x^8+1)+C$

(18)、$\int\frac{x^2+1}{x^4+1}dx$

$=\frac{1}{2}\int\left(\frac{1}{x^2+\sqrt{2}x+1}+\frac{1}{x^2-\sqrt{2}x+1}\right)dx$

$=\frac{1}{2}\int\left(\frac{1}{(x+\frac{\sqrt{2}}{2})^2+\frac{1}{2}}+\frac{1}{(x-\frac{\sqrt{2}}{2})^2+\frac{1}{2}}\right)dx$

$=\frac{1}{2}\times 2\int\left(\frac{1}{(\sqrt{2}x+1)^2+1}+\frac{1}{(\sqrt{2}x-1)^2+1}\right)dx$

$=\frac{1}{\sqrt{2}}arctan\left(\sqrt{2}x+1\right)+\frac{1}{\sqrt{2}}arctan\left(\sqrt{2}x-1\right)+C$

(19)、$\int\frac{\ln(x+1)-\ln x}{x(x+1)}dx$

$=\int\frac{\ln\frac{x+1}{x}}{x(x+1)}dx$

$=-\int\ln\frac{x+1}{x}\cdot\left[\frac{-1}{x(x+1)}\right]dx$

$=-\int\ln\frac{x+1}{x}\,d\ln\frac{x+1}{x}$

$=-\frac{1}{2}\left(\ln\frac{x+1}{x}\right)^2+C$

2、计算下列定积分

(1)、$\int\frac{1}{x^2(1-x^2)^{\frac{3}{2}}}dx$

令$x=\sin t$,t∈$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$，$\therefore$ 原式$=\int\frac{1}{sin^2t\,cos^3t}\cdot\cos t\,dt$

$=\int \frac{1}{sin^2t\,cos^2t}dt$

$=\int \frac{dt}{sin^2t}+\int \frac{dt}{cos^2t}$

$=-\cot t+\tan t+C$

因为$x=sint$,所以$cost=\sqrt{1-x^2}$. 所以原式$=-\frac{\sqrt{1-x^2}}{x}+\frac{x}{\sqrt{1-x^2}}+C$

(2)、$\int \frac{\sqrt{x^2-1}}{x^3}dx$

令$x=\frac{1}{cost}$,$\cos t=\frac{1}{x}$,$\sin t=\sqrt{1-\frac{1}{x^2}}$

原式$=\int \frac{\tan t}{\frac{1}{cos^3t}}\cdot\frac{\sin t}{\cos t}dt$

$=\int sin^2tdt=\int \frac{1-\cos 2t}{2}dt=\int 1dt-\int \frac{\cos 2t}{2}dt$

$=t-\frac{1}{4}\sin 2t+C$

$=arccos\frac{1}{x}-\frac{2}{x}\sqrt{1-\frac{1}{x^2}}+C$

(3)、$\int \frac{1}{x\sqrt{x^2-1}}dx$

令$x=\frac{1}{\cos t}$,则原式$=\int \frac{1}{\frac{1}{\cos t}\cdot\frac{\sin t}{\cos t}}\cdot\frac{\sin t}{cos^2t}dt$

$=\int 1dt=t+C=arccos\left|\frac{1}{x}\right|+C$

(4)、$\int \frac{1}{(x^2+a^2)^2}dx$

设$x=a\tan t$,原式$=\int \frac{\frac{a}{cos^2t}}{\left(\frac{a^2sin^2t}{cos^2t}+a^2\right)^2}dt$

$=\int \frac{cos^2t}{a^3}dt=\frac{1}{a^3}\int cos^2\,tdt=\frac{1}{a^3}\int \frac{1+\cos 2t}{2}dt$

$=\frac{1}{2a^3}\int 1dt+\frac{1}{2a^3}\int \cos 2tdt$

$=\frac{t}{2a^3}+\frac{\sin 2t}{4a^3}+C$

$=\frac{arctan\frac{x}{a}}{2a^3}+\frac{8xa^4}{x^2+a^2}+C$

(5)、$\int \frac{dx}{\sqrt{3+2x-x^2}}=\int \frac{dx}{\sqrt{4-(x-1)^2}}=\int \frac{1}{2\sqrt{1-\left(\frac{x-1}{2}\right)^2}}dx$

$=arcsin\left(\frac{x-1}{2}\right)+C$

(6)、$\int \frac{x^2}{\sqrt{9-x^2}}dx$

设$x=3sint$, 则原式$=\int\frac{9sin^2t}{\sqrt{9(1-sin^2t)}}\cdot 3costdt$

$$=\int\frac{9sin^2t\cdot 3cost}{3cost}dt=\int 9\,sin^2tdt$$

$$=9\int\frac{1-cos2t}{2}dt=\frac{9}{2}\left(\int 1dt-\int cos2tdt\right)$$

$$=\frac{9}{2}t-\frac{9}{4}sin2t+C=\frac{9}{2}t-\frac{x\sqrt{9-x^2}}{2}+C$$

因为$sint=\frac{x}{3}$，所以$cost=\frac{\sqrt{9-x^2}}{3}$

原式$=\frac{9}{2}arcsin\frac{x}{3}-\frac{x\sqrt{9-x^2}}{2}+C$

(7)、$\int\frac{\sqrt{x^2-4}}{x}dx$

令$x=\frac{2}{cost}$，则原式$=\int\frac{\sqrt{4\left(\frac{1}{cos^2t}-1\right)}}{\frac{2}{cost}}\cdot\frac{2sint}{cos^2t}dt$

$$=\int 2tan^2tdt=\int 2\frac{sin^2t}{cos^2t}dt=2\int\frac{1}{cos^2t}dt-2\int 1dt$$

$$=2tant-2t+C$$

因为$cost=\frac{2}{x}$，所以$sint=\frac{\sqrt{x^2-2}}{x}$，所以原式$=\frac{2\sqrt{x^2-2}}{x}\times\frac{x}{2}-2arccos\left|\frac{2}{x}\right|+C$

$$=\sqrt{x^2-2}-2arccos\left|\frac{2}{x}\right|+C$$

(8)、$\int\frac{x^3}{(1+x^2)^{\frac{3}{2}}}dx$

令$x=tanx$，$t\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$，则原式$=\int\frac{tan^3t}{\frac{1}{cos^3t}}\cdot\frac{1}{cos^2t}dt=\int\frac{sin^3t}{cos^3t}dt$

$$=\int\frac{sint(1-cos^2t)}{cos^3t}dt=\int\frac{sint}{cost}dt-\int sintdt$$

$$=\frac{1}{cost}+cost+C$$

因为$tanx=x$，所以$cost=\frac{1}{\sqrt{x^2+1}}$，所以原式$=\sqrt{x^2+1}+\frac{1}{\sqrt{x^2+1}}+C$

(9)、$\int\frac{1}{(1-x^2)^{\frac{3}{2}}}dx$

令$x=sint$，$t\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$，则原式$=\int\frac{1}{cos^3t}\cdot costdt=\int\frac{1}{cos^2t}dt=tant+C$

因为$sint=x$，所以$cost=\sqrt{1-x^2}$，所以原式$=\frac{x}{\sqrt{1-x^2}}+C$

(10)、$\int\frac{1}{(a^2+x^2)^{\frac{3}{2}}}dx$

设$x=atant$，$t\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$，所以原式$=\int\frac{1}{\left(a^2+a^2\frac{sin^2t}{cos^2t}\right)^{\frac{3}{2}}}\cdot\frac{a}{cos^2t}dt$

$$=\frac{1}{a^2}\int costdt=\frac{1}{a^2}sint+C$$

因为$tant=\frac{x}{a}$,$所以sint=\frac{x}{\sqrt{x^2+a^2}}$,$原式=\frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}+C$

(11)、$\int\frac{1}{x^2\sqrt{x^2+9}}dx$

令$x=3tant$,$t\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$,$则原式=\int\frac{1}{9tan^2t\cdot\frac{3}{cost}}\cdot 3\frac{1}{cos^2t}dt$

$$=\int\frac{1}{9tan^2t\cdot cost}dt$$

$$=\frac{1}{9}\int\frac{cost}{sin^2t}dt=-\frac{1}{9}\frac{1}{sint}+C$$

因为$tant=\frac{x}{3}$,$所以sint=\frac{x}{\sqrt{x^2+9}}$

所以原式=$-\frac{1}{9}\frac{\sqrt{x^2+9}}{x}+C$

习题 5.3

1.

(1) $\int \mathrm{x}\cos\mathrm{x}\,dx = \int \mathrm{x}\,d\sin\mathrm{x} = x\sin\mathrm{x} - \int \sin\mathrm{x}\,\mathrm{d}\,x$

$= x\sin\mathrm{x} + \cos\mathrm{x} + C$

(2) $\int \ln\mathrm{x}\,\mathrm{d}\,x = \int \mathrm{x}\ln\mathrm{x}\,dx$

$= x\ln\mathrm{x} - \int \mathrm{x}\,d\ln\mathrm{x}$

$= x\ln\mathrm{x} - \int \mathrm{x}\cdot\frac{1}{\mathrm{x}}dx$

$= x\ln\mathrm{x} - x + C$

(3) $\int \mathrm{x}^2\mathrm{e}^{\mathrm{x}}\mathrm{d}\,x = \int \mathrm{x}^2(\mathrm{e}^{\mathrm{x}})'\mathrm{d}\,x = \mathrm{x}^2\mathrm{e}^{\mathrm{x}} - \int \mathrm{e}^{\mathrm{x}}\cdot 2xdx$

$= \mathrm{x}^2\mathrm{e}^{\mathrm{x}} - \left(\mathrm{e}^{\mathrm{x}}\cdot 2\mathrm{x} - \int \mathrm{e}^{\mathrm{x}}\cdot 2\mathrm{dx}\right)$

$= \mathrm{e}^{\mathrm{x}}(\mathrm{x}^2 - 2\mathrm{x} + 2) + C$

(4) $\int \arcsin\mathrm{x}\,\mathrm{d}\,x = \int \mathrm{x}'\arcsin\mathrm{x}\,dx = x\arcsin\mathrm{x} - \int \mathrm{x}\cdot\frac{1}{\sqrt{1-\mathrm{x}^2}}dx$

$= x\arcsin\mathrm{x} + \sqrt{1-\mathrm{x}^2} + C$

(5) $\int \frac{\ln(\ln\mathrm{x})}{\mathrm{x}}dx = \int (\ln\mathrm{x})'\ln(\ln\mathrm{x})\,dx$

$= \ln\mathrm{x}\cdot\ln(\ln\mathrm{x}) - \int \ln\mathrm{x}\cdot\frac{\frac{1}{\mathrm{x}}}{\ln\mathrm{x}}dx$

$= \ln\mathrm{x}\cdot\ln(\ln\mathrm{x}) - \ln\mathrm{x} + C$

$$(6)\int e^{2x}\cos x\,dx=\int\frac{1}{2}(e^{2x})'\cos x\,dx$$

$$=\frac{1}{2}\left(e^{2x}\cos x+\int e^{2x}\sin x\,dx\right)$$

$$=\frac{1}{2}\left[e^{x}\cos x+\frac{1}{2}\left(e^{2x}\sin x-\int e^{2x}\cos x\,dx\right)\right]$$

移项可得$\displaystyle\int e^{2x}\cos x\,dx=\frac{1}{5}e^{2x}(\sin x+2\cos 2x)+C$

$$(7)\int x\sin x\cos x\,dx=\frac{1}{2}\int x\sin 2x\,dx=-\frac{1}{4}\int x(\cos 2x)'\,dx$$

$$=-\frac{1}{4}\left(x\cos 2x-\int\cos 2x\,dx\right)$$

$$=-\frac{1}{4}\left(x\cos 2x-\frac{1}{2}\sin 2x\right)+C$$

$$(8)\int x f''(x)\,dx=\int x\big(f'(x)\big)'\,dx$$

$$=xf'(x)-\int f'(x)\,dx$$

$$=xf'(x)-f(x)+C$$

$$(9)\int x\sin^2 x\,dx=\int x\frac{1-\cos 2x}{2}\,dx$$

$$=\frac{1}{2}\int x\left(x-\frac{1}{2}\sin 2x\right)'\,dx$$

$$=\frac{1}{2}\left[x^2-\frac{1}{2}x\sin 2x-\int\left(x-\frac{1}{2}\sin 2x\right)dx\right]$$

$$=\frac{1}{2}\left[x^2-\frac{1}{2}x\sin 2x-\frac{1}{2}x^2+\left(-\frac{1}{4}\cos 2x\right)\right]$$

$$=\frac{1}{4}x^2-\frac{x}{4}\sin 2x-\frac{1}{8}\cos 2x+C$$

$$(10)\int \mathrm{x}(\arctan \mathrm{x})^2 \mathrm{d}\,x$$

$$=\int \frac{1}{2}(\mathrm{x}^2)'(\arctan \mathrm{x})^2 \mathrm{d}\,\mathrm{x}$$

$$=\frac{1}{2}\left[\mathrm{x}^2 \arctan \mathrm{x})^2-\int 2\arctan \mathrm{x}\left(1-\frac{1}{1+\mathrm{x}^2}\right)\mathrm{dx}\right]$$

$$=\frac{1}{2}\mathrm{x}^2(\arctan \mathrm{x})^2-\int \mathrm{x}'\arctan \mathrm{x}\,dx+\frac{1}{2}[[(\arctan \mathrm{x})^2]'\mathrm{dx}$$

$$=\frac{1}{2}\mathrm{x}^2(\arctan \mathrm{x})^2-\left(\mathrm{x}\arctan \mathrm{x}-\int \frac{\mathrm{x}}{1+\mathrm{x}^2}\mathrm{dx}\right)+\frac{1}{2}(\arctan \mathrm{x})^2$$

$$=\frac{1+\mathrm{x}^2}{2}(\arctan \mathrm{x})^2-x\arctan \mathrm{x}+\sqrt{1+\mathrm{x}^2}+C$$

$$(11)\int \ln\left(\mathrm{x}+\sqrt{1+\mathrm{x}^2}\right)\mathrm{d}\,x$$

$$=\int \mathrm{x}'\ln\left(\mathrm{x}+\sqrt{1+\mathrm{x}^2}\right)dx$$

$$=x\ln\left(\mathrm{x}+\sqrt{1+\mathrm{x}^2}\right)-\int \frac{1+\frac{2\mathrm{x}}{2\sqrt{1+\mathrm{x}^2}}}{\mathrm{x}+\sqrt{1+\mathrm{x}^2}}\cdot x dx$$

$$=x\ln\left(\mathrm{x}+\sqrt{1+\mathrm{x}^2}\right)-\int \frac{\mathrm{x}}{\sqrt{1+1^2}}dx$$

$$=x\ln\left(\mathrm{x}+\sqrt{1+\mathrm{x}^2}\right)-\sqrt{1+\mathrm{x}^2}+C$$

$$(12)\int \frac{\mathrm{x}\cos \mathrm{x}}{\sin^3 \mathrm{x}}dx=\int -\mathrm{x}\cdot\frac{1}{2}\left(\frac{1}{\sin^2 \mathrm{x}}\right)'dx$$

$$=-\left(\frac{\mathrm{x}}{\sin^2 \mathrm{x}}-\int \frac{1}{\sin^2 \mathrm{x}}\mathrm{dx}\right)$$

$$=-\frac{\mathrm{x}}{2\sin^2 \mathrm{x}}-\frac{1}{2}\cot \mathrm{x}+C$$

$$(13)\int \sec^5 \mathrm{x}\,\mathrm{d}\,x=\int (\tan \mathrm{x})'\sec^3 \mathrm{x}\,dx$$

$= \tan x \sec^3 x - 3\int \tan x \sec^4 \sin x\, dx$

得①$4\int \sec^5 x\, \mathrm{d}x = \tan x \sec^3 x + 3\int \sec^3 x\, \mathrm{d}x$

由$\int \sec^3 x\, \mathrm{d}x = \tan x \sec x - \int \tan x \frac{\sin x}{\cos^2 x} dx$

得②$2\int \sec^3 x\, \mathrm{d}x = \tan x \sec x + \int \frac{1}{\cos x} dx$

$= \tan x \sec x + \ln|\sec x + \tan x| + C$

②代入①得

$$\int \sec^5 x\, \mathrm{d}x = \frac{1}{4}\tan x \sec^3 x + \frac{3}{8}\tan x \sec x + \frac{3}{8}\ln|\sec x + \tan x| + C$$

(14) $\int \frac{x^2 \arctan x}{1+x^2} dx$

$= \int \left(1 - \frac{1}{1+x^2}\right) \arctan x\, dx$

$= (x - \arctan x)\arctan x - \int (x - \arctan x)\frac{1}{1+x^2} dx$

$= (x - \arctan x)\arctan x - \frac{1}{2}\ln(1+x^2) + \int \frac{\arctan x}{1+x^2} dx$

由$\int \frac{\arctan x}{1+x^2} dx = (\arctan x)^2 - \int \frac{\arctan x}{1+x^2} dx$

得$\int \frac{\arctan x}{1+x^2} = \frac{1}{2}(\arctan x)^2 + C$

$$\int \frac{x^2 \arctan x}{1+x^2} dx = x \arctan x - \frac{1}{2}\ln(1+x^2) - \frac{1}{2}(\arctan x)^2 + C$$

2.对于正整数 n≥2,建立$I_n = \int \sin^n x\, \mathrm{d}x$的递推公式

$I_n = \int \sin^{n-1} x \cdot \sin x\, dx = -\int \sin^{n-1} x\, (\cos x)'\, \mathrm{d}x$

$$= -\sin^{n-1}x\cos x + \int \cos x\,(n-1)\sin^{n-2}x\cos x\,dx$$

$$= -\sin^{n-1}x\cos x + \int (1-\sin^2 x)(n-1)\sin^{n-2}x\,dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\left(\int \sin^{n-2}x\,\mathrm{d}x - \int \sin^{n}x\,\mathrm{d}x\right)$$

整理可得$I_n = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}I_{n-2}$

习题 5.4

(1) $\int \frac{x^3}{1+x}dx$

$= \int (x^2+1-x-\frac{1}{1+x})\,dx$

$= \frac{x^3}{3}-\frac{x^2}{2}+x-ln|x+1|+C$

(2) $\int \frac{x^5+x^4-8}{x^3-x}dx$

$= \int \frac{x^2(x^3-x)+x(x^3-x)+(x^3-x)+x^2+x-8}{x^3-x}dx$

$= \int (x^2+x+1+\frac{1}{x-1}-\frac{8}{x^3-x})\,dx$

$= \frac{x^3}{3}+\frac{x^2}{2}+x+8ln|x|-4\ln|x+1|-3\ln|x-1|+C$

(3) $\int \frac{x^3+1}{x^3-x^2}dx$

$= \int (1-\frac{1}{\mathrm{x}}-\frac{1}{\mathrm{x}^2}+\frac{2}{x-1})\,dx$

$= x+\frac{1}{x}+ln\frac{(\mathrm{x}-1)^2}{|x|}+C$

(4) $\int \frac{x^5}{(x-1)^2(x^2-1)}dx$

$= \int (x+2+\frac{\frac{1}{8}}{1+x}+\frac{\frac{31}{8}}{x-1}+\frac{\frac{9}{4}}{(x-1)^2}+\frac{\frac{1}{2}}{(x-1)^3})\,dx$

$= \frac{x^2}{2}+2x-\frac{1}{4(x-1)^2}-\frac{9}{4(x-1)}+\frac{31}{8}ln|x-1|+\frac{1}{8}\ln|x+1|+C$

(5) $\int \frac{x^4}{1+x^2}dx$

$= \int (x^2-1+\frac{1}{1+x^2})\,dx$

$= \frac{x^3}{3} - x + arctanx + C$

(6) $\int \frac{x^2}{1-x^4} dx$

$= \int (\frac{\frac{1}{4}}{1+x} + \frac{\frac{1}{4}}{1-x} - \frac{\frac{1}{2}}{1+x^2}) dx$

$= \frac{1}{4} ln\left|\frac{1+x}{1-x}\right| - \frac{1}{2} arctanx + C$

(7) $\int \frac{1}{(x+1)^2(x^2+1)} dx$

$= \int \left(\frac{\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{(x+1)^2} - \frac{\frac{1}{2}x}{1+x^2}\right) dx$

$= \frac{1}{2} ln|x+1| - \frac{1}{4}\ln(x^2+1) - \frac{1}{2(x+1)} + C$

(8) $\int \frac{x^3-x^2-x+3}{x^2-1} dx$

$= \int (x - 1 + \frac{1}{x-1} - \frac{1}{x+1}) dx$

$= \frac{x^2}{2} - x + ln\left|\frac{x-1}{x+1}\right| + C$

(9) $\int \frac{2x+2}{(1+x)^2(x-1)} dx$

$= \int (\frac{1}{x-1} - \frac{1}{x+1}) dx$

$= ln|x-1| - \ln|x+1| + C$

（10）

$$\int \frac{x^3+2x^2+1}{(x-1)(x-2)(x-3)^2} dx$$

$=\int \left(\frac{-1}{x-1} + \frac{17}{x-2} + \frac{-15}{x-3} + \frac{23}{(x-3)^2}\right) dx$

$$=-\ln|x-1|+17\ln|x-2|-15\ln|x-3|-\frac{23}{x-3}+c$$

（11）

$$\int\frac{x^3}{(x-1)^{100}}dx$$

$$=\int\frac{(x-1)^3+3(x-1)^2-3(x-1)-1}{(x-1)^{100}}dx$$

$$=\int\left(\frac{1}{(x-1)^{97}}+\frac{3}{(x-1)^{98}}+\frac{3}{(x-1)^{99}}+\frac{1}{(x-1)^{100}}\right)dx$$

$$=-\frac{1}{96(x-1)^{96}}-\frac{3}{97(x-1)^{97}}-\frac{3}{98(x-1)^{98}}-\frac{1}{99(x-1)^{99}}+c$$

（12）

$$\int\frac{1}{x(x^{10}+2)}dx$$

$$=\frac{1}{2}\int\left(\frac{1}{x}-\frac{x^9}{x^{10}+2}\right)dx$$

$$=\frac{1}{2}\ln|x|-\frac{1}{20}\ln(x^{10}+2)+c$$

T2（1）

令$\tan\frac{x}{2}=t$，$x=2\arctan t$， $dx=\frac{2}{1+t^2}dt$

$$\int\frac{1}{2\sin x-\cos x+5}dx$$

$$=\int\frac{1}{\frac{4t}{1+t^2}-\frac{1-t^2}{1+t^2}+5}\cdot\frac{2}{1+t^2}dt$$

$=\int \frac{1}{(3t+1)^2+(\sqrt{5})^2} d(3t+1)$

$=\frac{1}{\sqrt{5}} arctan \frac{3t+1}{\sqrt{5}} + c$

$=\frac{1}{\sqrt{5}} arctan \frac{3\, tan\frac{x}{2}+1}{\sqrt{5}} + c$

（2）

令$tan\frac{x}{2} = t$，$x = 2\,arctan\, t$，$dx = \frac{2}{1+t^2} dt$

$\int \frac{1}{(2+cos\, x)\, sin\, x} dx$

$=\frac{1}{3}\int \frac{1}{t^3+3t} d(t^3 + 3t)$

$=\frac{1}{3}\ln|t^3 + 3t| + c$

$=\frac{1}{3} ln \left|tan\frac{x}{2}\left(3 + tan^2\frac{x}{2}\right)\right| + c$

（3）

令$tan\frac{x}{2} = t$，$x = 2\,arctan\, t$，$dx = \frac{2}{1+t^2} dt$

$\int \frac{1+sin\, x}{sin\, x(1+cos\, x)} dx$

$=\int \left(\frac{1}{2}t + 1 + \frac{1}{2t}\right) dt$

$=\frac{1}{4}t^2 + t + \frac{1}{2} ln|t| + c$

$=\frac{1}{4}\tan^2\frac{x}{2} + \tan\frac{x}{2} + \frac{1}{2}\ln\left|\tan\frac{x}{2}\right| + c$

（4）

令$tan\frac{x}{2}=t$，$x=2\,arctan\,t$， $dx=\frac{2}{1+t^2}dt$

$$\int\frac{1}{sin\,x+tan\,x}dx$$

$=\int\left(-\frac{1}{2}t+\frac{1}{2t}\right)dt$

$=-\frac{1}{4}t^2+\frac{1}{2}ln|t|+c$

$=-\frac{1}{4}tan^2\frac{x}{2}+\frac{1}{2}ln\left|tan\frac{x}{2}\right|+c$

（5）

令$tan\,x=t$，$x=arc\,tan\,t$， $dx=\frac{1}{1+x^2}$

$$\int\frac{1}{(sin\,x+cos\,x)^2}dx$$

$=\int\frac{1}{1+sin\,2x}dx$

$=\int\frac{1}{1+\frac{2t}{1+t^2}}\cdot\frac{1}{1+t^2}dt$

$=\int\frac{1}{(t+1)^2}dt$

$=-\frac{1}{t+1}+c$

$=-\frac{1}{tan\,x+1}+c$

（6）

令$cos\,x=t$ ，$sin^2x=1-t^2$

$$\int\frac{1}{sin\,x\,cos^3x}dx$$

$$=-\int \frac{1}{sin^2 x\, cos^3 x} d(cos\, x)$$

$$=-\int \left(\frac{1}{t}+\frac{1}{t^3}+\frac{t}{1-t^2}\right) dt$$

$$=-ln|cos\, x|+\frac{1}{2\, cos^2 x}+ln|sin\, x|+c$$

（7）$\int \frac{\cos x}{1+\sin x} dx$

$$=\int \frac{1}{1+\sin x} d(\sin x+1)$$

$$=\ln(1+\sin x)+C$$

(8) 令$tan\frac{x}{2}=t$

$$\int \frac{1}{3+5\cos x} dx$$

$$=\frac{1}{4}\int \left(\frac{1}{2-t}+\frac{1}{2+t}\right) dt$$

$$=\frac{1}{4}\ln\left|\frac{2+\tan\frac{x}{2}}{2-\tan\frac{x}{2}}\right|+C$$

(9) 令$tan\frac{x}{2}=t$

$$\int \frac{1}{\sin 2x-2\sin x} dx$$

$$=-\frac{1}{4}\int \left(\frac{1}{t^3}+\frac{1}{t}\right) dt$$

$$=-\frac{1}{4}\left(-\frac{1}{2t^2}+\ln|t|\right)+C$$

$$=\frac{1}{8}cos^2\frac{x}{2}-\frac{1}{4}\ln\left|\tan\frac{x}{2}\right|+C$$

(10)$\int \frac{1}{sin^4x+cos^4x}dx$

$=\int \frac{sec^4x}{(tan^4x)+1}d\tan x$ 令$tan\, x = t$

$=\int \frac{1+t^2}{1+t^4}dt$

$=\frac{\sqrt{2}}{2}\arctan\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)+C$

$=\frac{\sqrt{2}}{2}\arctan\left(\frac{tan^2x-1}{\sqrt{2}\tan x}\right)+C$

3 (1)令 t=$\sqrt{\frac{x}{1-x}}, x = \frac{t^2}{1+t^2}$

$\int \frac{1}{x}\sqrt{\frac{x}{1-x}}dx$

$=\int \frac{2}{1+t^2}dt$

$=2arctant + C$

$=2arctan\sqrt{\frac{x}{1-x}} + C$

(2)$\int \frac{\sqrt{x}}{\sqrt[3]{x^2}-\sqrt[4]{x}}dx$, 令$t = \sqrt[12]{x}, x = t^{12}$

$=\int \frac{t^6}{t^8-t^3}dt^{12}$

$=12\int (\frac{t^4}{t^5-1} + t^4 + t^9)$dt

$=\frac{6}{5}x^{\frac{5}{6}} + \frac{12}{5}x^{\frac{5}{12}} + \frac{12}{5}\ln\left|x^{\frac{5}{12}} - 1\right| + C$

(3)$\int \frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}dx$

$=\int \frac{\left(1+\sqrt{1-x^2}\right)\left(1+\sqrt{1-x^2}\right)}{\left(1-\sqrt{1-x^2}\right)\left(1+\sqrt{1-x^2}\right)}dx$

$$= \int \frac{2-x^2+2\sqrt{1-x^2}}{x^2}dx$$

$=-\frac{2}{x}-x-2\int\sqrt{1-x^2}d\left(\frac{1}{x}\right)$

$=-\frac{2}{x}-x-\frac{2}{x}\sqrt{1-x^2}-2\int\frac{dx}{\sqrt{1-x^2}}$

$=-\frac{2+x^2}{x}-\frac{2}{x}\sqrt{1-x^2}-2arcsinx+C$

(4)$\int\sqrt{\frac{e^x-1}{e^x+1}}dx$，令$t=\sqrt{\frac{e^x-1}{e^x+1}}, dx=\frac{4t}{1-t^4}dt$

$=\int t\frac{4t}{1-t^4}dt$

$=2\int\left(\frac{1}{1-t^2}+\frac{1}{1+t^2}\right)dt$

$=\ln\left|\frac{1-t}{1+t}\right|+arctant+C$

$=\ln\left|\frac{1-\sqrt{\frac{e^x-1}{e^x+1}}}{1+\sqrt{\frac{e^x-1}{e^x+1}}}\right|+arctan\sqrt{\frac{e^x-1}{e^x+1}}+C$

(5)$\int\frac{1}{1+\sqrt[3]{x+1}}dx$，令$t=\sqrt[3]{x+1}, x=t^3-1$

$=3\int\left(t-1+\frac{1}{1+t}\right)dt$

$=\frac{3}{2}t^2-3t+3\ln|1+t|+C$

$=\frac{3}{2}\sqrt[3]{(x+1)^2}-3\sqrt[3]{x+1}+3\ln\left|1+\sqrt[3]{x+1}\right|+C$

(6)$\int\frac{x}{\sqrt{5+x-x^2}}dx$

$=\frac{2}{\sqrt{21}}\int\frac{x}{\sqrt{1-\left[\frac{2}{\sqrt{21}}\left(x-\frac{1}{2}\right)\right]^2}}dx$

$= \int x d\arcsin\frac{2x-1}{\sqrt{21}}$, 令$t = \arcsin\frac{2x-1}{\sqrt{21}}$

$= \left(\int (\frac{1}{2} + \frac{\sqrt{21}}{2}\sin t)\right)$dt

$= \frac{1}{2}t - \frac{\sqrt{21}}{2}\cos t + C$

$= -\sqrt{5 + x - x^2} + \frac{1}{2}\arcsin\frac{2x-1}{\sqrt{21}} + C$

第 5 章复习题

1. (1) $\int(\cos\frac{x}{2}-\sin\frac{x}{2})^2dx=$______________

原式$=\int(1-sinx)dx$

$=\int 1dx-\int sinxdx$

$=$ $x+\cos x+C$

(2) 若 a≠0，则$\int\frac{1}{x^2\sqrt{x^2+a^2}}dx=$______________

令 $x=a\tan t$

原式$=\int\frac{1}{a^3(tant)^2sect}\cdot a(sect)^2dt$

$=\frac{1}{a^2}\int\frac{sect}{(tant)^2}dt$

$=\frac{1}{a^2}\int\frac{1}{(sint)^2}dsint$

$=-\frac{x}{a^2\sqrt{x^2+a^2}}+C$

(3) $\int\frac{1+cosx}{x+sinx}dx=$______________

原式$=\int\frac{1}{x+sinx}d(x+sinx)$

$=\ln|x+\sin x|+C$

(4) $\int\frac{\sqrt{lnx}}{x}dx=$______________

原式$=\int\sqrt{lnx}\,dlnx$

$=\frac{2}{3}(lnx)^{\frac{3}{2}}+C$

2. (1) $\int\frac{arctanx}{x^2(1+x^2)}dx$

$=\int\left[arctanx(\frac{1}{x^2}-\frac{1}{x^2+1})\right]dx$

$=\int\frac{arctanx}{x^2}dx-\int\frac{arctanx}{x^2+1}dx$

$=-\int \arctan x \, d\frac{1}{x} - \int \arctan x \, d(\arctan x)$

$=-\frac{\arctan x}{x} + \int \frac{1}{x(x^2+1)} dx - \frac{(\arctan x)^2}{2}$

$=-\frac{\arctan x}{x} + \frac{1}{2}\int \frac{1}{x^2(x^2+1)} dx^2 - \frac{(\arctan x)^2}{2}$

$=-\frac{\arctan x}{x} - \frac{(\arctan x)^2}{2} + \frac{1}{2}\int \frac{1}{x^2} dx^2 - \frac{1}{2}\int \frac{1}{x^2+1} d(x^2+1)$

$=\frac{1}{2} ln \frac{x^2}{x^2+1} - \frac{\arctan x}{x} - \frac{(\arctan x)^2}{2} + C$

(2) $\int \frac{1}{(1-x)\sqrt{1-x^2}} dx$

令 $x=\sin t$，$t \in (-\frac{\pi}{2}, \frac{\pi}{2})$

原式$=\int \frac{1}{1-sint} dt$

$=\int \frac{1+sint}{(1-sint)(1+sint)} dt$

$=\int \frac{1+sint}{(cost)^2} dt$

$=\int \frac{1}{(\cos t)^2} dt + \int \frac{sint}{(cost)^2} dt$

$=\tan t + \frac{1}{\cos t} + C$

$=\frac{x+1}{\sqrt{1-x^2}} + C$

(3) $\int \frac{e^{arctanx}}{(1+x^2)^{\frac{3}{2}}} dx$

令$x = \tan t$，$t = \arctan x$

原式$=\int e^t \cos t dt$

$=e^t \cos t + \int e^t \sin t dt$

$=e^t \cos t + e^t \sin t - \int e^t \cos t dt$

原式$=\frac{e^t \cos t + e^t \sin t}{2} + C$

$=\frac{1+x}{2\sqrt{1+x^2}} e^{arctanx} + C$

(4) $\int \frac{x^2-1}{x\sqrt{x^4+3x^2+1}} dx$

$= \int \frac{1-\frac{1}{x^2}}{\sqrt{x^2+\frac{1}{x^2}+3}} dx$

$= \int \frac{1}{\sqrt{\left(x+\frac{1}{x}\right)^2+1}} d(x+\frac{1}{x})$

令$x+\frac{1}{x} = tant$, $\sec t = \sqrt{\left(x+\frac{1}{x}\right)^2+1}$

原式$= \int \frac{1}{\sqrt{\left(x+\frac{1}{x}\right)^2+1}} d(x+\frac{1}{x})$

$= \int \sec t \, dt$

$= \ln|\sec t + \tan t| + C$

$= \ln\left|\sqrt{\left(x+\frac{1}{x}\right)^2+1} + x + \frac{1}{x}\right| + C$

(5) $\int \frac{1}{(sinx)^2+3} dx$

$= \int \frac{(secx)^2}{(tanx)^2+3(secx)^2} dx$

$= \int \frac{1}{(tanx)^2+3(secx)^2} dtanx$

$= \frac{1}{x} \int \frac{1}{4(tanx)^2+3} d(2tanx)$

$= \frac{\sqrt{3}}{6} arctan(\frac{2tanx}{\sqrt{3}}) + C$

(6) $\int \frac{xe^x}{\sqrt{e^x-1}} dx$

令$\sqrt{e^x-1} = t$, $x = ln(t^2+1)$

原式$= 2\int ln(t^2+1)dt$

$= 2\text{t}ln(t^2+1) - 4\int \frac{t^2}{t^2+1} dt$

$= 2\text{t}ln(t^2+1) - 4\int 1\, dt + 4\int \frac{1}{t^2+1} dt$

$= 2\text{t}ln(t^2+1) - 4t + 4arctant + C$

$=2x\sqrt{e^x-1}-4\sqrt{e^x-1}+4arctan\sqrt{e^x-1}+C$

(7) 原式$=\int\frac{\sin^4 x}{\cos^4 x}dx$

$=\int\frac{\cos^4 x-2\cos^2 x+1}{\cos^4 x}dx$

$=\int 1dx\ -\int\frac{2}{\cos^2 x}dx\ +\int\frac{1}{\cos x^4}dx$

$=x-2tanx+\int(tan2x+1)dtanx$

$=x-2tanx+\frac{1}{3}tan3x+tanx+C$

$=x-tanx+\frac{1}{3}tan3x+C$

(8) 原式$=\int\frac{arcsinx}{x^2\sqrt{1-x^2}}dx+\int\frac{arcsinx}{\sqrt{1-x^2}}dx$

令 x=sint, t=arcsinx, dx=costdt($-\frac{\pi}{2}<t<\frac{\pi}{2}$)

则$\int\frac{arcsinx}{x^2\sqrt{1-x^2}}dx$

$=\int\frac{t}{\sin^2 t}dt$

$=-\int tdcott$

$=-tcott+\int cott\,dt$

$=-tcott+\int\frac{cosx}{sinx}dt$

$=-tcott+\int\frac{1}{sinx}dsint$

$=-tcott+\ln|sint|$

$=-arcsinx\cdot cot(arcsinx)+\ln|x|+C$

$\int\frac{arcsinx}{\sqrt{1-x^2}}dx=\int t dt=\frac{1}{2}t^2=\frac{1}{2}(arcsinx)^2$

综上，原式$=-arcsinx\cdot cot(arcsinx)+\ln|x|+\frac{1}{2}(arcsinx)^2+C$

（9）原式 $=\int\frac{xe^x+e^x-e^x}{(1+x)^2}dx$

$=\int\frac{(x+1)e^x}{(1+x)^2}dx-\int\frac{e^x}{(1+x)^2}dx$

$=\int \frac{e^x}{x+1}dx+\int e^x d\frac{1}{x+1}$

$=\int \frac{e^x}{x+1}dx+\frac{e^x}{x+1}-\int \frac{1}{x+1}de^x$

$=\frac{e^x}{x+1}+C$

(10) 原式$=\int \frac{x^4-x^2+1}{x^6+1}dx+\int \frac{x^2}{x^6+1}dx$

$=\int \frac{(x^2-1)^2}{x^6+1}dx+\frac{1}{3}\int \frac{1}{x^6+1}dx^3$

$=\arctan x+\frac{1}{3}\arctan x^3+C$

(11) 原式$=\int \frac{e^x}{\cos x+1}dx+\int \frac{\sin x}{\cos x+1}dx$

$=\int \frac{e^x}{2\cos^2\frac{x}{2}}dx+\int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\ e^x dx$

$=\int e^x d(\tan\frac{x}{2})dx+\ \int e^x\tan\frac{x}{2}dx$

$=e^x\tan\frac{x}{2}-\int \tan\frac{x}{2}e^x dx+\ \int e^x\tan\frac{x}{2}dx$

$=e^x\tan\frac{x}{2}+C$

(12) 原式$=\int e^{x\ln x}(\ln x+1)dx$

$=\int e^{x\ln x}d(x\ln x)$

$=e^{x\ln x}+C+$

$=x+C$

(13) 原式$=\int \frac{(x+a)\ln(x+a)+(x+b)\ln(x+b)}{(x+a)(x+b)}dx$

$=\int \frac{\ln(x+a)}{x+b}dx+\int \frac{\ln(x+b)}{x+a}dx$

$=\int \frac{\ln(x+a)}{x+b}dx+\ln(x+b)d\ln(x+a)$

$=\int \frac{\ln(x+a)}{x+b}dx+\ln(x+b)\cdot\ln(x+a)-\int \frac{\ln(x+a)}{x+b}dx$

$=\ln(x+a)\cdot\ln(x+b)+C$

3.(1)由题得：f(x)=$(\frac{\cos x}{x})'$ $=\frac{-x\sin x-\cos x}{x^2}$

$\int xf'(x)$dx =$\int x$df(x)

=xf(x) $-\int f(x)$dx

=$\frac{-x\sin x-\cos x}{x}-\frac{\cos x}{x}$+C

=-$\frac{-x\sin x+2\cos x}{x}$+C

(2)因$\int xf'(x)$dx=arcsinx+C,

则 f(x)=$\frac{1}{x\sqrt{1-x^2}}$

则$\int\frac{1}{f(x)}$dx=$\int x\sqrt{1-x^2}$dx

=$-\frac{1}{2}\sqrt{1-x^2}$d$(1-x^2)$

= $-\frac{1}{3}\sqrt{(1-x^2)^3}$

(3)设f^{-1}(x)=x，则 x=f(y)

$\int xf^{-1}(x)$dx=$\int y$df(y)

=yf(y)-F(y)+C

=xf^{-1}(x)-F(f^{-1}(x))+C

4.令 u=$\sin^2 x$,|u|≤1

所以原式即为 f ' (u)=1-u

两边取积分得 f(u)=u-$\frac{1}{2}u^2$+C

所以 f(x)=x-$\frac{1}{2}x^2$+C

5.令 t=x^2-1,则原式即为 f(t)=ln$\frac{t+1}{t-1}$

所以 $f[\varphi(x)]=\ln\frac{\varphi(x)+1}{\varphi(x)-1}=\ln x$

$\varphi(x)=\frac{x+1}{x-1}$

$\int\varphi(x)dx=\int\frac{x+1}{x-1}dx=\int 1+\frac{2}{x-1}dx=x+\ln(x-1)^2+C$

6. （1）$\int\min\{|x|,x^2\}dx=\begin{cases}\int xdx\\ \int x^2 dx\\ \int xdx\end{cases}=\begin{cases}-\frac{1}{2}x^2+C_1, x<-1\\ \frac{1}{3}x^3+C_2, |x|\le 1\\ \frac{1}{2}x^2+C_3, x>1\end{cases}$

∵$\min\{|x|,x^2\}$在定义域上连续，∴$\int\min\{|x|,x^2\}dx$ 在定义域上也连续

∴ $\lim\limits_{x\to -1^-}(-\frac{1}{2}x^2+C_1)=\lim\limits_{x\to -1^+}(\frac{1}{3}x^3+C_2)$ 得 $C_1=C_2+\frac{1}{6}$

同理 $C_3=C_2+\frac{1}{6}$，令 $C_2=C$

∴$\int\min\{|x|,x^2\}dx=\le$

(2). $\int\max\{1,x^2,x^3\}dx=\begin{cases}\int x^2dx\\ \int 1dx\\ \int x^3dx\end{cases}=\begin{cases}\frac{1}{3}x^3+C_1, x<-1\\ x+C_2, |x|\le 1\\ \frac{1}{4}x^4+C_3, x>1\end{cases}$

同（1）$C_1=C_2-\frac{2}{3}$，$C_3=C_2+\frac{3}{4}$，令 $C_2=C$

∴$\int\max\{1,x^2,x^3\}dx=\begin{cases}\frac{1}{3}x^3-\frac{2}{3}+C, x<-1\\ x+C\ , |x|<1\\ \frac{1}{4}x^4+\frac{3}{4}+C\ , x>1\end{cases}$

7.（1）由题 $y'=2x-1$

两边取积分得 $y=x^2-x+C$

又 x=1 时，y=0

∴带入得 C=0

∴y= x^2-x

（2）由题 y ' =$\frac{1}{x}$

两边取积分得 y=ln|x|+C;

又 x=e^2时，y=4

∴带入得 C=2

∴y=ln|x|+2

8.由题取 x=0, ξ=1;

则有 f(0+1)=f(1)=f(0)f(1)

∵f(1) ≠0, ∴f(0)=1;

f ' (x)= $\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

又∵$f(x+\Delta x)$=f(x)f(Δx)

∴f ' (x)= $\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x)[f(\Delta x)-1]}{\Delta x}$

$=f(x)\lim\limits_{\Delta x\to 0}\frac{f(0+\Delta x)-f(0)}{\Delta x}$

= $f(x)$ f ' (0)

综上 f ' (x)= $f(x)$ f ' (0)

令 y=f(x),则有$\frac{dy}{dx}=y$f ' (0)

当 y=0 时显然成立;

当 y≠0 时有$\frac{dy}{y}=\mathrm{f}'(0)\mathrm{dx}$

两边取积分得 ln|y|=f(0)

$\therefore f(x)=y=Ce^{f'(0)x}$