40
190

Assignment :- 1

Q1) 1.1) 1) $\quad S = -10, \quad G_1(0) = 1, \quad 0\,dB.$

2)

$$T(jw) = \frac{10}{jw + 10}$$

$$|T(jw)| = \frac{10}{\sqrt{100 + w^2}}$$

$100 \;\;\;\; =$

$$M(dB) = 20 - 10\log_{10}(100 + w^2) = 20 - 20\log_{10} w.$$

$$\text{Phase (Degrees)} = -\tan^{-1}\left(\frac{w}{10}\right)$$
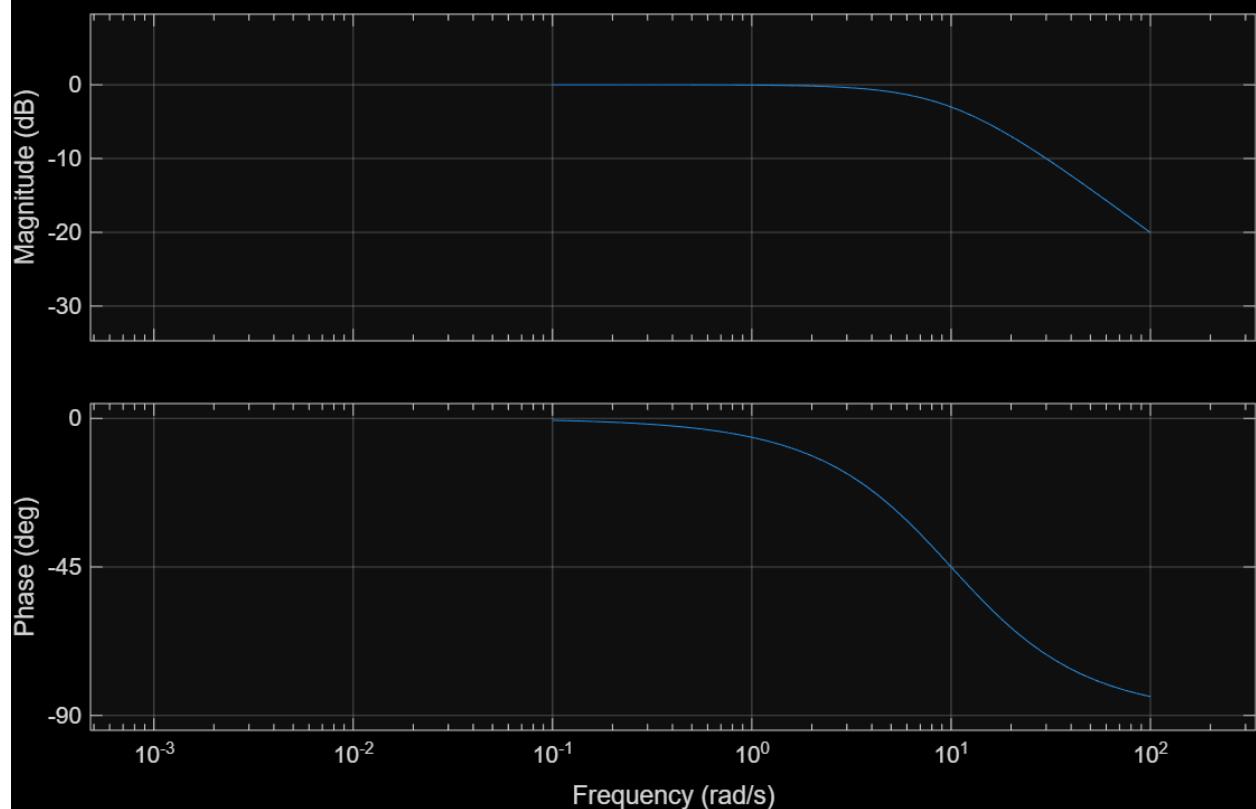
1.2) 1) Zero : $S = 2$, Pole : $S = -10$, $G_2(0) = -\frac{1}{5}$.

2) $\quad T(jw) = \dfrac{jw - 2}{jw + 10}$
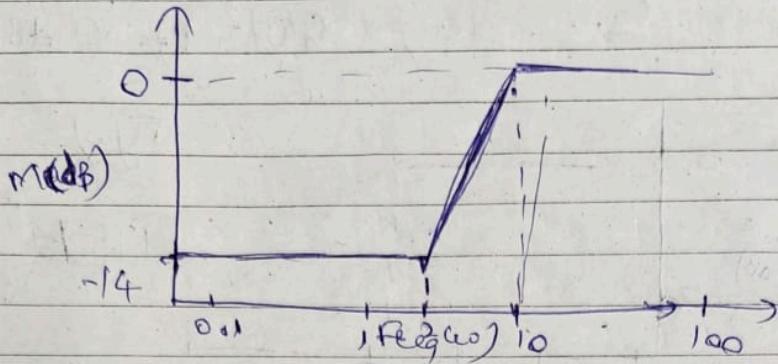
$$M(dB) = 20\log_{10}\left(\frac{\sqrt{4 + w^2}}{\sqrt{100 + w^2}}\right) = 20\log_{10}(w) - 20\log_{10}(w)$$

$$\text{At } w = 10, \quad M(dB) = 20\log_{10}\left(\frac{\sqrt{14}}{\sqrt{200}}\right)$$

$$= -2.76$$

Bode Plot for Problem A.1: G1(s)

At $\omega = 2$, $M(dB) = 20\log_{10}\sqrt{\dfrac{8}{104}} \approx -11dB.$
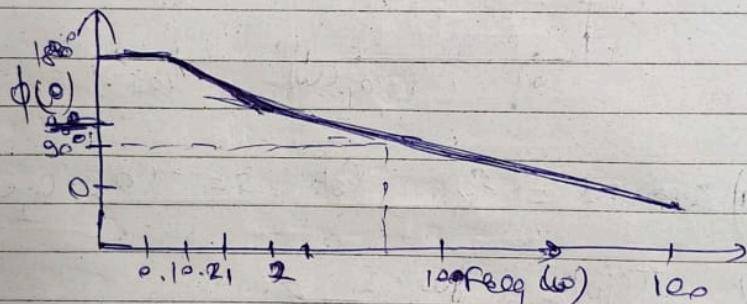
$$\text{Phase } (^\circ) = \tan^{-1}\left(\dfrac{-\omega}{2}\right) - \tan^{-1}\left(\dfrac{\omega}{10}\right).$$

$$\Rightarrow \quad 180 - \tan^{-1}\left(\dfrac{\omega}{2}\right) - \tan^{-1}\left(\dfrac{\omega}{10}\right).$$

At $\omega \to 0$ $\phi = 180^\circ$
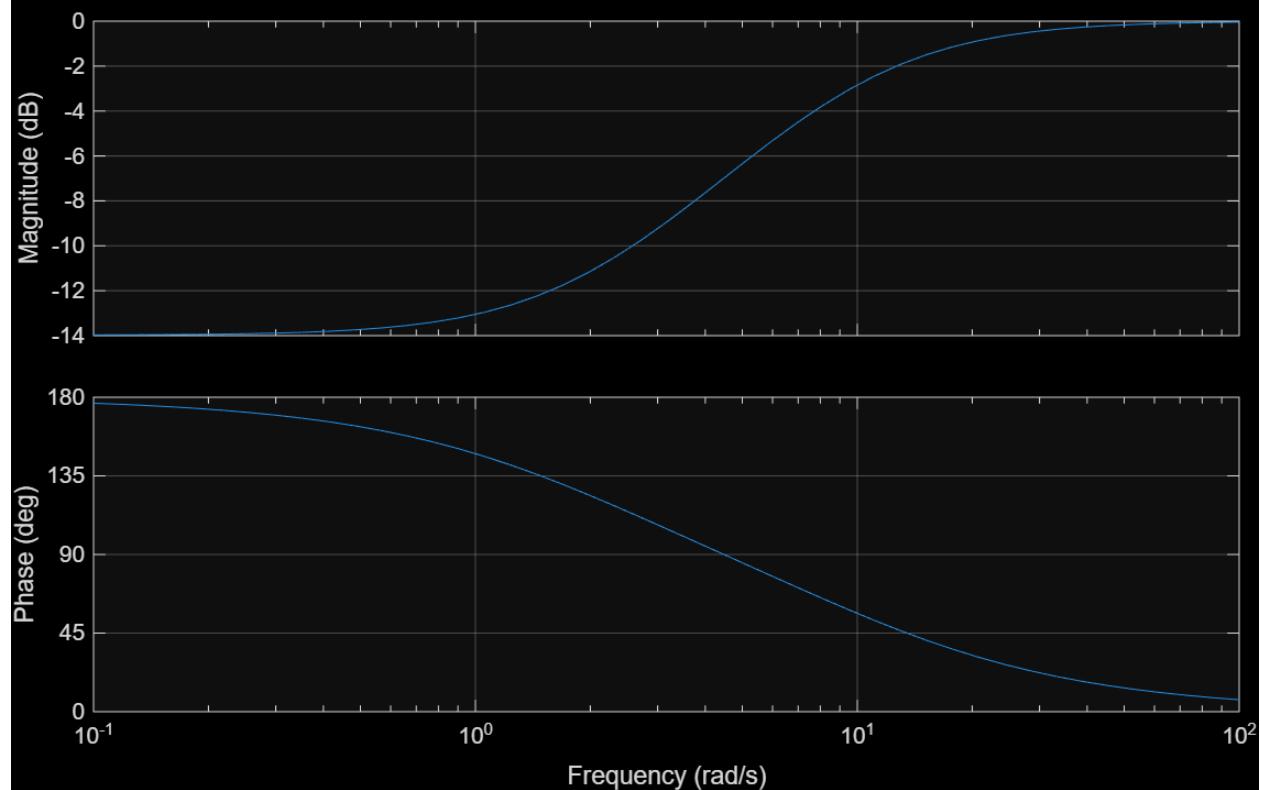
$\omega \to \infty$ $\phi = 0.$

$\omega = 2$ $\phi = 123.7^\circ$

4) A right half-plane zero subtracts phase (phase lagging) even though it increases the magnitude.

Bode Plot for Problem A.1: G2(s)

dB.

1.3) 1) $S_{1,2} = \dfrac{-10 \pm \sqrt{300}j}{2}$

$M(dB) = \dfrac{100}{-\omega^2 + 10j\omega + 100}$

$\Rightarrow M(dB) = 40 - 20\log_{10}\sqrt{(\omega^2 + 10j)^2 + 100\omega^2}$
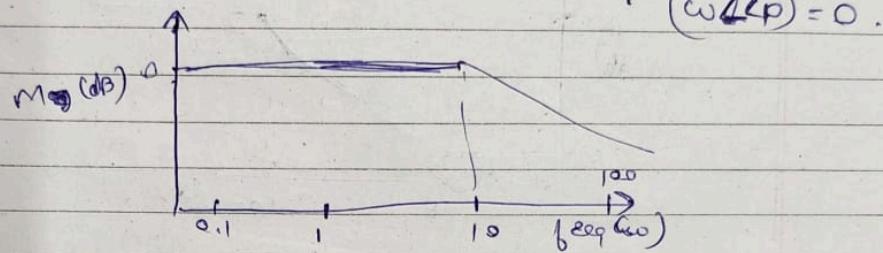
$=/ \quad 40 - 20\log_{10}\sqrt{(\omega^2 - 100)^2 + 100\omega^2}$

$\Rightarrow 40 - 10\log_{10}(\omega^2 - 100)^2 + 100\omega^2$

$= 40 - 10\log_{10}\omega^4 \quad \Rightarrow 40(1 - \log_{10}\omega)$
$(\omega >> 10)$

$\not R_e \, |S_{1,2}| = 10.$

$(\omega << p) = 0.$

Mag (dB)

0.1   1   10   freq (ω) 100

Phase (°) $= 0 - \tan^{-1}\left(\dfrac{100 - \omega^2}{10\omega}\right)^{-1}$

$\omega \to 0 \qquad$ Phase $= 0.$

$\omega \to \infty \qquad$ Phase $= -180°.$

$\omega = 10 \qquad$ Phase $= 90°.$

M(dB)

-90

-180

0.1   1   10   100 (ω)

Bode Plot for Problem A.1: G3(s)

$$10 \frac{(s+10)}{10(s+10)}$$

$$\frac{2}{100}$$
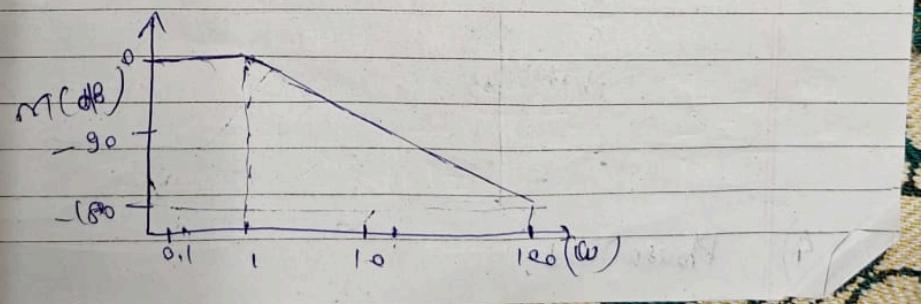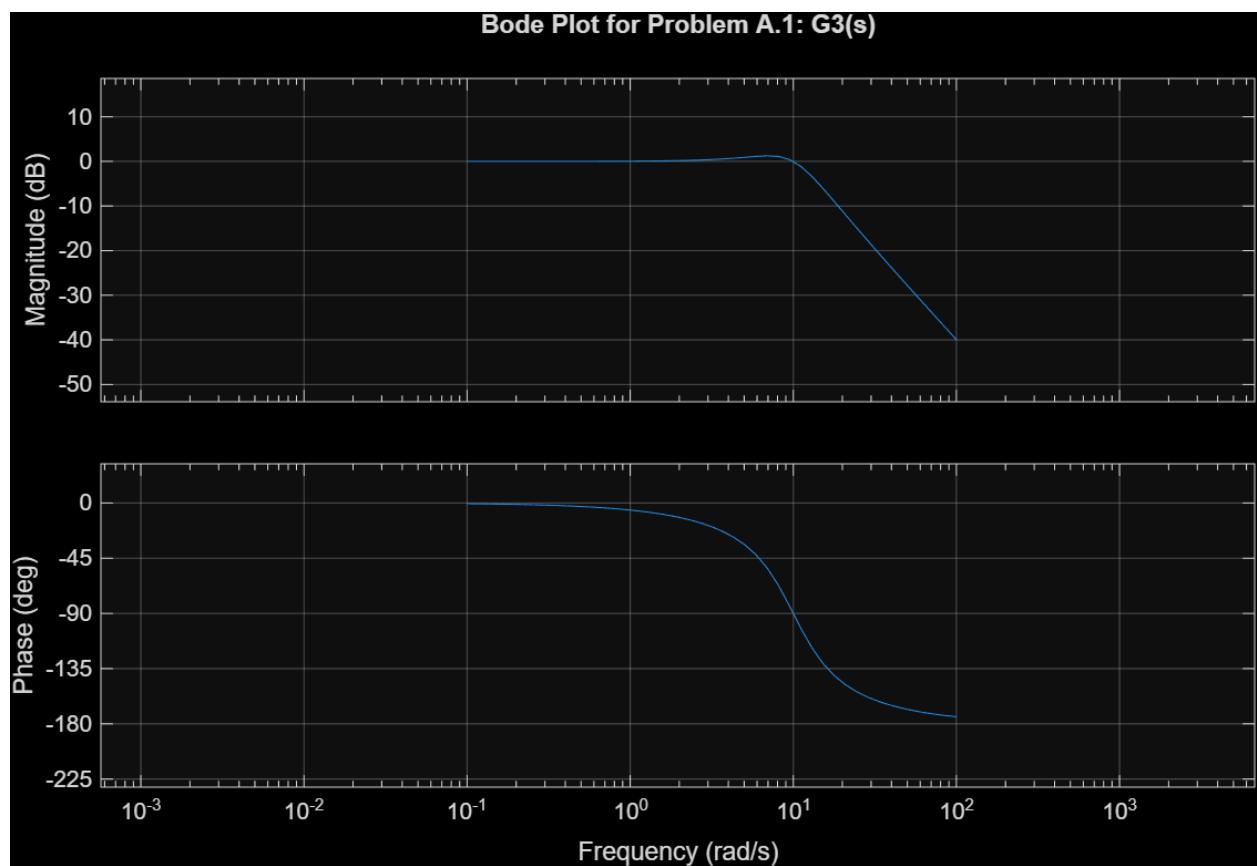
1.4) 1) zero $s = -10$, Pole $s = -100$,

2) $M(dB) = 20 + 20 \log_{10} \left(\sqrt{\frac{100 + \omega^2}{10^4 + \omega^2}} \right)$.
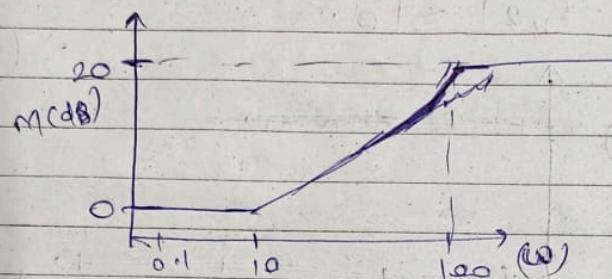
$= 20 + 10 \log_{10} \sqrt{100 + \omega^2} - 10 \log_{10} (10^4 + \omega^2)$

$\Rightarrow$ $\omega \to 0$ $M(dB) = 20 + 20 - 40 = 0$.

$\omega \to \infty$ $M(dB) = +20$.

$\omega = 10$ $M(dB) \simeq -0.9 \ 3.06$

$\omega = 100$ $M(dB) = 20$.
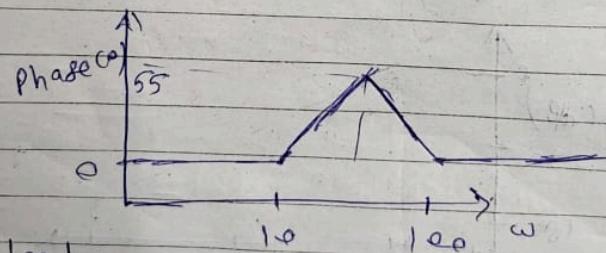
Phase $(°) = 0° + \tan^{-1} \left(\frac{\omega}{10} \right) - \tan^{-1} \left(\frac{\omega}{100} \right)$

$\omega \to 0$ Phase $= 0$,
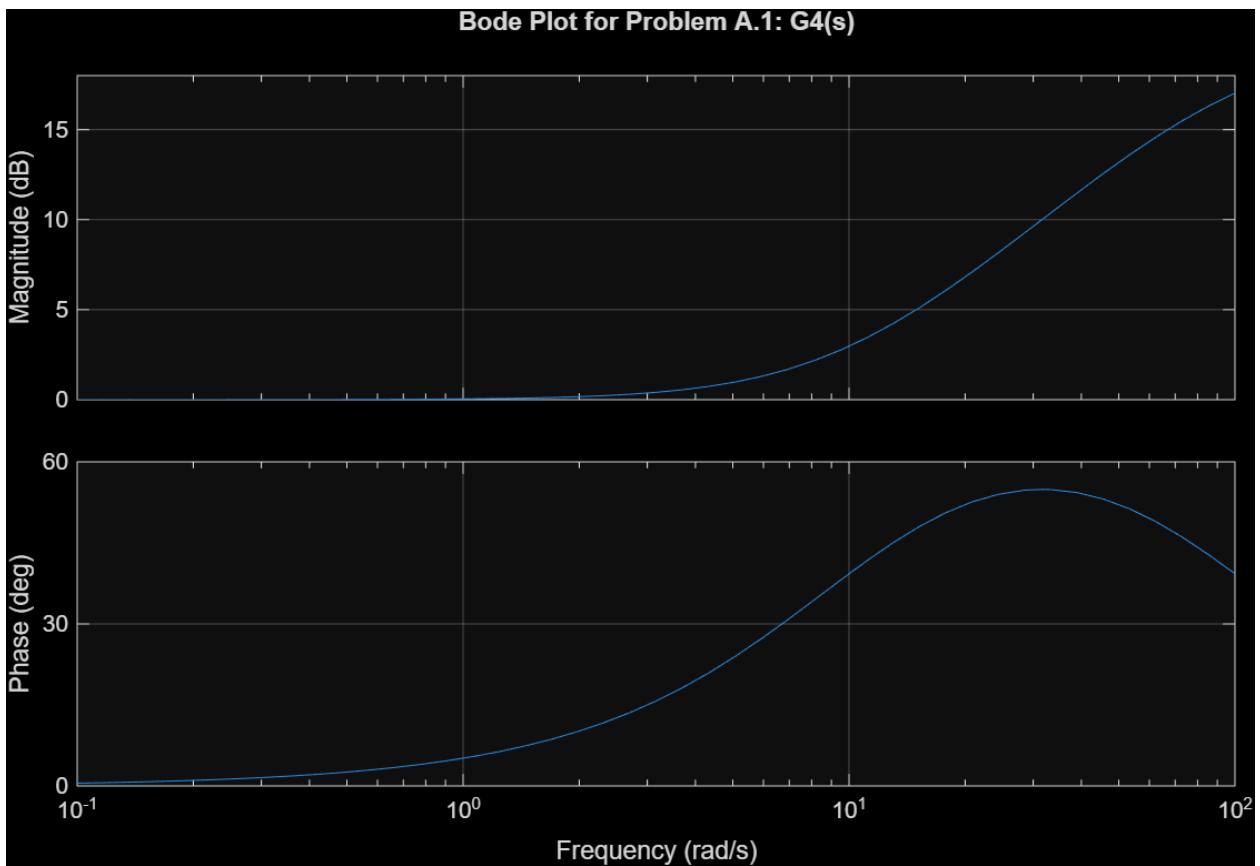
$\omega \to \infty$ Phase $= 0$.

$\omega = 10$ phase $= 39.3$,
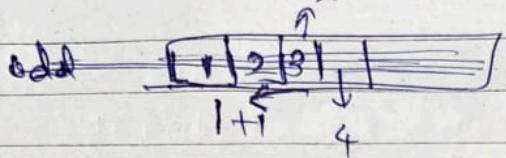
$\omega = 100$ Phase $= 39.3$.

4) Phase lead.

Bode Plot for Problem A.1: G4(s)

$$[F - kx(s) - csX(s)] = m[s^2X(s)]$$

$$G = \frac{-k \quad G - CS}{-ms^2 G}$$

$$G = \frac{1}{ms^2 + k + cs}$$

odd $\boxed{[1|2|3|}$

1+1

B.) 1) 1)

$kx \leftarrow$
$c\dot{x} \leftarrow$ $\boxed{\quad} \rightarrow f(t)$
$m\ddot{x}$

$-(kx + c\dot{x}) = f(t) = m\ddot{x}$
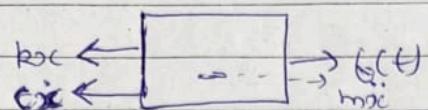
2)

$$F - (kX + csX) = ms^2X.$$
$$F = X(ms^2 + cs + k).$$
$$\frac{1}{ms^2 + cs + k} = G$$

2) 1)

$$G(s) = \frac{1}{s^2 + 4s + 16.}$$

2) Pole $s_{1,2} = \frac{-4 \pm 4\sqrt{3}\,j}{2} = -2 \pm 2\sqrt{3}\,j$

3)

$$M(dB) = 20\log_{10}\sqrt{\frac{1}{(16-\omega^2)^2 + 16\omega^2}}$$

$$= -10\log_{10}(16-\omega^2)^2 + 16\omega^2.$$

$\omega \to 0$ $M(dB) = -20\log_{10}16 = -80\log_{10}\frac{2}{4}$

$\omega \to \infty$ $M(dB) = -64\,\infty$

$\omega = 4\text{\o}$ $M(dB) = -20\log_{10}11 = -64$

$\omega = 4$ $M(dB) = (-24)$

$M(dB)\,24$
-64

$1 \quad 4 \quad 40 \quad 100\,\omega$

Phase
-90
-180

$0 \quad 1 \quad 4 \quad 40 \quad 100 \quad \omega$

Bode Plot for Problem B.2: G(s)