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HIDRAULIKA LANJUT
Tugas 4 - Mac Cormack Method

Persamaan Pengatur :

↳ Kontinuitas

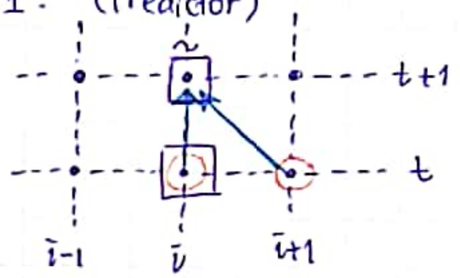
↳ $$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

↳ Momentum

↳ $$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + g \cdot A \frac{\partial}{\partial x}(h+z_0) + g \cdot \frac{Q|Q|n^2}{A R^{4/3}}$$

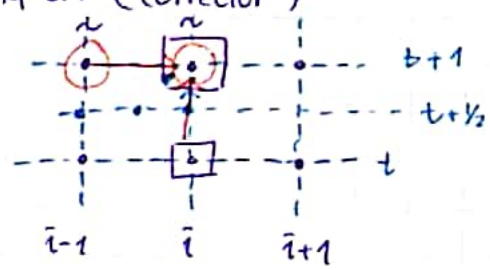
Skema Mac Cormack : [source: the ficalroom]

Step 1 : (Predictor)



FD scheme in x

Step 2 : (Corrector)



BD scheme in x with Δt/2

↳ Diskritisasi :

- Kontinuitas

Predictor : ↳ $$\frac{\overline{A_i^{t+1}} - A_i^t}{\Delta t} + \frac{Q_{i+1}^t - Q_i^t}{\Delta x} = 0$$

$$\leftrightarrow \quad \overline{A_i^{t+1}} = A_i^t - \frac{\Delta t}{\Delta x}\left(Q_{i+1}^t - Q_i^t\right) \quad \ldots (1)$$

Corrector ↳ $$\frac{A_i^{t+1} - A_i^{t+1/2}}{½ \cdot \Delta t} + \frac{\overline{Q_i^{t+1}} - \overline{Q_{i-1}^{t+1}}}{\Delta x} = 0$$

$$\leftrightarrow \quad A_i^{t+1} = A_i^{t+1/2} - \frac{\Delta t}{2\Delta x}\left(\overline{Q_i^{t+1}} - \overline{Q_{i-1}^{t+1}}\right) \quad \ldots (2)$$

dengan $$A_i^{t+1/2} = \frac{A_i^t + \overline{A_i^{t+1}}}{2}$$

↳ Momentum :

$$\underbrace{\frac{\partial Q}{\partial t}}_{I_1} + \underbrace{\frac{\partial}{\partial x}\left(\beta \cdot \frac{Q^2}{A}\right)}_{I_2} + \underbrace{g \cdot A \frac{\partial}{\partial x}(h+z_0)}_{I_3} + \underbrace{g \cdot \frac{Q|Q|n^2}{AR^{4/3}}}_{I_4} = 0$$

▸ Predictor :

$$\tilde{I_1} \rightarrow \frac{\partial Q}{\partial t} = \frac{\overline{Q_i^{t+1}} - Q_i^t}{\Delta t}$$

$$\tilde{I_2} \rightarrow \frac{\partial}{\partial x}\left(\beta \cdot \frac{Q^2}{A}\right) = \beta \cdot \left[\frac{\frac{(Q_{i+1}^t)^2}{A_{i+1}^t} - \frac{(Q_i^t)^2}{A_i^t}}{\Delta x}\right]$$

$$\tilde{I_3} \rightarrow g\, A \frac{\partial}{\partial x}(h+z) = g \cdot A_i^t \cdot \left[\frac{(h_{i+1}^t + z_{i+1}^t) - (h_i^t + z_i^t)}{\Delta x}\right]$$

$$\tilde{I_4} \rightarrow g \cdot \frac{Q|Q|n^2}{AR^{4/3}} = g \cdot \frac{Q_i^t\, |Q_i^t|\, n^2}{A_i^t\, (R_i^t)^{4/3}}$$

maka ↳ $\tilde{I_1} + \tilde{I_2} + \tilde{I_3} + \tilde{I_4} = 0$

$$\leftrightarrow \frac{\overline{Q_i^{t+1}} - Q_i^t}{\Delta t} + \tilde{I_2} + \tilde{I_3} + \tilde{I_4} = 0$$

$$\leftrightarrow \overline{Q_i^{t+1}} = Q_i^t - \Delta t\,(\tilde{I_2} + \tilde{I_3} + \tilde{I_4}) \quad \ldots \quad (3)$$

Scanned by CamScanner

- Corrector

↳ $I_1' \rightarrow \dfrac{\partial Q}{\partial t} = \dfrac{Q_i^{t+1} - Q_i^{t+1/2}}{1/2 \cdot \Delta t}$

↳ $I_2' \rightarrow \dfrac{\partial}{\partial x}\left(\beta \cdot \dfrac{Q^2}{A}\right) = \beta \left[\dfrac{\dfrac{(Q_i^{\overline{t+1}})^2}{A_i^{t+1}} - \dfrac{(Q_{i-1}^{\overline{t+1}})^2}{A_{i-1}^{t+1}}}{\Delta x} \right]$

↳ $I_3' \rightarrow g \cdot A \cdot \dfrac{\partial}{\partial x}(h+z) = g \cdot A_i^t \left[\dfrac{(h_i^{\overline{t+1}} + z_i^{\overline{t+1}}) - (h_{i-1}^{\overline{t+1}} + z_{i-1}^{\overline{t+1}})}{\Delta x} \right]$

↳ $I_4' \rightarrow g \cdot \dfrac{Q|Q|n^2}{A \cdot R^{4/3}} = g \cdot \dfrac{Q_i^t |Q_i^t| n^2}{A_i^t (R_i^t)^{4/3}}$

maka : $I_1' + I_2' + I_3' + I_4' = 0$

↳ $\dfrac{Q_i^{t+1} - Q_i^{t+1/2}}{1/2 \cdot \Delta t} + I_2' + I_3' + I_4' = 0$

$\leftrightarrow \quad Q_i^{t+1} = Q_i^{t+1/2} - \dfrac{\Delta t}{2}(I_2' + I_3' + I_4') = 0$

dengan $Q_i^{t+1/2} = \dfrac{Q_i^t + Q_i^{\overline{t+1}}}{2}$

Ringkasan Mac Cormack

- Predictor

(1) ↳ $\overline{A_i^{t+1}} = A_i^t - \frac{\Delta t}{\Delta x}\left(Q_{i+1}^t - Q_i^t\right)$

(3) ↳ $\overline{Q_i^{t+1}} = Q_i^t - \Delta t\left(\tilde{I}_2 + \tilde{I}_3 + \tilde{I}_4\right)$

- Corrector

(2) ↳ $A_i^{t+1} = A_i^{t+1/2} - \frac{\Delta t}{2\Delta x}\left(\overline{Q_i^{t+1}} - \overline{Q_{i-1}^{t+1}}\right)$

(4) ↳ $Q_i^{t+1} = Q_i^{t+1/2} - \frac{\Delta t}{2}\left(I_2 + I_3 + I_4\right)$

dengan $A_i^{t+1/2} = \frac{A_i^t + \overline{A_i^{t+1}}}{2}$; $Q_i^{t+1/2} = \frac{Q_i^t + \overline{Q_i^{t+1}}}{2}$

Rumus I_2^*, I_3^*, I_4^* ; cat: $\tilde{I}_4 = \dot{I}_4 = I_4$.

	~ (Predictor)	• (corrector)
I_2	$\beta\left[\frac{\frac{(Q_{i+1}^t)^2}{A_{i+1}^t} - \frac{(Q_i^t)^2}{A_i^t}}{\Delta x}\right]$	$\beta\left[\frac{\frac{(\overline{Q_i^{t+1}})^2}{\overline{A_i^{t+1}}} - \frac{(\overline{Q_{i-1}^{t+1}})^2}{\overline{A_{i-1}^{t+1}}}}{\Delta x}\right]$
I_3	$g\,A_i^t\left[\frac{(h_{i+1}^t + z_{i+1}^t) - (h_i^t + z_i^t)}{\Delta x}\right]$	$g\,A_i^t\left[\frac{(\overline{h_i^{t+1}} + \overline{z_i^{t+1}}) - (\overline{h_{i-1}^{t+1}} + \overline{z_{i-1}^{t+1}})}{\Delta x}\right]$
I_4	$g\,\frac{Q_i^t\,\lvert Q_i^t\rvert\,n^2}{A_i^t\,(R_i^t)^{4/3}}$	← sama