MATLAB Questions: Transfer Functions, Step Response, and Basic Controller Design

Q1. Understanding a First-Order Plant Using Step Response

Consider the first-order plant:

$$G(s) = \frac{4}{s+2}$$

1. Plot the unit step response in MATLAB.

2. Using the plot or `stepinfo`, determine:

   - Time constant τ
   - Rise time t_r
   - Settling time t_s
   - Final value (using Final Value Theorem)
   - Steady-state error e_{ss}

3. Compare MATLAB's final value with:

$$y_{ss} = \lim_{s \to 0} s \cdot G(s) \cdot \frac{1}{s}.$$

MATLAB starter code:

```
s = tf('s');
G = 4/(s+2);
step(G), grid on
stepinfo(G)
```

Q2. System Type, Step Error, and Final Value Theorem

Given the plant:

$$G(s) = \frac{10}{s(s+5)}$$

1. Identify the **system type** (count the number of integrators).

2. Using the Final Value Theorem, find the steady-state error to a unit step:

$$e_{ss} = \lim_{s \to 0} s \left(\frac{1}{s} - G(s)\frac{1}{s} \right).$$

3. Predict whether MATLAB's step response should reach 1, overshoot it, or settle below 1.

Q3. Required Specifications → Modify Transfer Function

Your goal is to design a first-order system that satisfies:

$$t_s < 1.2 \text{ seconds}, \qquad e_{ss} = 0.1$$

1. Using the first-order formulas:

$$t_s \approx \frac{4}{a}, \qquad e_{ss} = \frac{1}{1+K}$$

   determine:

   - the required pole location a,
   - the required static gain K.

2. Construct the modified plant:

$$G_{\text{new}}(s) = \frac{K}{s+a}.$$

3. Predict the shape of the step response before running MATLAB:

   - Should it be faster or slower than Q1?
   - Should the final value be higher or lower?

Q4. Designing a Simple Controller to Meet Specifications

You are given the following plant:

$$G(s) = \frac{3}{s+1}$$

You must design a simple controller:

$$C(s) = K(s+z)$$

to meet these desired characteristics:

$$t_s < 2 \text{ s}, \qquad M_p < 10\%, \qquad y_{ss} = 0.8$$

1. Using formulas from the class cheat sheet:

   - Choose a zero z that reduces rise time.
   - Choose a gain K that sets the desired steady-state value.
   - Estimate the resulting damping ratio ζ from the overshoot requirement.

2. Write the resulting closed-loop transfer function:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}.$$

3. Before using MATLAB, **predict qualitatively**:

   - Will adding the zero increase or decrease overshoot?
   - Will increasing K increase or decrease y_{ss}?
   - Will the response be faster than the original plant?

Q5. Ramp Tracking and System Type

Using the controller and closed-loop system from Q4:

$$r(t) = t \quad \text{(unit ramp)}$$

1. Determine the **system type** of the closed-loop system.

2. Using system type rules, predict whether the ramp error will be:

   - infinite,
   - finite non-zero,
   - or zero.

3. Verify using the Final Value Theorem for ramp input:

$$e_{ss}^{\text{ramp}} = \lim_{s \to 0} s \left(\frac{1}{s^2} - T(s)\frac{1}{s^2} \right).$$

4. Explain whether adding the zero at $(s + z)$ helps or hurts ramp tracking.