Assignment 1

1.1

1.2

- Time constant : $\frac{Y(s)}{X(s)} = \frac{4}{s+2} \Rightarrow Y(s) = \frac{1}{s}\frac{4}{s+2} = 2\left(\frac{1}{s} - \frac{1}{s+2}\right) \Rightarrow y(t) = 2(1-e^{-2t})\,u(t)$

  $\therefore$ Time constant $\tau = 1/2 = 0.5$ seconds
- Rise time : $t_r = 1.0985$ seconds
- Settling time : $t_s = 1.9560$ seconds
- Final value : $y(\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} G(s) = G(0) = 2$
- steady state error : $e_{ss} = x(\infty) - y(\infty) = \lim_{s\to 0} sX(s) - 2 = -1$

1.3 Matlab final value saturates at 2 and $\lim_{s\to 0} sY(s) = 2$ from final value thm

2.1 $G(s) = \frac{10}{s(s+5)}$ is of form $\frac{k}{s(s+a)} \Longrightarrow$ Type 1 : one integrator

2.2 $e_{ss} = \lim_{s\to 0} s\left(\frac{1}{s} - \frac{G(s)}{s}\right) = \lim_{s\to 0} 1 - G(s) = 1 - \lim_{s\to 0} G(s) = -\infty$

2.3 it should overshoot as e_{ss} is ∞

2.2 closed loop : $G \to \boxed{\frac{G}{G+1}}$

$e_{ss} = \lim_{s\to 0} 1 - \frac{G}{G+1} = \lim_{s\to 0} \frac{1}{G+1}$

$e_{ss} = \lim_{s\to 0} \frac{s(s+5)}{10+s(s+5)} = 0$

$e_{ss} = 0$

3.1

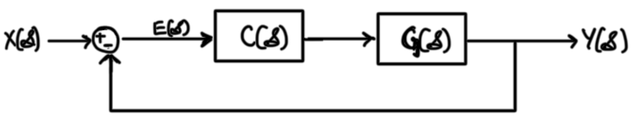
- $4/a < 1.2 \Longrightarrow a > 3.33$
- $\frac{1}{1+k} = \frac{1}{10} \Longrightarrow k = 9$

3.2 $G_{new}(s) = \frac{9}{s+4}$

3.3

- here time constant is $1/4 = 0.25 < 0.5 \Longrightarrow$ This is faster than Q1
- final value : $\lim_{s\to 0} sY(s) = G(s) = 9/4 = 2.25 > 2 \Longrightarrow$ higher than Q1

4.2



$C(s) = K(s+z)$

$G(s) = \frac{3}{s+1}$

$(X-Y)CG = Y \Longrightarrow T(s) = \frac{Y(s)}{X(s)} = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)}$

$T(s) = \frac{3k(s+z)}{s+1+3k(s+z)} \xrightarrow[k=4/3]{z=1} T(s) = \frac{4(s+1)}{5(s+1)} = \frac{4}{5} = 0.8$

4.1

- To reduce rise time, pick z so that $s+1$ in $G(s)$ denominator gets cancled $\Longrightarrow z = 1$
- $\lim_{s\to 0} T(s) = \frac{3k}{1+3k} = 0.8 \Longrightarrow 3k = 4 \Longrightarrow k = 4/3 = 1.33$
- $M_p = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) < 10\% \Longrightarrow \frac{\pi\zeta}{\sqrt{1-\zeta^2}} > \ln 10 \Longrightarrow \zeta > \frac{\ln 10}{\sqrt{(\ln 10)^2 + \pi^2}} = 0.5911$

4.3

- initially also there was no overshoot ($\because y(t) \propto (1+3e^{-4t})u(t)$)
  finally also there is no overshoot ($\because y(t) \propto u(t)$) } Proof

- $e_{ss} = T(s\to 0) = \dfrac{3kz}{1+3kz} \implies \dfrac{\partial}{\partial k} e_{ss} = \dfrac{3z}{(1+3kz)^2}$

  if $z > 0$: e_{ss} increases with an increase in k
  if $z < 0$: e_{ss} decreases with an increase in k (for us, $z=1$)

- originally, without controller : $T(s) = \dfrac{1}{1+G} = \dfrac{s+1}{s+4}$ (if closed loop)

  $Y(s) = \dfrac{T(s)}{s} = \dfrac{1}{4}\left(\dfrac{1}{s} + \dfrac{3}{s+4}\right) \implies y(t) = \dfrac{u(t)}{4}(1+3e^{-4t}) \implies \tau = 0.25\,sec$

  Now: if $z=1$, $T(s) = 4/s \implies Y(s) = \dfrac{4}{s}\dfrac{1}{s} \implies y(t) = \dfrac{4}{s}u(t) \implies \tau = 0\,sec$

  $\therefore$ This system with controller is faster

5.1 if open loop : Transfer fn $T(s) = C(s)G(s) = \dfrac{3k(s+z)}{s+1} \implies$ Type 0

if close loop : $T(s) = \dfrac{CG}{1+CG} = \dfrac{3k(s+z)}{s+1+3k(s+z)} \implies$ Type 0 (no integrators)

without controller : open loop : $T(s) = G(s) = \dfrac{1}{s+1} \implies$ Type 0

close loop : $T(s) = \dfrac{G}{1+G} = \dfrac{s+1}{s+4} \implies$ Type 0

5.2 without controller : $e_{ss} = \lim_{s\to 0} s\left(\dfrac{1}{s^2} - \dfrac{T(s)}{s^2}\right) = \lim_{s\to 0} \dfrac{1-\frac{s+1}{s+4}}{s} = \lim_{s\to 0}\dfrac{3}{s(s+4)}$

$\therefore e_{ss} \to \infty$ (cannot be tracked)

with controller : $e_{ss} = \lim_{s\to 0} \dfrac{1}{s} - \dfrac{3k(s+z)}{s(s+1+3k(s+z))} = \infty$ (cannot be tracked) ($z \neq 1/3k$)

5.3 Done previously

5.4 Adding zero at $(s+z)$ has no effect on tracking if $z \neq 1/3k$

if $z = 1/3k$: $e_{ss} = \left.\dfrac{1-3k(s+z)}{s(s+1+3k(s+z))}\right|_{s\to 0} = \dfrac{-3k}{2+(1+3k)s} = -\dfrac{3k}{2}$

if $z = 1/3k$, e_{ss} becomes finite (with controller)