SI-5232 : HIDROLIKA LANJUT — Taruma Sakti M (25017046)

Tugas 2 - Leap Frog

Persamaan Dasar

St-Venant Equation

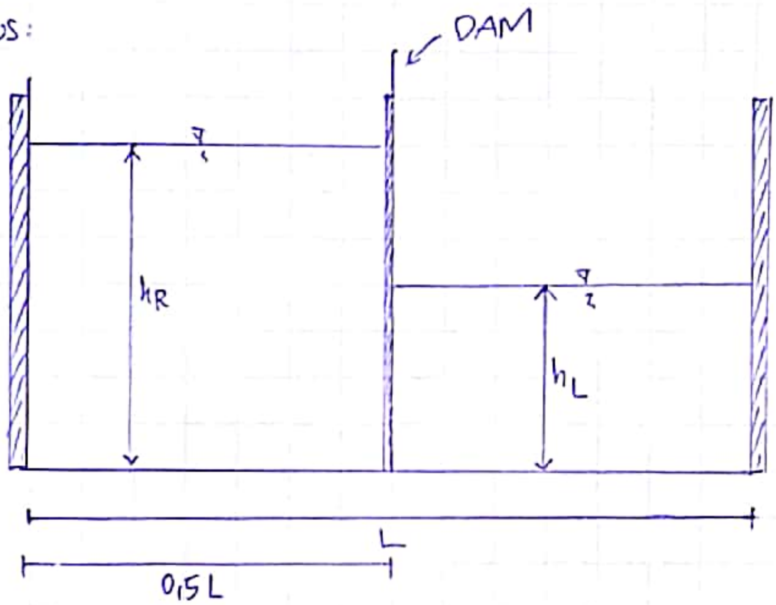
→ Kontinuitas :

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

→ Momentum

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + g \cdot A \frac{\partial (h+z_0)}{\partial x} + g \cdot \frac{Q|Q| n^2}{A R^{4/3}} = 0$$

Kasus:



Dik

$h_R = 1{,}5$ m

$h_L = 1$ m

$L = 54$ m

$n = 0{,}013$

$b = 1$ m

Simulasi waktu

$t = 10$ s

$\Delta t = 0{,}05$ s

$\Delta x = 0{,}5$ m

- Diskritisasi
  - Leap Frog (Beda Tengah)
    - → Kontinuitas

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{A_i^{t+1} - A_i^{t-1}}{2\Delta t} + \frac{Q_{i+1}^{t} - Q_{i-1}^{t}}{2\Delta x} = 0$$

$$\leftrightarrow A_i^{t+1} = A_i^{n-1} - \frac{\Delta t}{\Delta x}\left(Q_{i+1}^{t} - Q_{i-1}^{t}\right)$$

$$\leftrightarrow An(i) = A_0(i) - \frac{dt}{dx}\left(Q(i+1) - Q(i-1)\right)$$

Cat

$A^t = A$; $Q^t = Q$

$A^{t+1} = An$; $Q^{t+1} = Qn$

$A^{t-1} = A_0$; $Q^{t-1} = Q_0$

↳ Pers Momentum

$$\underbrace{\frac{\partial Q}{\partial t}}_{I_1} + \underbrace{\frac{\partial}{\partial x}\left(\beta \cdot \frac{Q^2}{A}\right)}_{I_2} + \underbrace{g \cdot A \frac{\partial(h+z_0)}{\partial x}}_{I_3} + \underbrace{g \frac{Q|Q|\,n^2}{A \cdot R^{4/3}}}_{I_4} = 0 \quad \ldots \quad (2)$$

Penyelesain masing' suku

I_1 ↳ $\frac{\partial Q}{\partial t} = \frac{Q_i^{t+1} - Q_i^{t-1}}{2\Delta t}$

I_2 ↳ $\frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) = \beta_i^t \cdot \left(\frac{\left(\frac{(Q_{i+1}^t)^2}{A_{i+1}^t}\right) - \left(\frac{(Q_{i-1}^t)^2}{A_{i-1}^t}\right)}{2\Delta x} \right) \leftrightarrow \beta \cdot \left(\frac{\left(\frac{Q(i+1)^2}{A(i+1)} - \frac{Q(i-1)^2}{A(i-1)}\right)}{2\,dx} \right)$

I_3 ↳ $g \cdot A \frac{\partial(h+z_0)}{\partial x} = g_i^t \; A_i^t \cdot \left(\frac{(h_{i+1}^t + z_{i+1}^t)}{2\Delta x} - \frac{(h_{i-1}^t + z_{i-1}^t)}{2\Delta x} \right)$

$\leftrightarrow = g \cdot A(i) \cdot \frac{1}{2\,dx} \left((h(i+1) + z(i+1)) - (h(i-1) + z(i-1)) \right)$

I_4 ↳ $g \cdot \frac{Q|Q|\,n^2}{A\,R^{4/3}} = g_i^t \cdot \frac{Q_i^t \, |Q_i^t| \, (n_i^t)^2}{A_i^t \, (R_i^t)^{4/3}} \leftrightarrow g \cdot \frac{Q(i) \cdot Abs(Q(i)) \cdot n^2}{A(i) \cdot (R(i))^{4/3}}$

maka pers (2) menjadi

↳ $\frac{Q_i^{t+1} - Q_i^{t-1}}{2\Delta t} + I_2 + I_3 + I_4 = 0 \quad \ldots$

$\leftrightarrow Q_i^{t+1} = Q_i^{t-1} - 2\Delta t\,(I_2 + I_3 + I_4) \leftrightarrow Qn(i) = Qo(i) - 2dt\,(I_2 + I_3 + I_4)$

Stoker's Solution (Analytical)

↳ $-8g \cdot h_R \; c_m^2 (\sqrt{g \cdot h_L} - c_m)^2 + (c_m^2 - g\,h_R)^2 (c_m^2 - g \cdot h_R) = 0$

↳ $c_m = \sqrt{g \cdot h_m}$