

Ex. 1.  $G_1(s) = \frac{10}{s+10} = \frac{1}{\left(1 + \frac{s}{10}\right)}$

① Poles = -10      corner frequency = 10

$$G_1(0) = \frac{10}{10} = 1$$

Ex 1

$$G(j\omega) = \frac{10}{j\omega + 10} = \frac{1}{1 + j\omega/10}$$

Corner frequency = 10 rad/s

$$\omega \gg 10$$

$$G(j\omega) = \frac{1}{j\omega/10} = \frac{10}{j\omega}$$

Magnitude (in dB)

$$= 20 [\log(10) - \log(j\omega)]$$

$$= 20 [1 - \log j\omega]$$

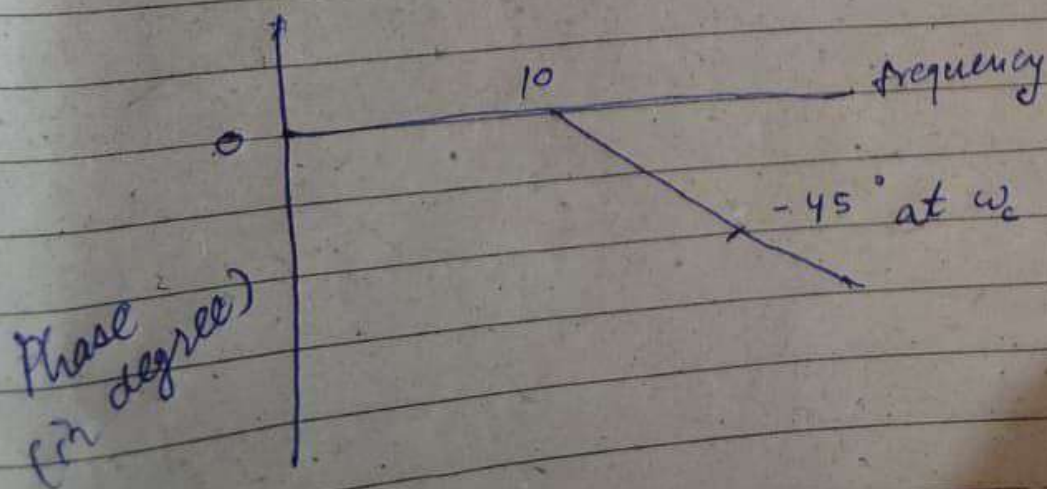
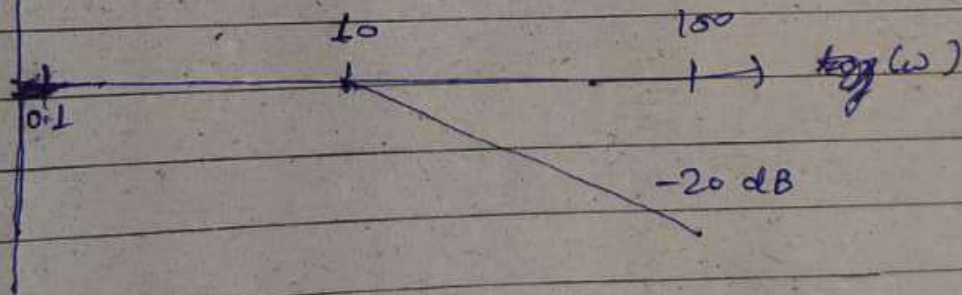
M (dB)

$$\omega \ll 10$$

$$G(j\omega) = \frac{1}{1 + 0} = 1$$

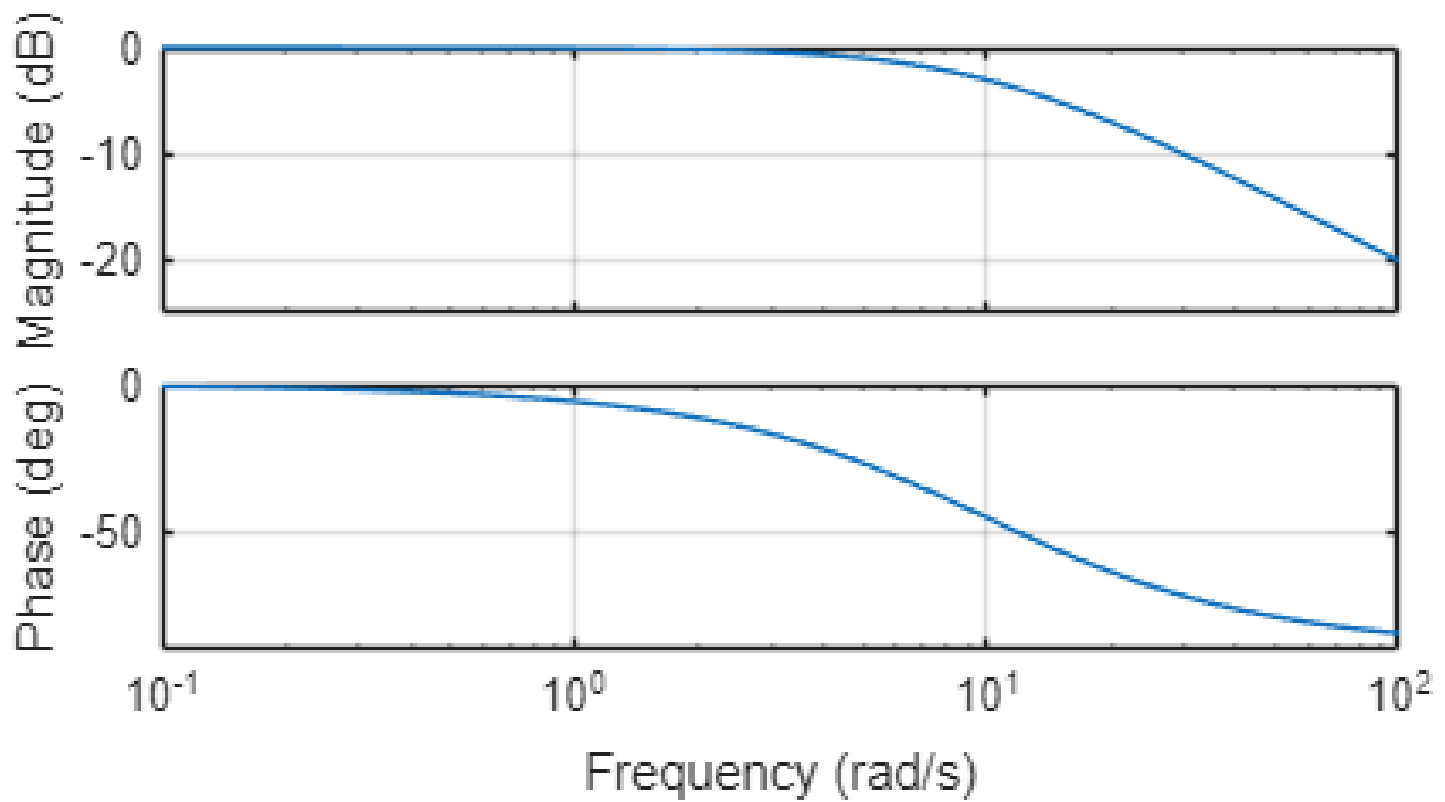
Magnitude (dB)

$$= 20 \log 1 = 0$$



```
s = tf('s');  
G1 = 10/(s + 10);  
  
w = logspace(-1, 2, 500); % Frequency range: 0.1 to 100 rad/s  
  
figure;  
bode(G1, w);  
grid on;  
title('Bode Plot of G_1(s) = 10/(s + 10)');
```

### Bode Plot of $G_1(s) = 10/(s + 10)$



$$1-2 \quad G_2(s) = \frac{s-2}{s+10} = \frac{1}{5} \left( \frac{\frac{s}{2}-1}{\frac{s}{10}+1} \right)$$

Zeros = 2      poles = -10

Cutoff freq = 2, 10

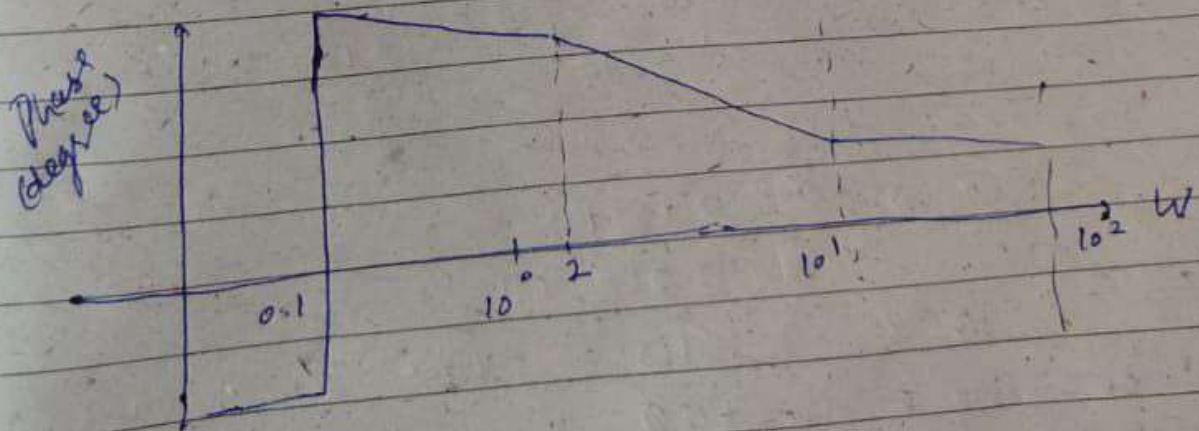
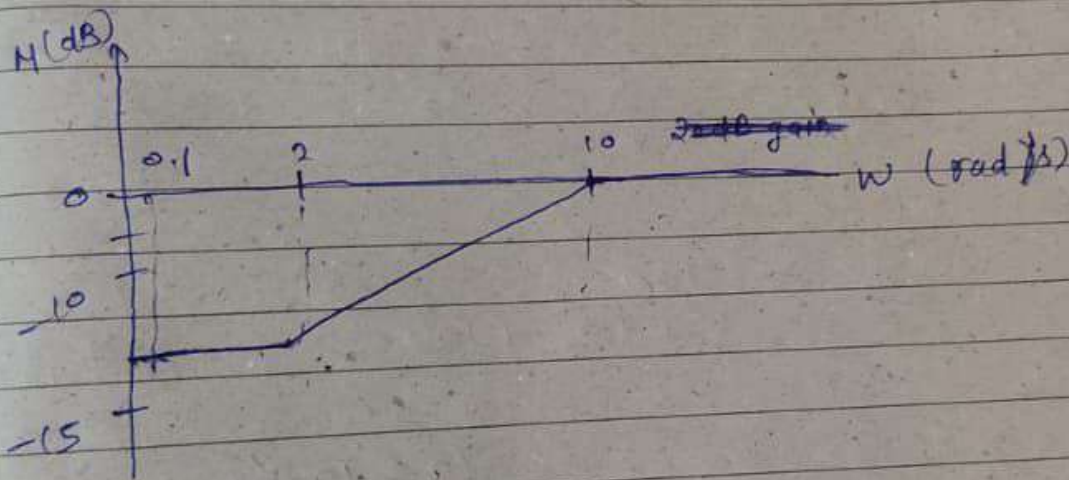
$$G_2(0) = \frac{-2}{10} = -5$$



1.2)  $G_2(s) = \frac{s-2}{s+10} = \frac{1}{5} \left( \frac{\frac{s}{2} - 1}{\frac{s}{10} + 1} \right)$

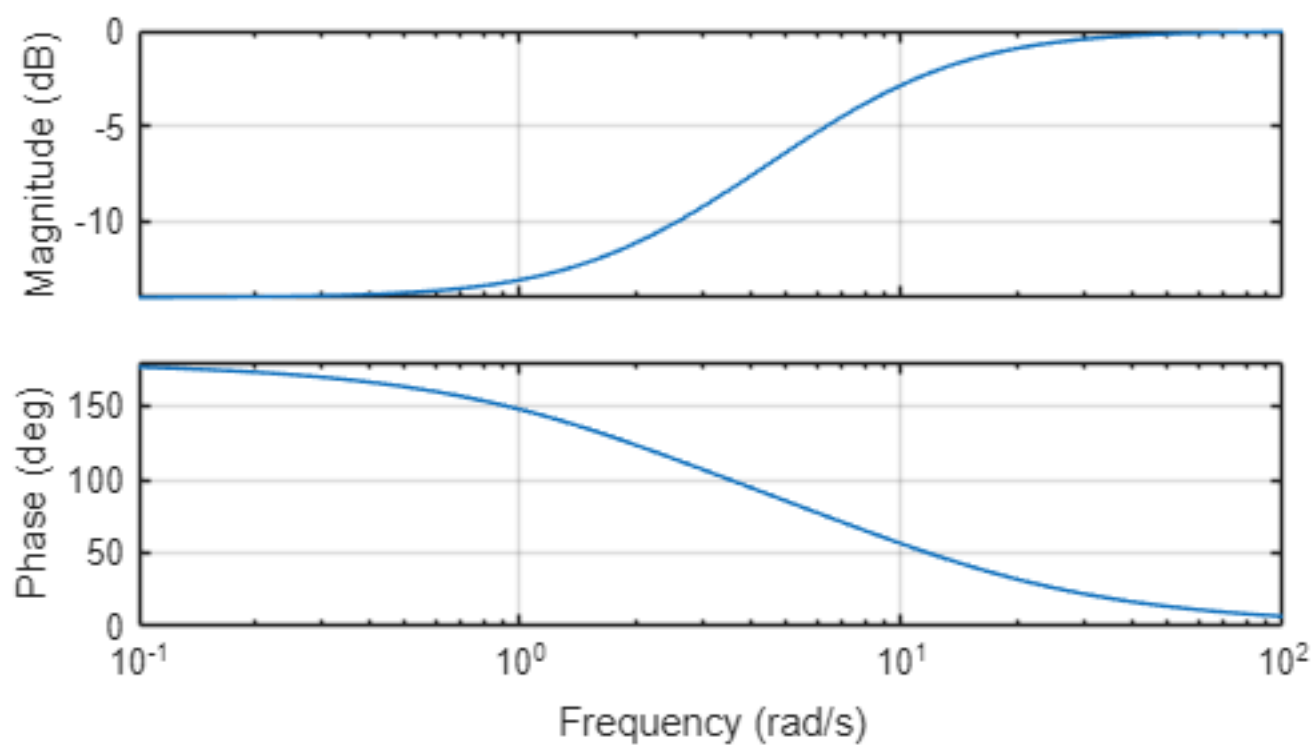
$\omega_c = 2, 10 \quad [\text{rad/s}]$

$\omega < 2$	$2 < \omega < 10$	$\omega > 10$
$G(s) = -1/5$	$G(s) = \frac{s}{2} \times \frac{1}{s} = \frac{s}{10}$	$G(s) = 1$
$M = 20 [\log(1) - \log 5]$	$M = 20 [\log s - 1]$	$M = 0$
$= -20 \log 5$	$M = -20 + 20 \log \omega$	



```
s = tf('s');  
G2 = (s-2)/(s + 10);  
  
w = logspace(-1, 2, 500); % Frequency range: 0.1 to 100 rad/s  
  
figure;  
bode(G2, w);  
grid on;  
title('Bode Plot of G_2(s) = (s-2)/(s + 10)');
```

Bode Plot of  $G_2(s) = (s-2)/(s + 10)$





2.d) A RHP (right half plane) zero contributes  $-90^\circ$  phase (instead of  $+90^\circ$  LHP zero), producing extra phase lag that worsens transient and feedback performance.

1.3

$$G_3(s) = \frac{100}{s^2 + 10s + 100} = \frac{1}{\frac{s^2}{100} + \frac{s}{10} + 1}$$

$$\text{Poles} = \frac{-10 \pm \sqrt{100 - 400}}{2} = \frac{-10 \pm 10\sqrt{3}i}{2} = -5 \pm 5\sqrt{3}i$$

$$\text{Poles} = -5 + 5\sqrt{3}i, -5 - 5\sqrt{3}i$$

1.3

$$G(j\omega) = \frac{100}{(j\omega)^2 + 10j\omega + 100} = \frac{1}{\left(\frac{j\omega}{10}\right)^2 + \frac{j\omega}{10} + 1}$$

$\omega_c = 10$

$\omega > 10$

$$|G(j\omega)| = \frac{1}{\left(\frac{\omega}{10}\right)^2}$$

$$\angle G(j\omega) = \frac{-100}{\omega^2}$$

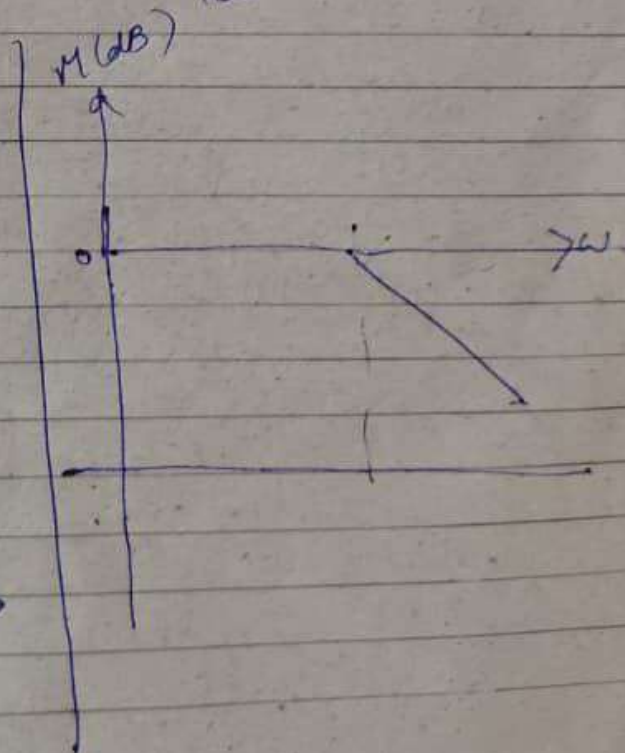
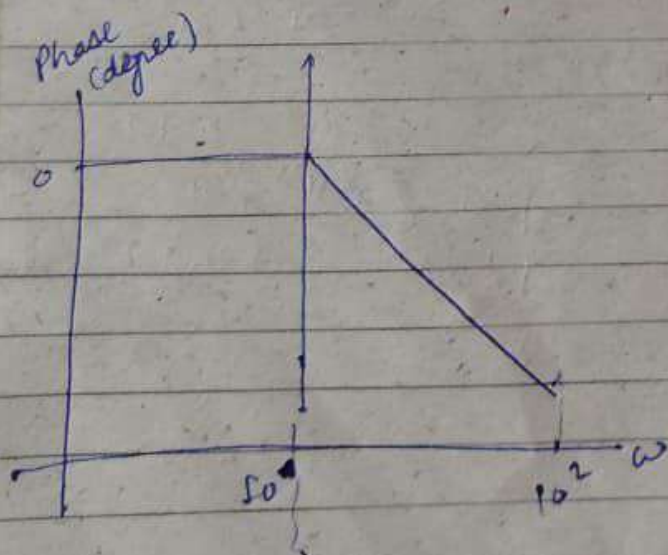
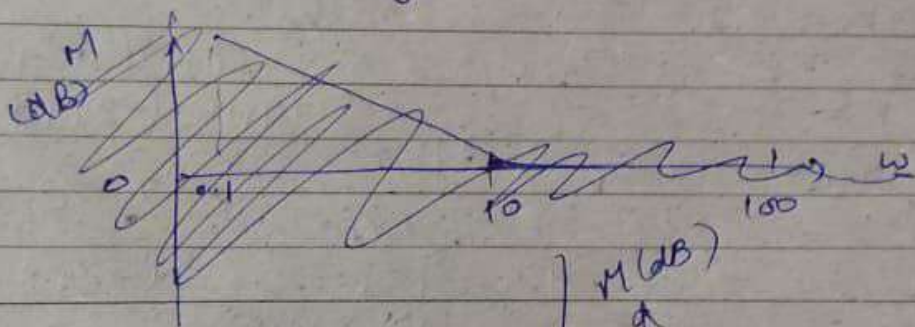
$$M = 20[2 - 2\log\omega]$$

$$M = 40 - 40\log\omega$$

$\omega < 10$

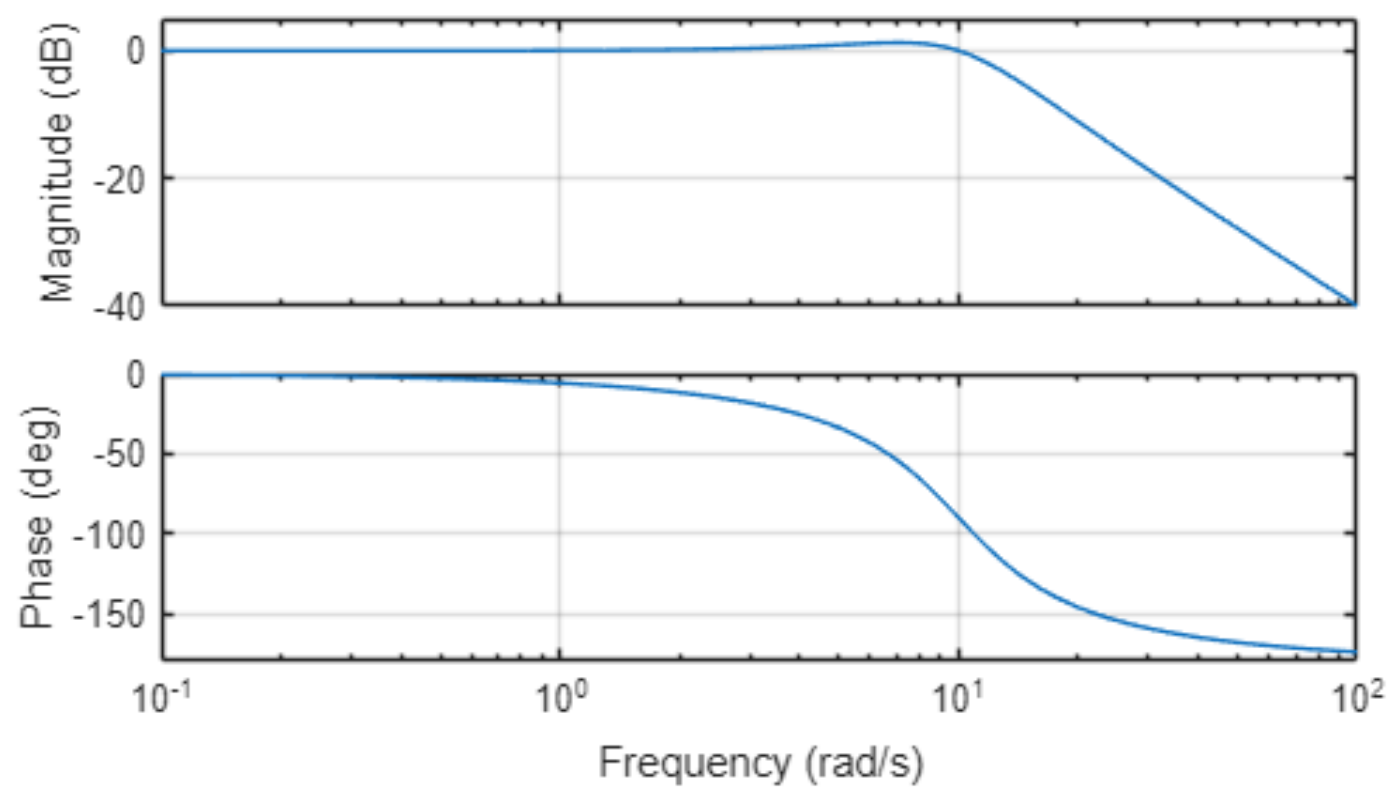
$$|G(j\omega)| = \frac{1}{1} = 1$$

$M = 0$



```
s = tf('s');  
G3 = 100/(s^2 +10*s+100);  
  
w = logspace(-1, 2, 500); % Frequency range: 0.1 to 100 rad/s  
  
figure;  
bode(G3, w);  
grid on;  
title('Bode Plot of G_3(s) = 100/(s^2 +10s+100)');
```

Bode Plot of  $G_3(s) = 100/(s^2 + 10s + 100)$



1.4

$$G_y(s) = \frac{0.1s + 1}{0.01s + 1}$$

Zeros = -10

Poles = -100

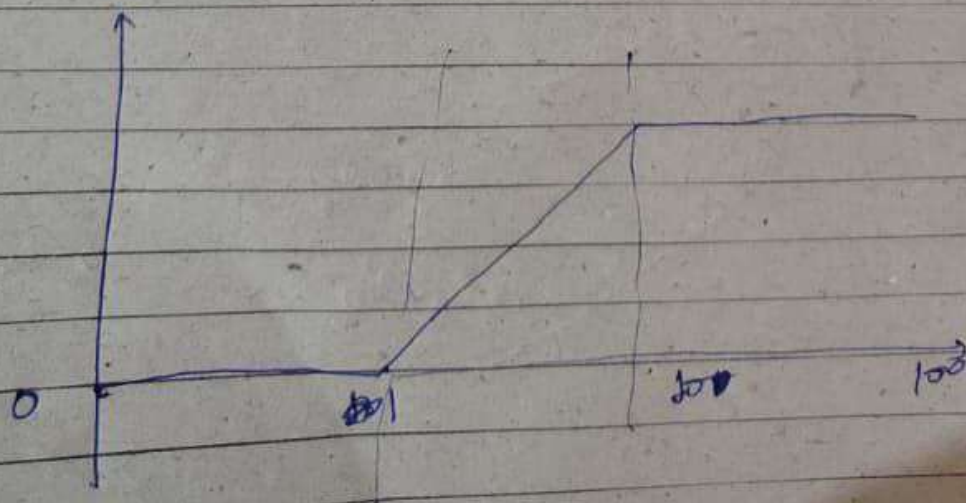
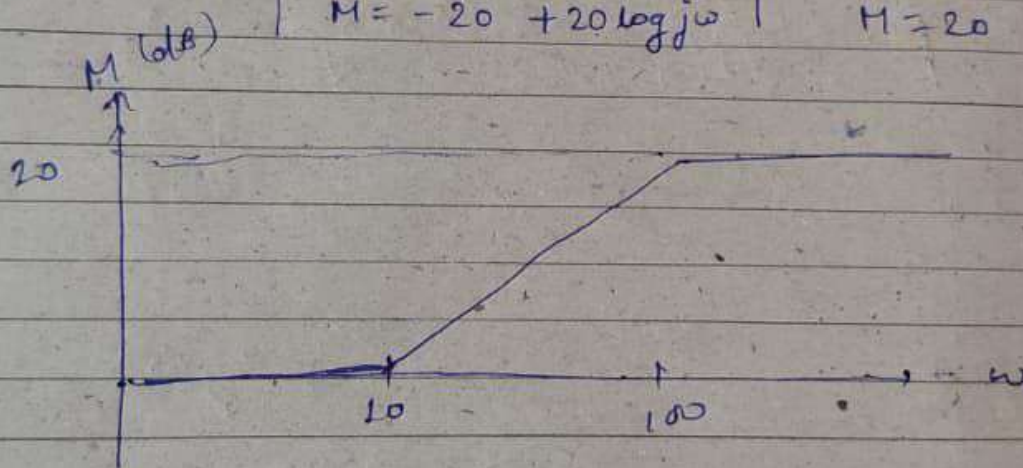
Cutoff freq = 10, 100



(1.4)  $G(j\omega) = \frac{j\omega}{10} + 1$

$\omega_c = 10, 100$

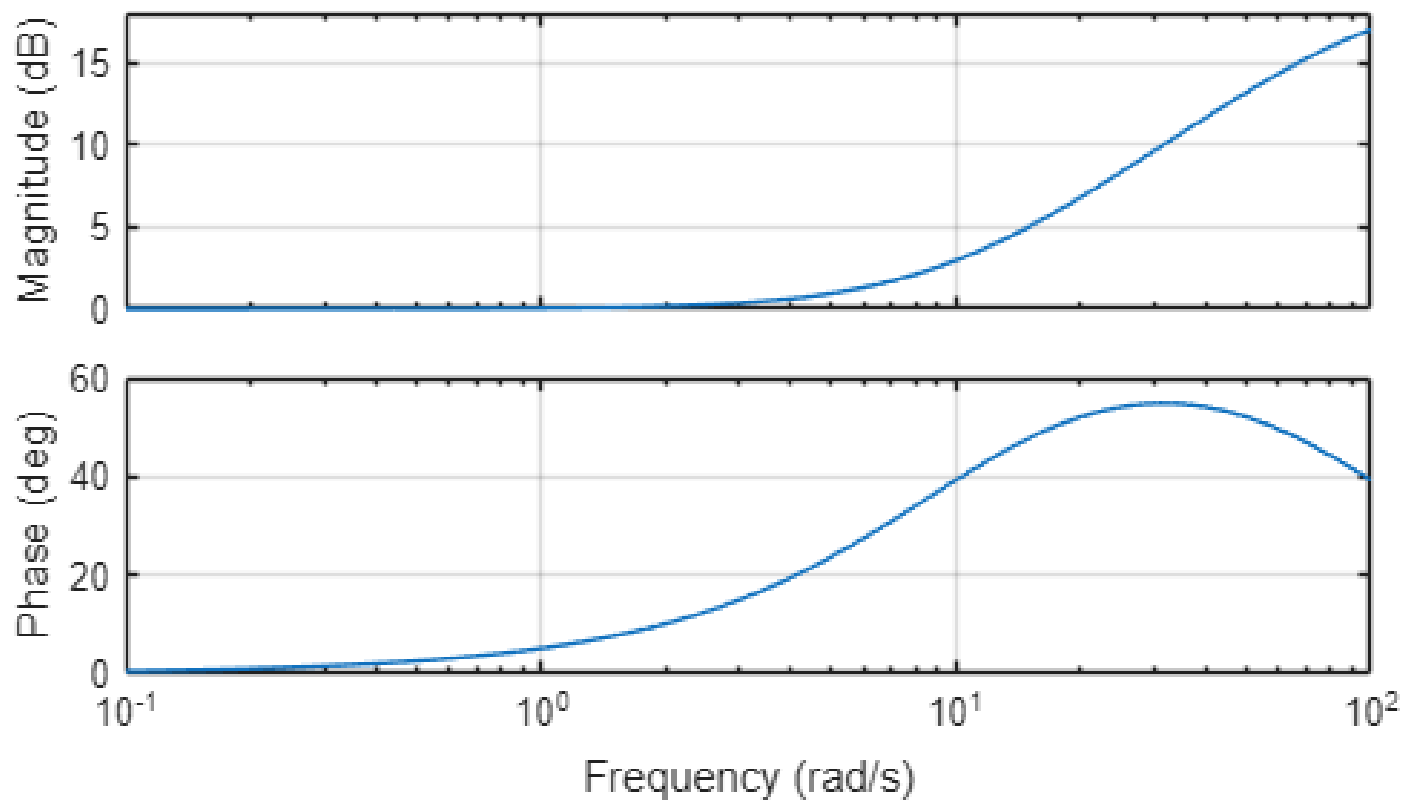
$\omega < 10$	$10 < \omega < 100$	$\omega > 100$
$G(j\omega) = 1$	$G(j\omega) = \frac{j\omega}{10}$	$G(j\omega) = 10$
$M = 0$	$M = 20 [\log j\omega - 1]$	$M = 20 \log 10$
	$M = -20 + 20 \log j\omega$	$M = 20$



4.d) Between the zero ( $\omega_z = 10$ ) and the pole ( $\omega_p = 100$ ) the LHP zero is contributing  $+90^\circ$  lead while the pole is  $-90^\circ$  lag has not yet kicked in, so the net effect is positive phase.

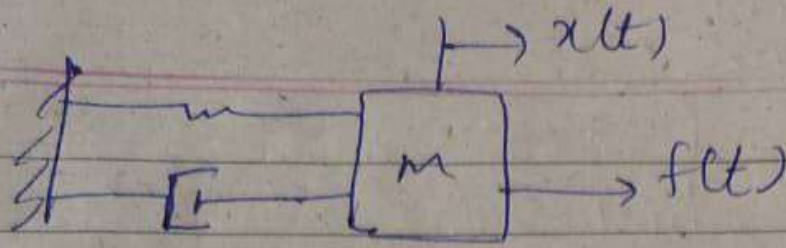
```
s = tf('s');  
G4 = (0.1*s+1)/(0.01*s+1);  
  
w = logspace(-1, 2, 500); % Frequency range: 0.1 to 100 rad/s  
  
figure;  
bode(G4, w);  
grid on;  
title('Bode Plot of G_4(s) = (0.1s+1)/0.01s+1');
```

**Bode Plot of  $G_4(s) = (0.1s+1)/0.01s+1$**





Q2/  
B.1



a) 
$$m \frac{d^2 x(t)}{dt^2} = F(t) - c \frac{dx(t)}{dt} - kx$$

$$\Rightarrow F(t) = m \frac{d^2 x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx$$

b) Laplace transform (zero ICS)  
Take Laplace (initial conditions = 0)

$$ms^2 X(s) + c s X(s) + k X(s) = F(s)$$

c) Transfer function  $G(s) = \frac{X(s)}{F(s)}$

$$G(s) = \frac{1}{ms^2 + cs + k}$$

Q.2

$$G(s) = \frac{1}{ms^2 + cs + k}$$

$$= \frac{1}{(1)s^2 + (4)s + 16}$$

$$G(s) = \frac{1}{s^2 + 4s + 16}$$

$$\text{Poles} = \frac{-4 \pm j\sqrt{48}}{2} = -2 \pm 2\sqrt{3}j$$

$$\text{DC gain} = G(0) = \frac{1}{16} = 0.0625$$

$$\omega_n = 4 \text{ rad/sec}$$



B)

$$G(j\omega) = \frac{1}{\left(\frac{j\omega}{4}\right)^2 + \frac{j\omega}{4} + 1}$$

$$\omega_c = 4 \text{ rad/sec}$$

$$\omega < 4$$

$$G(j\omega) = 1$$

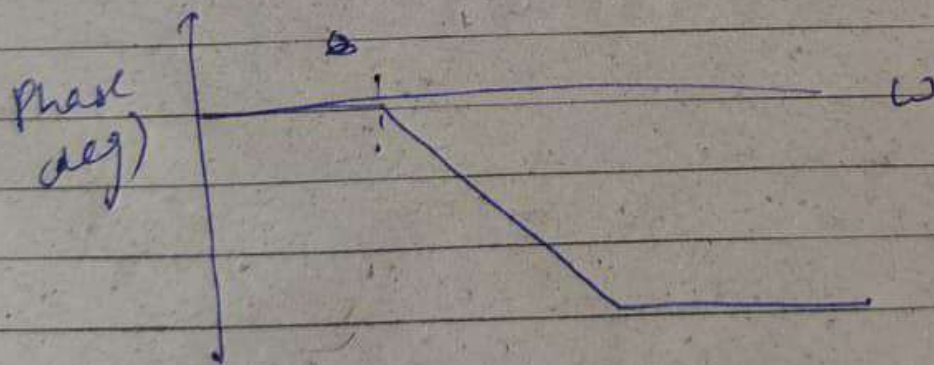
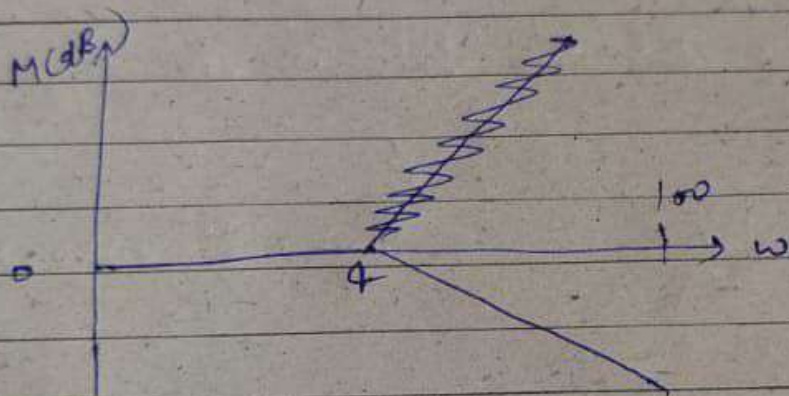
$$M = 0$$

$$\omega > 4$$

$$G(j\omega) = \frac{-16}{\omega^2}$$

$$M = 20 [\log 16 - 2 \log \omega]$$

$$M = 80 \log 2 - 40 \log \omega$$



```
s = tf('s');  
G = 1/(s^2+4*s+16);  
  
w = logspace(-1, 2, 500); % Frequency range: 0.1 to 100 rad/s  
  
figure;  
bode(G4, w);  
grid on;  
title('Bode Plot of  $G(s) = 1/(s^2+4s+16)$ ');
```

**Bode Plot of  $G(s) = 1/(s^2+4s+16)$**

