习题 6.2

1. （1）$F'(x)=\sqrt{1+x^2}\quad F'(0)=1$

（2）$F'(x)=2-\frac{1}{\sqrt{x}}<0\Rightarrow x<\frac{1}{4}$　区间为$\left(0,\frac{1}{4}\right)$

（3）$F'(x)=f(e^{-x})\cdot e^{-x}(-1)-f(x)=-f(e^{-x})\cdot e^{-x}-f(x)$

（4）令$\int_0^y e^{-t^2}dt+\int_0^x sin^2\,tdt=F(x)$

$F'(x)=e^{-y^2}y'+sin^2x=0\Rightarrow y'=-e^{y^2}sin^2x$

（5）因为$[-\pi,\pi]$关于原点对称 又$|sinx|$为偶函数

所以 原式$=2\int_0^{\pi}sinxdx=-2cosx|_0^{\pi}=4$

2. （1）原式$=\lim\limits_{x\to0}\frac{cosx^2}{1}=1$

（2）原式$=\lim\limits_{x\to0}\frac{2\int_0^x e^tdt\cdot e^x}{xe^{2x^2}}=\lim\limits_{x\to0}\frac{2\int_0^x e^tdt}{xe^{2x^2-x}}$

$=\lim\limits_{x\to0}\frac{2e^x}{e^{2x^2-x}+xe^{2x^2-x}\cdot(4x-1)}=\frac{2\times1}{1+0\times1\times(-1)}=2$

3. （1）$\int_0^1\sqrt{x}\left(1-\sqrt{x}\right)^2dx=\int_0^1\sqrt{x}\left(1+x-2\sqrt{x}\right)dx$

$=\int_0^1\left(\sqrt{x}+x^{\frac{3}{2}}-2x\right)dx=\left(\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}-x^2\right)\Big|_0^1$

$=\frac{2}{3}+\frac{2}{5}-1=\frac{1}{15}$

（2）原式$=\int_0^1\frac{-(x^2+1)+2}{1+x^2}dx=\int_0^1\left(-1+\frac{2}{1+x^2}\right)dx$

$=(-x+2arctanx)|_0^1=-1+2\times\frac{\pi}{4}=\frac{\pi}{2}-1$

（3）令　$1-x=t\Rightarrow x=1-t$

$dx=-dt\quad x|_0^1\to t|_1^0$

原式$\int_1^0 e^t(-dt)=\int_0^1 e^tdt=e^x|_0^1=e-1$

（4）原式$=\int_0^1\frac{\frac{1}{2}}{\sqrt{1-\left(\frac{x}{2}\right)^2}}dx=\int_0^1\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}d\left(\frac{1}{2}x\right)$

$=arcsin\frac{x}{2}\Big|_0^1=\frac{\pi}{6}-0=\frac{\pi}{6}$

（5）原式$=\int_{-1}^2\sqrt{2+x}\,d(x+2)=\frac{2}{3}(x+2)^{\frac{3}{2}}\Big|_{-1}^2$

$=\frac{2}{3}\times4^{\frac{3}{2}}-\frac{2}{3}=\frac{2}{3}\times7=\frac{14}{3}$

（6）原式$=\int_0^{\pi}\frac{1-cos2x}{2}dx=\int_0^{\pi}\frac{1}{2}dx-\frac{1}{4}\int_0^{\pi}cos2xd(2x)$

$=\frac{x}{2}\Big|_0^{\pi}-\left(\frac{1}{4}sin2x\right)\Big|_0^{\pi}=\frac{\pi}{2}-0=\frac{\pi}{2}$

（7）原式$=\int_0^{\frac{\pi}{4}}\left(\frac{sinx+cosx}{cosx}\right)^2dx=\int_0^{\frac{\pi}{4}}(tanx+1)^2\,dx$

$=\int_0^{\frac{\pi}{4}}(tan^2x+1+2tanx)\,dx$

$=\int_0^{\frac{\pi}{4}}(sec^2x+2tanx)\,dx$

$=(tanx-2ln|cosx|)\Big|_0^{\frac{\pi}{4}}=1-2ln\frac{\sqrt{2}}{2}$

$=1+\ln\left(\frac{\sqrt{2}}{2}\right)^{-2}=1+\ln 2$

（8）原式$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{2sin^2x}\,dx$

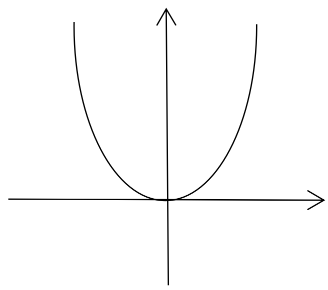
因为$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$关于原点对称，又$\sqrt{2sin^2x}$为偶函数

所以原式$=2\int_0^{\frac{\pi}{2}}\sqrt{2}sinx\,dx=-2\sqrt{2}cosx\Big|_0^{\frac{\pi}{2}}=2\sqrt{2}$

（9）原式$=\int_0^1x^2\,dx+\int_1^2 1dx$

$=\frac{x^2}{3}\Big|_0^1+x\Big|_1^2$

$=\frac{1}{3}+2-1=\frac{4}{3}$



（10）原式$=\int_0^1 0\,dx+\int_1^2 sinxdx+\int_2^3 2sinxdx+\int_3^{\pi}3sinxdx$

$=-cosx\Big|_1^2-2cosx\Big|_2^3-3cosx\Big|_3^{\pi}$

$=-cos2+cos1-2cos3+2cos2+3+3cos3$

$=cos1+cos2+cos3+3$

4. 解：$\begin{cases}dx=f(t)f'(t)dt\\dy=f^2(t)f'(t)dt\end{cases}$

$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=\frac{f^2(t)f'(t)}{f(x)f'(t)}=f(t)$

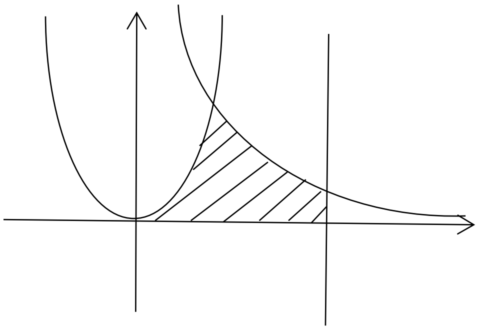
$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dt}\left(\frac{dy}{dx}\right)\times\frac{dt}{dx}=\frac{f'(t)}{f(t)f'(t)}=\frac{1}{f(t)}$

5. 证：$y'=xf(x)$

因为$f(x)$>0 当x>0 时，y'>0，$y\uparrow$

当x<0 时，y'<0，$y\downarrow$

所以当$x=0$时，y取最小值，得证



6. 解:$S=\int_0^1 x^2dx+\int_1^2\frac{1}{x}dx$

$=\frac{x^3}{3}\Big|_0^1+lnx|_1^2$

$=\frac{1}{3}+ln2$

7. （1）证：$\int_{-\pi}^{\pi}cosnxdx=\int_{-\pi}^{\pi}\frac{1}{n}cosnxd(nx)$

$=\frac{1}{n}sin(nx)|_{-\pi}^{\pi}=\frac{1}{n}[sinn\pi-sin(-n\pi)]=0$

所以得证

（2）证：$=\int_{-\pi}^{\pi}sinnxdx=\frac{1}{n}\int_{-\pi}^{\pi}\frac{1}{n}sinnxd(nx)$

$=-\frac{1}{n}cosnx|_{-\pi}^{\pi}=-\frac{1}{n}[cosn\pi-cos(-n\pi)]$

$=-\frac{1}{n}(cosn\pi-cosn\pi)=0$

所以得证

（3）证：$\int_{-\pi}^{\pi}cos^2nxdx=\int_{-\pi}^{\pi}\frac{1+cos(2nx)}{2}dx$

$=\frac{1}{2}\int_{-\pi}^{\pi}1dx+\frac{1}{2}\int_{-\pi}^{\pi}\frac{1}{2n}cos(2nx)d(2nx)$

$=\frac{1}{2}x\Big|_{-\pi}^{\pi}+\frac{1}{4n}sin(2nx)|_{-\pi}^{\pi}$

$=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)+0=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

所以得证

（4）证：$\int_{-\pi}^{\pi}sin^2nxdx=\int_{-\pi}^{\pi}\frac{1-cos(2nx)}{2}dx$

$=\int_{-\pi}^{\pi}\frac{1}{2}dx-\int_{-\pi}^{\pi}\frac{1}{4n}cos(2nx)d(2nx)$

$=\frac{x}{2}\Big|_{-\pi}^{\pi}-\frac{1}{4n}sin(2nx)|_{-\pi}^{\pi}$

$=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)-0=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

所以得证

8. （1）证：$\int_{-\pi}^{\pi} cosmxcosnxdx = \int_{-\pi}^{\pi} \frac{1}{2}[cos(m+n)x + cos(m-n)x]dx$（积化和差公式）

$= \frac{1}{2}\int_{-\pi}^{\pi} cos(m+n)xdx + \frac{1}{2}\int_{-\pi}^{\pi} cos(m-n)xdx$

$= \frac{1}{2(m+n)}\int_{-\pi}^{\pi} cos(m+n)xd(m+n)x + \frac{1}{2(m-n)}\int_{-\pi}^{\pi} cos(m-n)xd(m-n)x$

$= \frac{1}{2(m+n)} sin(m+nx)|_{-\pi}^{\pi} + \frac{1}{2(m-n)} sin(m-n)x|_{-\pi}^{\pi}$

$= 0 + 0 = 0$

所以得证

（2）证：$\int_{-\pi}^{\pi} sinmxsinnxdx = \int_{-\pi}^{\pi} \left[-\frac{1}{2}cos(m+n)x + \frac{1}{2}cos(m-n)x\right]dx$

$= -\frac{1}{2}\int_{-\pi}^{\pi} cos(m+n)xdx + \frac{1}{2}\int_{-\pi}^{\pi} cos(m-n)xdx$

$= -\frac{1}{2(m+n)}\int_{-\pi}^{\pi} cos(m+n)xd(m+n)x + \frac{1}{2(m-n)}\int_{-\pi}^{\pi} cos(m-n)xd(m-n)x$

$= -\frac{1}{2(m+n)} sin(m+nx)|_{-\pi}^{\pi} + \frac{1}{2(m-n)} sin(m-n)x|_{-\pi}^{\pi}$

$= 0 + 0 = 0$

所以得证

（3）证：①$m \neq n$时

$$\int_{-\pi}^{\pi} sinmxcosnxdx = \int_{-\pi}^{\pi} \frac{1}{2}[sin(m+n)x + sin(m-n)x]dx$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} sin(m+n)xdx + \frac{1}{2}\int_{-\pi}^{\pi} sin(m-n)xdx$$

$$= -\frac{1}{2(m+n)} cos(m+n)x\Big|_{-\pi}^{\pi} - \frac{1}{2(m-n)} cos(m-n)x|_{-\pi}^{\pi}$$

$$= -\frac{1}{2(m+n)}[cos(m+n)\pi - cos(-(m+n)\pi)]$$

$$-\frac{1}{2(m-n)}[cos(m-n)\pi - cos(-(m-n)\pi)]$$

$= 0 - 0 = 0$

②$m = n$时

$\int_{-\pi}^{\pi} sinmxcosnx\,dx = \frac{1}{2}\int_{-\pi}^{\pi} sin2mxdx = 0$（第 7.（2）的结论）