# ME 318M Homework #8

Name: Jingsi Zhou

UT EID: jz24729

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**Problem 1:**

MATLAB Code:

```
function value = f(x, y)
value = x^2 + y^2 - 16;

function value = g(x,y)
value = exp(x*1/3) + (1/3)*y - 1;


function out = Jacobian(x_guess, y_guess)
partial_circle_y = 2*y_guess;
partial_circle_x = 2*x_guess;
partial_curve_y = 1/3;
partial_curve_x = (1/3)*exp(x_guess*1/3);
out = [partial_circle_x, partial_circle_y; partial_curve_x, partial_curve_y];


function [out, count] = VectNewR(x_guess, y_guess)
x_0 = x_guess;
y_0 = y_guess;
count = 0;
while (sqrt((f(x_0, y_0))^2 + (g(x_0, y_0))^2) >= 10e-7) || (sqrt((x_0 -
x_nm1)^2 + (y_0 - y_nm1)^2) >= 10e-6)
    count = count + 1;
    A = [x_0; y_0];
    J = Jacobian(x_0, y_0);
    F = [f(x_0, y_0); g(x_0, y_0)];
    var_New = A - (inv(J))*F;
    x_nm1 = x_0;
    y_nm1 = y_0;
    x_0 = var_New(1);
    y_0 = var_New(2);
end
out = [x_0; y_0];
count = count;
```

Command Window:

```
>> [out, count]= VectNewR(4,-4)

out =
```

```
    2.2343
   -3.3178


count =

     5
```

I wrote a function to implement the Vectorial Newton-Raphson method. To do this, I wrote separate functions for the two given curves and a function to compute the Jacobian given an input. Then, the next x-value and y-value was found using the formula from class. The intersection of these two curves was found to be at x = 2.2343 and y = -3.3178.

**Problem 2:**

a) Not sure what this question was asking so I just linearized the given equations. However, I could've also written this question as [JacobianMatrix(x_0, y_0)]*[x_1 – x_0; y_1 – y_0] = - [equation1(x_0, y_0); equation2(x_0, y_0)]

Let $x_1 = x^2$
Let $x_2 = y^2$
Let $x_3 = x$
Let $x_4 = y$

s.t. must sqrt $x_1$ and $x_2$ when found to find x and y.

$$\frac{x_1}{34596} - \frac{x_2}{55404} - 1 = 0$$

$$\frac{x_2}{77841} - \frac{1000x_4}{77841} + \frac{250000}{77841} - \frac{x_1}{172159} + \frac{600x_3}{172159} - \frac{90000}{172159} - 1 = 0$$

$$\begin{bmatrix} \frac{1}{34596} & -\frac{1}{55404} & 0 & 0 \\ -\frac{1}{172159} & \frac{1}{77841} & \frac{600}{172159} & -\frac{1000}{77841} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1.688903 \end{bmatrix}$$

b)

$$f(x,y) = \frac{x^2}{186^2} - \frac{y^2}{300^2 - 186^2} = 1$$

$$g(x,y) \quad \frac{(y-500)^2}{279^2} - \frac{(x-300)^2}{500^2 - 279^2} = 1$$

$$\frac{x^2 - 600x + 90000}{172159} \qquad \frac{y^2 - 1000y + 250000}{77841}$$

$$f_x(x,y) = \frac{x}{17298}$$

$$f_y(x,y) = -\frac{y}{27702}$$

$$g_x(x,y) = -\frac{1}{172159}(2x - 600)$$

$$g_y(x,y) = \frac{1}{77841}(2y - 1000)$$

Jacobian =

$$\begin{bmatrix} \frac{x}{17298} & -\frac{y}{27702} \\ -\frac{1}{172159}(2x-600) & \frac{1}{77841}(2y-1000) \end{bmatrix}$$

c)

MATLAB code:

```
function value = f(x, y)
value = (x^2/186^2) + (y^2)/(300^2-186^2) - 1;

function value = g(x,y)
value = (y - 500)^2/(279^2) + (x-300)^2/(500^2-279^2) - 1;

function out = Jacobian(x_guess, y_guess)
partial_f_y = -y_guess/27702;
partial_f_x = x_guess/17298;
partial_g_y = 1/77841*(2*y_guess - 1000);
```

```
partial_g_x = -1/172159*(2*x_guess - 600);
out = [partial_f_x, partial_f_y; partial_g_x, partial_g_y];

function [out, count] = VectNewR(x_guess, y_guess)
x_0 = x_guess;
y_0 = y_guess;
count = 0;
while count < 10
    count = count + 1;
    A = [x_0; y_0];
    J = Jacobian(x_0, y_0);
    F = [f(x_0, y_0); g(x_0, y_0)];
    var_New = A - (inv(J))*F;
    x_nm1 = x_0;
    y_nm1 = y_0;
    x_0 = var_New(1);
    y_0 = var_New(2);
end
out = [x_0; y_0];
count = count;
```

Command Window:

```
>> [out, count]= VectNewR(4,-4)

out =

    -8.4286499e+26
    -2.6618638e+34


count =

    10
```

**Problem 3:**

| x Speed | 30 | 45 | 60 | 75 | 90 | 120 |
|---|---|---|---|---|---|---|
| $y_1$ Rdist | 5.4 | 8.9 | 11.2 | 14.1 | 16.2 | 22.9 |
| $y_2$ Bdist | 5.2 | 12.5 | 21.1 | 32.3 | 47.2 | 85.1 |

Reaction dist (linear)

| x | $y_1$ | $x^2$ | $xy_1$ |
|---|---|---|---|
| 30 | 5.4 | 900 | 162 |
| 45 | 8.9 | 2025 | 400.5 |
| 60 | 11.2 | 3600 | 672 |
| 75 | 14.1 | 5625 | 1057.5 |
| 90 | 16.2 | 8100 | 1458 |
| 120 | 22.9 | 14400 | 2748 |
| 420 | 78.7 | 34650 | 6498 |

$$m = \frac{6(6498) - (420)(78.7)}{6(34650) - (420)^2} = 0.18838$$

$$b = \frac{78.7 - m(420)}{6} = -0.07$$

$$y = 0.18838x - 0.07$$

Braking

$$a(30)^2 + b(30) + c = 5.2$$
$$a(45)^2 + b(45) + c = 12.5$$
$$a(60)^2 + b(60) + c = 21.1$$
$$a(75)^2 + b(75) + c = 32.3$$
$$a(90)^2 + b(90) + c = 47.2$$
$$a(120)^2 + b(120) + c = 85.1$$

$$\begin{bmatrix} 30^2 & 30 & 1 \\ 45^2 & 45 & 1 \\ 60^2 & 60 & 1 \\ 75^2 & 75 & 1 \\ 90^2 & 90 & 1 \\ 120^2 & 120 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5.2 \\ 12.5 \\ 21.1 \\ 32.3 \\ 47.2 \\ 85.1 \end{bmatrix}$$

A

$$y = 6.21 \cdot 10^{-3}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = A^{-1}(y)$$

$$\downarrow$$

$$A^T \cdot (A \cdot A^T)^{-1}$$

$$y = 0.0062116x^2 - 0.050037x + 1.53$$

@ 105 km/h → total stopping dist: 84.47m