习题 7.1

微分方程的阶：指方程中未知函数的最高阶导数的阶数

n 阶线性微分方程：方程$F(x,y,y',\cdots,y^{(n)})=0$的左端为$y,y',\cdots,y^{(n)}$用一次多项式

(1) $x^2y''-xy'+3y=\cos x$ 是二阶线性方程

(2) $x^2dx=y^3dy$

$x^2=y^3\frac{dy}{dx}$　　$y'y^3=x^2$　为一阶非线性方程

(3) $(1+y^2)y'''+6(y'')^2+3y=0$ 为三阶非线性方程

(4) $y''+\sin(x+y)=\sin x$ 为二阶非线性方程

(5) $y^{(m)}+y''+y=0$　为 m 阶线性方程

(6) $y''+P(x)y'+q(x)y=g(x)$ 为二阶线性方程

T2

(1) $y=x\tan\left(x+\frac{\pi}{6}\right)$　$y'=\tan\left(x+\frac{\pi}{6}\right)+x\frac{1}{cos^2\left(x+\frac{\pi}{6}\right)}$

$xy'=x^2+y^2+y$

$$xtan\left(x+\frac{\pi}{6}\right)+\frac{x^2}{cos^2\left(x+\frac{\pi}{6}\right)}=x^2+x^2tan^2\left(x+\frac{\pi}{6}\right)+xtan\left(x+\frac{\pi}{6}\right)$$

$$\frac{1}{cos^2\left(x+\frac{\pi}{6}\right)}=1+tan^2\left(x+\frac{\pi}{6}\right)$$

$$\frac{1}{cos^2\left(x+\frac{\pi}{6}\right)}-1=tan^2\left(x+\frac{\pi}{6}\right)$$

$$\frac{sin^2\left(x+\frac{\pi}{6}\right)+cos^2\left(x+\frac{\pi}{6}\right)-cos^2\left(x+\frac{\pi}{6}\right)}{cos^2\left(x+\frac{\pi}{6}\right)}=tan^2\left(x+\frac{\pi}{6}\right)$$

$tan^2\left(x+\frac{\pi}{6}\right)=tan^2\left(x+\frac{\pi}{6}\right)$　成立

(2) $y=5x^2+x$

$y'=10x+1$

$xy'=10x^2+x$　$2y+1=10x^2+2x+1$

$xy'\neq 2y+1$　不成立

(3) $y=C_1x+C_2x^2$

$y'=C_1+2C_2x$　$y''=2C_2$

$y''-\frac{2}{x}y'+\frac{2y}{x^2}$

$=2C_2-\frac{2}{x}(C_1+2C_2x)+\frac{2C_1+2C_2x^2}{x^2}$

$= 2C_2 - 4C_2 - \frac{2C_1}{x} + \frac{2C_1}{x} + 2C_2$

$= 0$　成立

(4)　$y = x \quad y' = 1$

$xy' = y\left(1 + ln\frac{y}{x}\right)$

$x = x(1 + ln\,1)$

$x = x$　成立

T3　$y = C_1 \cos x + C_2 \sin x \qquad y' = -\sin x\, C_1 + \cos x\, C_2$

$y'' = -C_1 \cos x - C_2 \sin x$

$y'' + y = -C_1 \cdot \cos x - C_2 \sin x + C_1 \cos x + C_2 \sin x = 0$

$\therefore y = C_1 \cos x + C_2 \sin x$ 是方程 $y'' + y = 0$ 的通解

$y|_{x=0} = 1 \Rightarrow C_1 \cos 0 + C_2 \sin 0 = C_1 = 1$

$y'|_{x=0} = 3 \Rightarrow -\sin 0\, C_1 + \cos 0\, C_2 = C_2 = 3$

$\therefore y = \cos x + 3\sin x$

T4

(1)　$y' = x^2$

(2)　$(X - x) + y'(Y - y) = 0$

线段 PQ 被 y 轴平分$\Rightarrow x_{中点} = 0$

$Q(-x, 0)$

$P(x, y)$的法线斜率为$-\frac{1}{y'}$

$\frac{y}{x+x'} = -\frac{1}{y'}$

$yy' + 2x = 0$

(3)　$\because$线段 MN 被点 P 平分

$\therefore M(2x, 0) \quad N(0, 2y)$

过点$P(x, y)$处的切线斜率为$k = \frac{0-2y}{2x-0} = \frac{-y}{x} = y'$

$-y = xy' \Rightarrow xy' + y = 0$

$$\begin{cases} xy' + y = 0 \\ y|_{x=1} = 2 \end{cases}$$