习题3.3

(1) 求函数 $y=x^2$ 在点 $x=1$ 的微分. 其中自变量的增量 Δx 分别如下:

(1) $\Delta x=0.1$　　(2) $\Delta x=0.01$　　(3) $\Delta x=0.001$

解: 由公式可得 $dy|_{x=1}=f'(1)\Delta x$. 分别代入 Δx.

可得 $\boxed{dy=0.2;\quad dy=0.02;\quad dy=0.002}$

(2) 求下列函数微分.

(1) $y=\frac{x^2-1}{x^2+1}$　　(2) $y=\tan x+\sec x$　　(3) $y=\arccos\frac{1}{x}$　　(4) $y=\arcsin\sqrt{1-x^2}$

(5) $y=\arctan\frac{x^2-1}{x^2+1}$　　(6) $y=(x^2+4x+1)(x^2-\sqrt{x})$

解: 由 $dy=f'(x)dx$ 可知分别求出 $f'(x)$ 即可得出结果.

(1) $y=\frac{x^2-1}{x^2+1}$　$\therefore y=\frac{x^2+1-2}{x^2+1}=1-\frac{2}{x^2+1}$　$\therefore y'=\left(0-\frac{-2\cdot 2x}{(x^2+1)^2}\right)=\frac{4x}{(x^2+1)^2}$

$\therefore \boxed{dy=\frac{4x}{(x^2+1)^2}dx}$

(2) $y=\tan x+\sec x$　$\therefore y'=\frac{\sin x}{\cos x}+\frac{1}{\cos x}$　$\therefore y'=\frac{\sin^2x+\cos^2x}{\cos^2x}+\frac{\sin x}{\cos^2x}=\sec^2x+\sec x\tan x$

$\therefore \boxed{dy=(\sec^2x+\sec x\tan x)dx}$

(3) $y=\arccos\frac{1}{x}$　$\therefore y'=\frac{-1}{\sqrt{1-\frac{1}{x^2}}}\cdot\left(\frac{1}{-x^2}\right)=\frac{1}{|x|\cdot\sqrt{x^2-1}}$

$\therefore \boxed{dy=\frac{1}{|x|\sqrt{x^2-1}}dx \quad 且(|x|>1)}$

(4) $y=\arcsin\sqrt{1-x^2}$　$\therefore y'=\frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}}\cdot\left(\frac{1}{2}\cdot\frac{}{\sqrt{1-x^2}}\right)\cdot(-2x)=\frac{1}{\sqrt{x^2}}\cdot\frac{-x}{\sqrt{1-x^2}}=\frac{-x}{|x|\sqrt{1-x^2}}$

$\therefore \boxed{dy=\frac{-x}{|x|\sqrt{1-x^2}}dx \quad 且(|x|<1)}$

(5) $y=\arctan\frac{x^2-1}{x^2+1}$　$\therefore y'=\frac{1}{1+\left(\frac{x^2-1}{x^2+1}\right)^2}\cdot\left(\frac{x^2-1}{x^2+1}\right)'$　由(1)知 $\left(\frac{x^2-1}{x^2+1}\right)'=\frac{4x}{(x^2+1)^2}$

$=\frac{1}{2x^4+2}\cdot 4x=\frac{2x}{x^4+1}$

$\therefore \boxed{dy=\frac{2x}{x^4+1}dx}$

(6) $y=(x^2+4x+1)(x^2-\sqrt{x})$

$\therefore y'=(2x+4)(x^2-\sqrt{x})+(x^2+4x+1)(2x-\frac{1}{2\sqrt{x}})=2x^3-2x^{\frac{3}{2}}+4x^2-4x^{\frac{1}{2}}+2x^5-\frac{1}{2}x^{\frac{5}{2}}+8x^2-2x^{\frac{1}{2}}+2x$

$-2x^{-\frac{1}{2}}$

$=4x^3+12x^2-\frac{5}{2}x^{\frac{3}{2}}+2x-6x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}$

$\therefore \boxed{dy=(4x^3+12x^2-\frac{5}{2}x^{\frac{3}{2}}+2x-6x^{\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}})dx}$

3. 求复合函数微分

(1) $y=\ln u,\ u=\sqrt{1+x^2}$　(2) $y=\arcsin u,\ u=\frac{1}{x}$　(3) $y=\arctan u,\ u=\sqrt{x}$　(4) $y=e^u,\ u=\sin x$

解: (1) $y=\ln\sqrt{1+x^2}$　$\therefore y'=\frac{1}{\sqrt{1+x^2}}\cdot\frac{1}{2\sqrt{1+x^2}}\cdot 2x=\frac{x}{1+x^2}$

$\therefore \boxed{dy=\frac{x}{1+x^2}dx}$

(2) $y=\arcsin\frac{1}{x}$　$\therefore y'=\frac{1}{\sqrt{1-(\frac{1}{x})^2}}\cdot(-\frac{1}{x^2})=\sqrt{\frac{x^2}{x^2-1}}\cdot(-\frac{1}{x^2})=\frac{|x|}{\sqrt{x^2-1}}\cdot\frac{1}{-|x|^2}$

$\therefore \boxed{dy=\frac{1}{|x|\sqrt{x^2-1}}dx\ (\frac{1}{\sqrt{x^4-x^2}}dx)}$　$=\frac{1}{|x|\cdot\sqrt{x^2-1}}$

(3) $y=\arctan\sqrt{x}$　$\therefore y'=\frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2(1+x)\sqrt{x}}$

$\therefore \boxed{dy=\frac{1}{2(1+x)\sqrt{x}}dx}$

(4) $y=e^{\sin x}$　$\therefore y'=e^{\sin x}\cdot\cos x$　$\therefore \boxed{dy=(e^{\sin x}\cdot\cos x)dx}$

4. 求下列各式近似值

(1) $\sqrt[3]{1.02}$　(2) $\ln 1.005$

解: 由微分定义: $\Delta y=f'(x)\Delta x+o(\Delta x)\ (\Delta x\to 0)$, 即 $f(x+\Delta x)-f(x)\approx f'(x)\Delta x$

$\therefore f(x+\Delta x)\approx f(x)+f'(x)\cdot\Delta x$

(1) 令 $\sqrt[3]{1.02}=\sqrt[3]{1+0.02}$　$\therefore f(x)=x^{\frac{1}{3}}$　$f'(x)=\frac{1}{3}x^{-\frac{2}{3}}$　$\Delta x=0.02$

$\therefore \boxed{f(1.02)\approx f(1)+f'(1)\cdot 0.02=1+\frac{1}{3}\times 0.02=1.006\approx 1.01}$

(2) 令 $\ln 1.005=\ln(1+0.005)$　$\therefore f(x)=\ln x$.　$f'(x)=\frac{1}{x}$　$\Delta x=0.005$

$\therefore \boxed{f(1.005)\approx f(1)+f'(1)\cdot 0.005=0+1\times 0.005=0.005}$