UNIT - I

(10)+5

IMAGE TRANSFORMS

Transform :- A transform is a mathematical tool that allows to move from one domain to another. (Ex :- time domain to freq. domain)

Image Transforms :- used in image processing & analysis.

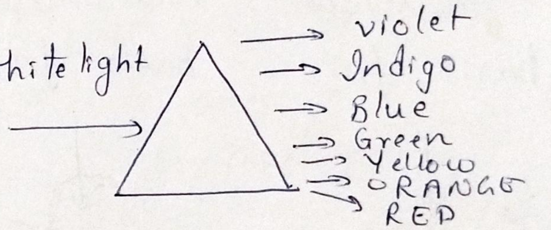
Need for Image Transforms :-

(1) Mathematical Convenience :-

convolution in time domain $\Leftrightarrow$ Multiplication in freq. domain

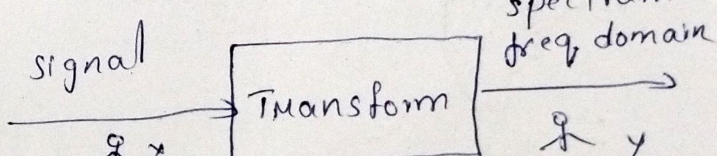
(2) To extract more information :-

Transforms allow us to extract information



Ex :- A person X sees white light on LHS of prism whereas a person Y sees combination of seven colours (VIBGYOR) on RHS.

Y gets more information compared to X.



Similarly a transform tool allows to extract more information. This allows to move from time domain to freq domain & vice versa.

(3) Applications of Image transforms:-

Real time applications

> Biometric finger print attendence

> Airport baggage screening system.

Classification of Image Transforms:-

Transforms are classified on the nature of basis function.

Transforms

Orthogonal sinusoidal basis fn.	Orthogonal Non sinusoidal basis fn	Basis fn depending on statistics of i/p signal	Directional transformation
→ Fourier transform.	→ Haar transform	→ KL transform	→ Hough transform
→ Discrete Fourier transform	→ Walsh "	→ SVD "	→ Random transform
→ Discrete sine transform	→ Hadamard "		→ Ridgelet transform
	→ Slant "		→ countourlet transform

DISCRETE FOURIER TRANSFORM (DFT)

DFT is used for discrete formulation of Fourier transform. The integration in Fourier Transform is replaced by summation.

One Dimensional DFT :-

The DFT for one dimensional function is given as

$$\boxed{F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x)\, e^{\frac{-j2\pi ux}{M}}} \quad — (1)$$

where $u = 0, 1, 2 \ldots M-1$

The inverse DFT is written as

$$\boxed{f(x) = \sum_{u=0}^{M-1} F(u)\, e^{\frac{+j2\pi ux}{M}}} \quad — (2)$$

Representing DFT in terms of real & imaginary terms

$$|F(u)| = \left[R^2(u) + I^2(u)\right]^{1/2} \quad — (3)$$

The phase angle $\phi = \tan^{-1} \frac{I(u)}{R(u)} \quad — (4)$

The power spectrum

$$P(U) = \left[R^2(u) + I^2(u)\right] \quad — (5)$$

Two Dimensional DFT :-

The DFT for a two dimensional array given by

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} \quad -(1)$$

where $u = 0, 1, 2, \ldots M-1$

$v = 0, 1, 2 \ldots N-1$

The two dimensional Inverse DFT (IDFT)

$$f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{+j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} \quad -(2)$$

If images are sampled in square array, then $M = N$. Therefore eq (1) & eq (2) can be represented as

$$\text{DFT} \rightarrow F(u,v) = \frac{1}{M^2} \sum_{x=0}^{M-1} \sum_{y=0}^{M-1} f(x,y)\, e^{-j2\pi\frac{(ux+vy)}{M}}$$

$$\text{IDFT} \Rightarrow f(x,y) = \sum_{u=0}^{M-1} \sum_{v=0}^{M-1} F(u,v)\, e^{j2\pi\frac{(ux+vy)}{M}}$$

Representation in complex form

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

$$\phi = \tan^{-1} \frac{I(u,v)}{R(u,v)} \quad \& \quad P(u) = \left[R^2(u,v) + I^2(u,v)\right]$$

PROPERTIES OF TWO DIMENSIONAL DFT :-

1) Separability
2) Periodicity & conjugate
(3) Distribution & scaling
(4) Translation
(5) Average value
6) Laplacian
7) Rotation
8) Convolution & correlation
9) Orthogonality

1) Separability :-

The 2D DFT is represented as

$$F(u,v) = \frac{1}{N} \sum_{x,y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux+vy}{N}\right)} \quad \text{—— (1)}$$

Rearranging this expression

$$F(u,v) = \frac{1}{N} \sum_{x=0}^{N-1} e^{\frac{-j2\pi ux}{N}} \sum_{y=0}^{N-1} f(x,y)\, e^{\frac{-j2\pi vy}{N}} \quad \text{—— (2)}$$

↓ outer summation ↓ Inner summation

→ Inner summation is taken from $y = 0 \to N-1$.
outer summation is taken from $x = 0 \to N-1$.

→ Inner summation gives fourier transformation of different rows.

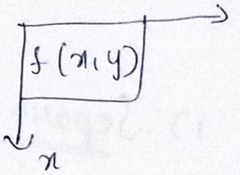
→ Outer summation gives fourier transformation of different coloumns.

$$\therefore \boxed{F(u) = \frac{1}{N}\sum_{x=0}^{N-1} e^{-\frac{j2\pi ux}{N}} F(x, v)} \quad -(3)$$

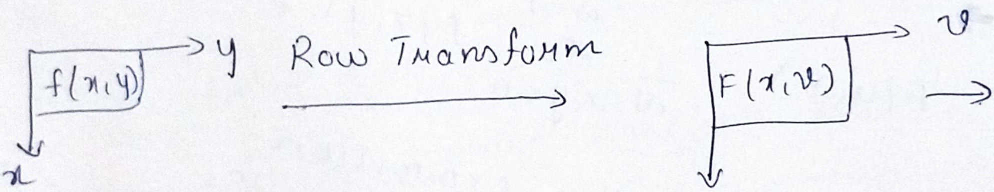
$$\text{where } F(x, v) = \sum_{y=0}^{N-1} f(x, y)\, e^{-\frac{j2\pi vy}{N}} \quad -(4)$$

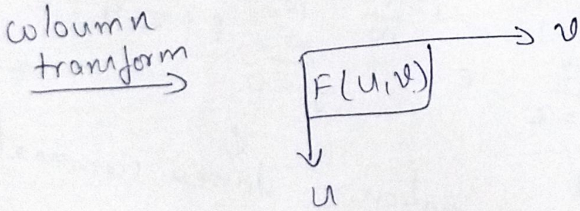
The entire procedure can be summarized as follows:

→ Consider i/p array given by f(x, y)



→ Perform row transformation for every row of i/p image.





→ Perform coloumn transform. o/p F(u, v) is obtained.

➜ Inverse Discrete Fourier transformation (IDFT) is also separable.
Perform IDFT on every row & then every coloumn
The o/p is in the form of f(x, y)

(3)

(2) Periodicity & Conjugate :-

(2)

→ Periodicity

Periodicity property ~~says~~ states that both DFT & IDFT are periodic with a period N.

$$F(u,v) = F(u+N, v) = F(u, v+N)$$

$$= F(u+N, v+N)$$

The 2D DFT for M = N array is

$$F(u,v) = \frac{1}{N}\sum_{x,y=0}^{N-1} f(x,y)\, e^{\frac{-j2\pi(ux+vy)}{N}} \quad —(1)$$

$$F(u+N, v+N) = \frac{1}{N}\sum_{x,y=0}^{N-1} f(x,y)\, e^{-j\frac{2\pi}{N}(ux+Nx+vy+Ny)}$$

$$= \frac{1}{N}\sum_{x,y=0}^{N-1} f(x,y)\, e^{-j\frac{2\pi}{N}(ux+vy)} \cdot \underbrace{e^{-\frac{j2\pi N(x+y)}{N}}}_{=1}$$

$= 1$ because exponent of some integer multiple of 2π

$$\Rightarrow F(u+N, v+N) = \frac{1}{N}\sum_{x,y=0}^{N-1} f(x,y) \cdot e^{\frac{-j2\pi(ux+vy)}{N}}$$

$$\Rightarrow \boxed{F(u+N, v+N) = F(u)}$$

→ Conjugate

The conjugate property states that if $f(x,y)$ is a real value function, the fourier transformation

$$\boxed{F(u,v) = F^*(-u,-v)}$$

$*$ - complex conjugate.

(* so for one dimensional signal.

$f(x) \iff F(u)$

$F(u) = F(u+N)$ - periodicity

$|F(u)| = |F(-u)|$ - conjugate *)

(3) <u>Distribution</u> & <u>Scaling</u> :-

Consider two signals $f_1(x,y)$ & $f_2(x,y)$

Fourier transform of →

$$\boxed{F\{f_1(x,y) + f_2(x,y)\} = F\{f_1(x,y)\} + F\{f_2(x,y)\}}$$

Hence 2D DFT & 2D IDFT are distributive.

If we consider multiplication

$$\boxed{F\{f_1(x,y) \cdot f_2(x,y)\} \neq F\{f_1(x,y)\} F\{f_2(x,y)\}}$$

∴ 2D DFT & 2D IDFT are not distributive over multiplication

<u>Scaling</u> :- Consider two scalar quantities a & b.

Given signal $f(x,y)$

$$a\{f(x,y)\} \iff aF(u,v)$$

$$f(ax, by) \iff \frac{1}{|ab|} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

(4) Average value :-

The average value of a 2D DFT is represented as

$$\boxed{f(x,y) = \frac{1}{N} F(0,0)}$$

(5) Translation :-

Consider an image represented by $f(x,y)$. This should be translated by (x_0, y_0)

i.e $f(x,y) \xrightarrow{x_0,\, y_0} f[(x-x_0),(y-y_0)]$

$\Rightarrow f(x-x_0, y-y_0) = F_t(u,v) = F(u,v)\, e^{-j\frac{2\pi}{N}(ux_0 + vy_0)}$ ↳ represents additional phase shift in Fourier spectrum ≈ 1.

$$\therefore \boxed{|F_t(u,v)| = |F(u,v)|}$$

Similarly

$F(u,v) \xrightarrow{u_0,\, v_0} F[(u-u_0),(v-v_0)]$

$\Rightarrow F(u-u_0, v-v_0) = f_t(x,y) = f(x,y)\, e^{+j\frac{2\pi}{N}(u_0x + v_0y)}$ ↳ ≈ 1.

$$\boxed{|f_t(x,y)| = |f(x,y)|}$$

6) <u>Rotation</u> :-

To explain rotation property, we use the polar co-ordinate system

i.e $x = r\cos\theta$ $y = r\sin\theta$

$u = \omega\cos\phi$ $v = \omega\sin\phi$

$\therefore f(x,y) \Rightarrow f(r,\theta)$

$F(u,v) \Rightarrow F(\omega,\phi)$

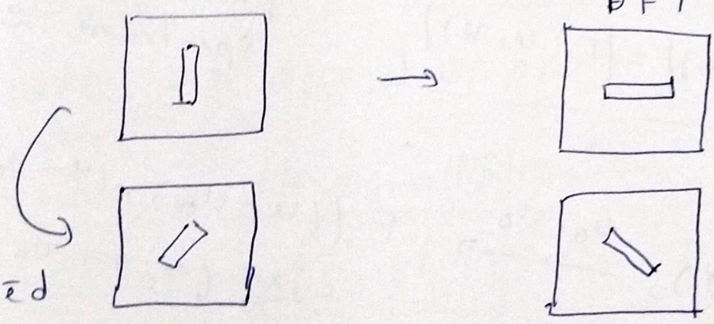
$$\boxed{f(r, \theta+\theta_0) \Longleftrightarrow F(\omega, \phi+\theta_0)}$$

In polar co-ordinate system, original signal is $f(r,\theta)$

If this is rotated by an angle θ, then rotated image is represented by $f(r, \theta+\theta_0)$ & fourier transform becomes $F(\omega, \phi+\theta_0)$

Ex :-

DFT



rotated by 45^0

$F(r, \theta+\theta_0)$ $F(\omega, \phi+\theta_0)$

7) Convolution & correlation :-

Convolution

consider two ~~signals~~ images $f(x)$ & $g(x)$ whose fourier transforms are $F(u)$ & $G(u)$.

$$f(x) \cdot g(x) \iff F(u) * G(u)$$

[$*$ - convolution]

$$f(x) * g(x) \iff F(u) \cdot G(u)$$

Convolution holds good for both one & two dimension.

Correlation :-

$$f(x, y) \circ g(x, y) \iff F^*(u, v) \cdot G(u, v)$$

[F^* - complex conjugate]

$$f^*(x, y) \cdot g(x, y) \iff F(u, v) \circ G(u, v)$$

8) Laplacian Property :-

For a 2D image $f(x, y)$ the laplacian property is defined as

$$\nabla^2 f(x, y) = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$$

The 2D-~~D~~FT for

$$F\{\nabla^2 f(x, y)\} = \nabla^2 \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v)\, e^{j2\pi(ux + vy)}\, du\, dv \right]$$

$$\frac{d^2 f}{dx^2} = \frac{d^2}{dx^2} \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v)\, e^{j2\pi(ux + vy)}\, du\, dv \right.$$

$$\frac{d^2 f}{dx^2} = j^2 4\pi^2 u^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v)\, du\, dv.$$

$$\Rightarrow \frac{d^2 f}{dx^2} = -4\pi^2 u^2 F(u,v)$$

Similarly, $\frac{d^2 f}{dy^2} = -4\pi^2 v^2 F(u,v)$

$$\nabla^2 f(x,y) = \boxed{\frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2} = -4\pi^2 (u^2 + v^2) F(u,v)}$$

Laplacian transform is used for outlining the edges of image.

(9) <u>Orthogonality</u> :-

The orthogonality property of a 2D - DFT is given as

$$\boxed{\frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} a_{u,v}(x,y)\, a_{u_0,v_0}(x,y) = \delta(u-u_0, v-v_0)}$$

where $\delta(u-u_0, v-v_0)$ is kronecker delta.

The orthogonality condition can be used to derive the formula for IDFT.