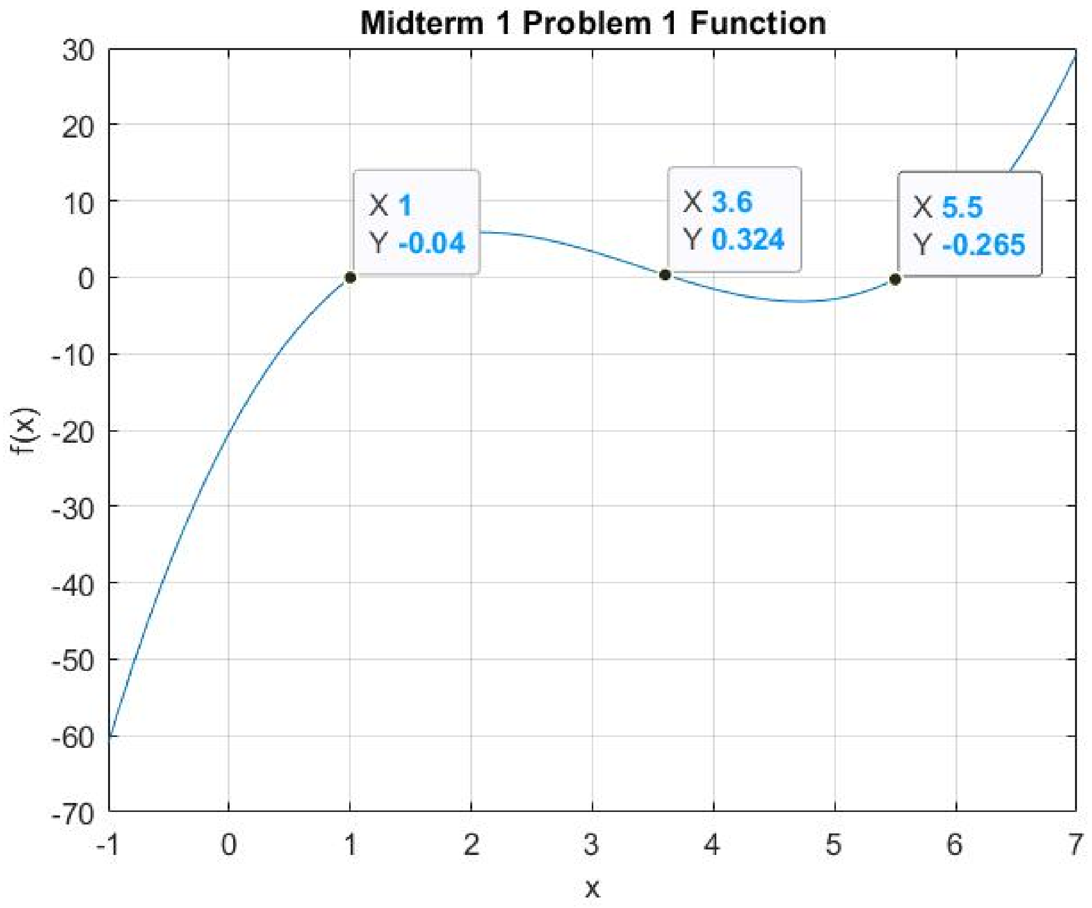
## Problem 1 (50 points):

The roots of this function were found to be 1.003312, 3.664579, and 5.532108. These roots were found using the False Position Method, followed by Newton Raphson's Method. In order to utilize False Position, an interval is required within which the root in question lies. These intervals were found by graphing the function in MATLAB. A MATLAB function for the given function was created and the outputs of this function, given an array of input, were graphed. After graphing the function, I used the MATLAB data cursor to select points that were close to the zeroes. This step is shown in the inserted graph. Also, as extra assurance, I selected x values where the function was increasing and decreasing and calculated the derivative at those points. In addition, I selected points on either side of the root in question and evaluated the function at the points. Because the derivative changes from positive to negative to positive, we know the function may cross the y-axis three times. Evaluating the function at points between the roots allows for us to say with certainty that there is a root, because the sign of the function changes when there is a root.

The False Position Method was utilized to narrow the root intervals. A 10-iteration function was used for this step. The root intervals are shown below in the MATLAB output section, while the function itself is pasted in the MATLAB code section. After finding the new x-value intervals, the function was evaluated at each of these points (so twice for every root). This step is shown below in the MATLAB output. The x-value that yielded a function value closer to 0 for each root interval is used in the next step, which is the Newton Raphson's Method. The code is displayed in the MATLAB code section of this problem. This function was used for each x-value to find the three roots of the function that satisfied the question—that is, f(x) evaluates to less than 10^-7. This verification step is used as the while loop condition for the Newton Raphson Method.

As for the reasoning behind using these methods, I used the False Position simply because I wanted to give myself more experience using this method over the Bisection Method. As for finding the roots, I used Newton Raphson's Method because the given function is simple enough where a derivative MATLAB function can easily be written and used within the code for Newton Raphson's Method.



**Root intervals:** [0.5, 1.5]; [3, 4]; [5, 6]

**MATLAB output:**

```
>> deriv_1 = f_deriv(1)

deriv_1 =

   12.1000

>> deriv_3 = f_deriv(3)

deriv_3 =

   -4.7000

>> deriv_6 = f_deriv(6)

deriv_6 =

   15.1000

>> val1 = displayf(0)

val1 =

    'Value of f(x): -20.3400000'
```

```
>> val2 = displayf(2)

val2 =

    'Value of f(x): 5.8600000'

>> val3 = displayf(5)

val3 =

    'Value of f(x): -2.8400000'

>> val4 = displayf(6)

val4 =

    'Value of f(x): 5.4600000'

>> root1 = FalsePostion(.5, 1.5);
iteration 1: 0.500000, 1.148988
iteration 2: 0.921185, 1.148988
iteration 3: 0.921185, 1.069026
iteration 4: 0.921185, 1.017132
iteration 5: 0.983454, 1.017132
iteration 6: 0.983454, 1.005311
iteration 7: 0.997639, 1.005311
iteration 8: 1.002618, 1.005311
iteration 9: 1.002618, 1.004365
iteration 10: 1.002618, 1.003752
The root of the equation is located between 1.002618 and 1.003752

>> root2 = FalsePostion(3, 4);
iteration 1: 3.000000, 3.685714
iteration 2: 3.470204, 3.685714
iteration 3: 3.617983, 3.685714
iteration 4: 3.664427, 3.685714
iteration 5: 3.664427, 3.679024
iteration 6: 3.664427, 3.674436
iteration 7: 3.664427, 3.671291
iteration 8: 3.664427, 3.669134
iteration 9: 3.664427, 3.667654
iteration 10: 3.664427, 3.666640
The root of the equation is located between 3.664427 and 3.666640

>> root2 = FalsePostion(5, 6);
iteration 1: 5.342169, 6.000000
iteration 2: 5.342169, 5.567258
iteration 3: 5.419187, 5.567258
iteration 4: 5.469852, 5.567258
iteration 5: 5.503181, 5.567258
iteration 6: 5.525106, 5.567258
iteration 7: 5.525106, 5.539529
iteration 8: 5.530042, 5.539529
iteration 9: 5.530042, 5.533288
iteration 10: 5.531152, 5.533288
```

```
The root of the equation is located between 5.531152 and 5.533288

>> displayf(1.003752)

ans =

    'Value of f(x): 0.0052979'

>> displayf(1.002618)

ans =

    'Value of f(x): -0.0083715'

>> displayf(3.666640)

ans =

    'Value of f(x): -0.0102379'

>> displayf(3.664427)

ans =

    'Value of f(x): 0.0007573'

>> displayf(5.533288)

ans =

    'Value of f(x): 0.0099863'

>> displayf(5.531152)

ans =

    'Value of f(x): -0.0080824'

>> Root1 = NewtonRhapson(1.003752)
iteration 1: 1.003312
iteration 2: 1.003312
Root = 1.003312

Root1 =

   1.003312310557692

>> Root2 = NewtonRhapson(3.664427)
iteration 1: 3.664579
Root = 3.664579

Root2 =

   3.664579368860922

>> Root3 = NewtonRhapson(5.531152)
iteration 1: 5.532109
iteration 2: 5.532108
Root = 5.532108

Root3 =

   5.532108316875385
```

**MATLAB code:**

*Function:*

```
function value = f(x)
value = x.^3 - 10.2*x.^2 + 29.5*x - 20.34;
```

*Derivative of function:*

```
function value = f_deriv(x);
value = 3*x.^2 - 20.4*x + 29.5;
```

*Display function value at given x:*

```
function out = displayf(x)
A = f(x);
out = sprintf('Value of f(x): %.7f', A);
```

*False Position Method Code:*

```
function Root1 = FalsePostion(xL, xR);

LeftSol = f(xL);
RightSol = f(xR);

if LeftSol*RightSol < 0
    LeftX = xL;
    RightX = xR;
    count = 0;
    while count < 10
        XApp = LeftX - ((RightX-LeftX)/(RightSol-LeftSol))*LeftSol;
        XApp_Sol = f(XApp);
        if LeftSol*XApp_Sol < 0
            RightX = XApp;
        else
            LeftX = XApp;
        end
        count = count + 1;
        fprintf(['iteration ', num2str(count)])
        fprintf(': %.6f', LeftX)
        fprintf(', %.6f \n', RightX)
    end
else
    disp(['There is no root between ', num2str(xL), ' and ', num2str(xR)])
end

Root1 = [];
fprintf('The root of the equation is located between %.6f', LeftX);
fprintf(' and %.6f \n', RightX);
```

*Newton Raphson's Method Code:*

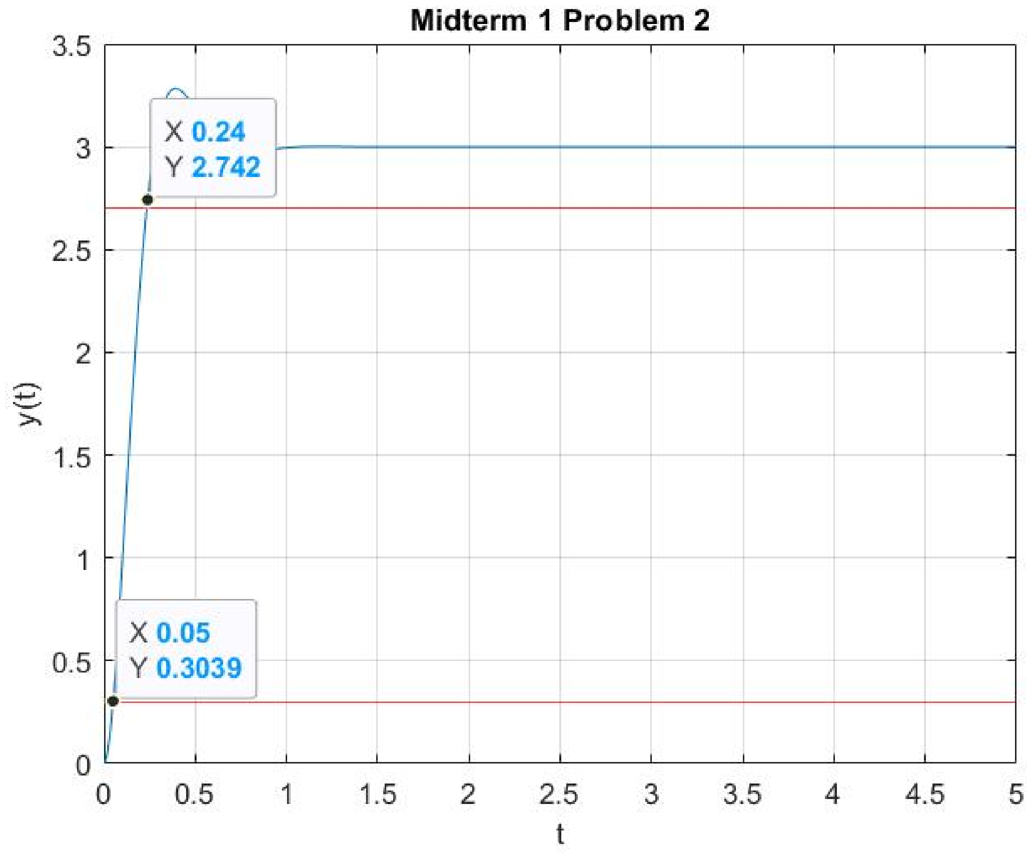
```
function Root = NewtonRhapson(x);

x_n = x;
Value_xn = f(x_n);
```

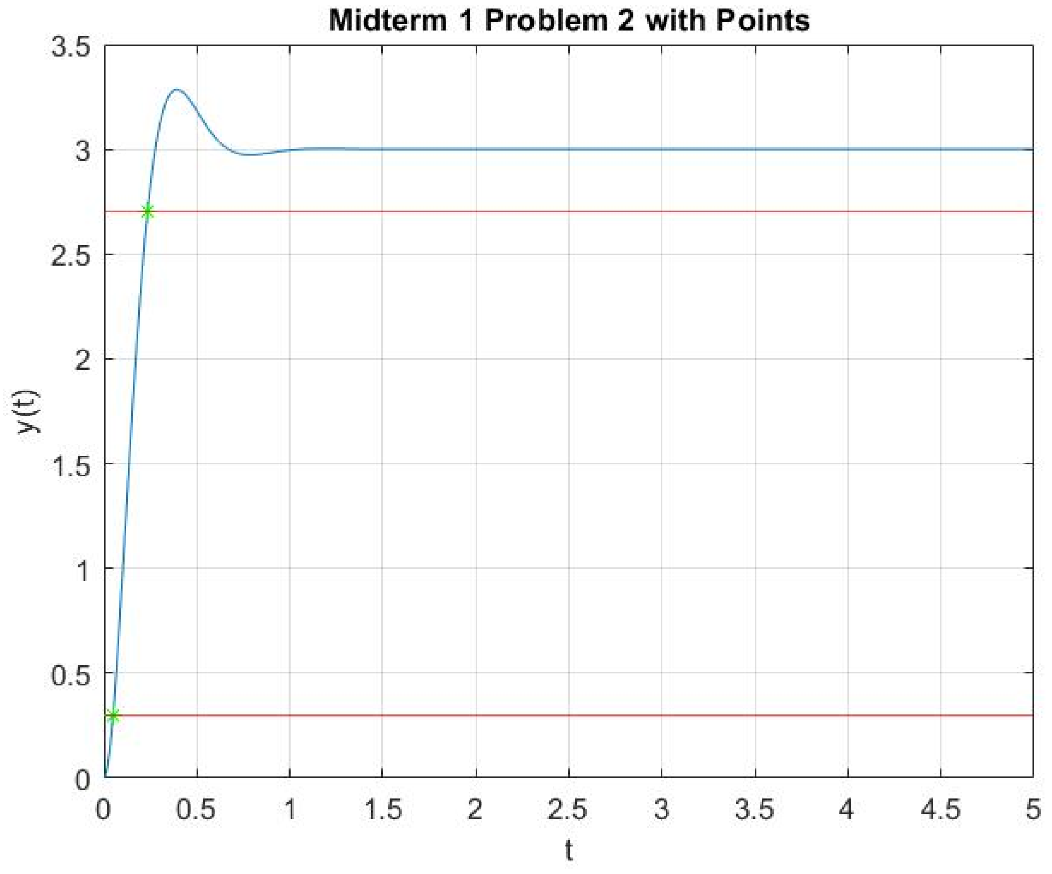
```
count = 0;

while abs(Value_xn) >= 10^(-7)
    Value_xn = f(x_n);
    Deriv_xn = f_deriv(x_n);
    x_np1 = x_n - (Value_xn/Deriv_xn);
    x_n = x_np1;
    count = count + 1;
    fprintf(['iteration ', num2str(count)])
    fprintf(': %.6f \n', x_n)
    Value_xn = f(x_n);
end

Root = x_n;
fprintf('Root = %.6f \n', x_n);
```

## Problem 2 (50 points):

The rise time of this system is estimated to be 0.18541.

For this problem, I started by making a function for the system in MATLAB. Then, this system was graphed using MATLAB. For extra visualization, I also added a y = 0.3 and y = 2.7 line in red. To find the rise time of the system, I used the Bisection Method for root finding. Instead of using Newton Raphson or the Secant method, I allowed the root intervals for the Bisection Method to converge by increasing the number of loops within the Bisection Method function. Using the data cursor tool to estimate the t values that would yield y(t) values close to 0.3 and 2.7, I then used these intervals in the Bisection Method function. Because there was no y = 0 axis for this graph, I did some simple arithmetic to establish arbitrary axes for each root interval. By subtracting 0.3 or 2.7 from the y(t) values for each root interval, I was able to use the root finding method because doing so would "push" the y = 0.3 or y = 2.7 axis to a new y = 0 axis. After using the Bisection Method (see MATLAB output), I took these final t values and evaluated y(t) at these values. This was done using the displayy function (see MATLAB code and output) to print the values at the necessary precision. Because I used enough iterations of Bisect function (see MATLAB code), the t values converged enough where  I could use either t value of the output interval because they were the same due to the accuracy of the sensor.


Midterm 1 Problem 2
X 0.24
Y 2.742
X 0.05
Y 0.3039
y(t)
t
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Midterm 1 Problem 2 with Points
y(t)
t
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

**Root Intervals:** [0.03, 0.07]; [0.23, 0.24]

**<u>MATLAB output:</u>**

```
>> root1 = Bisect(0.03, 0.07)
iteration 1: 0.03000, 0.05000
iteration 2: 0.04000, 0.05000
iteration 3: 0.04500, 0.05000
iteration 4: 0.04750, 0.05000
iteration 5: 0.04875, 0.05000
iteration 6: 0.04938, 0.05000
iteration 7: 0.04938, 0.04969
iteration 8: 0.04953, 0.04969
iteration 9: 0.04961, 0.04969
iteration 10: 0.04961, 0.04965
iteration 11: 0.04963, 0.04965
iteration 12: 0.04964, 0.04965
iteration 13: 0.04964, 0.04964
iteration 14: 0.04964, 0.04964
iteration 15: 0.04964, 0.04964
iteration 16: 0.04964, 0.04964
iteration 17: 0.04964, 0.04964
iteration 18: 0.04964, 0.04964
iteration 19: 0.04964, 0.04964
iteration 20: 0.04964, 0.04964
The root of the equation is located between 0.04964 and 0.04964

>> root2 = Bisect(0.23, 0.24)
iteration 1: 0.23500, 0.24000
iteration 2: 0.23500, 0.23750
iteration 3: 0.23500, 0.23625
iteration 4: 0.23500, 0.23562
iteration 5: 0.23500, 0.23531
iteration 6: 0.23500, 0.23516
iteration 7: 0.23500, 0.23508
iteration 8: 0.23504, 0.23508
iteration 9: 0.23504, 0.23506
iteration 10: 0.23504, 0.23505
iteration 11: 0.23504, 0.23505
iteration 12: 0.23504, 0.23505
iteration 13: 0.23504, 0.23505
iteration 14: 0.23504, 0.23505
iteration 15: 0.23504, 0.23505
iteration 16: 0.23505, 0.23505
iteration 17: 0.23505, 0.23505
iteration 18: 0.23505, 0.23505
iteration 19: 0.23505, 0.23505
iteration 20: 0.23505, 0.23505
The root of the equation is located between 0.23505 and 0.23505

>> out1 = displayy(0.04964)

out1 =

    'Value of y(t): 0.300000'

>> out2 = displayy(0.23505)
```

```
out2 =

    'Value of y(t): 2.700043'

>> 0.23505 - 0.04964

ans =

   0.18541
```

**MATLAB code:**

*Function y(t):*

```
function value = y(t)
if t >= 0
value = 3.*(1-exp(-6.*t).*(cos(8.*t)+(3/4).*sin(8.*t)));
end
```

*Graph function:*

```
x = 0:0.005:5;
val = y(x);
plot(x, val)
grid on
hold on
yline( 2.7, '-r')
yline(0.3, '-r')
hold off
xlabel('t')
ylabel('y(t)')
title('Midterm 1 Problem 2')
```

*Bisection Method:*

```
function [Root1, NumIterations] = Bisect(xL, xR);

LeftSol = y(xL) - 0.3;
RightSol = y(xR) - 0.3;

if LeftSol*RightSol < 0
    LeftX = xL;
    RightX = xR;
    count = 0;
    while count < 20
        MidX = (LeftX + RightX)/2;
        Middle = y(MidX) - 0.3;
        if Middle*(f(LeftX) - 0.3) < 0
            RightX = MidX;
        else
            LeftX = MidX;
        end
        count = count + 1;
        fprintf(['iteration ', num2str(count)])
        fprintf(': %.5f', LeftX)
        fprintf(', %.5f \n', RightX)
    end
else
    disp(['There is no root between ', num2str(xL), ' and ', num2str(xR)])
end
```

```
Root1 = [];
fprintf('The root of the equation is located between %.5f', LeftX);
fprintf(' and %.5f \n', RightX);
NumIterations = count;
```

*Value of y(t):*

```
function out1 = displayy(x)
A = y(x);
out1 = sprintf('Value of y(t): %.6f', A);
```