

习题 5.1

1.C

解析: $F(x)$ 仅为 I 区间内 $f(x)$ 的原函数, 非整个区间 $f(x)$ 的原函数, 故 C 错误。

2.

$$(1) \int f(x) dx = C \Rightarrow C' = (\int f(x) dx)' = 0 = f(x)$$

$$(2) \quad (3) \text{ 区间 } I \text{ 需连续, 并非整个定义域内} \quad \text{例 } f(x) = \frac{1}{x}$$

(3) 定义 5.1.1: 设函数 $f(x)$ 在某区间 I 上有定义, 如果存在可导函数 $F(x)$, 使得对 I 内每一点 x , 都有 $F'(x) = f(x)$ 或 $dF(x) = f(x) dx$, 则称 $F(x)$ 为 $f(x)$ 在区间 I 上的一个原函数。

3.

$$(1) \int (3x^3 - 5x^2 + \frac{3}{x^2}) dx = \int (3x^3) dx - \int (5x^2) dx + \int (\frac{3}{x^2}) dx = \frac{3}{4}x^4 - \frac{5}{3}x^3 - \frac{3}{x} + C$$

$$(2) \int \sqrt{x} \sqrt{x} \sqrt{x} dx = \int \sqrt{x} \sqrt{x \cdot x^{\frac{1}{2}}} dx = \int \sqrt{x} \sqrt{x^{\frac{3}{2}}} dx = \int \sqrt{x \cdot x^{\frac{3}{4}}} dx = \int x^{\frac{7}{8}} dx = \frac{8}{15} x^{\frac{15}{8}} + C$$

$$(3) \int (2 \tan x + 3 \cot x)^2 dx = \int (4 \tan^2 x + 12 \tan x \cdot \cot x + 9 \cot^2 x)^2 dx \\ = \int \left[4 \left(\frac{1 - \cos^2 x}{\cos^2 x} \right) + 12 + 9 \left(\frac{1 - \sin^2 x}{\sin^2 x} \right) \right] dx = \int \left(4 \frac{1}{\cos^2 x} + 9 \frac{1}{\sin^2 x} - 1 \right) dx \\ = 4 \tan x - 9 \cot x - x + C$$

$$(4) \int e^{3x} (3^x - e^{-2x}) dx = \int e^{3x} \cdot e^{x \ln 3} dx - \int e^x dx \\ = \int e^{(ln 3 + 3)x} dx - \int e^x dx = \frac{e^{(ln 3 + 3)x}}{ln 3 + 3} - e^x + C \\ = \frac{e^x \cdot 3^x}{3 + ln 3} - e^x + C$$

$$(5) \int \left(\frac{1}{x} - \frac{3}{\sqrt{1-x^2}} \right) dx = \ln|x| - 3 \arcsin x + C$$

$$(6) \int \frac{\sqrt{x-2} \sqrt[3]{x^2+1}}{4\sqrt{x}} dx = \int \frac{\sqrt{x-2} \sqrt[3]{x^2+1}}{2} d\sqrt{x}$$

$$\text{令 } t = \sqrt{x}$$

$$\text{原式} = \int \frac{t}{2} dt - \int t^{\frac{4}{3}} dt + \frac{1}{2} \int dt$$

$$= \frac{x}{4} - \frac{3}{7} x^{\frac{7}{6}} + \frac{\sqrt{x}}{2} + C$$

$$(7) \int \frac{2^{x-1} - 5^{x-1}}{10^x} dx = \int \frac{1}{2} \left(\frac{1}{5} \right)^x dx - \int \frac{1}{5} \left(\frac{1}{2} \right)^x dx = \frac{1}{5 \cdot 2^x \ln 2} - \frac{1}{2 \cdot 5^x \ln 5} + C$$

$$(8) \int \frac{(1-x)^2}{x(1+x^2)} dx = \int \frac{x^2+1-2x}{x(1+x^2)} dx = \int \frac{1}{x} dx - \int \frac{2}{x^2+1} dx = \ln|x| - 2 \arctan x + C$$

$$(9) \int \frac{x^2}{1+x^2} dx = \int \frac{x^2+1-1}{1+x^2} dx = \int dx - \int \frac{1}{1+x^2} dx = x - \arctan x + C$$

$$(10) \int \frac{1+\cos^2 x}{1+\cos 2x} dx = \int \frac{2-\sin^2 x}{2\sin^2 x} dx = -\frac{1}{2} \int dx + \int \frac{1}{\sin^2 x} dx = \cot x - \frac{1}{2} x + C$$

4. 解：由题意得 $f'(x) = \frac{2}{\sqrt{1-x^2}}$

$$f(x) = \int f'(x) dx = 2 \arcsin x + C$$

$$\text{因为 } f\left(\frac{1}{2}\right) = 0 \text{ 得 } C = -\frac{\pi}{3}$$

$$\text{所以 } f(x) = 2 \arcsin x - \frac{\pi}{3}$$

5. 解：由题意得 $x = \int v dt = t^3 - t + C \text{ m}$

$$\text{因为在 } x(t) \text{ 中, } x(1) = 10 \text{ m}$$

$$\text{所以 } C = 10$$

$$\text{所以当 } t = 3 \text{ 时 } x(3) = 34 \text{ m}$$

6. 证明：因为 $\int f(x) dx = F(x) + C$

$$\text{所以 } F'(x) = f(x) \quad F'(ax+b) = af(ax+b)$$

$$\text{对两边积分得 } F(ax+b) = a \int f(ax+b) dx + C$$

$$\text{因为 } C \in \mathbb{R}$$

$$\text{所以 } \frac{1}{a} F(ax+b) + C = \int f(ax+b) dx$$