EE324 Control Systems Lab

Problem sheet 10

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Question 1

Given the following state space system

$$X = AX+BU$$

$$Y = CX+DU$$

Part(i)

Taking A,B,C,D and T as follows

$$A = \begin{matrix} 1 & 2 & 5 \\ 1 & 3 & 4 \\ 6 & 1 & 2 \end{matrix}, B = \begin{matrix} 1 \\ 2 \\ 1 \end{matrix}, C = \begin{matrix} 1 & 1 & 3 \end{matrix}, D = 2, T = \begin{matrix} 1 & 0 & 0 \\ 1 & 2 & 4 \\ 3 & 0 & 2 \end{matrix}$$

Initially G(s) is:

$$G1 (s) = D + C(sI - A)^{\wedge -1}B$$

$$G1(s) = 2s^3-6s^2-35s+60/s^3-6s^2-25s+39$$

After transforming A, B and C, I obtained the same transfer function G1(s) = G2(s)

A = T^-1AT, B = T^-1B, C = CT

$$G2(s) = 2s^3-6s^2-35s+60/s^3-6s^2-25s+39$$

Part (ii)

Poles of G1s = G2s were calculated as: 8.419, -3.678 and 1.259.Eigenvalues of A were same as that of T^-1AT, which were: 8.419, -3.678 and 1.259. Therefore, here we -1 observe that eigen values of A are the poles of G1s = G2s .

Part (iii)

Taking a proper transfer function G s (from part(i)) as follows)

$$Gbp (s) = (s+1)(s+5)/(s+3)(s+2)$$

value of D is non-zero in case of biproper transfer functions, whereas in case of strictly proper transfer function, D = 0. This can be understood as below. I observed that:

$$\lim_{s\to\infty}(D + C(sI – A)^{\wedge}-1 B) = D$$

And for a strictly proper transfer function, as degree of denominator is greater that numerator, hence

$$\lim_{s\to\infty}G(s) = 0$$

$$\lim_{s\to\infty}(D+C(sI-A)^{\wedge}-1B)=D$$

$$D = 0$$

Scilab Code:

```
clear;
clc;

s=poly(0,'s');
A = [1,2,5;
    1,3,4;
    6,1,2];
I = eye(3,3);
B = [1;
    2;
    1];
C = [1,1,3];
D = 0*eye(1,1);
T = [1,0,0;
    1,2,4;
    3,0,2];

G1 = D + C*(inv(s*I-A))*B;
evals1 = spec(A);

// Checking G(s) after modifying A, B and C
A = inv(T)*A*T;
B = inv(T)*B;
C = C*T;

G2 = D + C*(inv(s*I-A))*B;

// Eigenvalues of A
evals2 = spec(A);
poles_G = roots(G1.den);

// D value
G_bp = ((s+1)*(s+5))/((s+3)*(s+2));
G_sp = (s+1)/((s+3)*(s+2));

sysGbp = syslin('c',G_bp);
sysGsp = syslin('c',G_sp);

[A1,B1,C1,D1] = abcd(sysGbp);
[A2,B2,C2,D2] = abcd(sysGsp);
```

Question 2

Given the transfer function G(s):

$$G(s) = (s+3)/(s^2+5s+4)$$

We get state space realization as follows:

$$X = AX+BU$$
$$Y = CX+DU$$

Since values in A,B,C and D were preferred to be integers and scilab's tf2ss was giving float values as entries of A,B,C, and D I switched to MATLAB.

MATLAB Code:

```
clear;
clc;
syms s;
s=tf('s');
G1 = (s+3)/((s+1)*(s+4));
 [n1,d1] = tfdata(G1,'v');

[A1,B1,C1,D1] = tf2ss(n1,d1)

 G2 = (s+1)/(s^2+5*s+4);
[n2,d2] = tfdata(G2,'v');
[A2,B2,C2,D2] = tf2ss(n2,d2)
```

Question 3

The Transfer function G(s) is written as :

$$G(s) = D+C(sI-A)^{-1}B$$

Choosing A as follows:

$$A = \begin{matrix} a1 & 0 & 0 \\ 0 & a2 & 0 \\ 0 & 0 & a3 \end{matrix}$$

we observe that eigen values of A are {a1, a2, a3 } and $(sI - A)^{-1}$

$$(sI - A)^{-1} = \begin{matrix} 1/(s-a1) & 0 & 0 \\ 0 & 1/(s-a2) & 0 \\ 0 & 0 & 1/(s-a3) \end{matrix}$$

Here we see that the term if the element in the kth row of B or kth column of C is , then the term 1/(s-ak) will vanish from the product in term

Hence pole ak will no longer be a pole of G(s).

Case 1: an entry in B is 0

$$A = \begin{matrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{matrix},\ B = \begin{matrix} 2 \\ 0 \\ 6 \end{matrix},\ C = \begin{matrix} 1 & 4 & 1 \end{matrix},\ D=0$$

The transfer function obtained for above choice is:

G(s) = 8s-16/s^2-6s+5

which has poles at s = 1, 5. Thus the pole at s = 3 has been cancelled which is the second entry in diagonal matrix A. Corresponding to the 2 position B's entry was 0, hence the second entry in A (i.e 3) is no longer a pole of G(s).

Case2: an entry in C is 0

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix},\ B = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix},\ C = \begin{bmatrix} 0 & 4 & 1 \end{bmatrix},\ D=0$$

The transfer function obtained for above choice is:

G(s) = 10s-38/s^2-8s+15

which has poles at s = 3, 5. Thus the pole at s = 1 has been cancelled which is the first entry in diagonal matrix A. Corresponding to the 1 position C's entry was 0, hence the first entry in A (i.e 1) is no longer a pole of G(s).

Scilab Code:

```
clear;
clc;

s=poly(0,'s');
A = [1,0,0;
   0,3,0;
   0,0,5];

I = eye(3,3);

B = [2;
   0;
   6];

C = [1,4,1];
D = 0*eye(1,1);

G1 = D + C*(inv(s*I-A))*B;

A = [1,0,0;
   0,3,0;
   0,0,5];

I = eye(3,3);

B = [2;
   1;
   6];

C = [0,4,1];

D = 0*eye(1,1);

G2 = D + C*(inv(s*I-A))*B;
```

Question 4

According to the given conditions we have and assuming D as 0 wlog:

$$A = \begin{matrix} a1 & a2 & 0 \\ 0 & a3 & a4 \\ 0 & 0 & a5 \end{matrix}, \; B = \begin{matrix} b1 \\ b2 \\ b3 \end{matrix}, \; C = c1 \quad c2 \quad c3, \; D=0$$

The expression of G(s) simplifies as:

$$G(s) = C(sI - A)^{-1} B$$

Here I observed that poles of the system are at s = a1 , a3 and a5 , if there's no cancellation from a zero factor in numerator. There can be 3 possible cases of diagonal entries getting repeated

Case i: If we have a1 = a3, here we have pole/zero cancellation when b2(a3 - a5) + b3a4 = 0 or a2 = 0

Case ii: If we have a3 = a5,.here we have pole/zero cancellation when c2 (a3 - a1) + c1a2 = 0 or a4 =0

Case iii: If we have a5 = a1 , here we have pole/zero cancellation when either a2 = 0 or a4 = 0 or both are zero

Taking an example from each of the above cases:

Case1:

$$A = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 6 \end{matrix}, \; B = \begin{matrix} 1 \\ 3 \\ 5 \end{matrix}, \; C = 1 \quad 2 \quad 3$$

Poles and zeros of G(s) are obtained as: poles = {1, 1, 6} and zeros = {0.7727, 1}

Therefore, pole/zero cancellation happens as denominator and numerator have a common (s-1) factor.

Case2:

$$A = \begin{matrix} 1 & 5 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{matrix}, \; B = \begin{matrix} 1 \\ 3 \\ 5 \end{matrix}, \; C = 1 \quad 2 \quad 3$$

Poles and zeros of G(s) are obtained as: poles = {1, 6, 6} and zeros = {0.5455, 6}

Therefore, pole/zero cancellation happens as denominator and numerator have a common (s-6) factor.

Case3:

$$A = \begin{matrix} 2 & 5 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{matrix}, \; B = \begin{matrix} 1 \\ 3 \\ 5 \end{matrix}, \; C = 1 \quad 2 \quad 3$$

Poles and zeros of G(s) are obtained as:

poles = {2, 2, 3} and zeros = {2, 2.0455}

Therefore, pole/zero cancellation happens as denominator and numerator have a common (s-2) factor

MATLAB Code:

```
clear;
clc;
 % Case i
A = [1, 0, 0; 0, 1, 4; 0, 0, 6];
B = [1; 3; 5];
C = [1, 2, 3];
D = 0;
[n1,d1] = ss2tf(A,B,C,D);
poles_1 = roots(d1);
 zeros_1 = roots(n1);
 % Case ii
A = [1, 5, 0; 0, 6, 0; 0, 0, 6];
B = [1; 3; 5];
C = [1, 2, 3];
D = 0;
 [n2,d2] = ss2tf(A,B,C,D);
poles_2 = roots(d2);
 zeros_2 = roots(n2);
% Case iii
A = [2, 5, 0; 0, 3, 0; 0, 0, 2];
 B = [1; 3; 5];
C = [1, 2, 3];
D = 0;
[n3,d3] = ss2tf(A,B,C,D);
 poles_3 = roots(d3);
 zeros_3 = roots(n3);
```