Transfer Functions and Bode Plots in the Laplace Domain

Assignment 0

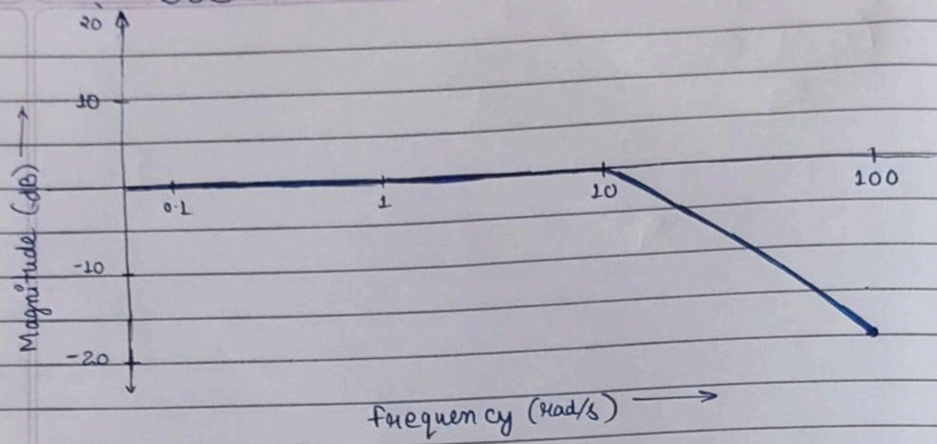
1) Part A: Transfer Functions and Bode Plots

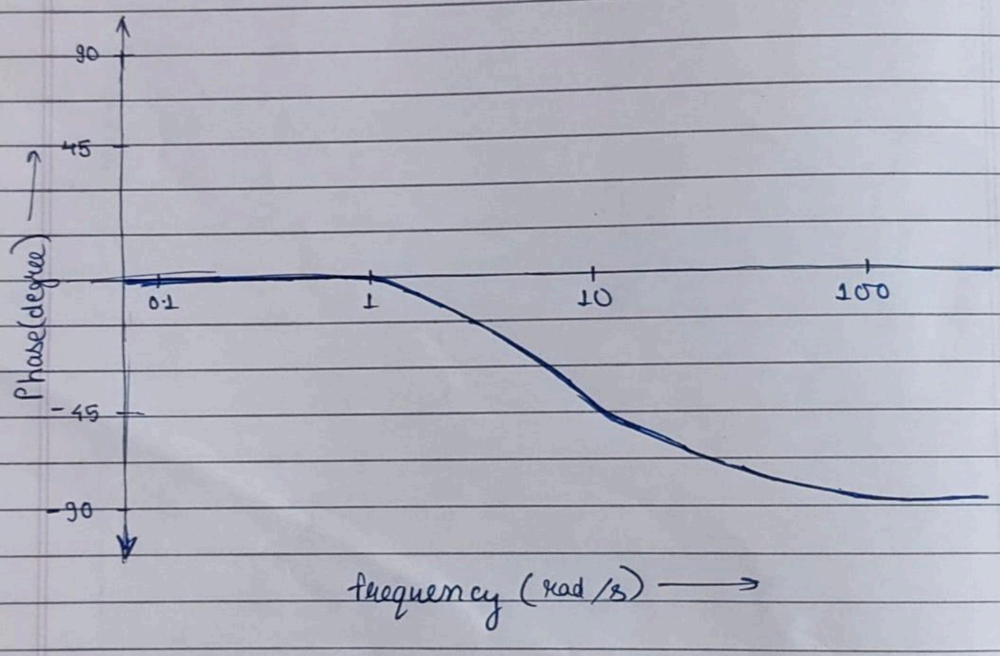
1.1 Problem A.1: $G1(s) = 10/(s+10)$

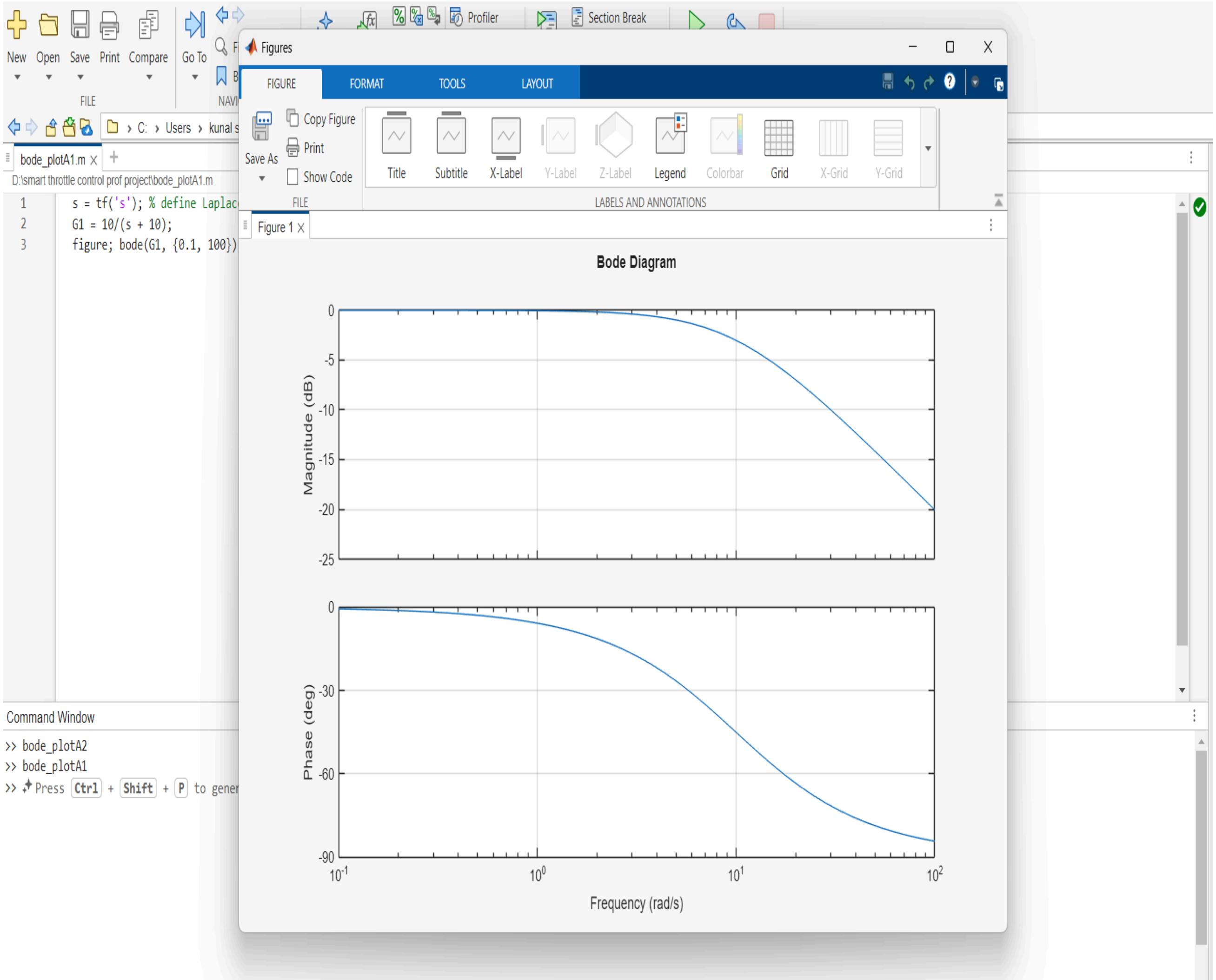
Q) Find the pole and DC gain:
Answer: Pole: $s=-10$
DC gain: $G1(0)=1$

2. Sketch the asymptotic Bode magnitude and phase plots for $\omega \in [0.1,100]$ rad/s.
3. Attach: (a) Hand-sketched asymptotic Bode plots (magnitude and phase).
(b) Screenshot of MATLAB/Octave/Python Bode plots for G1(s)

BODE PLOT Q.A1



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LABELS AND ANNOTATIONS
bode_plotA1.m
D:\smart throttle control prof project\bode_plotA1.m
s = tf('s'); % define Laplac
G1 = 10/(s + 10);
figure; bode(G1, {0.1, 100})
Figure 1
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
Command Window
>> bode_plotA2
>> bode_plotA1
>> Press Ctrl + Shift + P to gener

1.2 Problem A.2 : **$G2(s) = (s-2)/(s+10)$**

1. Find the zero, pole, and DC gain:

Answer : zero at s = -2 rad/s.

pole at s = -10 rad/s.

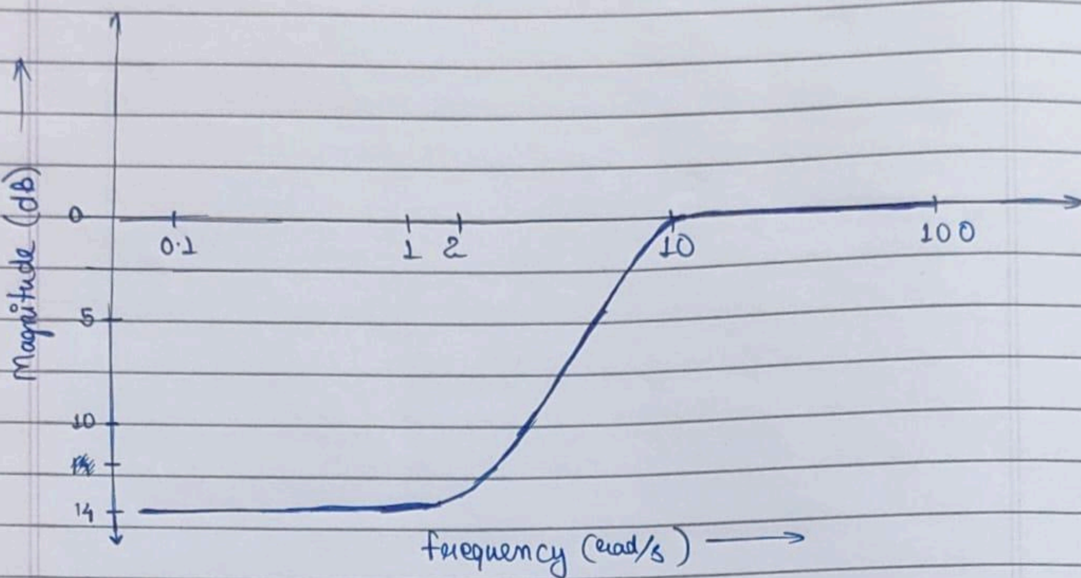
DC gain G2(0) = 2/10 = 0.2

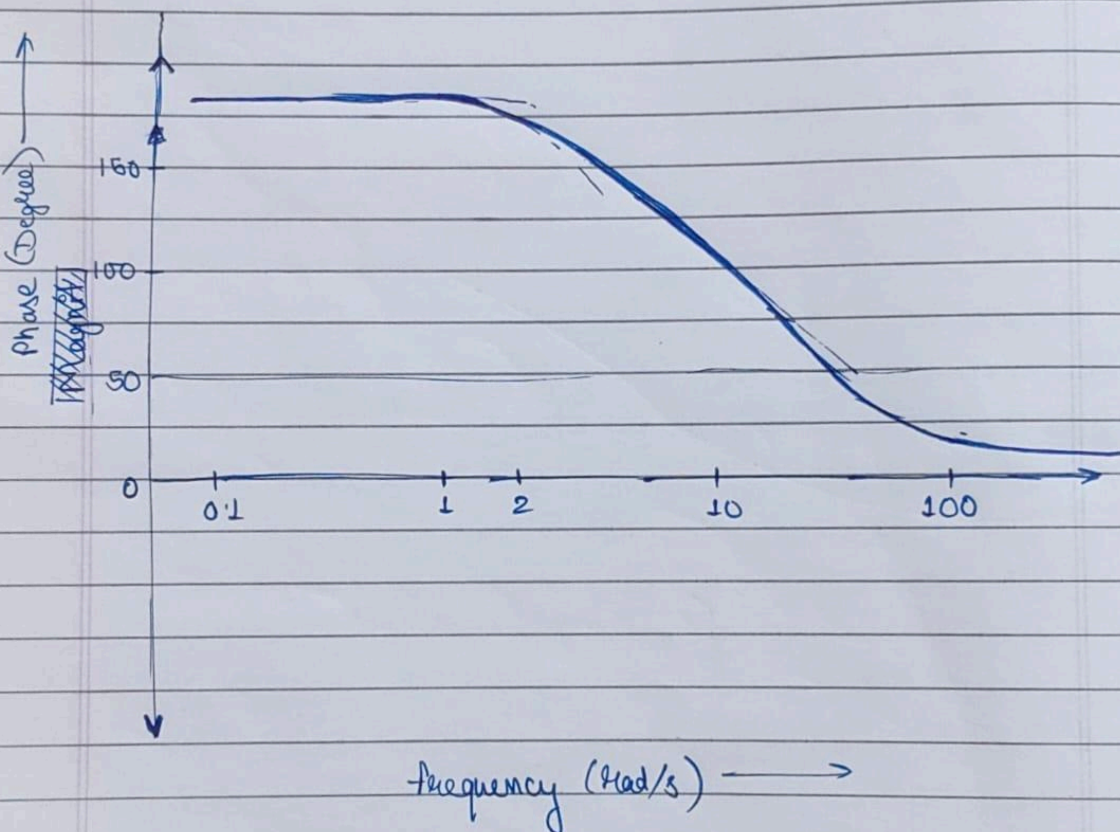
2. Sketch the asymptotic Bode magnitude and phase plots for $\omega \in [0.1,100]$ rad/s.

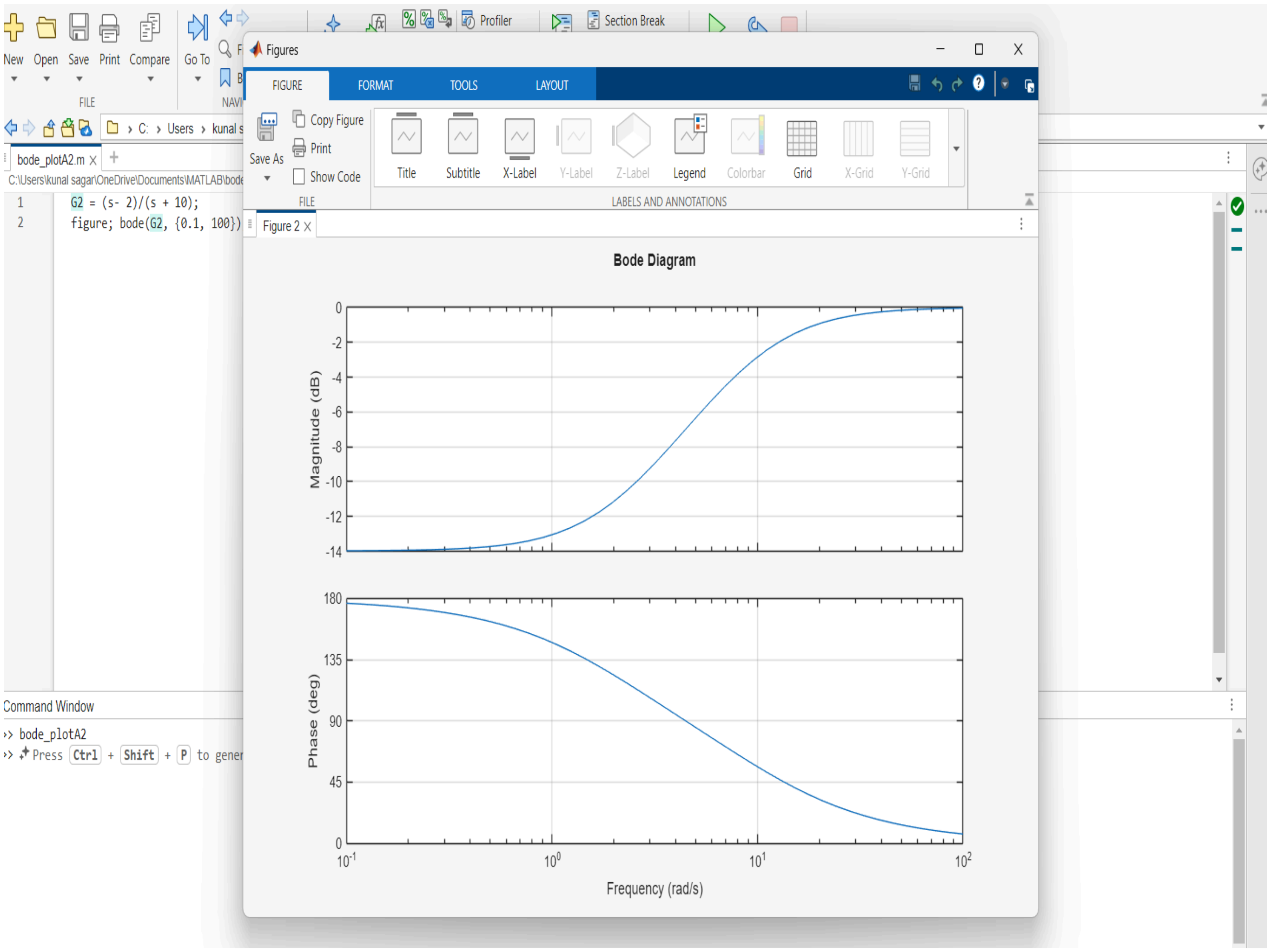
3. Attach: (a) Hand-sketched asymptotic Bode plots (magnitude and phase).

(b) Screenshot of MATLAB/Octave/Python Bode plots for G2(s).

BODE PLOT Q.A-2





Figures
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LABELS AND ANNOTATIONS
bode_plotA2.m
G2 = (s- 2)/(s + 10);
figure; bode(G2, {0.1, 100})
Figure 2
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
Command Window
>> bode_plotA2
Press Ctrl + Shift + P to gener
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Section Break

1.3 Problem A.3: **$G(3) = 100/(s^2 + 10s + 100)$**

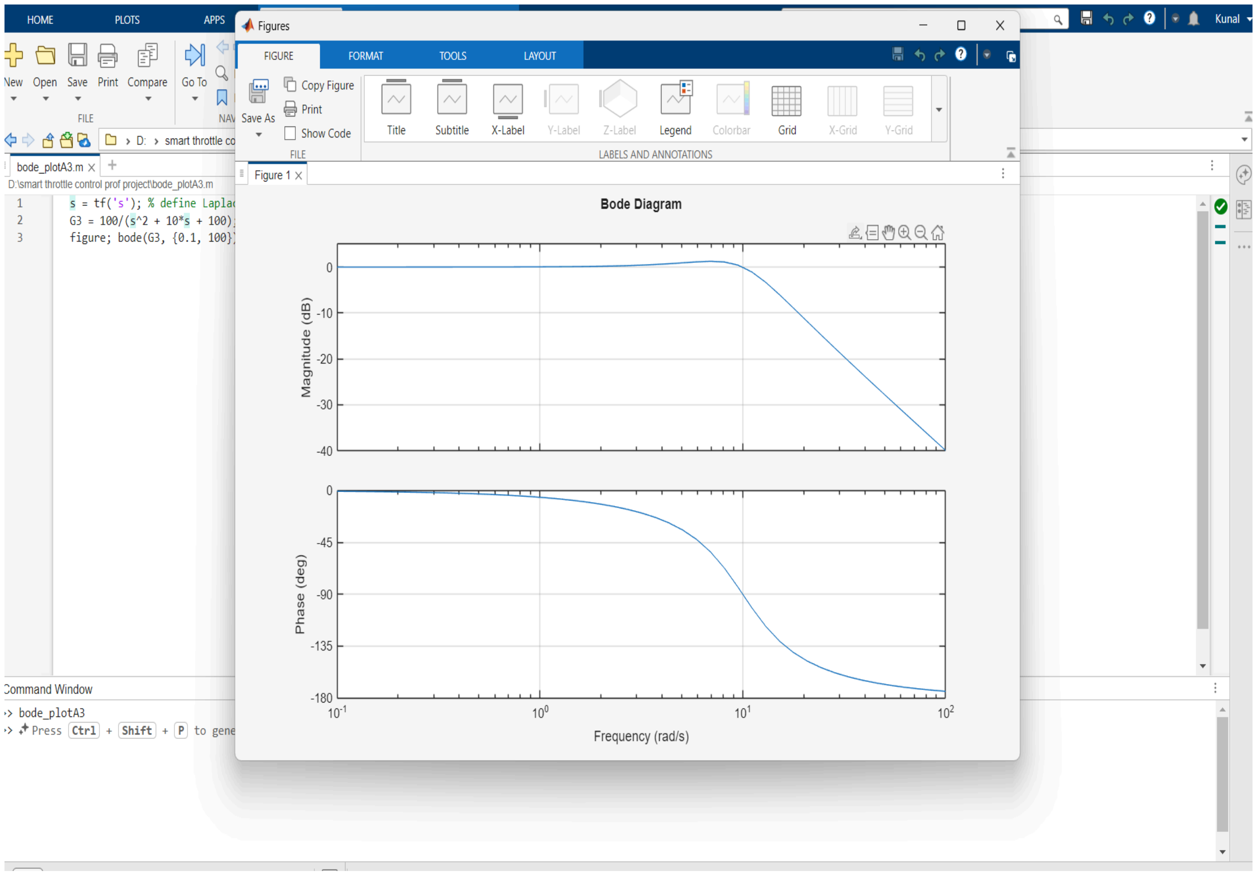
1.Find the poles

Answer : we get poles when we put $s^2 + 10s + 100$

$S_{1,2} =$ $-5 + 5j\sqrt{3}$, $-5 - 5j\sqrt{3}$

2. Sketch the asymptotic Bode magnitude and phase plots for $\omega \in [0.1,100]$ rad/s.
3. Attach: (a) Hand-sketched asymptotic Bode plots (magnitude and phase).

   (b) Screenshot of MATLAB/Octave/Python Bode plots for G3(s).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LABELS AND ANNOTATIONS
Figure 1
bode_plotA3.m
D:\smart throttle control prof project\bode_plotA3.m
s = tf('s'); % define Laplac
G3 = 100/(s^2 + 10*s + 100)
figure; bode(G3, {0.1, 100}
Command Window
>> bode_plotA3
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
0
-10
-20
-30
-40
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}

1.4 Problem A.4 : **G4(s) = (0.1s+1)/(0.01s+1)**

1. Find the zero and pole:

Answer : The zero occurs at s = -10 rad/s.

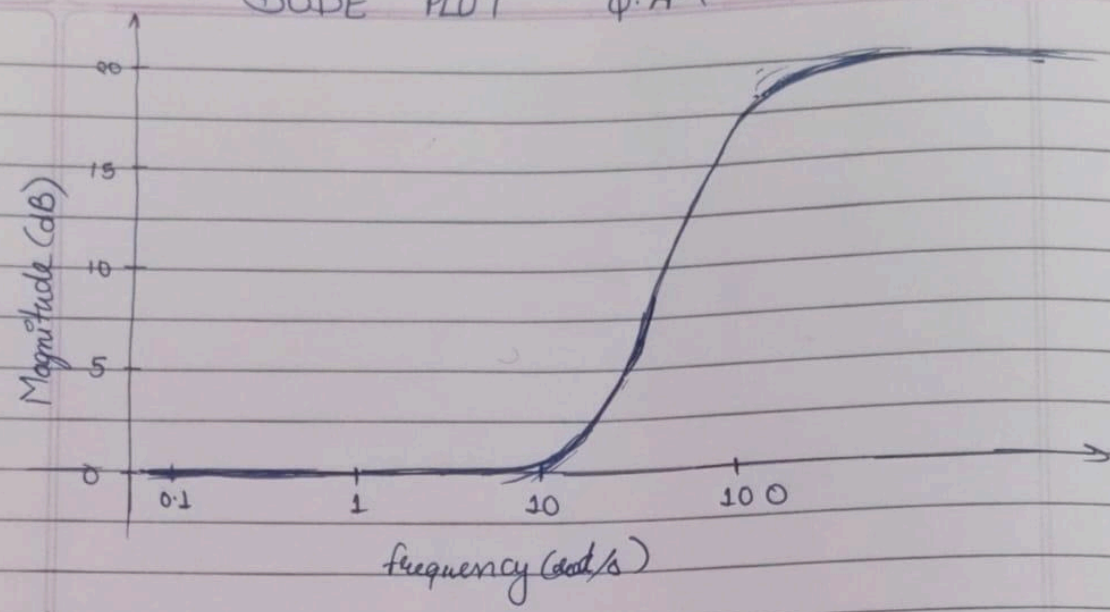
The pole occurs at s = -100 rad/s

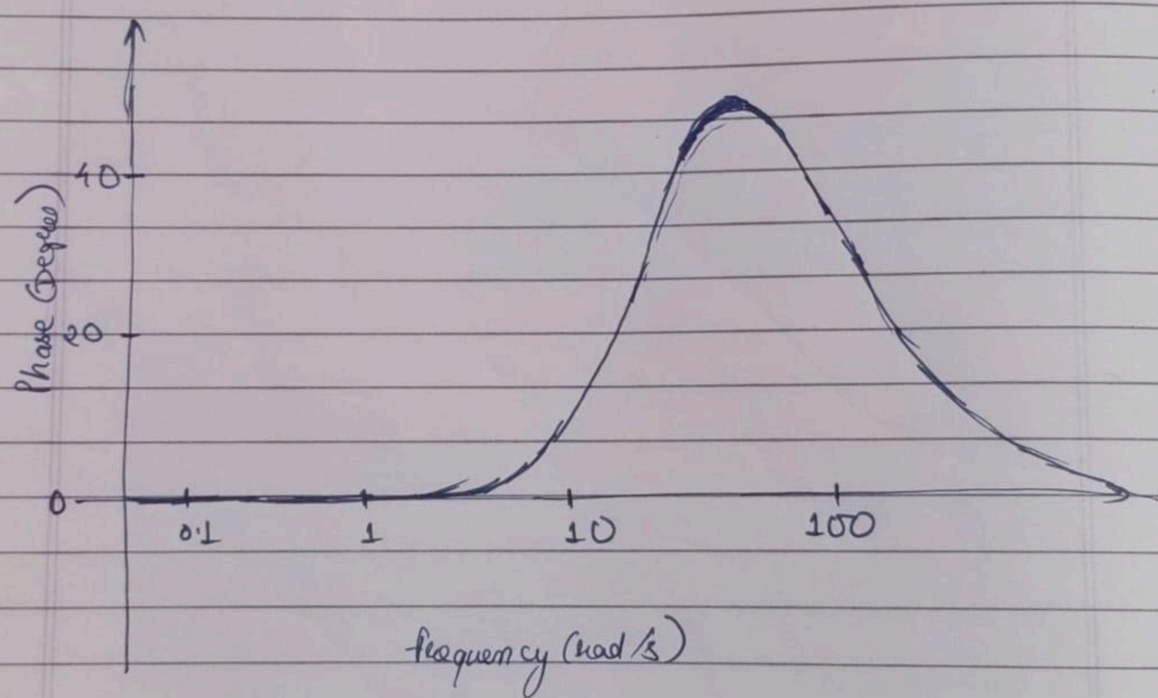
2. Sketch the asymptotic Bode magnitude and phase plots for $\omega \in [0.1,100]$ rad/s.

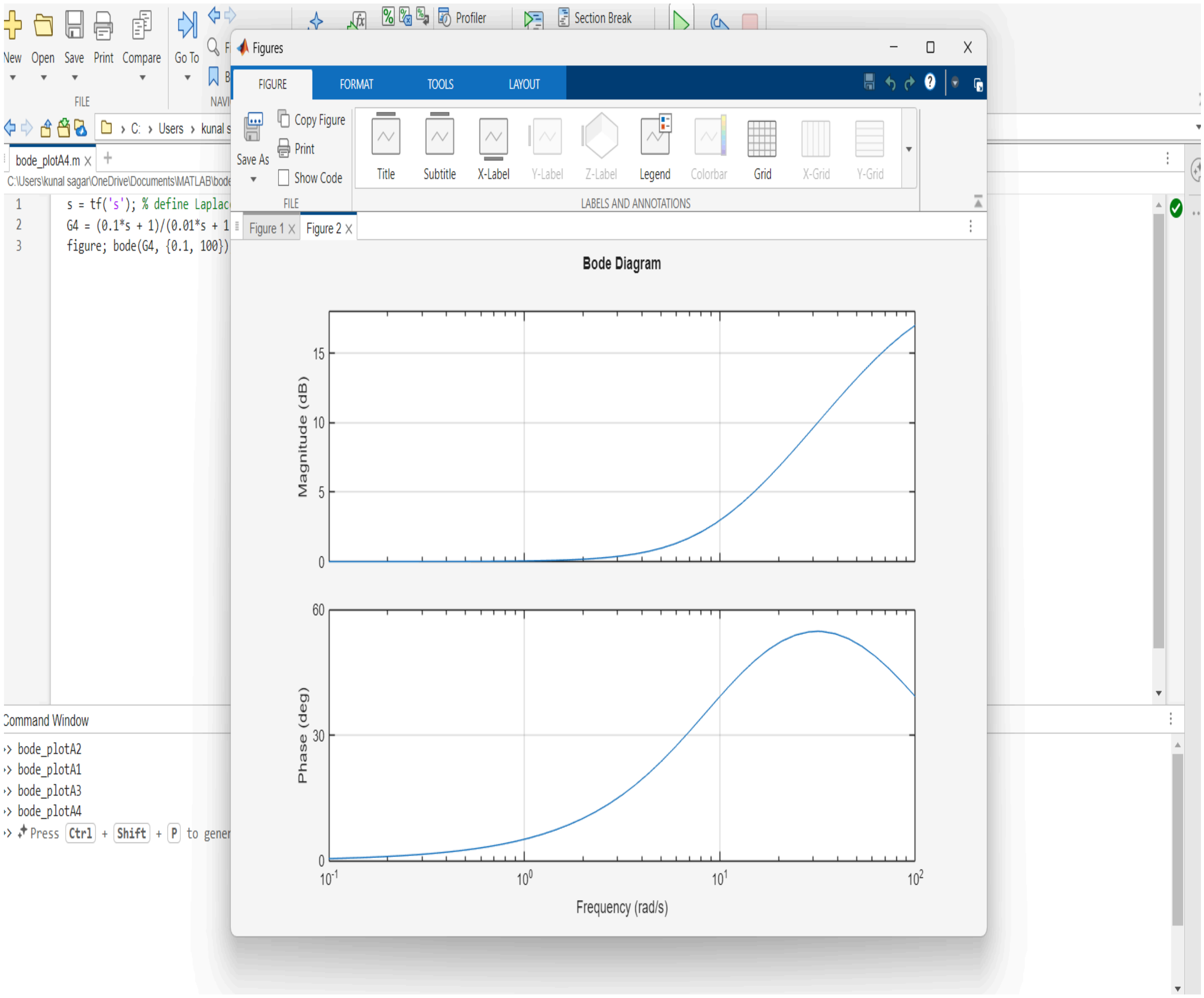
3. Attach: (a) Hand-sketched asymptotic Bode plots (magnitude and phase).

(b) Screenshot of MATLAB/Octave/Python Bode plots for G4(s).

BODE PLOT Q.A-4







4. Very short question: Around the frequency between the zero and pole, does G4(s) tend to add positive phase (phase lead) or negative phase (phase lag)?

Answer : G4(s) adds positive phase (phase lead) between the zero and pole frequencies.

2) Part B: Mass–Spring–Damper Transfer Function

B.1 Derive the Differential Equation and Transfer Function

1. Using Newton's second law and the diagram, write the differential equation relating x(t) and F(t):

Answer : in the diagram we can clearly see that,

1) Force f(t) is applied on the box on right
2) Spring of spring constant k pulls the the box to left with with force kx(since spring force is proportional to x)
3) Damper with damping constant c applies force of cx' (damping force is proportional to velocity)
4) Net force can be made equal to mx''.

   Therefore by using Newton's second law we get:

$$\mathbf{mx'' = f - kx - cx'}$$

2. Assume zero initial conditions and apply the Laplace transform to obtain an equation in X(s) and F(s):Answer: assuming initial conditions to be zero and applying laplace equation in the above equation, we get,

$$\mathbf{m[s^2X(s) - sx(0) - x'(0)] = F(s) - kX(s) - c[sX(s) - x(0)]}$$

Now x(0) = 0 and x'(0) = 0

$$\mathbf{ms^2X(s) = F(s) - kX(s) - csX(s)}$$

$$\mathbf{F(s) = ms^2X(s) + kX(s) + csX(s)}$$

3. Derive the transfer function: G(s) = X(s)/ F(s)

Answer :

$$\mathbf{F(s) = X(s)[ms^2 + k + cs]}$$

$$\mathbf{X(s)/F(s) = 1/[ms^2 + k + cs]}$$

Thus, $\mathbf{G(s) = 1/[ms^2 + k + cs]}$

B.2 Numerical Example and Bode Plots

For: m=1kg, d=4N·s/m, c=16 N/m

1. Write the numerical transfer function G(s):

Answer : plugging the given number m=1,*d*=4, c=16 into the general results from B.1 we get:

$$\mathbf{G(s) = 1/(s^2 + 4s + 16)}$$

2.Find the poles

Answer : $s^2+4s+16=0$

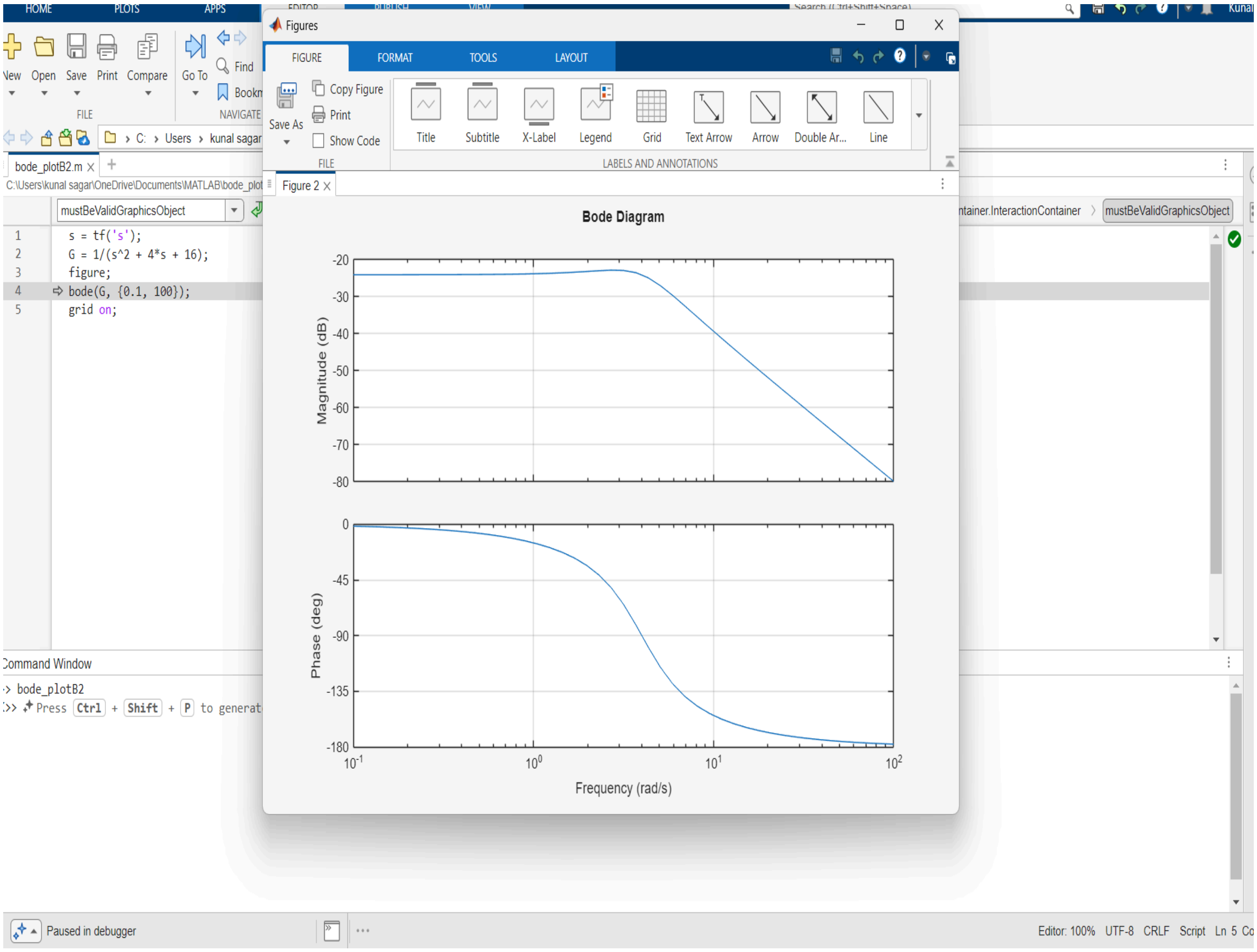
$$S_{1,2} = -2 + j\sqrt{12}\ ,\ -2 - j\sqrt{12}$$

3. Sketch the asymptotic Bode magnitude and phase plots for $\omega \in [0.1,100]$ rad/s.

4. Generate Bode plots in MATLAB/Octave/Python and attach:

(a) Hand-sketched asymptotic Bode plots (magnitude and phase).

(b) Screenshot of MATLAB/Octave/Python Bode plots for this numerical example.


bode_plotB2.m
s = tf('s');
G = 1/(s^2 + 4*s + 16);
figure;
bode(G, {0.1, 100});
grid on;
Command Window
bode_plotB2
Figures
Figure 2
Bode Diagram
Magnitude (dB)
-20
-30
-40
-50
-60
-70
-80
Phase (deg)
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)
Paused in debugger