UNIT – I

INTRODUCTION TO DIGITAL IMAGE PROCESSING

WHAT IS DIGITAL IMAGE PROCESSING

In Digital Image Processing we have three terms

- Processing - It is a series of actions that lead to some result
- Image - Image means the pictures
- Digital - It describes electronic technology that stores and process data in either positive or non-positive form. Requires digital computer

Hence the Processing Of Images which are digital in nature by using Digital Computer is called Digital Image Processing.

WHY DIGITAL IMAGE PROCESSING

1. Improvement or enhancement of quality of image for human perception

Image enhancement employs methods capable of enhancing pictorial information for human interpretation and analysis. The applications are

1) ***Noise filtering*** – Removing noise from image by using different techniques
2) ***Content enhancement*** – Two types of content enhancement can be done
    - Contrast enhancement – Increasing the brightness or darkness of an image
    - Deblurring - Blurring occurs because of two reasons- (a) if the focus is not adjusted properly (b) if there is a motion blur (due to handshake)
3) ***Remote sensing*** – aerial images taken from satellite

2. Image processing for autonomous machine application

- Applications in industries, particularly quality control in assembly automation etc

3. Efficient storage and transmission

Certain properties are used to minimize disk space and bandwidth

ORIGINS OF DIGITAL IMAGE PROCESSING

Newspaper Industry: Digital used to transmit news paper pictures were first sent by submarine cable between London and New York. The time taken for transmission of picture is one week. Later, Bartlane cable picture transmission system is used in early 1920s. The time taken for transmission is less than 3 hours to transport across Atlantic. This printing method was abandoned towards the end of 1921

Bartlane system was capable of coding 5 distinct brightness levels. This was increased to 15 levels by 1929. The advantage is better tonal quality and resolution. For the next 35 years research was done to improve the quality of the image. Image processing techniques have increased only after the advancements in the following:

Computer Advances in Digital Image Processing

- the invention of the transistor by Bell Labs. in 1948
- the development of the high-level programming languages
- the invention of the IC at Texas Instruments in 1958
- the development of operation systems in the early 1960s
- the development of microprocessor by Intel in the early 1970s
- introduction by IBM of the personal computer in 1981
- Large Scale IC in the late 1970s
- VLSI in the 1980s, ULSI present
- IC Technology, Mass storage and display systems
- Computers have powerful processing capability to process images

The first computers powerful enough to carry out meaningful Image processing tasks appeared in 1960s.
Advantages: Used to correct distortions in the image, to enhance and store images in many applications. Digital image processing techniques began in 1960s and early 1970s

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APPLICATIONS OF DIGITAL IMAGE PROCESSING

The simplest way to develop basic understanding of image processing is to categorize images according to their energy sources

Energy sources:

1. electromagnetic spectrum
2. Acoustic
3. Ultrasonic
4. Electronic beams

I. Electromagnetic Spectrum:

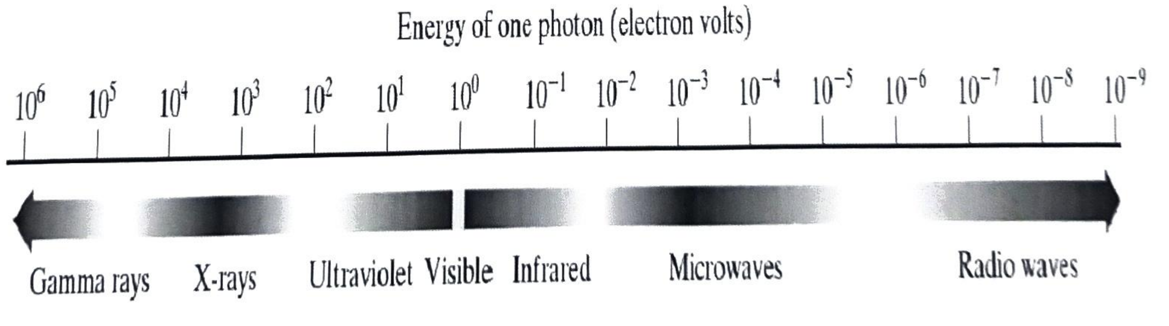


FIGURE 1.5 The electromagnetic spectrum arranged according to energy per photon.

- EM waves are sinusoidal waves that propagate with different wavelengths. They are a stream of mass less particles travelling with speed of light. Each mass less particle has certain bundle of energy and each bundle of energy is called a ***photon.*** The bands are shown shaded to convey the fact that bands of EM spectrum are distinct but rather transit smoothly from one to another

1. Gamma-Ray Imaging:

A) Nuclear medicine

Bone scan:

Inject a patient with a radioactive isotope that emits gamma rays. As it decays images are produced from emissions collected by gamma ray detectors. It is used to identify infections and tumors

PET scan (positron emission tomography) –

A **positron** is a particle of matter with same mass as an electron, but opposite charge. Patient is given radioactive isotope that emits positrons as it decays. When positron meets electron, both are annihilated and two gamma rays are given off. These gamma rays are detected and tomographic image is created .

B) Astronomical observations:

Cygnus loop: It is a star in the constellation which exploded about 15,000 yrs ago, generating a superheated stationary gas cloud. It glows in a spectacular array of colours.

2. X-Ray Imaging:

Oldest source of radiation used for imaging

Applications:

1. Medical diagnostics
2. Industry
3. Astronomy
4. Others

How X-Rays Are Generated:

X-rays are generated by using X-ray tube. Heating cathode releases free electrons Electrons travel towards anode. When electron strikes nucleus, energy is released in the form of X-rays. Energy of X-rays is controlled by voltage applied across anode. The no. of X-rays generated is controlled by current applied to filament of cathode

- *Chest X-Ray:*

  A patient is placed between x-ray source and film sensitive to X-ray energy. The intensity of X-rays is modified by absorption as they pass through patient. The resulting energy falling on the film develops it. **Digital radiography** is used to obtain digital images Digital X-ray sensors are used instead of photographic film. Advantage is time efficiency

- *Aniography –*

  X-ray study of blood vessels to obtain their images is called angiography. The procedure *is called aniogram. Figure represents Aortic aniogram .*

- *Computerized Axial Tomography(Cat Scan):* Revolutionized medicine since 1970s

- *Electronic Circuit Boards are* used to examine the flaws in manufacturing such as missing components or broken traces

- *Cygnus Loop*: using X-ray imaging

3. **Imaging in the Ultraviolet Band:**
Lithography. Industrial inspection. Lasers. Biological imaging, astronomical observations. Microscopy. Fluorescence microscopy is a branch of study of materials made of fluorescence either in natural Form or when treated with chemicals capable of fluorescing. Ex: images of normal and smut (**Smut**- disease that Occurs in onions, grasses. corn. Cereals etc

4. **Imaging in the Visible and Infrared Bands**

Band No.	Name	Wavelength (μm)	Characteristics and Uses
1	Visible blue	0.45–0.52	Maximum water penetration
2	Visible green	0.52–0.60	Good for measuring plant vigor
3	Visible red	0.63–0.69	Vegetation discrimination
4	Near infrared	0.76–0.90	Biomass and shoreline mapping
5	Middle infrared	1.55–1.75	Moisture content of soil and vegetation
6	Thermal infrared	10.4–12.5	Soil moisture; thermal mapping
7	Middle infrared	2.08–2.35	Mineral mapping

Applications:
- Light microscopy
- Astronomy
- Remote sensing
- Industry
- Law enforcement

5. **Imaging in the Microwave Band**
<u>Radar Imaging:</u> Used where data can be collected over any region at any time regardless of weather or ambient lighting conditions. Some radar waves penetrate clouds and under certain conditions can also see through vegetation, ice and dry sand. Imaging radar works like a flash camera and take a snap shot image. Instead of camera lens, radar uses an antenna and digital computer processing to record its images.

6. Imaging in the Radio Band

- *Medicine*

1. *Magnetic Resonance Imaging(MRI)* – *Anatomy*
2. *Electro Cardiography (ECG)* - *Heart*
3. *Electro Encephalography(EEG)* - *Electrical Activity Of Brain*
4. *Electro Myography(EMG)* - *Skeletal Muscles*
5. *Computer Aided Imaging* - *Tumors*

- *Astronomy*

II. ACOUSTICS:

1. Geological explorations - mineral and oil
2. Industry
3. Medicine

OFFICE AUTOMATION:

1. Optical character recognition
2. Document processing
3. Cursive script recognition
4. Logo
5. icon recognition,
6. identification of address area on envelop etc

INDUSTRIAL AUTOMATION:

1. Automatic inspection system - non- destructive testing
2. Automatic assembling - process related to VLSI manufacturing
3. Probotics, oil and natural gas exploration -process related to PCB checking
4. Seismography - process control applications

REMOTE SENSING:

1. Natural resources survey and management
2. Estimation related to hydrology, forestry, mineralogy, urban planning, environment and pollution control, cartography,
3. Registration of satellite images with terrain maps
4. Monitoring traffic along roads, docks and airfields

SCIENTIFIC APPLICATIONS:

High energy physics and bubble chamber and other forms of track analysis etc

CRIMINOLOGY:

Finger print identification
Human face registration and matching
Forensic investigation etc

ASTRONOMY AND SPACE APPLICATIONS:

Restoration of images suffering from geometric and photometric distortions
Computing close-up picture of planetary surfaces etc

BIO-MEDICAL APPLICATIONS:

Electro cardiography (ECG) - HEART
Electro Encephalography (EEG) – electrical activity of brain
Electro Mayography (EMG) – skeletal muscles
Computer Aided Tomography (CAT)
Magnetic Resonance Imaging (MRI)
Positron Emission Tomography (PET)
Cytology – study of microscopic cells
Histology - anatomical study of the microscopic structure of animal and plant tissues
Stereology - determination of the three-dimensional structure of objects based on two-Dimensional views of them.
Automated Raiology
Pathology
X-Ray Image Analysis
Mammograms
Cancer Smears
Screening Of Plant Samples
3d Reconstrution And Analysis
Single Photon Emitted Computer Tomography(Spect)
Mass Screening Of Medical Images For Detection Of Various Diseases

- **METEROLOGY:**

  Short Term Weather Forecasting
  Long Term Climatic Change Detection From Satellite
  Remote Sensing Data Cloud Pattern Analysis

- **INFORMATION TECHNOLOGY:**

  Facsimile Image Transmission
  Videotex
  Video Conferencing
  Video Phones Etc

- **ENTERTAINMENT AND CONSUMER ELECTRONICS:**
  - HDTV
  - Multimedia
  - Video-Editing Etc

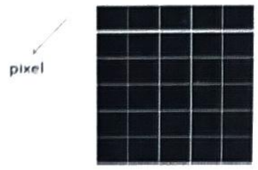
- **PRINTING AND GRAPHICS ARTS:**
  - Colour Fidelity In Desktop Publishing
  - Art Conservation And Distribution

- **MILITARY APPLICATIONS:**
  - Missile Guidance And Detection
  - Target Identification
  - Navigation Of Pilotless Vehicle
  - Reconnaissance (Exploration Or Investigation)
  - Range Finding Etc

WHAT IS AN **IMAGE:**

An image is a two dimensional function f(x,y). x and y are spatial coordinates. Amplitude of f is called the intensity or gray level. x, y, f(x,y) are all finite and discrete. An Image is composed of finite number of elements. The elements are referred as

- Picture elements
- Image elements
- Pels
- Pixels – most widely used



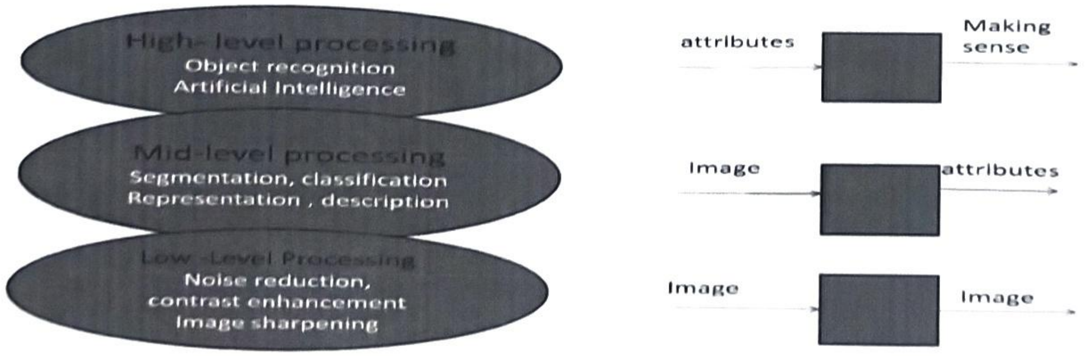
WHAT IS A DIGITAL IMAGE?

If x, y and the amplitude values of f are finite and discrete quantities, we call the image as a digital image. A digital image is composed of a finite number of elements called pixels (image elements or picture elements or pels), each of which has a particular location and value.

COMPUTER VISION

Computer vision is concerned with making computers "understand" what is contained in an image and other functions of human vision. It is sometimes considered to be a topic of artificial intelligence. However, it is not possible to state exactly where image processing ends and computer vision begins.

Image processing to computer vision



1) ***Low level Processing:*** In low level processing, input is an image and output is also image. In this stage enhancement of the image is done. The noise is filtered and image smoothing and sharpening is done. The contrast levels are also enhanced.

2) Mid level processing: In Mid level processing, input is image and output obtained contains attributes of image. (*Attributes: edges, identification of objects in image, countours etc*) In this level segmentation of image, classifying the objects of image , their representation and description is given.

3) High level processing: In this stage inputs are attributes of image and output is making sense of an image. In this stage, object recognition is done by using artificial intelligence

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FUNDAMENTAL STEPS IN DIGITAL IMAGE PROCESSING

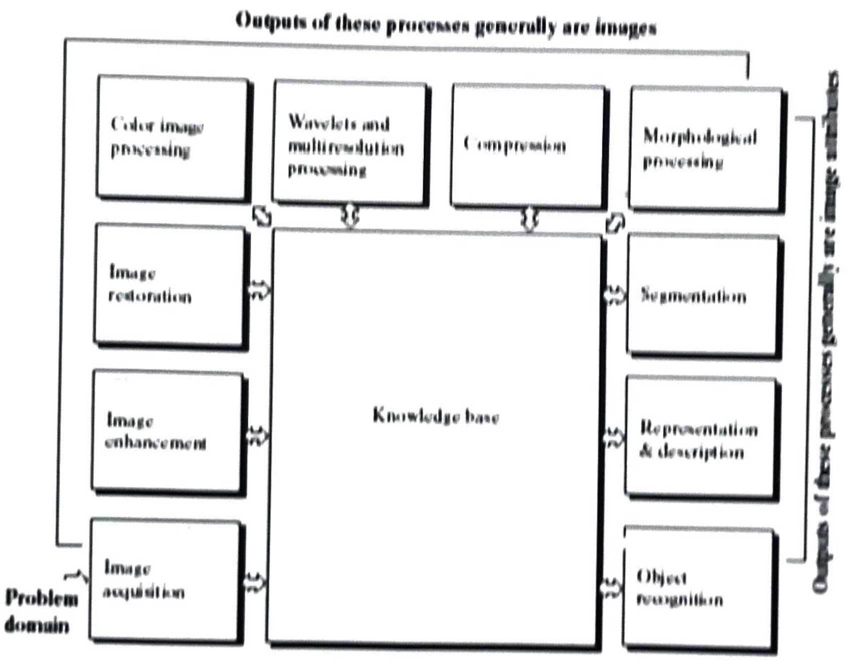


Image acquisition: The image to be captured is considered as problem domain. The image can be acquired by using a camera or a line scan. It involves pre-processing techniques such as scaling.

Image Enhancement: Enhancement is a subjective process in which the features of the image are enhanced. Image enhancement improves the contrast, reduces noise and deblurs the image

Image Restoration: It is an objective process in which the appearance of the image can be improved. The reasons for degradation of an image can be analyzed and eliminated using mathematical and probabilistic techniques. The level of restoration depends on human preferences.

Colour Image Processing: This techniques preserves the colour information in an image. Colour image processing restores the natural colours and characterisitics in the image. These techniques have gained importance due to images on internet

Wavelets and multiresolution: It deals with techniques involved in representing images in various resolutions. It uses Wavelet transforms to compress, transmit and analyze images.
Multiresolution unifies techniques from signal processing, digital speech recognition and pyramidal image processing

Multiresolution – represents signals in various angles of resolution

Image Compression: Compression reduces the storage space. The transmission bandwidth can be reduced. The compressed images are saved in the form of JPEG files (Joint Photographers experts group)

Morphological Processing: It deals with required tools for extraction of image components used for Representation and description of shape. Inputs are images and outputs are attributes

Segmentation: It partitions the image into small blocks or segments. Basically, there are two types of segmentation techniques

Rugged segmentation – long process but more accurate results
Weak segmentation - Quick process but poor results

Representation and Description: Representation describes the variation in images. It separates one image to another. There are two types of representations:
Boundary description - focuses on external properties like Shaping of corners
Regional description - focuses on internal properties like Texture, contrast etc
Description means Feature selection i.e., describing the features of an image and extracting the attributes. (Extracting attributes – differentiating one class of objects to other)

Object Recognition: Assigns label to object based on description

Knowledge base: All the basic knowledge required to solve the given problem domain is available in the knowledge base. It also controls the interaction between modules

COMPONENTS OF AN IMAGE PROCESSING SYSTEM

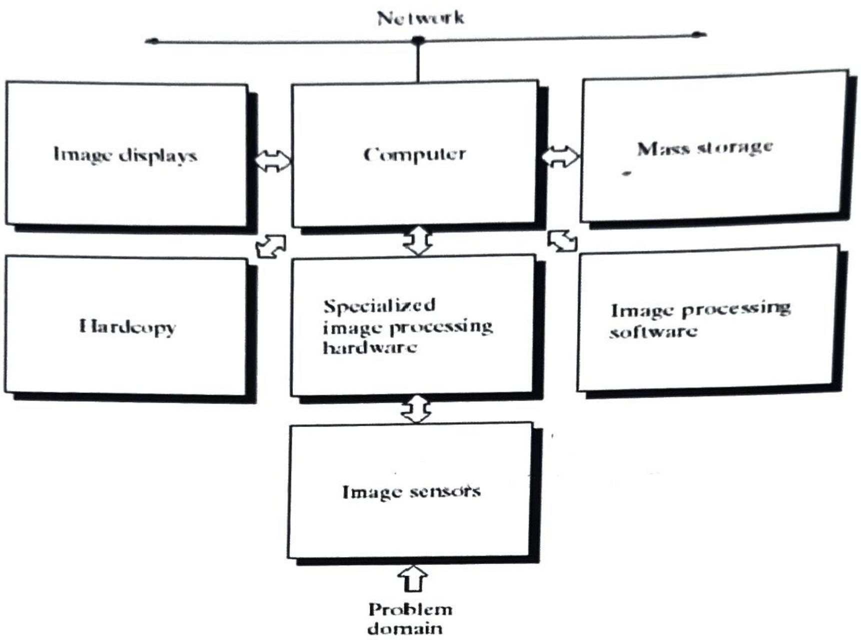


Image Sensors: These are the physical devices which senses the energy radiated from object Light energy is converted into electrical energy

Specialized Image Processing Hardware: It consists of a digitizer that converts sensed images into digital format. The Hardware circuitry is used to perform Arithmetic and logical operations on an image. The hardware is often called front end system. It gives fast outputs.

Hardcopy: Hard copy can be taken by using Laser printer, Film cameras, Heat sensitive devices(facsimile, Optical devices,)CD ROM , DVD, RAM disks

Image Displays: Display of images can be done using Colour TV monitors. Monitors are driven by output of images or graphic display cards

Network: A network is a medium used to receive or send images. Optical fiber and broad band technologies are good for networking.

Computer: A general purpose computer can be used

Mass Storage: Image compression techniques are used for mass storage purpose to save disk space. There are two types of storage techniques:

- Short term storage– computer memory – used during processing of images
- Online storage – frame buffers are used for fast recall of images
- Archival storage –infrequent use. Not frequently used images are stored in archives
- Storage is measured in bytes, kilo, mega, giga and tera bytes

Image processing software: Specialized hardware modules are used to perform specific tasks based on programs. Sophisticated software allows integration of modules and general purpose commands are used from at least one computer language

VISUAL PERCEPTION OF ELEMENTS

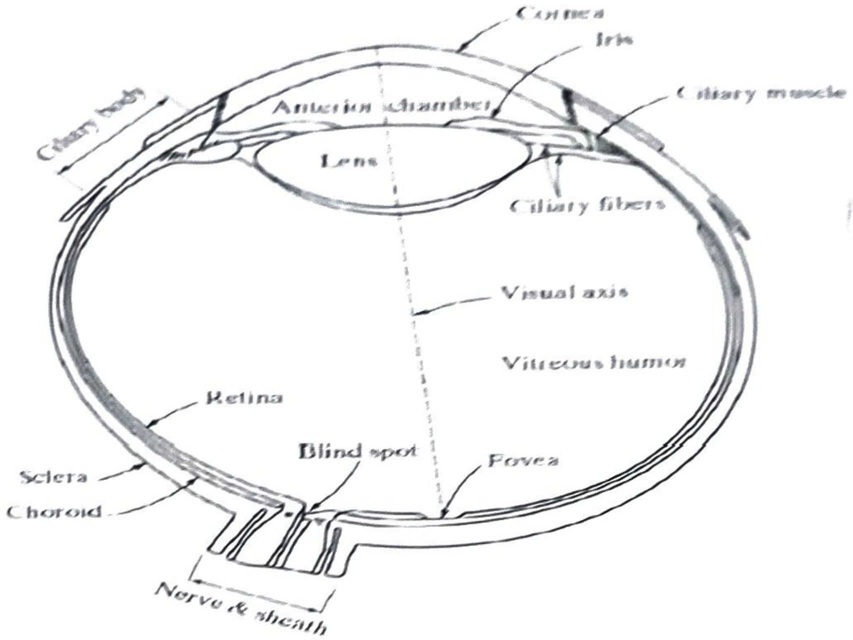


Figure shows the structure of human eye. The shape of the eye is a sphere with a diameter of 20mm. The eye contains three Membranes

- Chornea and sclera
- Choroid
- retina

Cornea: It is the outer cover of the eye and is a tough transparent tissue

Sclera: It is the opaque membrane and makes the eye appear spherical in shape (Optic Globe)

Choroid: It is alayer below sclera. It contains a network of blood vessels. Blood vessels serve as nutrition to the eye. Any injury to choroid restricts the flow of blood to the eye. It has two parts

- Ciliary body
- Iris diaphram

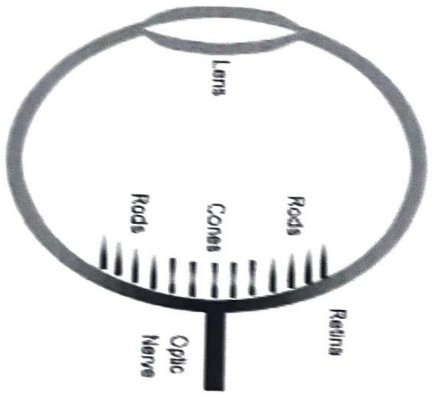
Iris helps eye to contract or expand to control amount of light. The pupil of iris changes in diameter from 2 to 8 mm. The front portion of iris contains visible pigment and back of iris contain black pigment

Lens is made of concentric layers of fibrous cells and is suspended by fibers attached to ciliary body. It contains 60-70%water, 6% fat and more protein . The lens is coloured by light yellow pigmentation and increases with age. Excessive clouding causes cataract in eye. The eye absorbs 8% of the visible light spectrum. Excessive absorption of light damages the eye

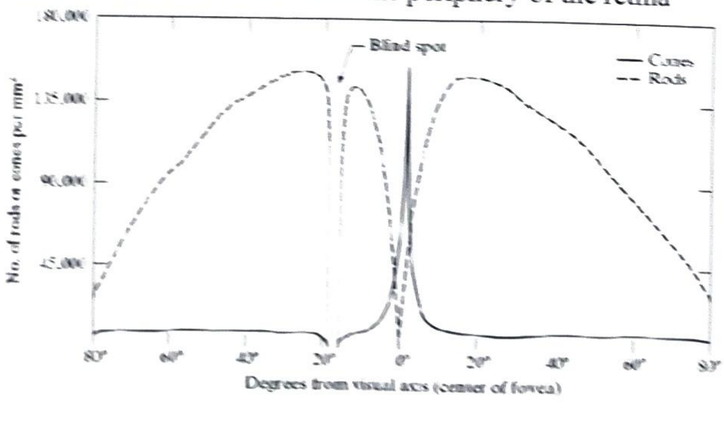
Retina: Max. no. of cones are located at center portion of retina . This portion is called fovea and is the inner most membrane of the eye. When light is focussed, object from outside the eye is imaged on retina and gives the overall picture. The retina consists of two receptors

Rods are large in number i.e., 75 to 150 million distributed over retina surface. It gives overall picture view. Rods are sensitive to low illumination. Rod vision is called *scotopic vision or dim light vision*

Cones: Cones exist between 6-7 million in number. Cones lies in central portion of retina called fovea. Cones are highly sensitive to colour. It provides provides fine details also. Cone vision is called *photopic or bright light vision*



Blind Spot: Absence of receptors results in blind spot. Figure shows density of rods and cones for the cross section of right eye passing through region of optic nerve from the eye. Absence of receptors in this area results in blind spot. Except this region distribution of receptors is radially symmetric about the fovea. Cones are more dense in the center of the retina. Rods increase in density from the center out to approximately 20° off the axis and then decrease in out to the extreme periphery of the retina



II – IMAGE FORMATION IN THE EYE

Eye lens is more flexible than ordinary lens. The shape of the lens is controlled by ciliary body. If object is far, then the ciliary body muscles allow lens to be flattened. If object is near, then the muscles allow lens to be thicker. The focul length is the distance between the center of the lens and retina. The focul length varies b/w 17mm to 14mm according to refractive power of lens. If object is away than 3mm, refractive power is low. If object is closer than 3mm, refractive power is high

- Ex: if observer is viewing a tree of height 15m which is at a distance of 100m. If 'h' is the height in mm of that object in retinal image, then 15/100 = h/17 or h = 2.55mm
- Retinal image is reflected primarily in the area of fovea

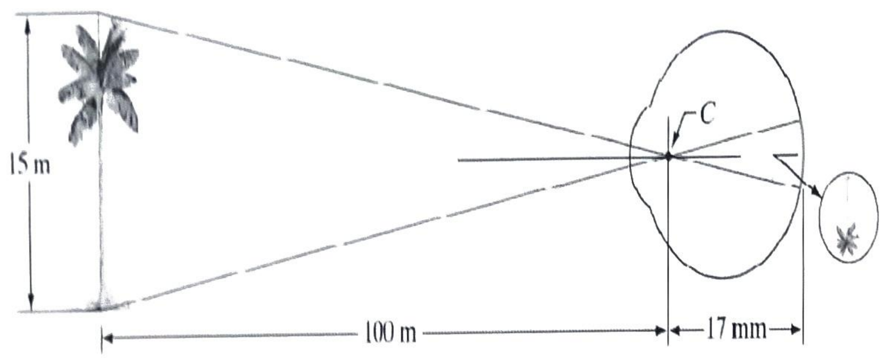
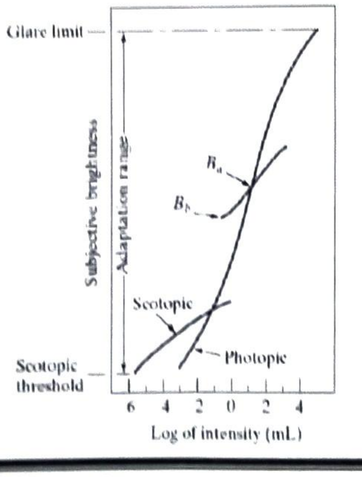


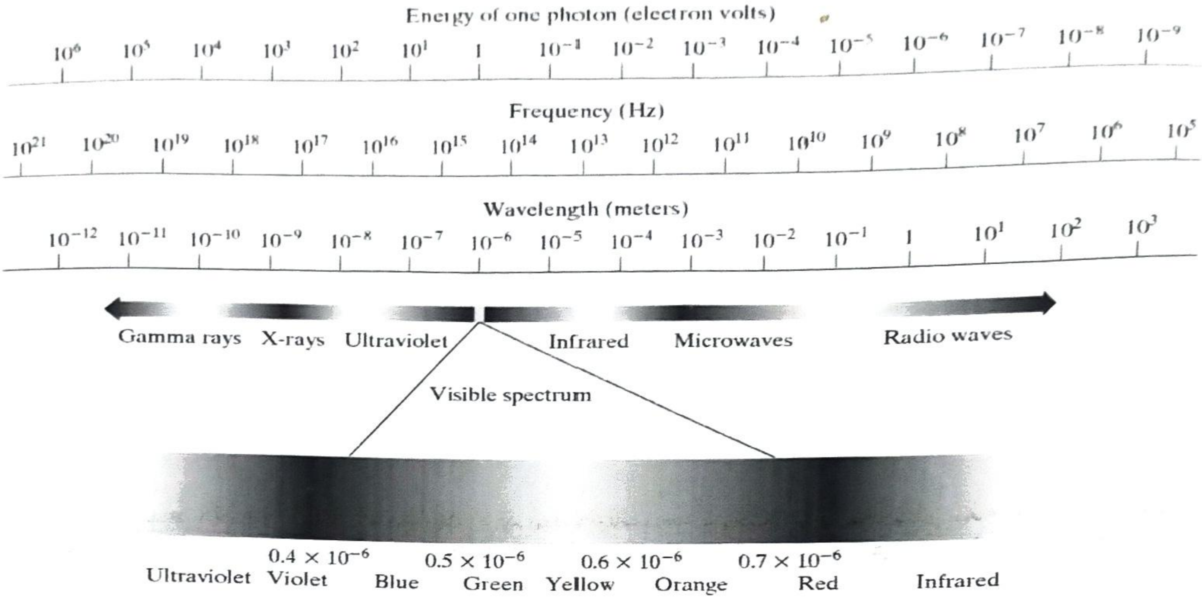
FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point C is the optical center of the lens.

II – BRIGHTNESS ADAPTATION OF DISCRIMINATION

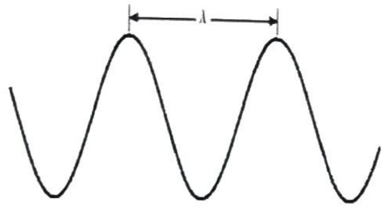
The ability of eye to discriminate b/w different intensity levels is important. Human eye accepts light intensity of the order of 10^{10} from scotopic threshold to glare limit. Subjective brightness is the intensity as perceived by the human visual system. Current sensitivity level of visual level system is called brightness adaption level. The short intersecting curve represents the range of light the eye can perceive. B_a represents the brightness adaption level. B_b represents all blacks. Long solid curve represents the range of intensities to which the visual system can adapt. In photopic vision alone, the range is about 10^6 . Transition from scotopic to photopic vision is gradual over the approximate range from 0.001 to 0.1 millilambert .



LIGHT AND ELECTROMAGNETIC SPECTRUM



EM waves can be conceptualized as propagating sinusoidal waves of varying wavelengths and can be thought of as a stream of mass less particles travelling with speed of light. Each massless particle has certain bundle of energy. Each bundle of energy is called a photon. The bands are shown shaded to convey the fact that bands of EM spectrum are distinct but rather transit smoothly from one to another



Since $E = h\nu$ and $\lambda = C/\nu$, higher the frequency smaller will be the wavelength Radio waves have photons with low energy. Gamma rays have photons with highest energy. Hence gamma rays are more dangerous to living organisms. Visible band of EM spectrum ranges from .43(violet) to 0.79 (red) micro meters. Colour spectrum is divided into six broad categories - Violet, blue, green, yellow, orange, red. Each range blends smoothly to the next. No colour ends abruptly.

MONOCHROMATIC LIGHT is Light without colour. It ranges from black to shades of grey to white.

CHROMATIC LIGHT It represents the colour ranges throughout EM spectrum from 0.43-0.79 micro meters. The quality of chromatic source is defined by three quantities

Radiance – amount of energy that flows from light source (watts)

luminescence - amount of energy observer perceives from light source (lumens – lm)

Brightness – subjective descriptor of light perception

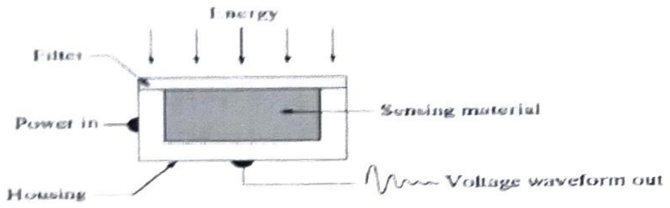
IMAGE SENSING AND ACQUISITION

Images are generated by combination of *"illumination"* from source and *reflection or absorption* of energy from that source by the elements of the image

- *Illumination source* – radar, infrared, ultra sound or X-ray energy
- Scene or image elements must be familiar objects – rock formations, human brain, image of sun etc
- *Reflected image* – light reflected from plane surface – any picture
- *Transmitted or absorbed image* – X-rays passing through a patients body

IMAGE SENSORS:

- Single imaging sensors
- Line sensors
- Array sensors
- Input – light energy
- Output – electrical energy and then digitized



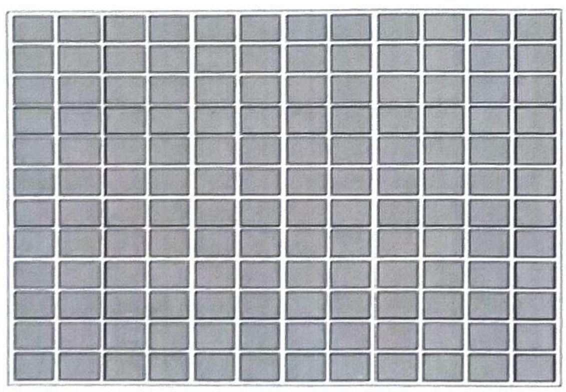
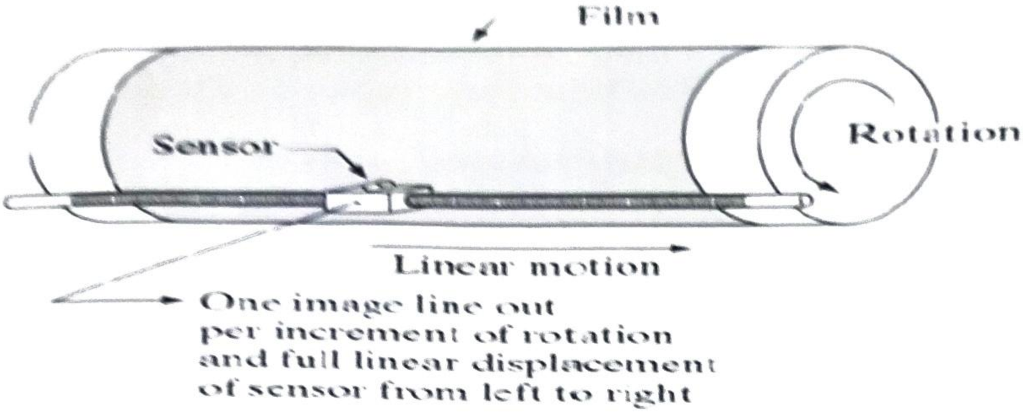


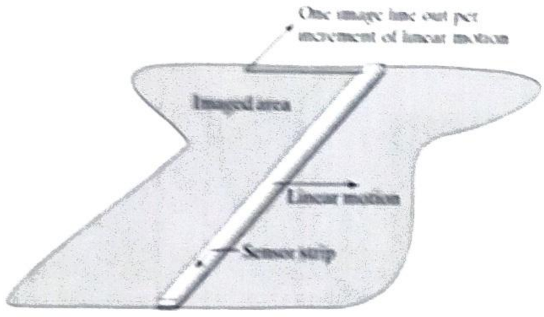
Image acquisition using single sensor: A Photodiode is used as a sensor. The input is light energy and o/p is electrical energy. Filter improves selectivity (ex: green filter favors green band of color spectrum. In order to generate 2D image, displacement should be along both x (sensor) and y(image) axis. Film negative is mounted on a mechanical drum which provides rotation along y-axis. A sensor is mounted on the lead screw which provides movement along x-axis

The advantage of single sensors are Low cost, Not suitable for high resolution pictures, Image acquisition is slow



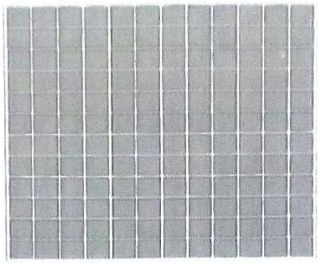
2. Sensor strips:

This consists of an arrangement of sensors in the form of strip (4000 or more line sensors). The image is represented in gray colour. The strip is arranged above the imaged area. It provides imaging elements in one direction and gives details of one line in the image at a time. Airborne imaging applications, medical and industrial image applications



Sensor strips mounted in ring configuration are used in medical Imaging to obtain cross sectional (slice) images of 3-D objects

3. Sensor Arrays: Sensors are arranged in the form of a 2D array. Electromagnetic and ultra sonic devices are arranged in this format. CCD sensors found in digital cameras. The number of elements is 4000 x 4000 or more. It is used to convert a continuous image into a digital image. It contains an array of light sensors



HOW A DIGITAL IMAGE IS FORMED:

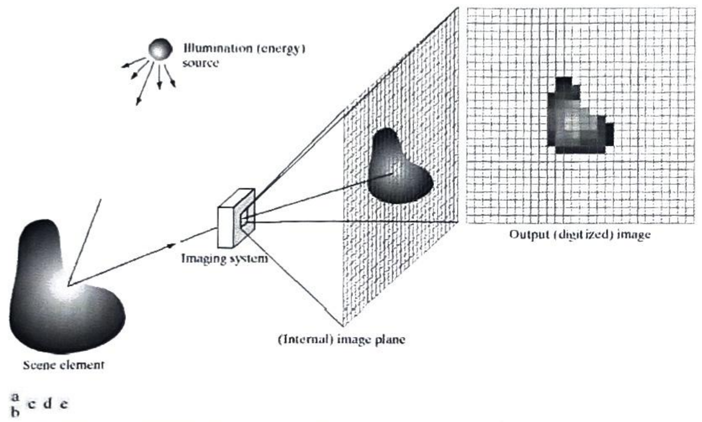


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

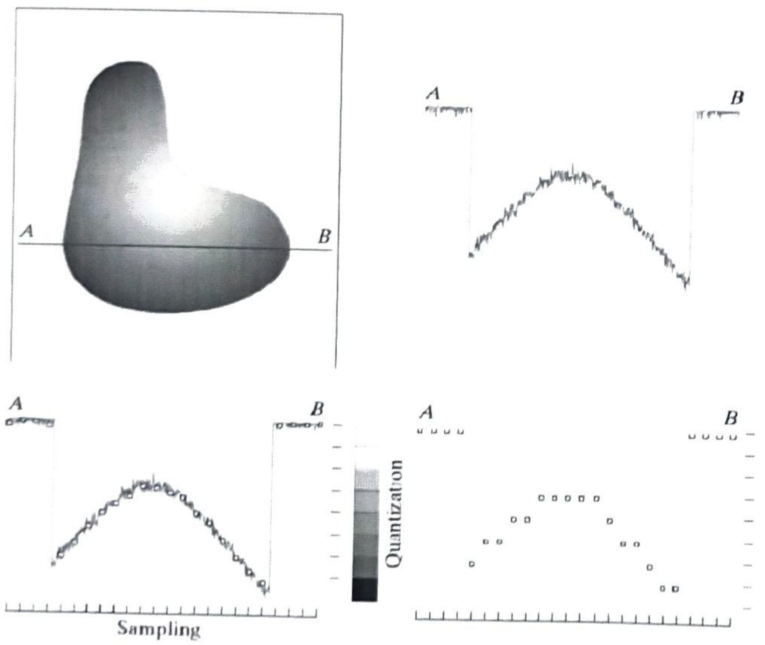
An image is defined as a two-dimensional function f(x, y), where x and y are the spatial (plane) coordinates, and the amplitude of f at any pair of coordinates (x, y) is called the intensity (brightness or gray level) of the image at that point. If x, y and the amplitude values of f are finite and discrete quantities, we call the image as a digital image. A digital image is composed of a finite number of elements called pixels (image elements or picture elements or pels), each of which has a particular location and value. Formation of a digital image using by an image processing system is shown in bellow figure.

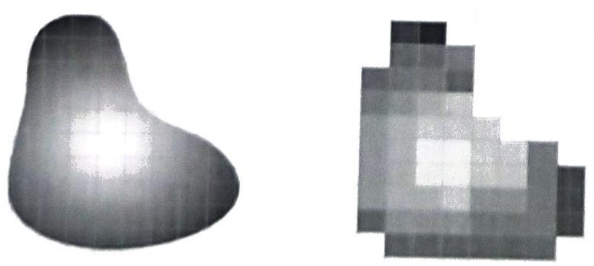
Illumination energy : It is source of light energy that falls on an object or scene

Source element: It is the element that gets illuminated due to light energy
Imaging system: It captures reflected light and converts it into electrical energy which is continuous in nature. It consists of a digitizer which converts analog signal to digital form

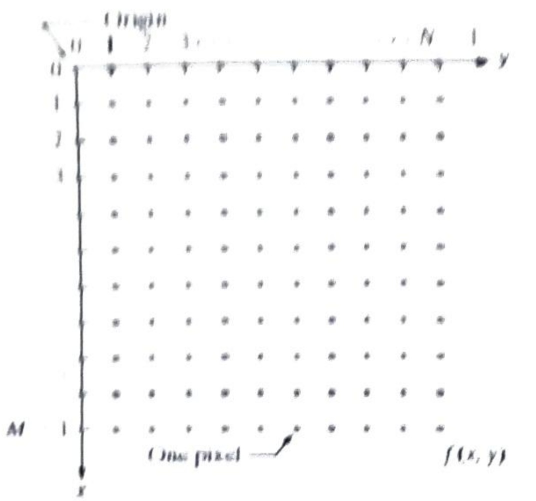
IMAGE SAMPLING AND QUANTIZATION

An image may be continuous w.r.t x, y coordinates and in amplitude. Digitization of image involves two process sampling and quantization. Fig1 represents a continuous image with different shades from black to white. Consider digitizing of one line AB. Fig 2: represents amplitudes (shades of gray along the length of line) of line AB. From figure we can observe that the edges are dark and towards centre the brightness is more. Fig 3: represents equally spaced samples along both the axis and the intensity scale divided to eight discrete intervals ranging from black to white. Fig 4: represents digital samples resulting from both sampling and quantization. Hence from an image, the elements can be converted into digital form by sampling and quantization techniques.





HOW TO REPRESENT DIGITAL IMAGES



Assume image f(x,y). The sampled & quantized image results in digital image. Assume Digital image has M rows and N columns. The values of the co-ordinates (x,y) now become discrete quantities. Consider co-ordinates of origin f(x,y) as (0,0). Next co-ordinates are referred as (0,1). The complete MxN digital image can be written in matrix form as follows

$$f(x,y) = \begin{vmatrix} f(0,0) & f(0,1) & f(0,N-1) \\ f(1,0) & f(1,1) & f(1,N-1) \\ & & \\ f(M-1,0) & f(M-1,1) & f(M-1,N-1) \end{vmatrix}$$

Right side of this equation is a digital image

Element of this matrix array is called image element, picture element, pixel or pel

- It is also advantageous to use traditional matrix notation to represent image and its elements

$$A = \begin{vmatrix} a(0,0) & a(0,1) & a(0,N-1) \\ a(1,0) & a(1,1) & a(1,N-1) \\ & & \\ a(M-1,0) & a(M-1,1) & a(M-1,N-1) \end{vmatrix}$$

Aij = f(x=i, y=j) = f(i,j).

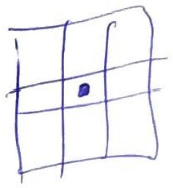
Hence eqns in both slides are identical

- The no. of intensity levels in an image is represented as a power of 2. These discrete levels are equally spaced. The no. of discrete gray levels allowed for each pixel is L
  $L = 2^k$; K- quantized number
- The discrete levels are integers in the interval [0, L-1]
- The no. of bits "b" required to store an image is $b = M \times N \times K$
  $b = N^2K$

SPATIAL RESOLUTION:

- Spatial resolution is a measure of the smallest visible detail of image. It can be stated in no. of ways. Intensity resolution is the no. of bits used to quantize intensity. It is measured in dots(pixels) per unit distance - used in printing. In U. S this is measured as dots per inch (dpi).

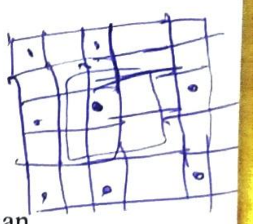
IMAGE INTERPOLATION: Interpolation is a basic tool used for zooming, shrinking, rotating and geometric corrections. Interpolation is the process of using known data to estimate values at unknown locations



Bilinear interpolation:
Four nearest neighbours are used to estimate the intensity at a given location. Let (x,y) denote the coordinates of the location to which we want to assign an intensity value. Let v(x,y) denote that intensity value. The assigned value is calculated using the equation

$$v(x,y) = ax + by + cxy + d$$

The four coefficients are determined from four equations in four unknowns that can be written using the four nearest neighbours of point (x,y). It gives better results.



Bicubic interpolation: **It** involves sixteen nearest neighbours of a point. The intensity value assigned to point (x,y) is obtained using the equation

$$v(x,y) = \sum_{i=0}^{j=3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

Where sixteen coefficients are determined from sixteen equations in sixteen unknowns that can be determined using sixteen nearest neighbours of point (x,y)

BASIC RELATIONSHIP BETWEEN PIXELS:

Neighbours: A pixel *p* at coordinates (*x,y*) has four *horizontal* and *vertical* neighbors whose coordinates are given by:

$$(x+1,y), (x-1, y), (x, y+1), (x,y-1)$$

	(x, y-1)	
(x-1, y)	*P (x,y)*	(x+1, y)
	(x, y+1)	

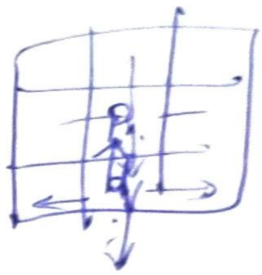
The 4-*neighbors* of *p*, are denoted by $N_4(p)$. Each pixel is one unit distance from (x,y). Some of the neighbors of p lie outside the digital image if (x,y) is on the border of the image.

The four *diagonal* neighbors of *p* have coordinates:
(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1) and are denoted by $N_D(p)$.

(x-1, y+1)		(x+1, y-1)
	P (x,y)	
(x-1, y-1)		(x+1, y+1)

These points, together with the 4-neighbors, are called the 8-neighbors of p, denoted by $N_8(p)$.

(x-1, y+1)	(x, y-1)	(x+1, y-1)
(x-1, y)	*P (x,y)*	(x+1, y)
(x-1, y-1)	(x, y+1)	(x+1, y+1)



Adjacency and Connectivity: Let *V*: a set of intensity values used to define adjacency and connectivity. In a binary image, *V* = {1}, implies adjacency of pixels with value 1. In a gray-scale image, *V* contains more elements, for example, *V* = {180, 181, 182, …, 200}. If the possible intensity values 0 – 255, *V* set can be any subset of these 256 values.

Types of Adjacency
1. 4-adjacency
2. 8-adjacency
3. m-adjacency(mixed)

m-adjacency:
Two pixels *p* and *q* with values from *V* are m-adjacent if :
- *q* is in $N_4(p)$ or
- *q* is in $N_D(p)$
- the set $N_4(p) \cap N_4(q)$ has no pixel whose values are from V (no intersection)

Mixed adjacency is a modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used.

0	1	1		0	1	1		0	1	1
0	1	0		0	1	0		0	1	0
0	0	1		0	0	1		0	0	1

Arrangement of pixels **8-adjacent** ***m*-adjacency.**

Digital Path: A digital path (or curve) from pixel p (x,y) to pixel. $q(s,t)$ is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1), ..., (x_n, y_n) where $(x_0,y_0) = (x,y)$ and $(x_n, y_n) = (s,t)$ and pixels (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$

- n is the length of the path
- If $(x_0,y_0) = (x_n, y_n)$, the path is closed.
- We can specify 4-, 8- or m-paths depending on the type of adjacency specified.

Connectivity:
Let S represent a subset of pixels in an image. Two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S. For any pixel p in S, the set of pixels that are connected to it in S is called a *connected component* of S. If it only has one connected component, then set S is called a *connected set*.

Regio **:** Let R be a subset of pixels in an image, we call R a region of the image if R is a connected set.

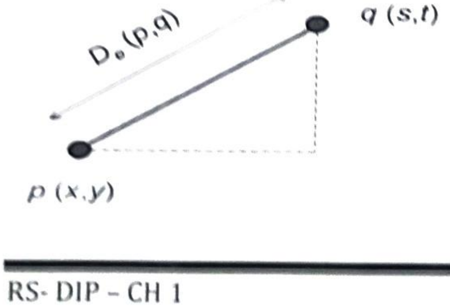
Boundary: The *boundary* (also called *border* or *contour*) of a region R is the set of pixels in the region that have one or more neighbors that are not in R. If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns in the image. This extra definition is required because an image has no neighbors beyond its borders

DISTANCE MEASURES:
Given pixels p, q and z with coordinates (x, y), (s, t), (u, v) respectively, the distance function D has following properties:

a. $D(p, q) \ge 0$ $[D(p, q) = 0$, iff $p = q]$
b. $D(p, q) = D(q, p)$
c. $D(p, z) \le D(p, q) + D(q, z)$

The *Euclidean Distance* between p and q is defined as:

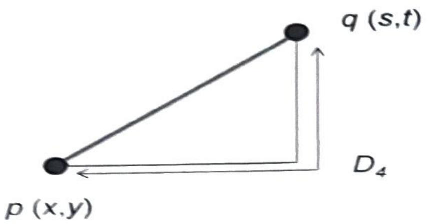
$$D_e(p,q) = [(x - s)^2 + (y - t)^2]^{1/2}$$



- *City block distance:* The D_4 *distance* between p and q is defined as:

$$D_4(p,q) = |x-s| + |y-t|$$

Pixels having distance D4 from (x,y) less than or equal to some value or r form diamond centered at (x,y)

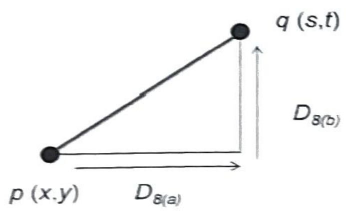


		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Chess board distance: The D_8 *distance* (between p and q is defined as:

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

Pixels having a D_8 distance from (x,y), less than or equal to some value r form a square Centered at (x,y)



2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

$D_8 = \max(D_{8(a)}, D_{8(b)})$

is defined as the shortest m-path between the points.

Distance Measures:

In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.

INTRODUCTION TO MATHEMATICAL OPERATIONS

Array Vs Matrix Operation:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product operator

$$A .* B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix product operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Linear vs Nonlinear operation:

$$H[f(x,y)] = g(x,y)$$

$$H\left[a_i f_i(x,y) + a_j f_j(x,y)\right]$$

$$= H\left[a_i f_i(x,y)\right] + H\left[a_j f_j(x,y)\right]$$ Additivity

$$= a_i H\left[f_i(x,y)\right] + a_j H\left[f_j(x,y)\right]$$ Homogeneity

$$= a_i g_i(x,y) + a_j g_j(x,y)$$

H is said to be a **linear operator.** H is said to be a **nonlinear operator** if it does not meet the above qualification.

Arithmetic Operations:

Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

s(x,y) = f(x,y) + g(x,y)
d(x,y) = f(x,y) – g(x,y)
p(x,y) = f(x,y) × g(x,y)
v(x,y) = f(x,y) ÷ g(x,y)

Set and Logical Operations:

Let A be the elements of a gray-scale image. The elements of A are triplets of the form (x, y, z), where x and y are spatial coordinates and z denotes the intensity at the point (x, y).

$$A = \{(x,y,z) \mid z = f(x,y)\}$$

The complement of A is denoted A^c

$$A^c = \{(x,y,K-z) \mid (x,y,z) \in A\}$$

$K = 2^k - 1; k$ is the number of intensity bits used to represent z

The union of two gray-scale images (sets) A and B is defined as the set

$$A \cup B = \{\max_z(a,b) \mid a \in A, b \in B\}$$

TABLE 2.2
Affine transformations based on Eq. (2.6-23)

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v\cos\theta - w\sin\theta$ $y = v\cos\theta + w\sin\theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$