

**Discrete Mathematics and Algorithms (CSE 611)**  
**Monsoon 2018**  
**Assignment Set 5**  
**Total Marks: 100**

Course Instructor: Dr. Ashok Kumar Das

**P, NP, NP-hard and NP-completeness**

1. Is the following Boolean formula satisfiable? If so, find its all satisfying assignments.

$$(x \vee y) \wedge (x \vee \bar{y}) \wedge (\bar{x} \vee y) \wedge (\bar{x} \vee \bar{y})$$

2. Let IS-HAM-CYCLE denote the computational problem that, given an undirected graph  $G$ , decides whether  $G$  contains just those edges necessary to form a Hamiltonian cycle in  $G$  (no more, no less). Prove or disprove: IS-HAM-CYCLE is NP-Complete.
3. Prove that P is closed under concatenation.
4. A **triangle** in an undirected graph is a 3-clique. Show that TRIANGLE is in P, where  $\text{TRIANGLE} = \{\langle G \rangle \mid G \text{ contains a triangle}\}$ .
5. Prove that NP is closed under union.
6. Prove or disprove that NP is closed under complement.
7. Let  $\text{HALF-CYCLE} = \{\langle G \rangle \mid G \text{ is an undirected graph having a complete subgraph with at least } \frac{n}{2} \text{ nodes, where } n \text{ is the number of nodes in } G\}$ . Show that HALF-CYCLE is NP-complete.
8. Let  $\text{MODEXP} = \{\langle a, b, c, p \rangle \mid a, b, c, \text{ and } p \text{ are binary integers such that } a^b \equiv c \pmod{p}\}$ . Show that  $\text{MODEXP} \in \text{P}$ .
9. A **2-cnf formula** is an AND of clauses where each clause is an OR of at most two literals. Let  $2\text{SAT} = \{\langle \phi \rangle \mid \phi \text{ is a satisfiable 2cnf-formula}\}$ . Prove or disprove that  $2\text{SAT} \in \text{NP}$ .
10. If  $G$  is an undirected graph, a **vertex cover** of  $G$  is a subset of the nodes where every edge of  $G$  touches one of those nodes. The vertex cover problem asks for the size of the smallest vertex cover. Let  $\text{VERTEX-COVER} = \{\langle G, k \rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover}\}$ . Prove that VERTEX-COVER is NP-complete.