1.

(11) $\int x\cos x\,dx = \int x\,d\sin x = x\sin x - \int \sin x\,dx$
$= x\sin x + \cos x + C$

(12) $\int \ln x\,dx = \int x'\ln x\,dx = x\ln x - \int x\,d\ln x$
$= x\ln x - \int x\cdot\frac{1}{x}\,dx$
$= x\ln x - x + C$

(13) $\int x^2 e^x\,dx = \int x^2 (e^x)'\,dx = x^2 e^x - \int e^x\cdot 2x\,dx$
$= x^2 e^x - (e^x\cdot 2x - \int e^x\cdot 2\,dx)$
$= e^x(x^2 - 2x + 2) + C$

(14) $\int \arcsin x\,dx = \int x'\arcsin x\,dx = x\arcsin x - \int x\cdot\frac{1}{\sqrt{1-x^2}}\,dx$
$= x\arcsin x + \sqrt{1-x^2} + C$

(15) $\int \frac{\ln(\ln x)}{x}\,dx = \int (\ln x)'\ln(\ln x)\,dx = \ln x\cdot\ln(\ln x) - \int \ln x\cdot\frac{\frac{1}{x}}{\ln x}\,dx$
$= \ln x\cdot\ln(\ln x) - \ln x + C$

(16) $\int e^{2x}\cos x\,dx = \int \frac{1}{2}(e^{2x})'\cos x\,dx = \frac{1}{2}(e^{2x}\cos x + \int e^{2x}\sin x\,dx)$
$= \frac{1}{2}[e^{2x}\cos x + \frac{1}{2}(e^{2x}\sin x - \int e^{2x}\cos x\,dx)]$
移项可得 $\int e^{2x}\cos x\,dx = \frac{1}{5}e^{2x}(\sin x + 2\cos 2x) + C$

(17) $\int x\sin x\cos x\,dx = \frac{1}{2}\int x\sin 2x\,dx = -\frac{1}{4}\int x(\cos 2x)'\,dx$
$= -\frac{1}{4}(x\cos 2x - \int \cos 2x\,dx)$
$= -\frac{1}{4}(x\cos 2x - \frac{1}{2}\sin 2x) + C$

(18) $\int x f''(x)\,dx = \int x(f(x))'\,dx = xf(x) - \int f(x)\,dx$
$= xf(x) - f(x) + C$

(19) $\int x\sin^2 x\,dx = \int x\frac{1-\cos 2x}{2}\,dx$
$= \frac{1}{2}\int x(x - \frac{1}{2}\sin 2x)'\,dx$
$= \frac{1}{2}[x^2 - \frac{1}{2}x\sin 2x - \int (x - \frac{1}{2}\sin 2x)\,dx]$
$= \frac{1}{2}[x^2 - \frac{1}{2}x\sin 2x - \frac{1}{2}x^2 + (\frac{1}{4}\cos 2x)] + C$
$= \frac{1}{4}x^2 - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + C$

(10) $\int x(\arctan x)^2\,dx$
$= \int \frac{1}{2}(x^2)'(\arctan x)^2\,dx = \frac{1}{2}[x^2(\arctan x)^2 - \int 2\arctan x(1 - \frac{1}{1+x^2})\,dx]$
$= \frac{1}{2}x^2(\arctan x)^2 - \int x'\arctan x\,dx + \frac{1}{2}\int [(\arctan x)^2]'\,dx$
$= \frac{1}{2}x^2(\arctan x)^2 - (x\arctan x - \int \frac{x}{1+x^2}\,dx) + \frac{1}{2}(\arctan x)^2$
$= \frac{1+x^2}{2}(\arctan x)^2 - x\arctan x + \frac{1}{2}\ln(1+x^2) + C$

(11) $\int \ln(x+\sqrt{1+x^2})\,dx$

$= \int x' \ln(x+\sqrt{1+x^2})\,dx$

$= x\ln(x+\sqrt{1+x^2}) - \int \frac{1+\frac{2x}{2\sqrt{1+x^2}}}{x+\sqrt{1+x^2}} \cdot x\,dx$

$= x\ln(x+\sqrt{1+x^2}) - \int \frac{1}{\sqrt{1+x^2}}\,dx$

$= x\ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + C$

(12) $\int \frac{x\cos x}{\sin^3 x}\,dx$

$= \int -x \cdot \frac{1}{2}\left(\frac{1}{\sin^2 x}\right)'\,dx$

$= -\left(\frac{x}{\sin^2 x} - \int \frac{1}{\sin^2 x}\,dx\right)$

$= -\frac{x}{2\sin^2 x} - \frac{1}{2}\cot x + C$

(13) $\int \sec^5 x\,dx$

$= \int (\tan x)' \sec^3 x\,dx$

$= \tan x \sec^3 x - 3\int \tan x \sec^4 \sin x\,dx$

$\Rightarrow$ ① $4\int \sec^5 x\,dx = \tan x \sec^3 x + 3\int \sec^3 x\,dx$

$\int \sec^3 x\,dx = \tan x \sec x - \int \tan x \frac{\sin x}{\cos^2 x}\,dx$

$\Rightarrow$ ② $2\int \sec^3 x\,dx = \tan x \sec x + \int \frac{1}{\cos x}\,dx = \tan x \sec x + \ln|\sec x + \tan x| + C$

③ 代入①中可得 $\int \sec^5 x\,dx = \frac{1}{4}\tan x \sec^3 x + \frac{3}{8}\tan x \sec x + \frac{3}{8}\ln|\sec x + \tan x| + C$

(14) $\int \frac{x^2 \arctan x}{1+x^2}\,dx$

$= \int \left(1 - \frac{1}{1+x^2}\right)\arctan x\,dx$

$= (x - \arctan x)\arctan x - \int (x - \arctan x)\frac{1}{1+x^2}\,dx$

$= (x - \arctan x)\arctan x - \frac{1}{2}\ln(1+x^2) + \int \frac{\arctan x}{1+x^2}\,dx$

$\int \frac{\arctan x}{1+x^2}\,dx = (\arctan x)^2 - \int \frac{\arctan x}{1+x^2}\,dx$

$\Rightarrow \int \frac{\arctan x}{1+x^2} = \frac{1}{2}(\arctan x)^2 + C$

$\int \frac{x^2 \arctan x}{1+x^2}\,dx = x\arctan x - \frac{1}{2}\ln(1+x^2) - \frac{1}{2}(\arctan x)^2 + C$

2. 对于正整数 $n \geq 2$，建立 $I_n = \int \sin^n x\,dx$ 的递推公式

$I_n = \int \sin^{n-1} x \cdot \sin x\,dx$

$= -\int \sin^{n-1} x (\cos x)'\,dx$

$= -\sin^{n-1} x \cos x + \int \cos x (n-1)\sin^{n-2} x \cos x\,dx$

$= -\sin^{n-1} x \cos x + \int (1-\sin^2 x)(n-1)\sin^{n-2} x\,dx$

$= -\sin^{n-1} x \cos x + (n-1)\left(\int \sin^{n-2} x\,dx - \int \sin^n x\,dx\right)$

整理可得 $I_n = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n} I_{n-2}$