5.1 答案详解

1. C

解析：$F(x)$ 仅为 I 区间内 $f(x)$ 的原函数，非整个区间 $f(x)$ 的原函数，故 C 错误.

2.

(1) $\int f(x)\,dx = C \Rightarrow C' = \left(\int f(x)\,dx\right)' = 0 = f(x)$

(2)(3) 区间 I 需连续，并非整个定义域内　例 $f(x)=\frac{1}{x}$.

(4) 定义 5.1.1：设函数 $f(x)$ 在某个区间 I 上有定义，如果存在可导函数 $F(x)$，使得对任意一点 x，都有 $F'(x)=f(x)$ 或 $dF(x)=f(x)\,dx$，则称 $F(x)$ 为 $F(x)$ 在区间 I 上的一个原函数

3. (1) $\int (3x^3-5x^2+\frac{3}{x^2})dx = \int (3x^3)dx - \int (5x^2)dx + \int (\frac{3}{x^2})dx = \frac{3}{4}x^4 - \frac{5}{3}x^3 - \frac{3}{x} + C$

(2) $\int \sqrt{x\sqrt{x\sqrt{x}}}\,dx = \int \sqrt{x\sqrt{x\cdot x^{\frac{1}{2}}}}\,dx = \int \sqrt{x\sqrt{x^{\frac{3}{2}}}}\,dx = \int \sqrt{x\cdot x^{\frac{3}{4}}}\,dx = \int x^{\frac{7}{8}}dx = \frac{8}{15}x^{\frac{15}{8}} + C$

(3) $\int (2\tan x+3\cot x)^2 dx = \int (4\tan^2 x + 12\tan x\cdot\cot x + 9\cot^2 x)^2 dx$

$= \int \left[4\left(\frac{1-\cos^2 x}{\cos^2 x}\right) + 12 + 9\left(\frac{1-\sin^2 x}{\sin^2 x}\right)\right] dx = \int \left(4\frac{1}{\cos^2 x} + 9\frac{1}{\sin^2 x} - 1\right) dx = 4\tan x - 9\cot x - x + C$

(4) $\int e^{3x}(3^{x}-e^{-2x})dx = \int e^{3x}\cdot e^{x\ln 3}dx - e^{x} = \int e^{(\ln 3+3)x}dx - e^{x}$

$= \frac{e^{(3+\ln 3)x}}{3+\ln 3} - e^{x} + C = \frac{e^{x}\cdot 3^{x}}{3+\ln 3} - e^{x} + C$

(5) $\int (\frac{1}{x} - \cancel{3\arcsin x}\frac{3}{\sqrt{1-x^2}})dx = \ln|x| - 3\arcsin x + C$

(6) $\int \frac{\sqrt{x} - 2\sqrt[3]{x^2} + 1}{4\sqrt{x}}dx = \int \frac{\sqrt{x} - 2\sqrt[3]{x^2}+1}{2}d\sqrt{x} \overset{t=\sqrt{x}}{=} \int \frac{t}{2}dt - \int t^{\frac{4}{3}}dt + \frac{1}{2}\int dt$

$= \frac{x}{4} - \frac{3}{7}x^{\frac{7}{6}} + \frac{\sqrt{x}}{2} + C$

(7) $\int \frac{2^{x-1} - 5^{x-1}}{10^{x}}dx = \int \frac{1}{2}(\frac{1}{5})^{x}dx - \int \frac{1}{5}(\frac{1}{2})^{x}dx = \frac{1}{5\cdot 2^{x}\ln 2} - \frac{1}{2\cdot 5^{x}\ln 5} + C$

(8) $\int \frac{(1-x)^2}{x(1+x^2)}dx = \int \frac{x^2+1-2x}{x(1+x^2)}dx = \int \frac{1}{x}dx - \int \frac{2}{x^2+1}dx = \ln|x| - 2\arctan x + C$

(9) $\int \frac{x^2}{1+x^2}dx = \int \frac{x^2+1-1}{1+x^2}dx = \int dx - \int \frac{1}{1+x^2}dx = x - \arctan x + C$

(10) $\int \frac{1+\cos^2 x}{1-\cos 2x} dx = \int \frac{2-\sin^2 x}{2\sin^2 x} dx = -\frac{1}{2}\int dx + \int \frac{1}{\sin^2 x} dx = -\cot x - \frac{1}{2}x + C$

4. 解：由题意得 $f'(x) = \frac{2}{\sqrt{1-x^2}}$ $f(x) = \int f'(x) dx = 2\arcsin x + C$

$\because f(\frac{1}{2}) = 0$ 得 $C = -\frac{\pi}{3}$ $\therefore f(x) = 2\arcsin x - \frac{\pi}{3}$

5. 解：由题意得 $x = \int v dt = t^3 - t + C$ m $\because$ 在 $x(t)$ 中，$x(1) = 10m$

$\therefore C = 10m$ $\therefore$ 当 $t=3$ 时 $x(3) = 34m$.

5、证明：$\because \int f(x)dx = F(x) + C$ $\therefore F'(x) = f(x)$ $F'(ax+b) = a f(ax+b)$

对两边积分得 $F(ax+b) = a\int f(ax+b)dx + C$ $\because C \in R$ $\therefore \frac{1}{a}F(ax+b) + c = a\int f(ax+b)dx$