Name - Tarun Upadhyay
Roll No. - 1961181
Sec - B

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Assignment - 1

Ans 1 → Asymptotic notations are the mathematical notations used to describe the running time of an algo when the i/p tends towards a particular value or a limiting value.

There are mainly 3 asymptotic notation.

a) Big - O notation ⟹
It represents the upper bound of running time of an algo. This notation is called as upper bound of the algo. or a worst case of an algo.
$O(g(n)) = \{ f(n)$ there exist positive constants $c \in n$ such that $0 \le f(n) \le g(n)$ for all $n \ge n_0$ where $c > 0$ and $n > n_0$.

Eg.
$f(n) = 3\log n + 100$
$g(n) = \log n$
$3\log n + 100 <= c * \log(n)$
$c = 1 < 0$ and $n \ge 2$
(undefined at $n = 1$)

(b) Big omega (Ω) notation ⇒

It represents the lower bound of the running time of an algo. This notation is known as lower bound of an algo, or best case of an algo.

$\Omega(g(n)) = \{ f(n) :$ there exist +ive constant c & n_0 such that $0 \le cg(n) \le f(n) \forall n, n \ge n_0$

e.g.,

$f(n) = 3n+2$

$cg(n) \le f(n)$

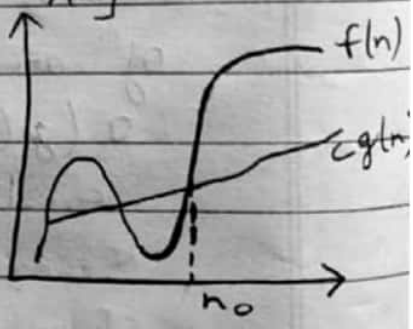
[c = constant, $g(n) = n$]

$cn \le 3n+2$

$cn - 3n \le 2$

$n(c-3) \le 2$

$n \le \frac{2}{c-3}$



if we assume $c = 4$, then $n_0 = 2$

$c = 4, n_0 = 2$

(c) Theta (Θ) notation ⇒

It encloses the function from above & below. Since, it represents the upper & lower bound of running time of an algo.

This is known as tight bounds of an algo, or an average case

Of algo

$\theta(g(n)) = \{ f(n)$ there exits positive constant c_1, c_2 & n_0 such that $0 \le c_1 * g(n) \le f(n) \le c_2 * g(n)$ $\forall n > n_0$

e.g.

$f(n) = 5n^3 + 16n^2 + 3n + 8$

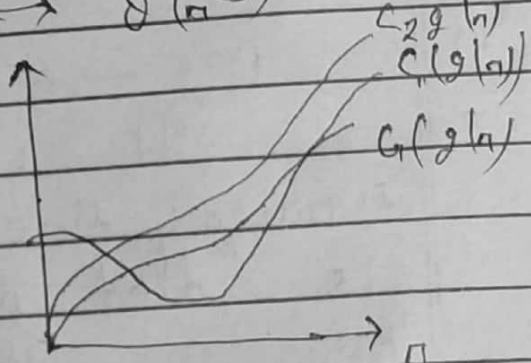
$5n^3 \le (n^3 + 16n^2 + 3n + 8) n^3$

$5n^3 \le (5 + 16 + 3 + 8) n^3$

$5n^3 \le f(n) \le 32n^3$

$c_1 = 5, \; c_2 = 32, \; n_0 = 1$

$f(n) \leftrightarrow \theta(n^3)$



Ans 2 $\Rightarrow$ $i = 1, 2, 4, 8, 16, \ldots, K^n$

$G_n = a_n^{n-1}$

$(n+1 = 1(2)^{K-1}$

$n = 2^{K-1}$

$\log_2 n = (K-1) \log_2 2$

$K = \log_2 n + 1$

$\theta(n) = \log n$

Ans 3 $\Rightarrow$ $T(n) = 3T(n-1)$

$T(n-1) = 3T(n-2)$

$T(n) = 3 \times 3T(n-2)$

$T(n-2) = 3T(n-3)$

$T(n) = 3 \times 3 \times 3T(n-3)$

$T(n) = 3^3 T(n-3)$

$T(n-3) = 3T(n-4)$

$T(n) = 3^3 \times 3T(n-4)$

$T(n) = 3^4 \times T(n-4)$

⋮

⋮

General form –

$T(n) = 3T(n-i) \quad \ldots (i) \qquad [T(0) = 1]$

$T(n-i) = T(0)$

$n - i = 0$

$n = i$

Putting $n = i$ in eq. (i) :

$T(n) = 3^n T(n-n)$

$T(n) = 3^n T(0) \qquad [T(0) = 1]$

$T(n) = 3^n$

$T(n) = O(3^n)$

Ans 4 ⇒ $T(n) = 2T(n-1) - 1$

$T(n-1) = 2T(n-2) - 1$

$T(n) = 2 \times (2T(n-2) - 1) - 1$

$T(n) = 2^2 \cdot T(n-2) - 2 - 1$

$T(n-2) = 2T(n-3) - 1$

$T(n) = 2^2(2T(n-3)-1) - 2 - 1$

$T(n) = 2^3 T(n-3) - 2^2 - 2 - 1$

$T(n-3) = 2T(n-4) - 1$

$T(n) = 2^3(2T(n-4)-1) - 2^2 - 2 - 1$

$T(n) = 2^4 T(n-4) - 2^3 - 2^2 - 2 - 1$

⋮

General form -

$T(n) = 2^i T(n-i) - (2^{i-1} + 2^{i-2} + \dots + 1)$

$T(n-i) = T(0)$

$n - i = 0$

$n = i$

$T(n) = 2^n T(0) - (1 + 2 + 2^2 + 2^3 + \dots 2^{n-1})$

$[T(0) = 1]$

$T(n) = 2^n (1) - (1 + 2 + 2^2 + \dots 2^{n-1})$

$T(n) = 2^n - 1 \frac{(2^{n-1})}{2-1}$

$T(n) = 2^n - 2^{n-1} + 1$

$T(n) = 2^{n-1}(2-1) + 1$

$T(n) = 2^{n-1} + 1$

$T(n) = O(2^n)$

Pass #	No. of steps (k)	s	i
0		0	1

1	1	1
2	3	2
3	6	3
4	10	4
5	15	5
6	21	6
⋮	⋮	⋮
K step	n	

$T(n) = O(K)$

$= 0, 1, 3, 6, 10, \ldots n$

$S_n = 1 + 3 + 6 + 10 + 15 + \ldots + n$

$S_n \quad 1 + 3 + 6 + 10 + \ldots + (n-1) + n$

$0 = 1 + 2 + 3 + 4 + 5 + \ldots - n$

$n = 1 + 2 + 3 + 4 + \ldots K$ step

$n = \frac{K}{2}[2(1) + (K-1)1]$

$2n = K[2 + K - 1]$

$2n = K^2 + K$

$2n = (K + \frac{1}{2})^2 - (\frac{1}{2})$

$2n + (\frac{1}{2})^2$

$(K + \frac{1}{2})^2$

$K + \frac{1}{2} = \sqrt{2n + (1/2)^2}$

$$K = \sqrt{2n + (1/2)^2}$$
$$= \frac{1}{2}$$

$$T(n) = T(K)$$
$$T(n) = T\left(\sqrt{2n + (1/2)^2} - 1/2\right)$$
$$T(n) = O\sqrt{n}$$

Ans 6 ⇒ Since, i is moving from 1 to $\sqrt{n}$ with linear growth so

$$T(n) = O(\sqrt{n})$$

Ans 7 ⇒ $O(n \log n \log n)$

$$O(n (\log n)^2$$

Ans 8 ⇒ $T(n) = T(n-1) + n^2$

$$T(n) = T(n-2) + n^2 + (n-1)^2$$
$$T(n) = T(n-3) + n^2 + (n-1)^2 + (n-2)^2$$

⋮

General Term -

$$T(n) = T(n-i) + n^2 + (n-1)^2 + (n-2)^2 + \dots (n-i)^2$$
$$T(n-i) = T(1)$$
$$n = i + 1$$
$$n - 1 = i$$

$T(n) = T(n-(n-1)) + n^3 + (n-1)^2 + (n-2)^2 + \ldots + (n-(n-1))^2$

$T(n) = T(1) = n^2 + (n-1)^2 + (n-2)^2 + \ldots + 1^2$

$T(n) = 1 + 1^2 + 2^2 + 3^2 + \ldots + n^2$

$T(n) = \frac{n(n+1)(2n+1)}{6}$

$T(n) = O(n^3)$

Ans 9 ⇒ $O(n\sqrt{n})$

Ans 10 ⇒ If $c > 1$ then the exponential c^n for outgrows any term, so that answer is:

n^k is $O(c^n)$

Ans 12 ⇒ $T(n) = T(n-1) + T(n-2) + c$

$T(n-2) \approx T(n-1)$

$T(n) = 2T(n-1) + c$

$T(n-1) = 2T(n-2) + c$

$T(n) = 2(2T(n-2) + c) + c$

$T(n) = 2^2 T(n-2) + 2c + c$

$T(n-2) = 2T(n-3) + c$

$T(n) = 2^3 (2T(n-3) + c) + 2c + c$

$T(n) = 2^3 T(n-3) + 2^2 c + 2c + c$

⋮

General Term –

$$T(n) = 2^i T(n-i) + (2^0 + 2^1 + 2^2 + \dots + 2^{i-1})c$$

$$n - i = 0$$

$$n = i$$

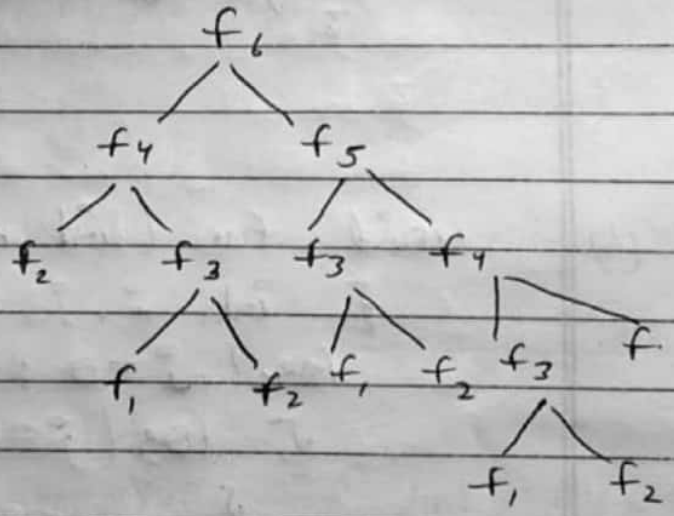
$$T(n) = 2^n T(0) + (2^0 + 2^1 + 2^2 + \dots 2^{n-1})c$$

$$T(n) = 2^n (1) + 2^0 \frac{(2^{n-1} - 1)}{2-1} c$$

$$T(n) = 2^n (1+c) - c$$

$$T(n) = O(2^n)$$

fib. (6)



The max depth is proportional to N, hence the space complexity of fibonacci recursive is O(n).

Ans 12 ⇒ i = 0, 1, 3, 6, 10, 15, ...

j = 0, 1, 2, 3, 4, 5, 6, ...

So, i will go on till n & general formula for K^{th} term is $n = \frac{K(K+1)}{2}$

$\therefore$ T.C = $O(\sqrt{n})$

Ans 13 ⇒

```
void fun()
{
  int i, j;
  for(i=1; i<=n; i++)
  {
    for(j=0; j<=n; j=j*2)
      printf("*");
      printf("\n");
  }
}
```

(b)

```
void fun(int n)
{ int i, j, k;
  for(i=0; i<=n; i++)
  { for(j=0; j<=n; j++)
    {
      for(k=0; k<=n; k++)
      printf("*");
    }
  }
}
```

(c)

```
void Sieve of Eratosthenes(int n)
{
  bool prime[n+1];
  memset(prime, true, sizeof
```

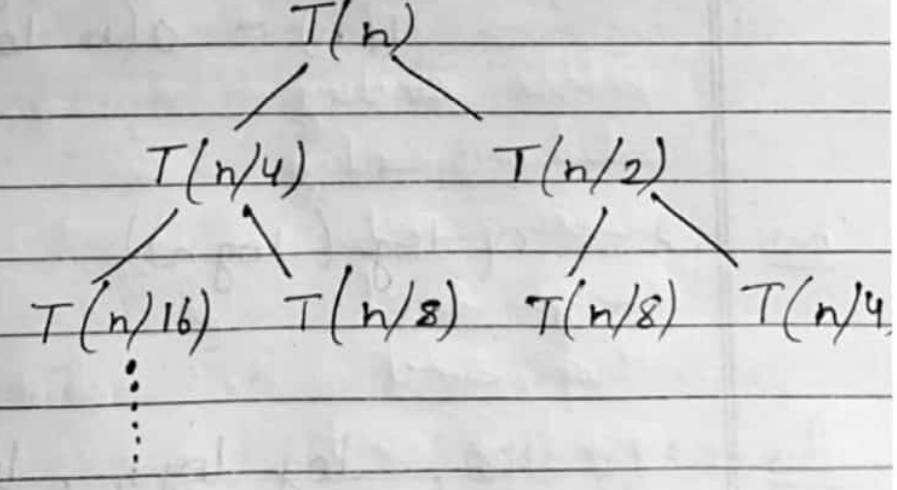
```
(prime));
for (int p=2 ; p*p <=n ; p++)
{
    if (prime[p] = true)
    {
    for (int i = p*p ; i<=n ;
            i+ = p)
        prime[i] = false ;
    }
}
    for (int p=2 ; p <=n; p++)
        if(prime[p])
            cout <<p<< endl;
    }
```

Ans 14 ⇒ $T(1) = c$

$n = n/2$

$T(n/2) = T(n/8) + T(n/4) + c(n^2/4)$

$T(n) = T(n/4) + 2T(n/16) + c(n^2/16 + n^2/4 + n^2)$



$$T(n) = c\left[n^2 + \frac{5n^2}{16} + \frac{25n^2}{256} + \ldots\right]$$

$$T(n) = n^2 c\left[1 + \frac{5}{16} + \frac{5^2}{16^2} + \ldots\right]$$

$$T(n) = O(n^2)$$

Ans 15) for $i = 1$, inner loop is executed n times

for $i = 2$, inner loop is executed $n/2$ times

for $i = 3$, inner loop is executed $n/3$ times

⋮

for $i = n$, inner loop is executed n/n times

Total time $= n + n/2 + n/3 + \ldots n/2$

$= n(1 + 1/2 + 1/3 + \ldots 1/n)$

$= n \log n$

$T(n) = O(n \log n)$

Ans 16) $O(\log(\log n))$

Ans 18) (a) 100, $\log \log n$, $\log n$, $\text{root } n$

$n, n\log n, n^2, 2^n, 2^{2n}, 4^n, n!$

(b) $1, \log(\log(n)), \sqrt{\log n}, \log n, \log(2n)$, $\log(n!), 2\log(n), n, 2n, 4n, n\log(n), n^2, 2(2^n), n!$

(c) $96, \log_8 n, \log_2 n, \log(n!), 5n, n\log_6 n, n\log_2 n, 8n^2, 7n^3, 8^{2n}, n!$

Ans 19 ⇒ Linear Search (A, Key)

```
comp ← 0 , f ← 0
for i = 1 to A.length
    comp ← comp + 1
    if A[i] == Key
    print "Element found"
        f = 1
    if f == 0
    print "Element not found"
    print comp
```

Ans 20 ⇒ Iterative method of insertion sort

```
INSERTION_SORT (A)
for j = 2 to A.length
    Key = A[j]
    i = j - 1
    while i > 0 and A[i] ≠ Key
        A[i+1] = A[i]
```

$j = i - 1$

$A[i+1] = Key$

Recursive Method of insertion sort

INSERTION_SORT (A, n)

if $n \leq 1$

return

INSERTION_SORT (A, n-1)

Key = Arr[n-1] ;

$j = n-2$;

while $j \geq 0$ and $A[j] > Key$

$A[j+1] = A[j]$

$j = j - 1$

$A[j+1] = Key$

Insertion sort considers one input element per iteration & produces a partial solution without considering future elements thats why it is called online sorting

Other sorting algorithm that ha been discussed in lectures are -

Bubble sorting
Selection sorting
Quick sort
Merge sort
Heap sort
Counting sort

Ans 21 ⇒

	Best case	Average case	Worst case
Bubble sort	$\Omega(N)$	$\theta(N^2)$	$O(N^2)$
Selection sort	$\Omega(N^2)$	$\theta(N^2)$	$O(N^2)$
Insertion sort	$\Omega(N)$	$\theta(N^2)$	$O(N^2)$
Merge sort	$\Omega(N\log N)$	$\theta(N\log N)$	$O(N\log N)$
Heap sort	$\Omega(N\log N)$	$\theta(N\log N)$	$O(N\log N)$
Quick sort	$\Omega(N\log N)$	$\theta(N\log N)$	$O(N^2)$
Counting sort	$\Omega(N+K)$	$\theta(N+K)$	$O(N+K)$

Ans 22 ⇒

	In Place	Stable	Online
Bubble sort	Yes	Yes	Yes
Insertion sort	Yes	Yes	Yes
Selection sort	Yes	No	Yes
Merge sort	No	Yes	Yes
Quick sort	Yes	No	Yes
Heap sort	Yes	No	Yes
Count sort	No	Yes	Yes

Ans 23 ⇒ Linear Search –

```
Linear_SEARCH (A, Key)
    found ← 0
    for i = 1 to N
        if A[i] == Key
            found ← 1
```

```
        print " Element found "
        break
    if found == 0
    print " Element not found "
    Time complexity - O(n)
    Space complexity - O(1)
```

Binary Search (Iterative)

```
BINARY_SEARCH ( A, beg, end, key)
    while beg ≤ end
        mid = beg + (end-beg)/2
            if mid == key
                return mid
            if A[mid] < key
                beg = mid + 1
            if A[mid] > key
                end = mid - 1
        return -1
```

Time complexity - $O(\log_2 n)$
Space complexity - $O(1)$

Binary Search (Recursion)

```
BINARY_SEARCH (A, beg, end, key)
    if end ≥ beg
        mid = ( beg + end)/2
        if A[mid] == item
            return mid + 1
        else if A[mid] < item
```

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return BINARY SEARCH (A, mid+1, end, key)

else

return BINARY_SEARCH(A, beg, mid - 1, end)

return -1

Time complexity = $O(\log n)$

Space complexity = $O(1)$

Ans 24 $\Rightarrow T(n) = T(n)2 + C$