Part A: Transfer Functions and Bode Plots

1.1 Problem A.1: $G_1(s) = \frac{10}{s+10}$

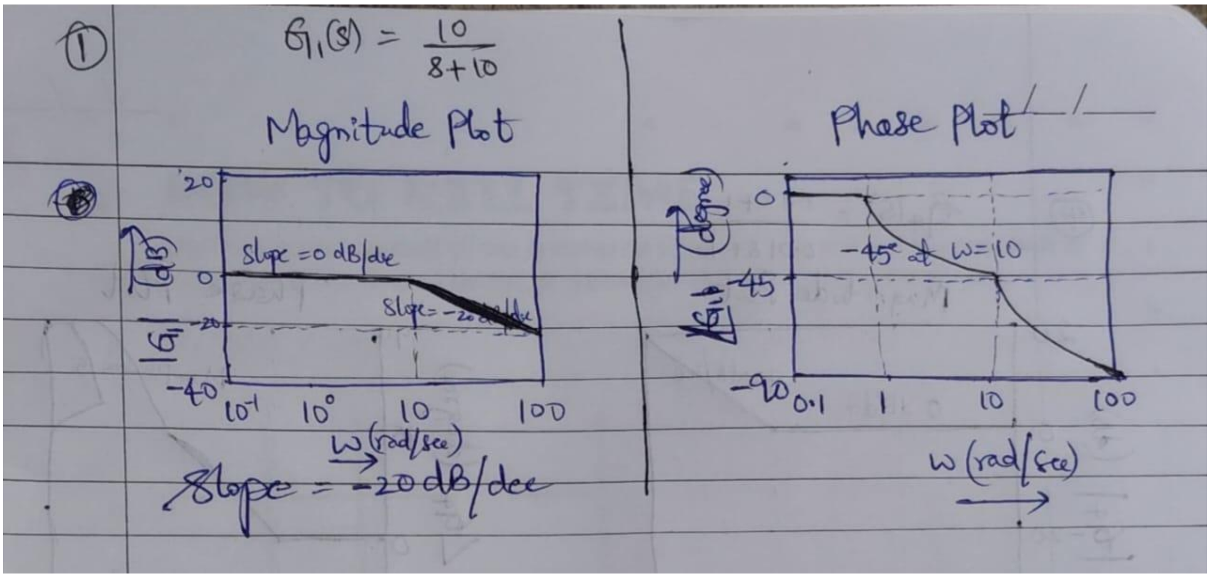
1. Pole and DC gain:

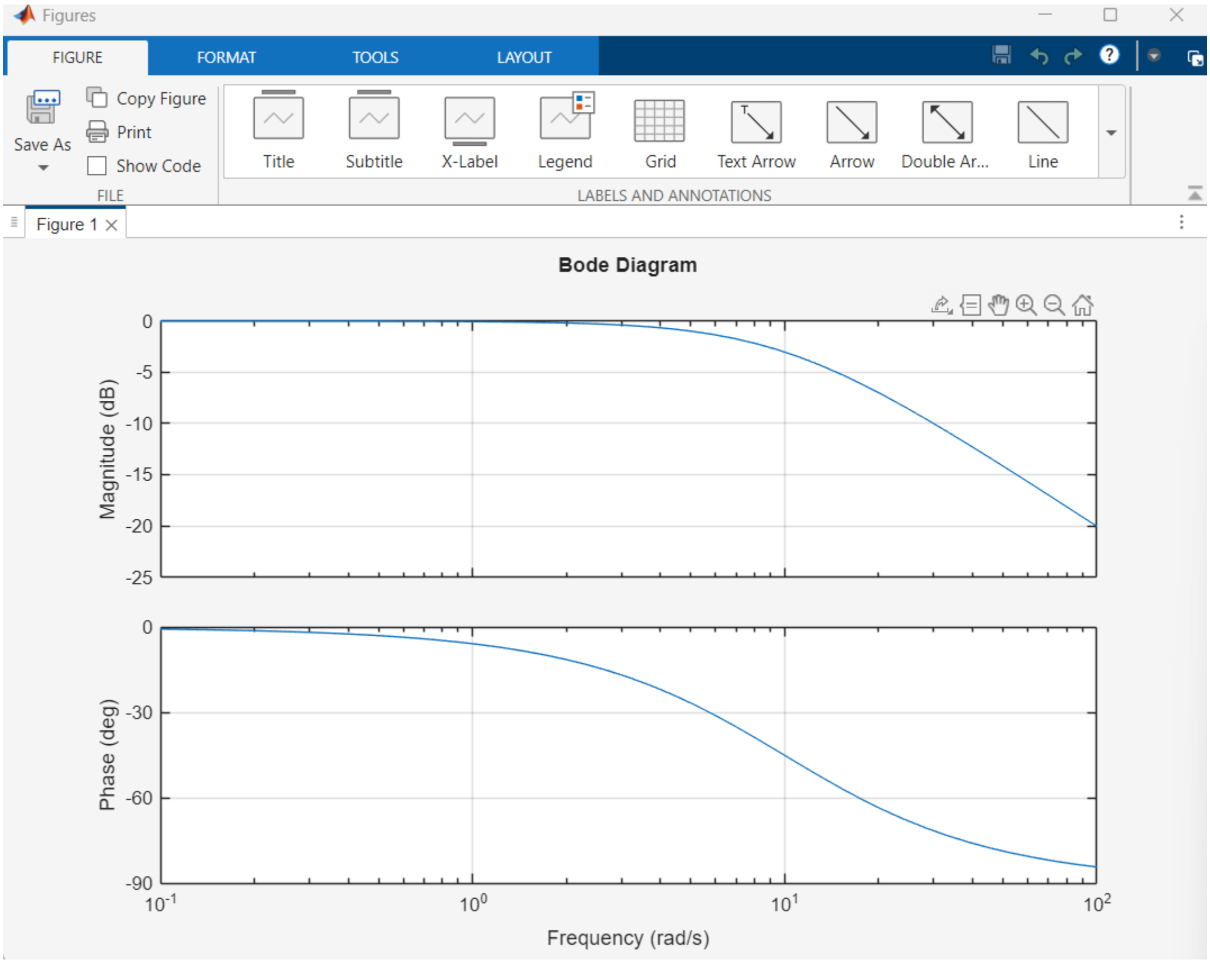
$$\text{Pole: } s = -10$$
$$G_1(0) = \frac{10}{0+10} = 1 \quad \text{(DC gain } 1 \Rightarrow 0 \text{ dB)}$$

2. Asymptotic Bode sketch reasoning:

This is a **first-order low-pass filter** with corner frequency $\omega_c = 10$ rad/s.

- **Magnitude plot**:
  - For $\omega < 10$, flat at 0 dB.
  - For $\omega > 10$, slope = -20 dB/decade.
- **Phase plot**:
  - Low freq → 0°
  - At $\omega = 10$ → $-45°$
  - High freq → $-90°$
  - Phase changes mostly between 1 rad/s and 100 rad/s.





1.2 Problem A.2: $\boldsymbol{G_2(s) = \frac{s-2}{s+10}}$

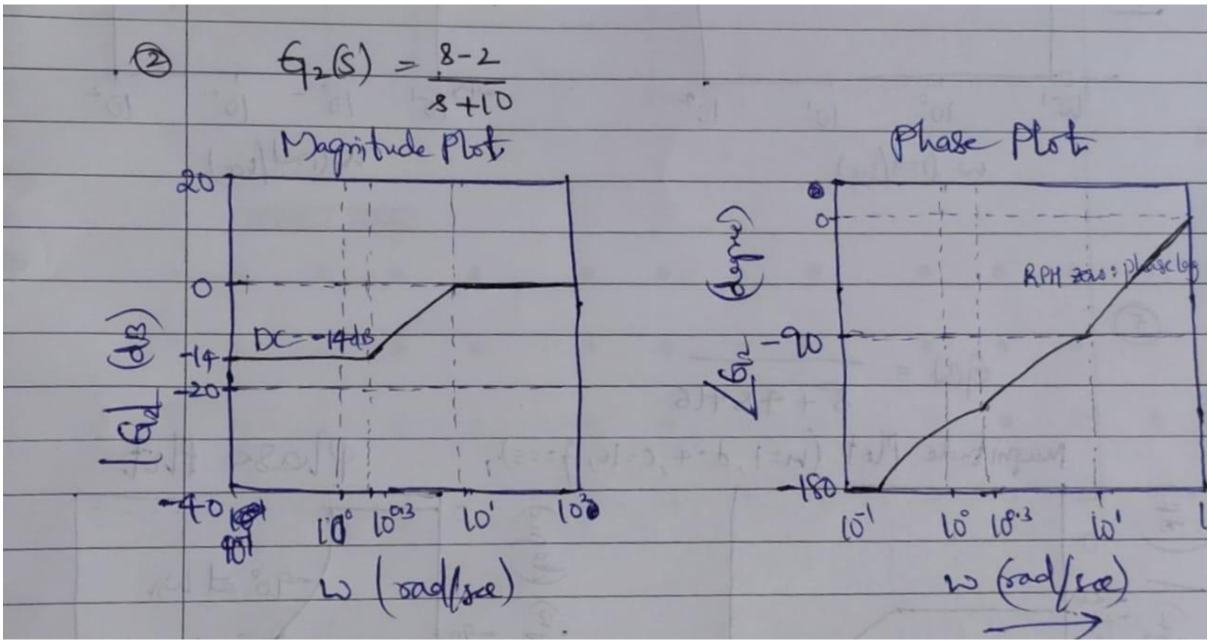
1. Zero, pole, DC gain:

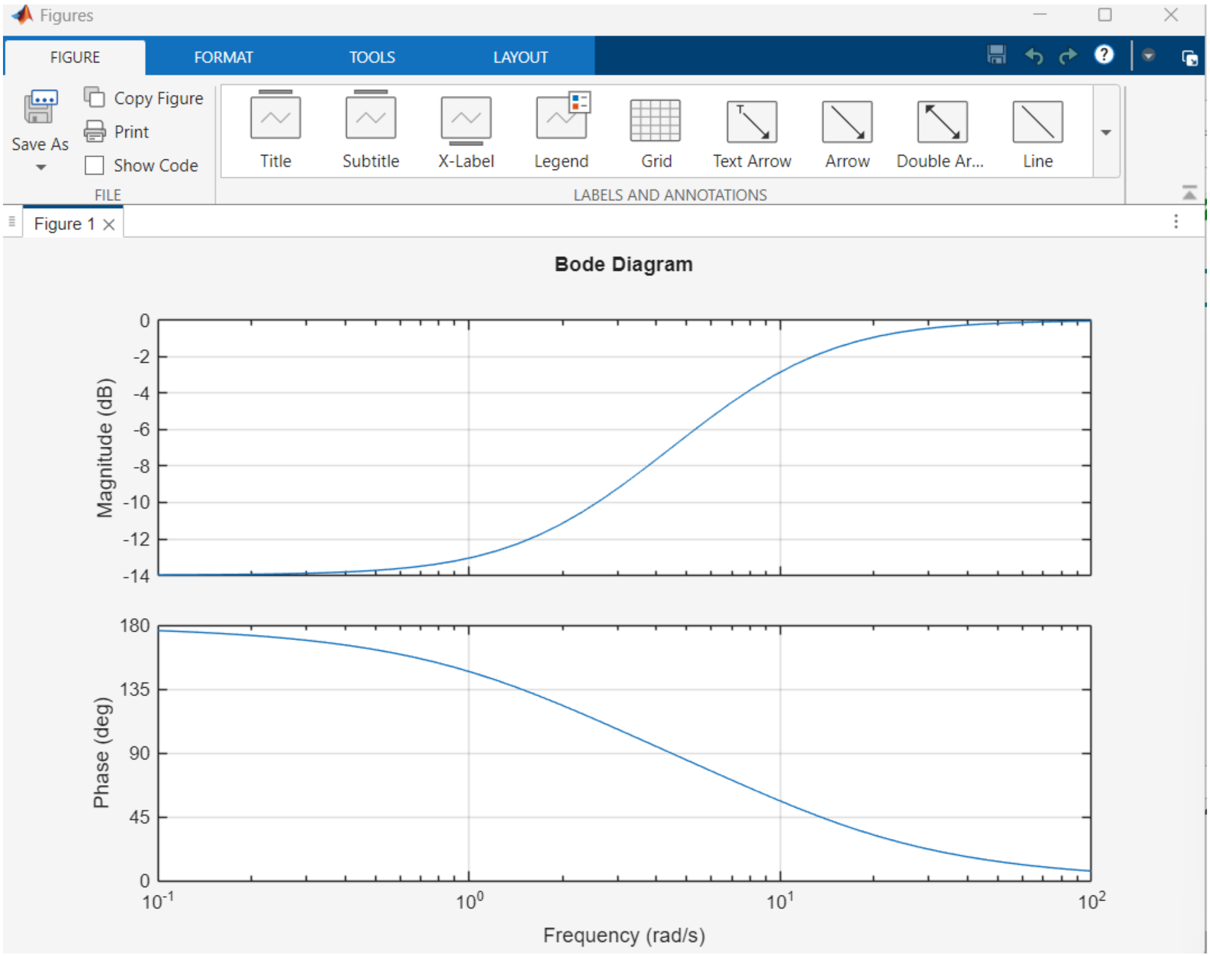
$$\text{Zero: } s = 2(\text{RHP zero at } +2)$$
$$\text{Pole: } s = -10$$
$$G_2(0) = \frac{-2}{10} = -0.2$$

Magnitude in dB: $20\log_{10}(0.2) \approx -14$ dB

2. Asymptotic Bode sketch reasoning:

- **Magnitude**:
  - For $\omega < 2$: flat at -14 dB.
  - Between 2 and 10: slope $+20$ dB/dec (due to zero).
  - For $\omega > 10$: slope 0 dB/dec (pole cancels zero slope).
- **Phase**:
  - Start: DC gain negative → phase = $-180°$ (because $-0.2 = 0.2\angle -180°$).
  - RHP zero at $\omega = 2$ adds phase **lag** from $-180°$ down toward $-270°$.
  - Pole at $\omega = 10$ adds further phase lag toward $-360°$ (or $0°$ if modulo 360).
  - Final high-frequency phase: $0°$.





4. Very short question:

An RHP zero causes **phase lag** (negative phase shift) as frequency increases, opposite to an LHP zero's phase lead.

1.3 Problem A.3: $\boldsymbol{G_3(s) = \frac{100}{s^2+10s+100}}$

1. Poles:

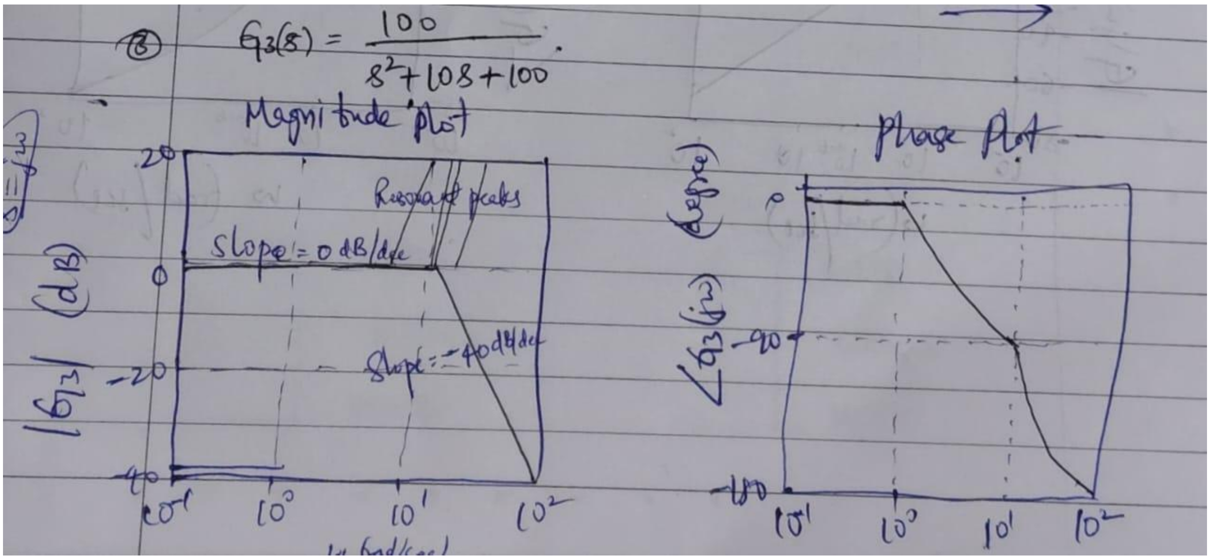
$$s^2 + 10s + 100 = 0$$

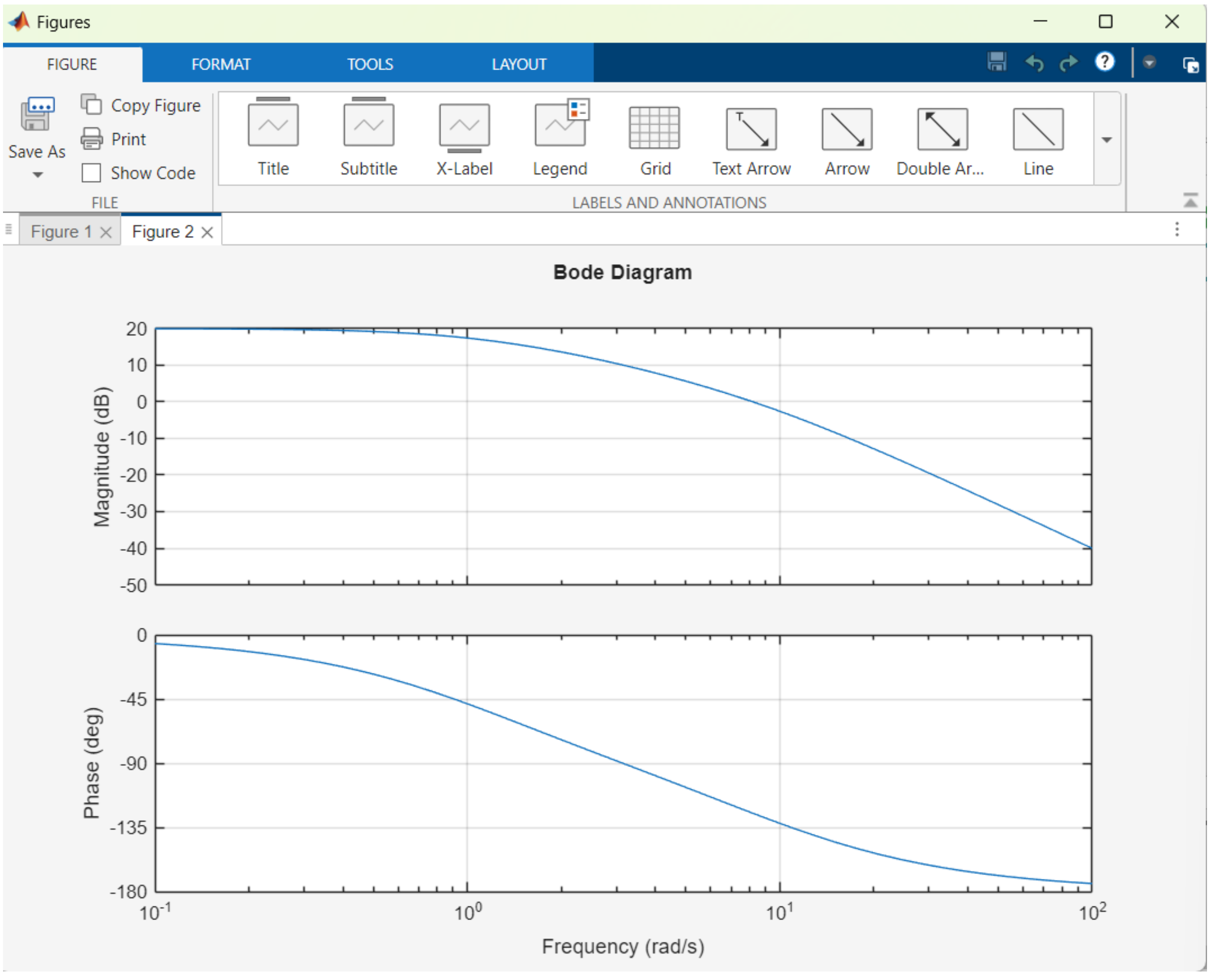
$$s = \frac{-10 \pm \sqrt{100 - 400}}{2} = \frac{-10 \pm j\sqrt{300}}{2} = -5 \pm j5\sqrt{3}$$

Natural frequency $\omega_n = 10$ rad/s, damping ratio $\zeta = 0.5$.

2. Asymptotic Bode sketch reasoning:

- **Magnitude**:
  - Low freq: 0 dB.
  - At $\omega_n = 10$: slope changes by -40 dB/dec.
  - Resonant peak due to $\zeta < 0.707$.
- **Phase**:
  - Start: $0°$
  - At $\omega = \omega_n$: $-90°$
  - High freq: $-180°$





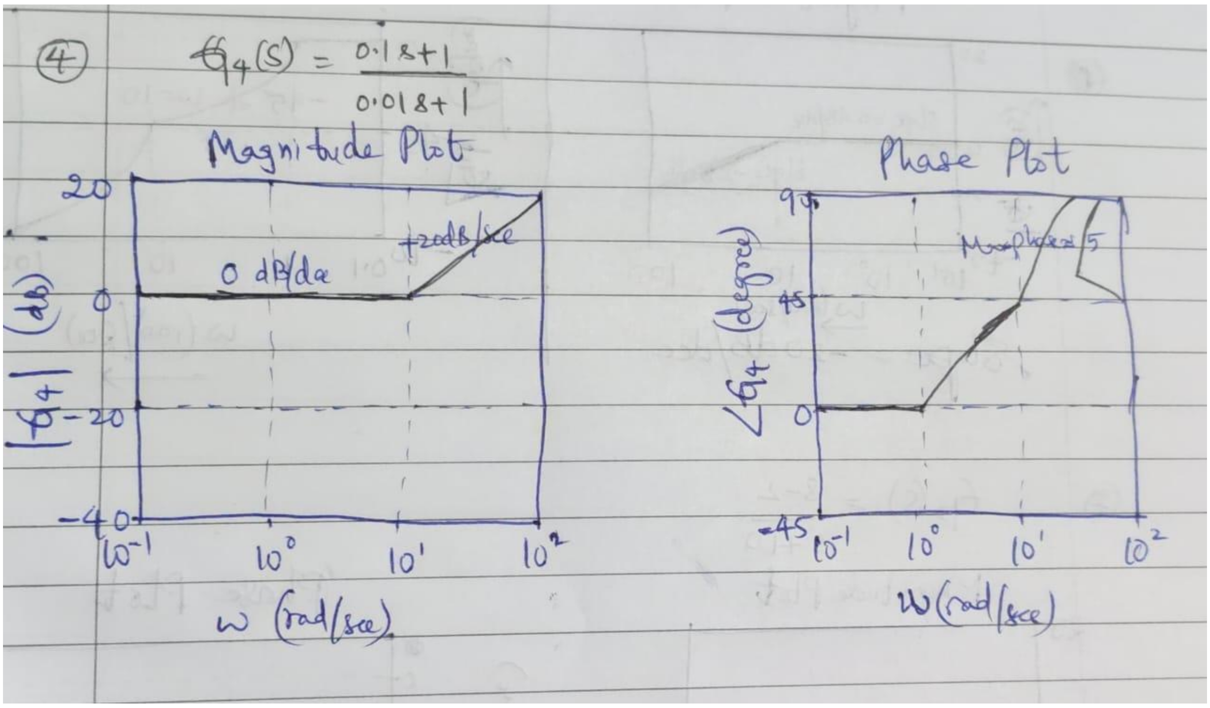
1.4 Problem A.4: $G_4(s) = \frac{0.1s+1}{0.01s+1}$

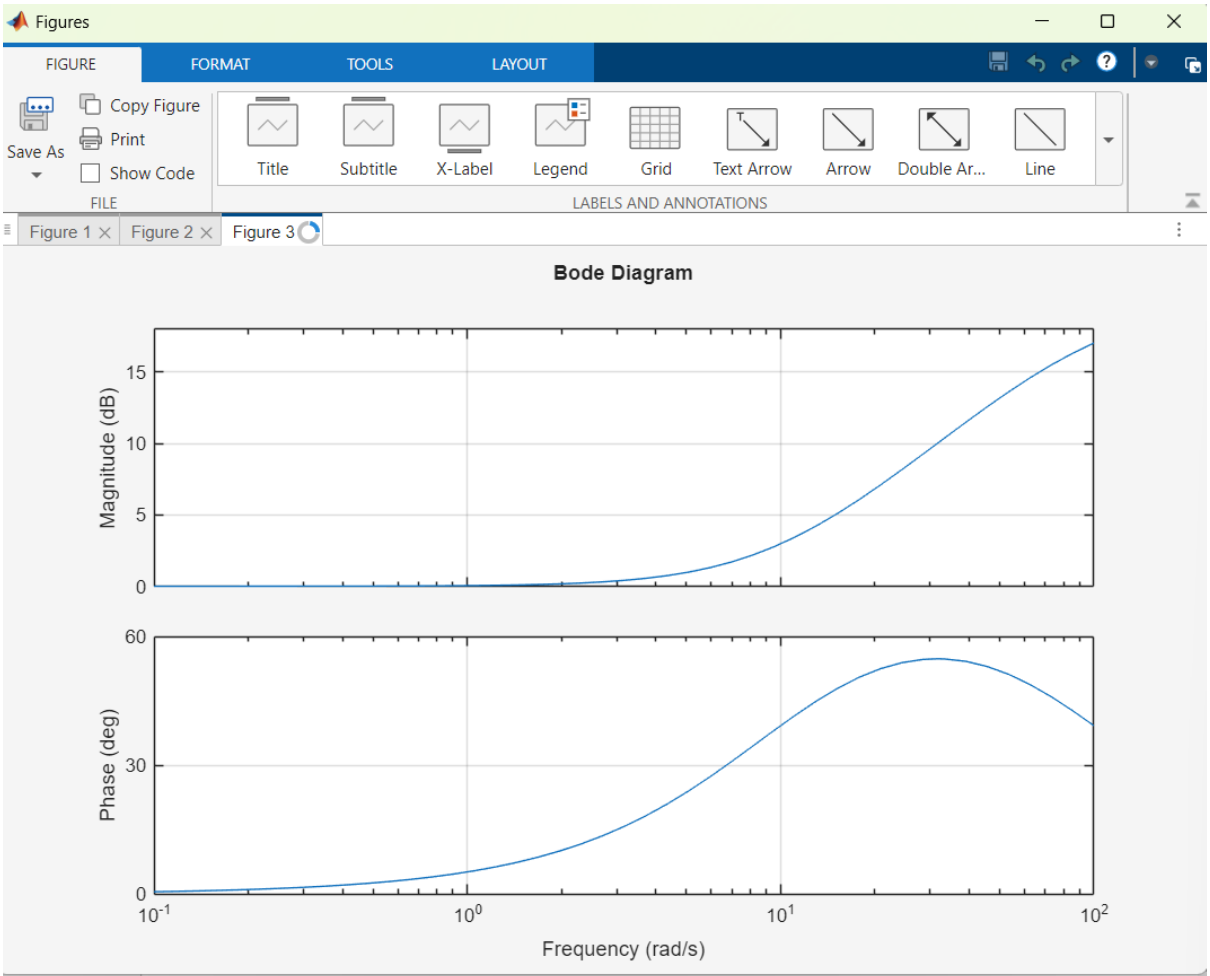
1. Zero and pole:

Zero: $0.1s + 1 = 0 \rightarrow s = -10 \rightarrow \omega_z = 10$ rad/s
Pole: $0.01s + 1 = 0 \rightarrow s = -100 \rightarrow \omega_p = 100$ rad/s

2. Asymptotic Bode sketch reasoning:

- **Magnitude**:
  - DC gain = 1 → 0 dB.
  - From $\omega_z = 10$: slope $+20$ dB/dec.
  - From $\omega_p = 100$: slope changes to 0 dB/dec.
- **Phase**:
  - Zero adds $+90^\circ$ lead.
  - Pole adds -90° lag and Maximum lead occurs between 10 and 100 rad/s.


(4)
$G_4(s) = \frac{0.1s+1}{0.01s+1}$
Magnitude Plot
0 dB/dec
+20dB/sec
$|G_4|$ (dB)
20
0
-20
-40
10^{-1}
10^{0}
10^{1}
10^{2}
ω (rad/sec)
Phase Plot
$\angle G_4$ (degree)
90
45
0
-45
10^{-1}
10^{0}
10^{1}
10^{2}
ω (rad/sec)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TOOLS
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Legend
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Line
LABELS AND ANNOTATIONS
Figure 1
Figure 2
Figure 3
Bode Diagram
Magnitude (dB)
15
10
5
0
Phase (deg)
60
30
0
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

4. Very short question:

Around the frequency between the zero and pole, $G_4(s)$ tends to add **positive phase (phase lead)**.

Part B: Mass-Spring-Damper Transfer Function

2.1 Derive the Differential Equation and Transfer Function

From Newton's second law:

$$m\ddot{x} = F(t) - d\dot{x} - cx$$
$$m\ddot{x} + d\dot{x} + cx = F(t)$$

where, $\ddot{x} = \frac{d^2x}{dt^2}$

$$\dot{x} = \frac{dx}{dt}$$

Laplace transform with zero initial conditions:

$$(ms^2 + ds + c)X(s) = F(s)$$
$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + ds + c}$$

2.2 Numerical Example: $\boldsymbol{m = 1, d = 4, c = 16}$

1. Numerical transfer function:

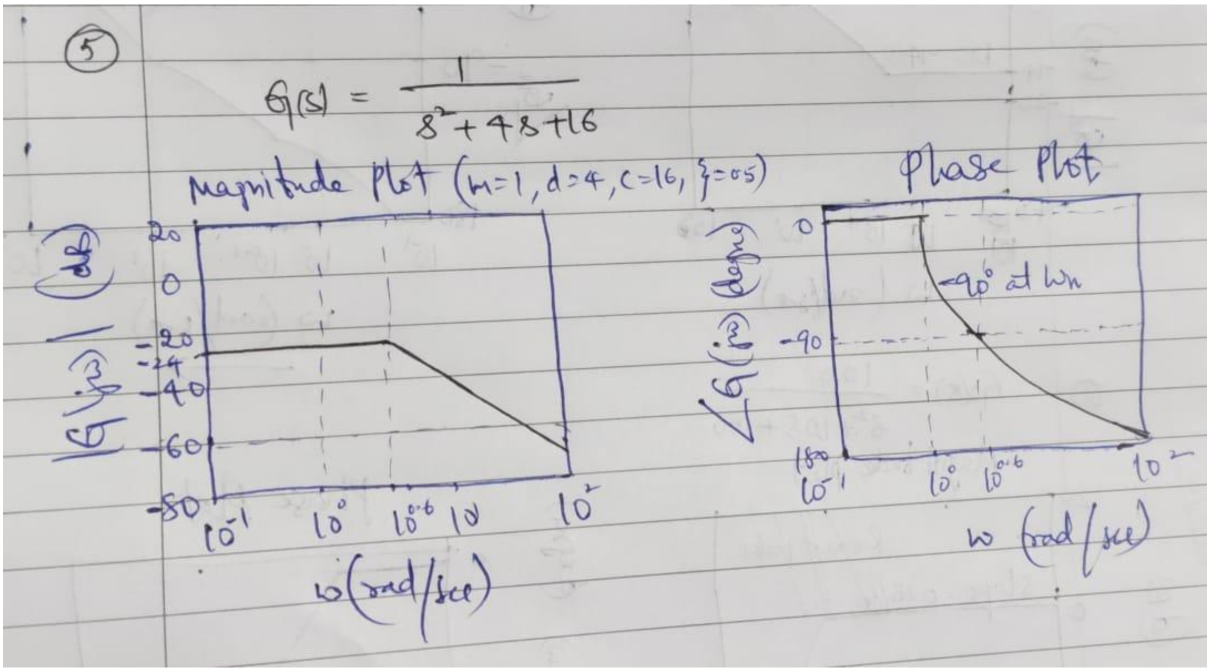
$$G(s) = \frac{1}{s^2 + 4s + 16}$$

2. Poles:

$$s^2 + 4s + 16 = 0$$
$$s = \frac{-4 \pm \sqrt{16-64}}{2} = \frac{-4 \pm j\sqrt{48}}{2} = -2 \pm j2\sqrt{3}$$
$$\omega_n = \sqrt{16} = 4 \text{ rad/s}, \zeta = \frac{4}{2 \cdot 4} = 0.5$$

3. Asymptotic Bode sketch reasoning:

- **Magnitude**:
  - DC gain: $|G(0)| = 1/16 \rightarrow 20\log_{10}(1/16) \approx -24$ dB
  - Corner at $\omega_n = 4$ rad/s: slope -40 dB/dec.
  - Resonant peak due to $\zeta = 0.5$.
- **Phase**:
  - Start: $0°$
  - At ω_n: $-90°$
  - High freq: $-180°$



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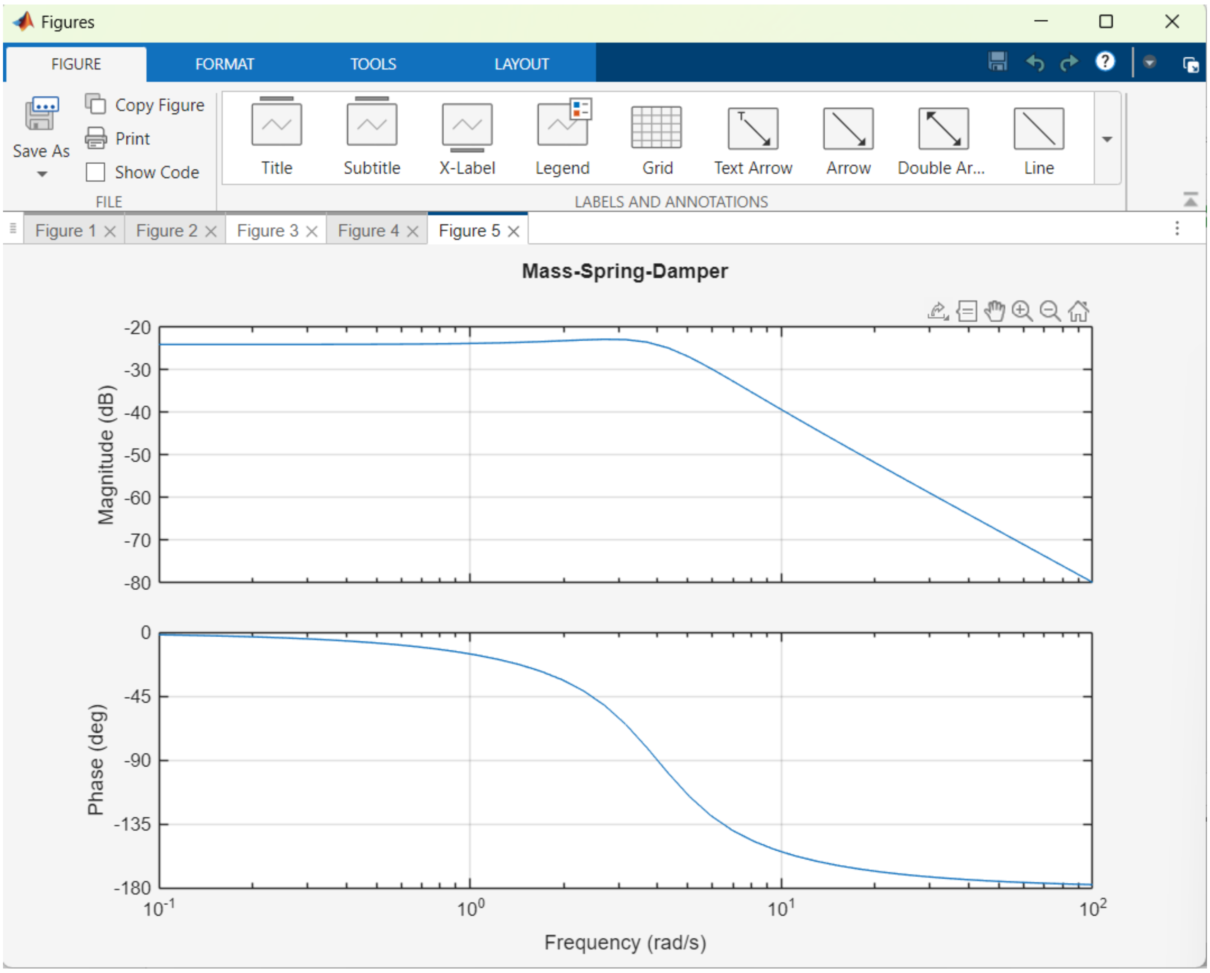

Optional MATLAB Code Used for Bode Plots (for reference)

matlab

```
s = tf('s');

G1 = 10/(s+10);

figure; bode(G1, {0.1, 100}); grid on; title('G1');

G2 = (s-2)/(s+10);

figure; bode(G2, {0.1, 100}); grid on; title('G2');
```

```
G3 = 100/(s^2 + 10*s + 100);

figure; bode(G3, {0.1, 100}); grid on; title('G3');


G4 = (0.1*s+1)/(0.01*s+1);

figure; bode(G4, {0.1, 100}); grid on; title('G4');


% Part B

m=1; d=4; c=16;

G = 1/(m*s^2 + d*s + c);

figure; bode(G, {0.1, 100}); grid on; title('Mass-Spring-Damper');
```