RS - DIP - VI - M - I

MORPHOLOGICAL IMAGE PROCESSING — UNIT - 6

Morphology :-

It is a branch of biology that deals with the form & structure of animals & plants.

Mathematical Morphology :-

It is a tool for extracting image components, that are useful in the representation & description of region shape (shape analysis) both in binary & gray scale images.

Uses :-

→ Image pre-processing
> Noise removal, shape simplification

→ Enhancement of object structure
> skeletonizing, thinning, thickening, convex hu

→ Object Segmentation

→ Quantitative description of objects area, perimet

BASIC CONCEPTS OF SET THEORY :

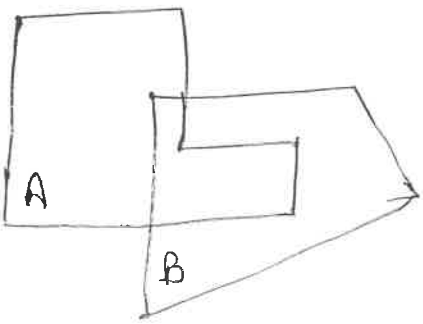
The language of mathematical morphology is Set Theory. It is an unified & powerful approach numerous image processing problems. In binary images, the set elements are members of the 2-D integer space $- Z^2$, where each element (x, y), is a co-ordinate of a black (or white) pixel in the image.

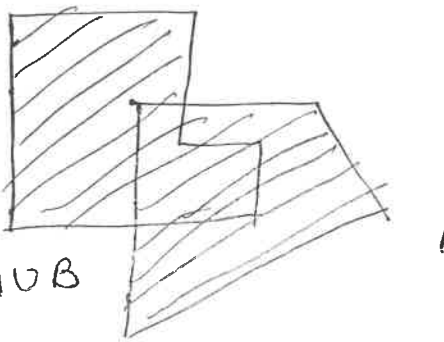
- Let the elements of set A be $\{a_1, a_2\}$. then we write $a \in A$.
- If a is not an element of 'A', we write $a \notin A$
- The set with no elements is called null set or empty set & denoted by symbol ϕ.
- A set is specified by the contents of two brac $\{\cdot\}$

The elements of the sets with which we are concerned are the co-ordinates of pixels represen objects or other features of an image.

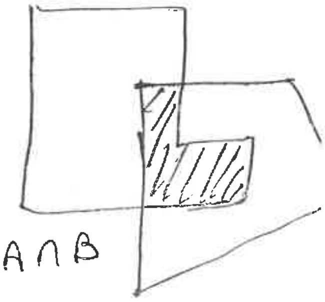
- If every element of set A is also an element of set B, then A is said to be subset of B denoted as $A \subseteq B$
- The union of two sets A & B is denoted as $C = A \cup B$
- The intersection of two sets A & B is denoted as $C = A \cap B$.
- Two sets A & B are said to be disjoint or mutually exclusive if they have no element in common $A \cap B = \phi$

- The difference of two sets A & B is denoted as A − B, is defined as
$$A - B = \{ w / w \in A,\ w \notin B \} = A \cap B^c$$
- The reflection of set B, denoted by $\hat{B}$ is defined as $\hat{B} = \{ w / w = -b, \text{ for } b \in B \}$
- The translation of set A by point $z = \{z_1, z_2\}$ is denoted (A_z) is $(A_z) = \{ c / c = a + z, \text{ for } a \in A \}$

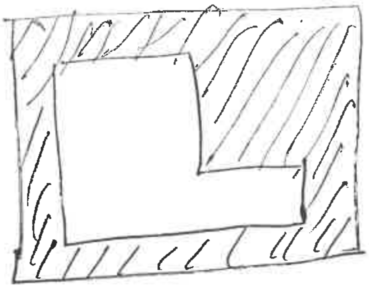




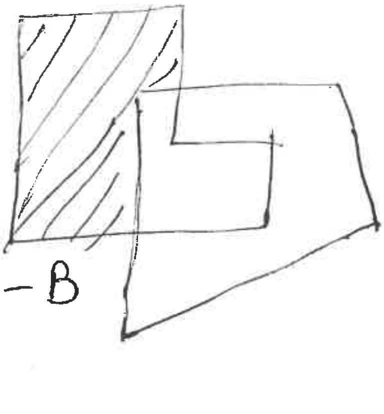
$A \cup B$



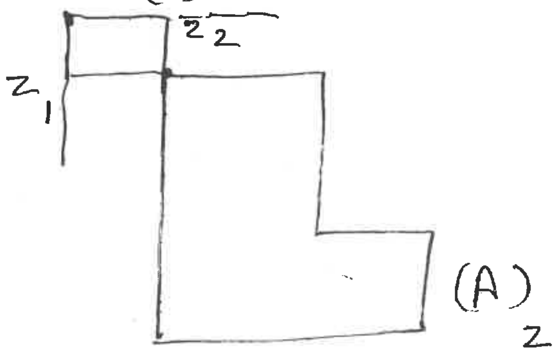
$A \cap B$



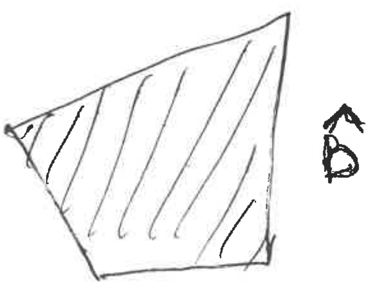
$(A)^c$



$A - B$



reflection



translation

BINARY IMAGES

Logic Operations Involving binary pixels in an image :-

The principal logic operations used in image processing are AND, OR, NOT (COMPLEMENT) these operations are functionally complete & are performed on a pixel by pixel basis. Other important logic operations are XOR, NAND. logic operations are just a private case for a binary set operations., such as : AND - Intersection OR - union, NOT - complement

Table represents the truth tables of logic operations

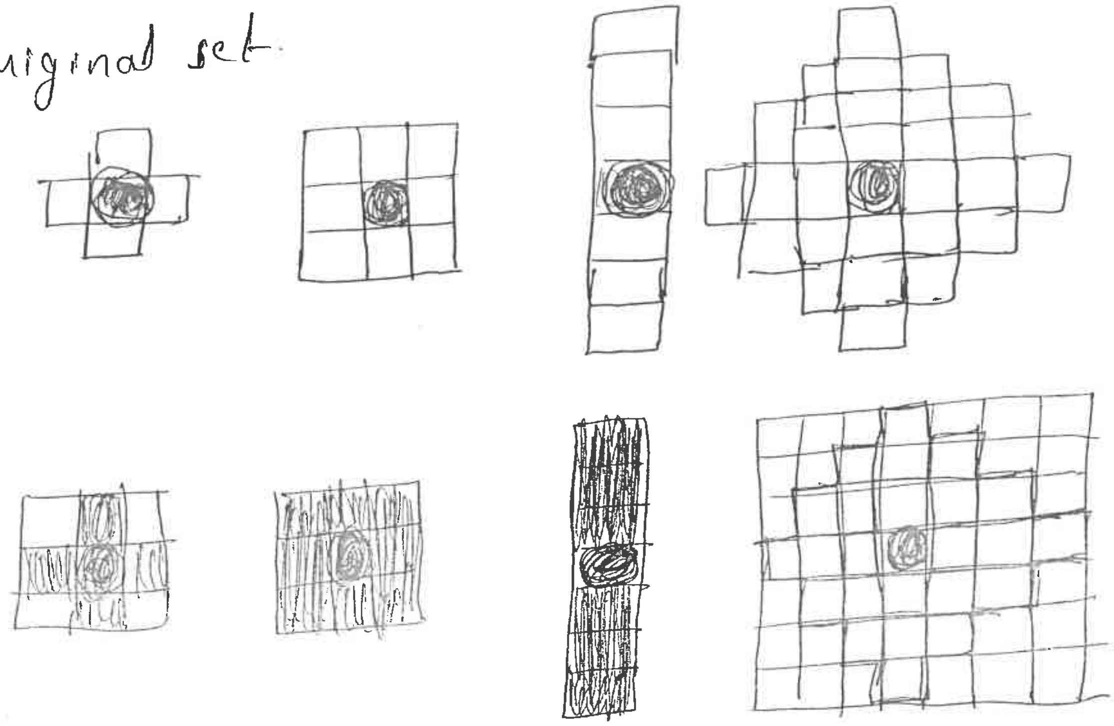
A	B	A AND B	A OR B	A XOR B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

Here black indicates binary image pixel 1
white indicates binary image pixel 0.

A AND B (A·B) & A ∩ B,
A OR B (A+B) & A ∪ B.
} can be used interchangeably

STRUCTURING OF ELEMENTS:-

Fig shows simple set o structuring elements Computer Implementation requires that set A be converted to rectangular array by adding back-ground elements. The background border is made large enough to accomodate the entire structuring element when its origin is on the border of the original set.



MORPHOLOGICAL OPERATORS:

There are basically two types of morphological operators

DILATION - GROW

EROSION - SHRINK

1) <u>DILATION</u> :- Dilation is used for expanding an element A by using structuring element B.
Dilation of A by B is defined by the following eq

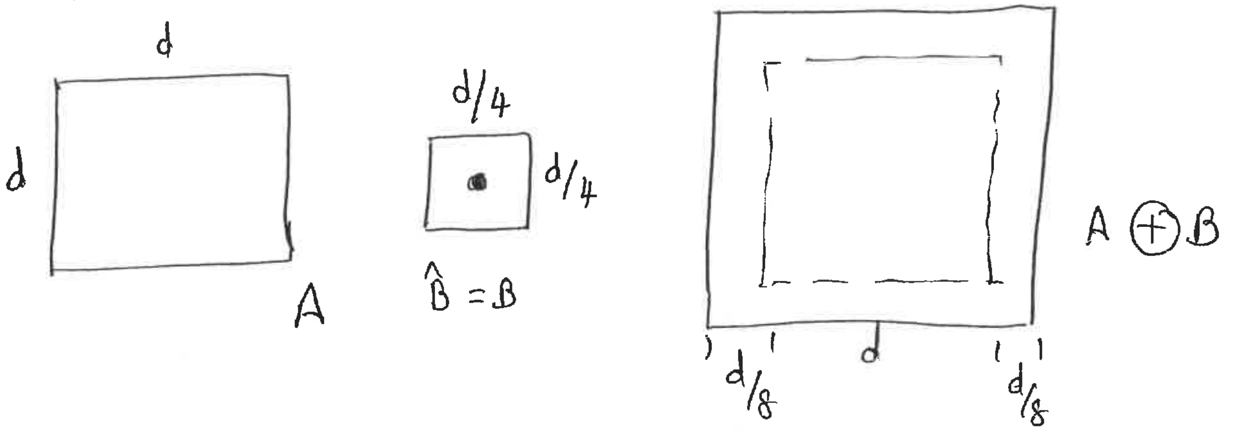
$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq 0 \}$$

The eq indicates that the dilation of A by B is the set of all points z such that reflection of B about its origin, translated by z, is combined in A

The dilation of A by B is the set of all displacements z, such that $\hat{B}$ & A overlap by atleast one element. Based on this interpretation the above eq is written as

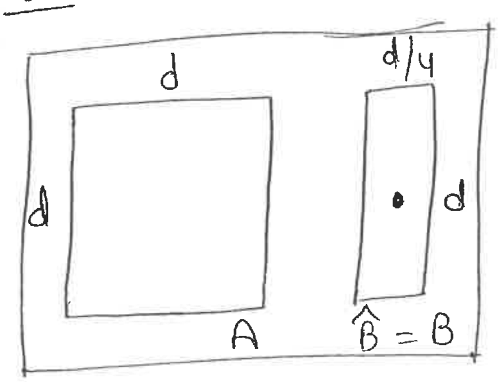
$$A \oplus B = \{ z \mid [(\hat{B})z \cap A] \subset A \}$$

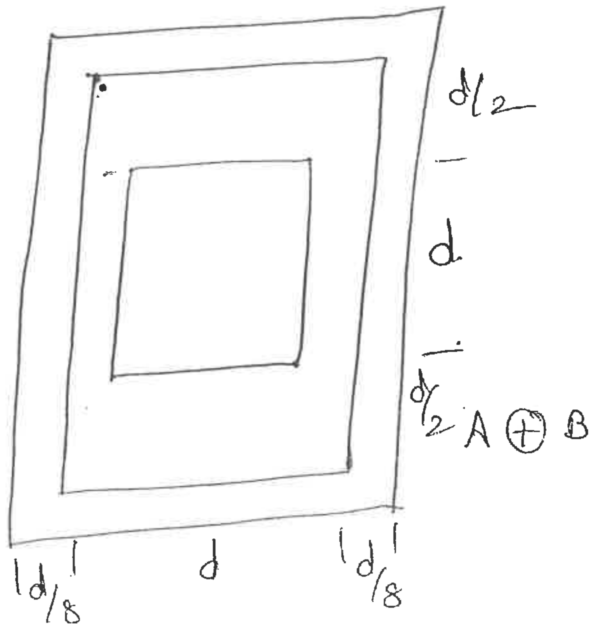
Ex :- Fig (a) represents a set A. Fig (b) represents a square structuring element. Fig (3) represents dilation of A by B.



The dashed line shows original set for reference the solid line shows the limit beyond which any further displacements of the origin would cause the intersection to be empty. All points inside the boundary constitute the dilation of A by B. Dilation is used for bridging the gaps. The maximum length of the breaks is known to be two pixels.

Ex:-





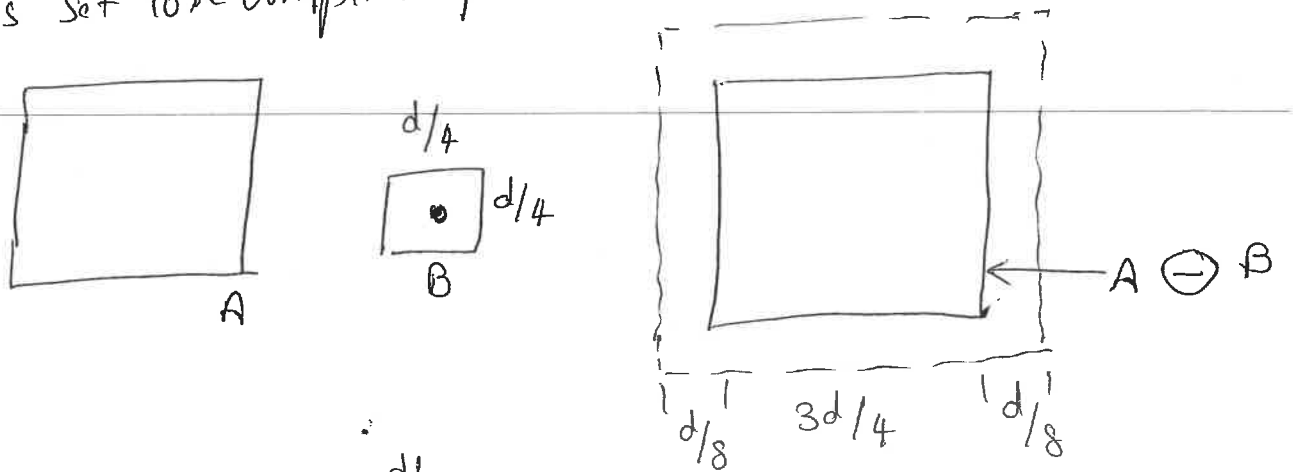
EROSION :-

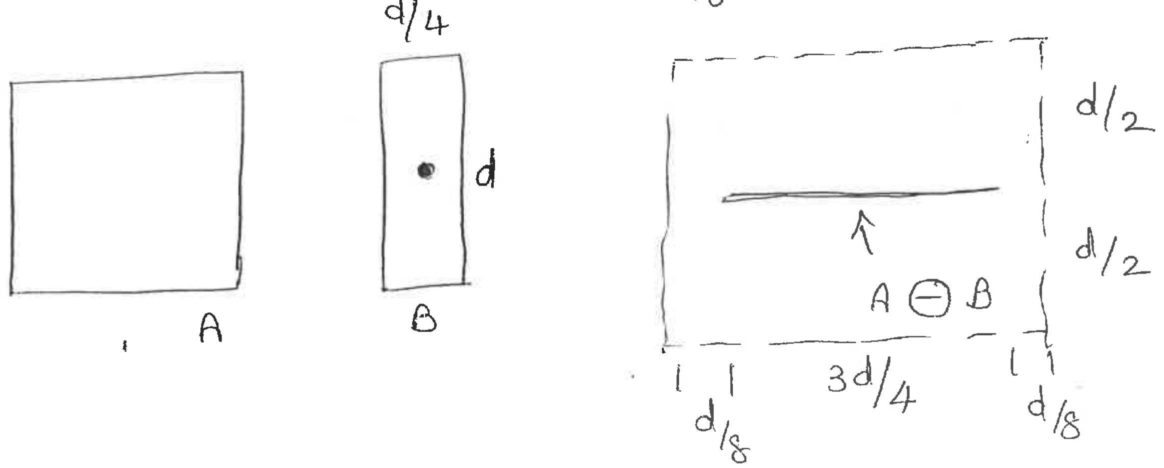
Erosion is used for shrinking of element A by using element B. Erosion for sets A & B in Z^2 is defined by the following equation

$$A \ominus B = \{ z \mid [(B)_z \subseteq A \}$$

This eq. indicates that the erosion of A by B is the set of all points z such that B, translated by z, is combined in A.

Ex :- Set A is shown by dashed line. The boundary of the shaded region shown the limit beyond which further displacement of the origin 'B' would cause this set to be completely contained in A.





Duality b/w dilation & erosion :-

Dilation & erosion are duals of each other w.r.t set complementation & reflection.

i.e $$(A \ominus B)^c = A^c \oplus \hat{B}$$

Eq indicates that erosion of A by B is the complement of the dilation of A^c by B

One of the simplest uses of erosion is for eliminating irrelevant details from a binary image.

Proof :-
Starting with the definition of erosion, it follows that $(A \ominus B)^c = \{ z \mid (B)_z \subseteq A \}^c$

if set B is contained in set A, then $(B)_z \cap A^c = \phi$ in which case the preceeding expression becomes
$(A \ominus B)^c = \{ z \mid (B_z) \cap A^c = \phi \}^c$

But the complement of the set of z's that satisfy $(B)_z \cap A^c \neq \phi$

Therefore
$(A \ominus B)^c = \{ z \mid (B)_z \cap A^c = \phi \} = \hat{A} \oplus \hat{B}$

OPENING & CLOSING

Opening - smoothes contours, eliminates protrusions

Closing - smoothes sections of contours, fuses narrow breaks & long thin gulfs, eliminates small holes & fill gaps in contours.

These operations are dual to each other

These operations can be applied few times, but has effect only once

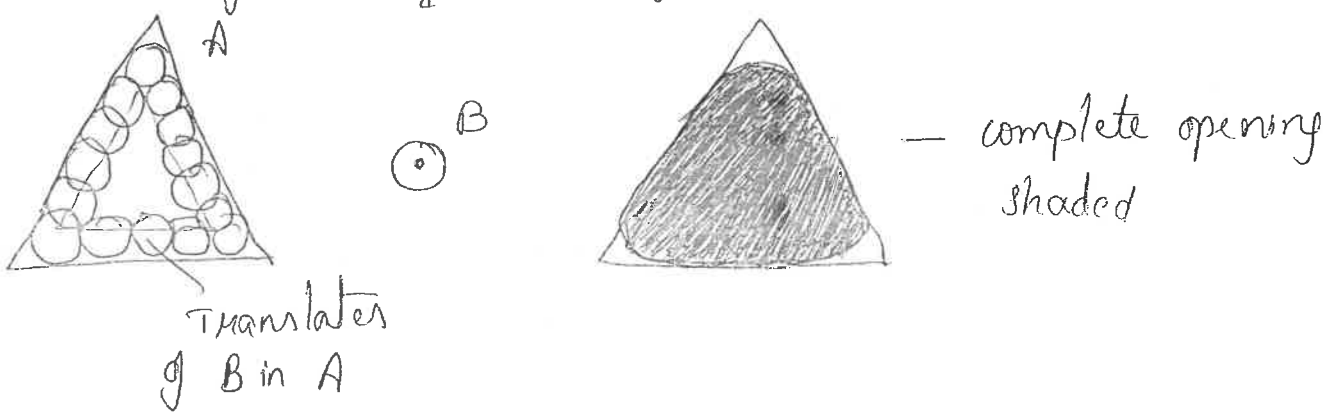
Opening :-

The opening of set 'A' by structuring element 'B', denoted as $A \circ B$ is defined as

$$A \circ B = (A \ominus B) \oplus B$$

Thus the "Opening A by B is the erosion of A by B followed by a dilation of the result by B"

Fig shows the geometric interpretation of opening operation. We can view structuring element 'B' as a 'rolling ball'. The boundary of $A \circ B$ is then established by the points in B that reaches the farthest into the boundary of A as B is rolled around the inside of this ~~of~~ boundary.



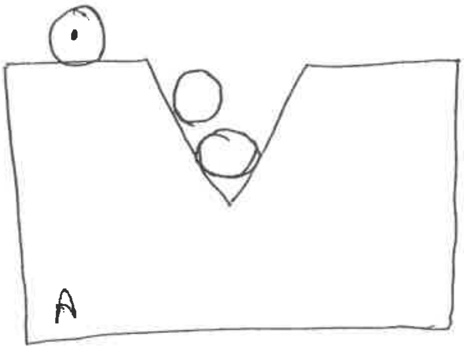
Closing :-

Closing is opposite to opening. It generally fuse narrow breaks & long thin gulfs. It eliminates small holes & fills gaps in contours.

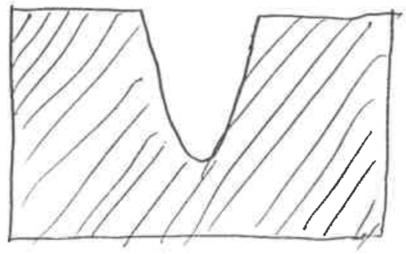
The "closing" of a set A by structuring element 'B' is denoted by $A \bullet B$. It is given by the formula

$$A \bullet B = (A \oplus B) \ominus B$$

Closing of A by B is simply dilation of A by B followed by the erosion of the result by B.
The geometric representation of closing is same as opening. But now the structuring element is rolled outside boundary of A



$A \bullet B$



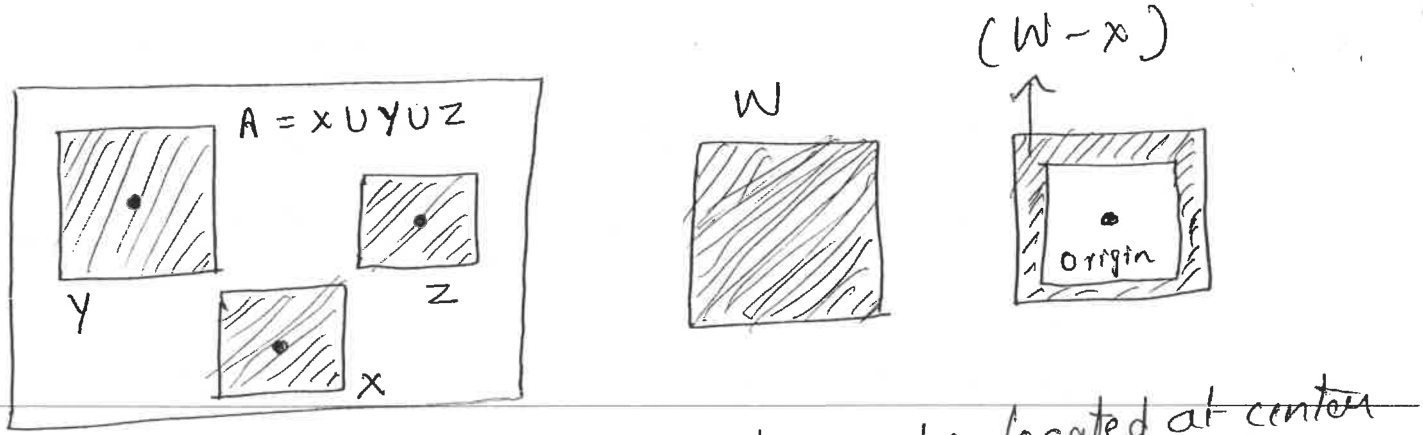
Relation between Opening & Closing :-

The opening & closing are duals of each other w.r.t set complementation & reflection i.e.,

$$(A \bullet B)^c = (\hat{A^c} \circ \hat{B})$$

HIT-OR-MISS TRANSFORMATION :-

It is a basic tool that is used for shape detection. This hit-or-miss transformation is used for finding a particular location of shape with the consideration of background also.



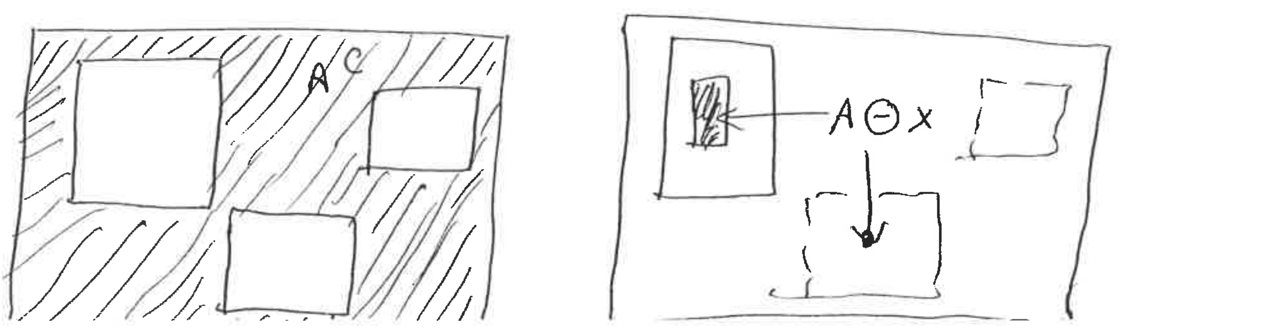
Let the origin of each shape be located at center of gravity. If we want to find the location of a shape, say -X, at (larger) image, say -A:

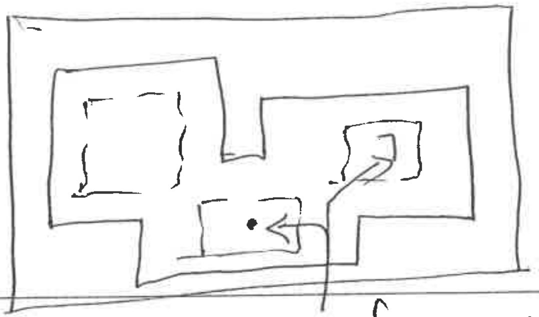
- Let X be enclosed by a small window say -W. the local background of x w.r.t W is defined as the set difference (W - x)

Apply erosion operator of A by X, to obtain the location of X, such that X is completely contained in A. This can be viewed geometrically as the set of all locations of the origin of X at which X found a match (hit) in A.

'A' consists of only three disjoint sets X, Y & Z.

Fig shows complement of A by the local background set (W - x). Outer shaded region is part of erosion.





$(A \ominus X) \cap (A^c \ominus (W - X))$

$A^c \ominus (W - X)$

If B denotes the set composed of X & its background, the match/hit (or set of matches/hits) of B in A is

$$A \circledast B = (A - X) \cap [A^c \ominus (W - X)]$$

consider generalized notation $B = (B_1, B_2)$

$B_1 = X$; $B_2 = (W - X)$

The above eq contains all the origin points at which simultaneously, B_1 found a hit in complement of A.

$$A \circledast B = (A \ominus B_1) - (A \oplus B_2)$$

BASIC MORPHOLOGICAL ALGORITHMS:-

The morphological algorithms are useful for

1) boundary extraction
2) Region filling
3) Extraction of connected components
4) Convex hull
5) Thinning
6) Thickening (7) Skeleton

1) Boundary Extraction :-

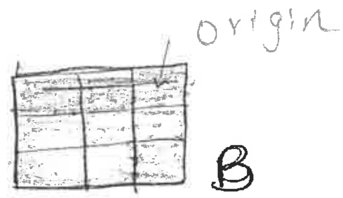
-> the boundary is denoted by $\beta(A)$

-> First erode A by B, then make set difference b/w A & erosion.

-> The thickness of the contour depends on the size of constructing object B.

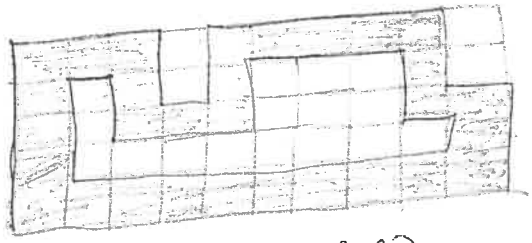
$$\boxed{\beta(A) = A - (A \ominus B)}$$

A



B

$A \ominus B$



$\beta(A)$

2) Region Filling :-

This algorithm is based on a set of dilations, complementation & intersections. 'P' is a point inside the boundary with a value of '1'. Now fill the entire region with 1s.

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

The process stops when $X(k) = X_{k-1}$

(3) Extraction of connected components :-

This algorithm extracts a component by selecting a point on a binary object A. It works similar to region filling, but this time we use in the conjunction the object A, instead of its complement

$$X_k = (X_{k-1} \oplus B) \cap A$$

(4) Convex hull :-

A set A is said to be convex if a straight line segment joining any two points in A lies entirely within A. The convex hull H of set S is the smallest convex set containing S. Convex deficiency is the set difference H - S. It is useful for object description

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \qquad i = 1, 2, 3, 4 \quad k = 1, 2, 3 \ldots \quad D^i = X_k^i$$

with $X_0^i = A$. when procedure converges ($X_k^i = X_{k-1}^i$),

$\therefore$ Convex hull $C(A) = \bigcup_{i=1}^{4} D^i$

(5) Thinning :- The thinning of a set A by a structuring element B, can be defined by term of hit & miss transform:

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

A more useful expression for thinning A symmetrical is based on a sequence of structuring elements:

$\{B\} = \{B^1, B^2, B^3 \ldots\ldots B^n\}$

where B^i is a rotated version of B^{i-1}. Using this concept we define thinning by a sequence of structuring elements:

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

(b) Thickening :-

Thickening is a morphological dual of thinning. Thickening can be defined as a sequential operation

$$A \odot B = A \cup (A \circledast B)$$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

The structuring elements used for thickening have the same form as in thinning, but with all 1's & 0's interchanged.

A separate algorithm is often used for thickening. Instead of the usual procedure is to thin the background of the set in question & then complement the result.

i.e to thicken set A → $C = C'$
thin C
Form C^c.

Depending on the nature of A, this procedure may result in some disconnected points. Therefore thickening by this procedure usually requires a simple post-processing step to remove disconnected points.

7) <u>Skeleton:-</u>

The skeleton $S(A)$ of a set A is intuitively defined as follows:

a) If z is a point of $S(A)$ & $D(z)$ is the largest disk centered in z & contained in A, this disk is called "maximum disk".

b) The disc $(D)_z$ touches the boundary of A at two or more different places.

The skeleton of A is defined by terms of erosion & openings:

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$

where B is the structuring element & $(A \ominus kB)$ indicates k successive erosions of A:

$$(A \ominus kB) = (\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B$$

k times & K is the last iterative step before A erodes to an empty set.

$S(A)$ can be obtained as union of skeleton subsets $S_K(A)$.

$$K = \max\{k \mid (A \ominus kB) \neq \phi\}$$

A can also be reconstructed from subsets $S_K(A)$ by using the eq

$$A = \bigcup_{K=0}^{K} (S_K(A) \oplus KB)$$

where $(S_K(A) \oplus KB)$ denotes k successive dilations of $S_K(A)$ i.e

$$(S_K(A) \oplus KB) = ((\cdots((S_K(A) \oplus B) \oplus B \oplus \cdots) \oplus$$

GRAY-SCALE IMAGES :-

In gray scale images, on the contrary to binary images, we deal with digital image functions of the form $f(x,y)$ as an input image & $b(x,y)$ as a structuring element.

$f(x,y)$ & $b(x,y)$ are functions that assign gray level value to each distinct pair of co-ordina

For ex: the domain of gray values can be 0-255, whereas 0 is black, 255 is white

<u>Dilation - Gray Scale :-</u>

Dilation of 'f' by a flat structuring element 'b' at any location (x, y) is defined as the max. value of the image in the window outlined by $\hat{b}$ when the origin of $\hat{b}$ is at (x, y)

Equation for gray scale dilation is

$$[f \oplus b](x, y) = \max_{(s, t \in b)} \{ f(x-s, y-t) \}$$

D_f & D_b are domains of 'f' & 'b'

the reflection of structured element

$$\hat{b} = b(-x, -y)$$

<u>Erosion - Gray Scale :-</u>

Erosion of 'f' by a flat structuring element 'b' at any location (x, y) is defined as the minimum value of the image in the window

∴ Gray scale erosion is defined as

$$[f \ominus b](x, y) = \min_{(s, t \in b)} \{ f(x+s, y+t) \}$$

where (s + x), (t + y) have to be in the domain of 'f' & x, y have to be in the domain of 'b'.

Opening & Closing : Gray Scale

The algorithms for opening & closing of gray scale images are similar to binary algorithms

The eq. for opening is expressed as

$$f \circ b = (f \ominus b) \oplus b$$

In opening of gray scale image, we remove small light details, while overall gray levels & larger bright features are relatively undisturbed.

The eq for closing is expressed as

$$f \bullet b = (f \oplus b) \ominus b$$

In closing of gray scale image, we remove small dark details, while overall gray levels & larger dark features are relatively undisturbed

Duality b/w opening & closing :-

$$(f \bullet b)^c = f^c \circ \hat{b}$$

$$(f \circ b)^c = f^c \bullet \hat{b}$$

Some Applications of Gray-scale Morphology:-

(1) Morphological Smoothing:-

It is performed by applying opening algorithm followed by closing on an image. The net result of these two operations is to remove or attenuate both bright & dark artifacts or noise.

(2) Morphological Gradient:-

Dilation & erosion are used to compute the morphological gradient of an image, denoted by g

$$g = (f \oplus b) - f(\ominus b)$$

It is used to highlight sharp gray level transitions in the i/p image.

(3) Top-hat Morphology:-

It is denoted by $h = f - (f \circ b)$.

The shape of the structuring element may be cylindrical or parallel piped function with a flat top. It is useful for enhancing detail in the presence of shading.

(4) Textural Segmentation:- This is used to find the boundary between different image regions based on their textural content

The algorithm consists of the following steps;

→ Close the I/P image by using successively larger structuring elements

→ Perform single opening

→ use simple threshold that yields the boundary between the textural regions

(8) <u>Granulometry</u> :-

It is the field that helps in determining the size of distribution of particles in an image. If the particles are lighter than background, we can use a <u>morphological</u> <u>approach</u> to determine size distribution & construct a histogram.