Tugas 03 - Hidrolika Lanjut

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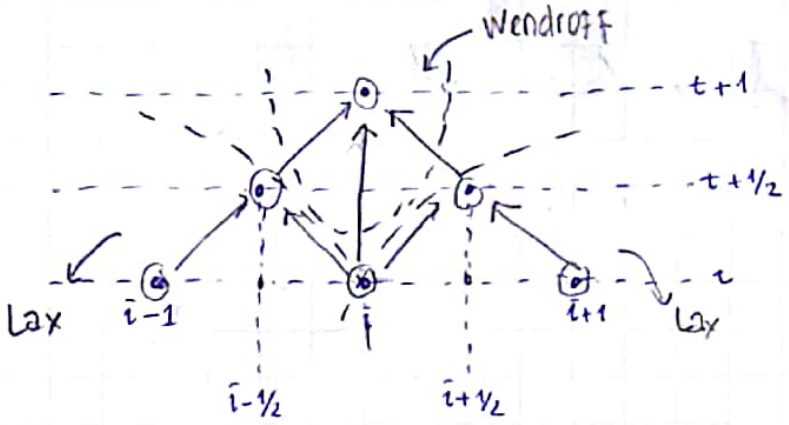
Skema Lax - Wendroff




Persamaan Pengatur:

→ Kontinuitas

↳ $\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$

→ Momentum

↳ $\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + g.A\frac{\partial(h+z_0)}{\partial x} + g\frac{Q|Q|n^2}{AR^{4/3}} = 0$

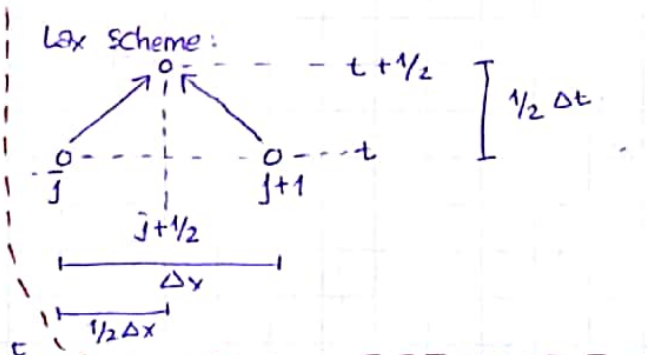
• Diskritisasi Persamaan

→ Kontinuitas

Lax ↳ $\frac{\partial A}{\partial t} = -\frac{\partial Q}{\partial x}$

(i+½) ↳ $\frac{A_{i+1/2}^{t+1/2} - A_{i+1/2}^{t}}{\frac{1}{2}\Delta t} = -\frac{Q_{i+1}^{t} - Q_i^{t}}{\Delta x}$

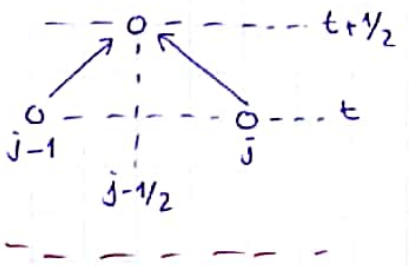
↳ $A_{i+1/2}^{t+1/2} = A_{i+1/2}^{t} - \frac{\Delta t}{\Delta x}\frac{Q_{i+1}^{t} - Q_i^{t}}{2}$



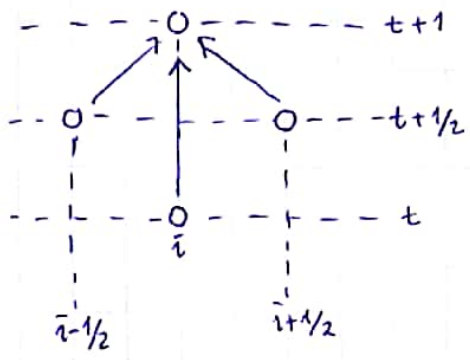

(1)

(i-½) ↳ $\frac{A_{i-1/2}^{t+1/2} - A_{i-1/2}^{t}}{\frac{1}{2}\Delta t} = -\frac{Q_i^{t} - Q_{i-1}^{t}}{\Delta x}$

↳ $A_{i-1/2}^{t+1/2} = A_{i-1/2}^{t} - \frac{\Delta t}{\Delta x}\frac{Q_i^{t} - Q_{i-1}^{t}}{2}$ ... (2)



Wendroff:




→ Kontuinitas:

↳ $\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \leftrightarrow \frac{\partial A}{\partial t} = -\frac{\partial Q}{\partial x}$

$\leftrightarrow \frac{A_i^{t+1} - A_i^t}{\Delta t} = -\frac{Q_{i+1/2}^{t+1/2} - Q_{i-1/2}^{t+1/2}}{\Delta x}$

$\leftrightarrow A_i^{t+1} = A_i^t - \frac{\Delta t}{\Delta x}\left(Q_{i+1/2}^{t+1/2} - Q_{i-1/2}^{t+1/2}\right) \quad \dots (3)$

Momentum:

$\rightarrow \underbrace{\frac{\partial Q}{\partial t}}_{I_1} + \underbrace{\beta \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)}_{I_2} + \underbrace{g\, A \frac{\partial (h+z_0)}{\partial x}}_{I_3} + \underbrace{g \frac{Q|Q| n^2}{A R^{4/3}}}_{I_4} = 0$

Lax:

Right $\rightarrow \frac{\partial Q}{\partial t} = \frac{Q_{i+1/2}^{t+1/2} - Q_{i+1/2}^{t}}{\frac{1}{2}\Delta t} = \frac{Q_{i+1/2}^{t+1/2} - Q_{i+1/2}^{t}}{\frac{1}{2}\Delta t}$

$I_2^R \rightarrow \beta \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = \beta\left[\frac{\frac{(Q_{i+1}^t)^2}{A_{i+1}^t} - \frac{(Q_i^t)^2}{A_i^t}}{\Delta x}\right]$

$I_3^R \rightarrow g\, A \frac{\partial (h+z_0)}{\partial x} = g\, A_i^t \frac{(h_{i+1}^t + z_{0i+1}^t) - (h_i^t + z_{0i}^t)}{\Delta x}$

$I_4^R \rightarrow g \frac{Q|Q| n^2}{A R^{4/3}} = g \frac{Q_i^t |Q_i^t| n^2}{A_i^t (R_i^t)^{4/3}}$

(R) $\rightsquigarrow$ ↳ $I_1^R + I_2^R + I_3^R + I_4^R = 0$

$\leftrightarrow \frac{Q_{i+1/2}^{t+1/2} - Q_{i+1/2}^t}{\frac{1}{2}\Delta t} + \beta\left[\frac{\frac{(Q_i^t)^2}{A_{i+1}^t} - \frac{(Q_i^t)^2}{A_i^t}}{\Delta x}\right] + I_3^R + I_4^R = 0$

$\leftrightarrow Q_{i+1/2}^{t+1/2} = Q_{i+1/2}^t - \frac{1}{2}\Delta t\left(I_2^R + I_3^R + I_4^R\right) \quad \dots (4)$

Left $\rightarrow \dfrac{\partial Q}{\partial t} = \dfrac{Q_{i-1/2}^{t+1/2} - Q_{i-1/2}^{t}}{1/2\,\Delta t} = I_1^L$

$I_2^L \rightarrow \beta \dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{A}\right) = \beta\left[\dfrac{\left(\dfrac{(Q_i^t)^2}{(A_i^t)} - \dfrac{(Q_{i-1}^t)^2}{(A_{i-1}^t)}\right)}{\Delta x}\right]$

$I_3^L \rightarrow g\,A\,\dfrac{\partial(h+z_0)}{\partial x} = g\,A_i^t\,\dfrac{(h_i^t + z_{0i}^t) - (h_{i-1}^t - z_{0i-1}^t)}{\Delta x}$

$I_4^L \rightarrow g\,\dfrac{Q|Q|n^2}{A\,R^{4/3}} = g\,\dfrac{Q_i^t\,|Q_i^t|\,n^2}{A_i^t\,(R_i^t)^{4/3}}$

(L) $\rightsquigarrow \hookrightarrow I_1^L + I_2^L + I_3^L + I_4^L = 0$

$\leftrightarrow \dfrac{Q_{i-1/2}^{t+1/2} - Q_{i-1/2}^{t}}{1/2\,\Delta t} = -(I_2^L + I_3^L + I_4^L)$

$\leftrightarrow Q_{i-1/2}^{t+1/2} = Q_{i-1/2}^{t} - \dfrac{1}{2}\Delta t\,(I_2^L + I_3^L + I_4^L) \quad \cdots (5)$

- Wendroff:

$I_1^W \rightarrow \dfrac{\partial Q}{\partial t} = \dfrac{Q_i^{t+1} - Q_i^t}{\Delta t}$

$I_2^W \rightarrow \beta\dfrac{\partial}{\partial x}\left(\dfrac{Q^2}{A}\right) = \left[\dfrac{\left(\dfrac{(Q_{i+1/2}^{t+1/2})^2}{A_{i+1/2}^{t+1/2}} - \dfrac{(Q_{i-1/2}^{t+1/2})^2}{A_{i-1/2}^{t+1/2}}\right)}{\Delta x}\right]$

$I_3^W \rightarrow g\,A\,\dfrac{\partial(h+z_0)}{\partial x} = g\,A_i^t\,\dfrac{(h_{i+1/2}^{t+1/2} + z_{0i+1/2}^{t+1/2}) - (h_{i-1/2}^{t+1/2} + z_{0i-1/2}^{t+1/2})}{\Delta x}$

$I_4^W \rightarrow g\,\dfrac{Q|Q|n^2}{A\,R^{4/3}} = g\,\dfrac{Q_i^t\,|Q_i^t|\,n^2}{A_i^t\,(R_i^t)^{4/3}}$

(W) $\rightsquigarrow \hookrightarrow I_1^W + I_2^W + I_3^W + I_4^W = 0$

$\leftrightarrow \dfrac{Q_i^{t+1} - Q_i^t}{\Delta t} = -(I_2^W + I_3^W + I_4^W) = 0$

$\leftrightarrow Q_i^{t+1} = Q_i^t - \Delta t\,(I_2^W + I_3^W + I_4^W) = 0 \quad \cdots (6)$

# RINGKASAN Lax-Wendroff

Persamaan

Kontinuitas

↳ Right → (1) ↳ $A_{i+1/2}^{t+1/2} = A_{i+1/2}^{t} - \frac{\Delta t}{\Delta x}\frac{Q_{i+1}^{t} - Q_{i}^{t}}{2}$

Left → (2) ↳ $A_{i-1/2}^{t+1/2} = A_{i-1/2}^{t} - \frac{\Delta t}{\Delta x}\frac{Q_{i}^{t} - Q_{i-1}^{t}}{2}$

Wendroff → (3) ↳ $A_{i}^{t+1} = A_{i}^{t} - \frac{\Delta t}{\Delta x}\left(Q_{i+1/2}^{t+1/2} - Q_{i-1/2}^{t+1/2}\right)$

Momentum

↳ Right → (4) ↳ $Q_{i+1/2}^{t+1/2} = Q_{i+1/2}^{t} - \frac{1}{2}\Delta t\left(I_2^R + I_3^R + I_4^R\right)$

Left → (5) ↳ $Q_{i-1/2}^{t+1/2} = Q_{i-1/2}^{t} - \frac{1}{2}\Delta t\left(I_2^L + I_3^L + I_4^L\right)$

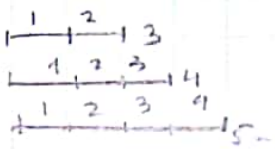
Wendroff → (6) ↳ $Q_{i}^{t+1} = Q_{i}^{t} - \Delta t\left(I_2^W + I_3^W + I_4^W\right)$

karena tidak ada turunan dalam $I_4^R, I_4^L, I_4^W$ → maka $I_4 = I_4^R = I_4^L = I_4^W$

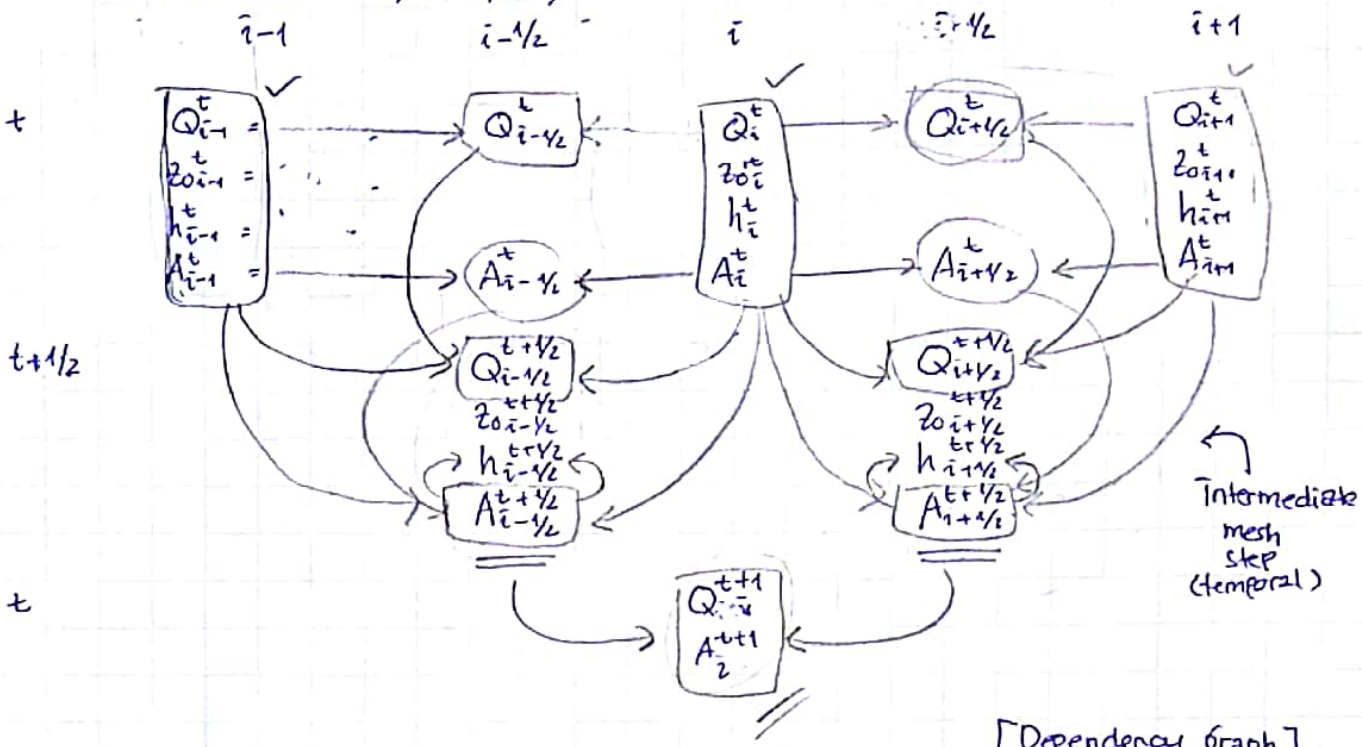
nilai $I_2^x, I_3^x, I_4^x$

| | R | L | W |
|---|---|---|---|
| $I_2$ | $\beta\left[\frac{\frac{(Q_{i+1}^{t})^2}{A_{i+1}^{t}} - \frac{(Q_{i}^{t})^2}{A_{i}^{t}}}{\Delta x}\right]$ | $\beta\left[\frac{\frac{(Q_{i}^{t})^2}{A_{i}^{t}} - \frac{(Q_{i-1}^{t})^2}{A_{i-1}^{t}}}{\Delta x}\right]$ | $\beta\left[\frac{\frac{(Q_{i+1/2}^{t+1/2})^2}{A_{i+1/2}^{t+1/2}} - \frac{(Q_{i-1/2}^{t+1/2})^2}{A_{i+1/2}^{t+1/2}}}{\Delta x}\right]$ |
| $I_3$ | $g\bar{A}^{t}\left[\frac{(h_{i+1}^{t} + zo_{i+1}^{t})}{\Delta x} - \dots \dots \frac{(h_{i}^{t} + zo_{i}^{t})}{\Delta x}\right]$ | $g A_{i}^{t}\left[\frac{(h_{i}^{t} + zo_{i}^{t})}{\Delta x} - \dots \dots \frac{(h_{i-1}^{t} + zo_{i-1}^{t})}{\Delta x}\right]$ | $g A_{i}^{t}\left[\frac{(h_{i+1/2}^{t+1/2} + zo_{i+1/2}^{t+1/2})}{\Delta x} - \dots \dots \frac{(h_{i-1/2}^{t+1/2} + zo_{i-1/2}^{t+1/2})}{\Delta x}\right]$ |
| $I_4$ | sama → | $\frac{g\, Q_{i}^{t}\lvert Q_{i}^{t}\rvert n^2}{A_{i}^{t}\,(R_{i}^{t})^{4/3}}$ | ← sama |

Dalam Pemrogramannya:




Pemisalan (Variable : Q, A, h, z)




[Dependency Graph]

nilai $Q^t_{i-1/2}$ didapat dari $\frac{Q^t_i + Q^t_{i-1}}{2}$

nilai $Q^t_{i+1/2} = \frac{Q^t_i + Q^t_{i+1}}{2}$ ; karena z tidak dipengaruhi waktu

maka $zo^{t+1/2}_{i-1/2} \approx zo^t_{i-1/2} = \frac{zo^t_i + zo^t_{i-1}}{2}$