EE324 Control Systems Lab

Problem sheet 4

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Question 1

1.a The three blocks are in series, so simply a product of three transfer functions leads to equivalent transfer function along forward path. Using standard equation for feedback we get the transfer function of the system

$$G(s) = s + 5 + a/s + 11s + 30$$

Scilab Code for the same:

```
s = poly(0, 's');
S_1 = 1/(s^2);
S_2 = (50*s)/(s^2+s+100);
S_3 = s-2;
G = S_1*S_2*S_3
T = G/(1+G);
```

$$\frac{-100 + 50s}{-100 + 150s + s^2 + s^3}$$

Figure 1: Input-Output Transfer Function obtained for System A

1.b We have the system with transfer function:

Upon simplifiying the block diagram and writing the equations we get:

$$sC(s) = (R(s) - 2sC(s)) /(s^2 + 1 /s)$$

$$C(s) (2s^3 + s + 2) = R(s^2+ 1 /s)$$

Scilab Code for the same:

```
s = poly(0, 's');
S_1 = s^2 + (1/s);
S_2 = 2*s^3 + s + 2;
T = S_1/S_2;
```

$$\frac{1 + s^3}{2s + s^2 + 2s^4}$$

Figure 2: Input-Output Transfer Function for Part b

1.c Upon simplifying the block diagram and writing the equations we get:

$$2s(R(s) - C(s)) - 5C(s) + (3s^2 / (s + 1)(R(s) - C(s)) = (s + 1) C(s)$$

which finally leads to:

$$C(s) (3s + 6 + (3s^2/ (s + 1) = R(s) (3s^2/ (s + 1) + 2s)$$

Scilab Code for the same:

```
s = poly(0, 's');
S_1 = 2*s + ((3*s^2)/(s+1));
S_2 = ((3*s^2)/(s+1)) + 6 + 3*s;
T = S_1/S_2;
```

Using Scilab, we obtain:

```
                        2
0.3333333s + 0.8333333s
------------------------
                  2
     1 + 1.5s + s
```

Question 2

2 The transfer of the system is given by:

$$H(s, K) = KG(s)/ 1 + KG(s)$$

$$\text{where, } G(s) = 10 /s(s + 2)(s + 4)$$

2a

For K = 5, we obtain the closed-loop transfer function as: (K value can be changed as required)

Scilab Code for the same:

```
s = poly(0, 's');
G = 10/(s*(s+2)*(s+4));
K = 5;
C_tf = (G*K)/(1+(G*K));
C_tf = syslin('c',C_tf);
disp(C_tf);
```

```
        50
-----------------
            2   3
50 + 8s + 6s + s
```

2b

For plotting the loccii of the closed-loop poles we use evans() function from Scilab, with maximum gain value (K) as 100. The following plot is obtained:

Scilab Code:

```
s = poly(0, 's');
G = 10/(s*(s+2)*(s+4));
G = syslin('c',G);
evans(G, 100);
sgrid('red');
```

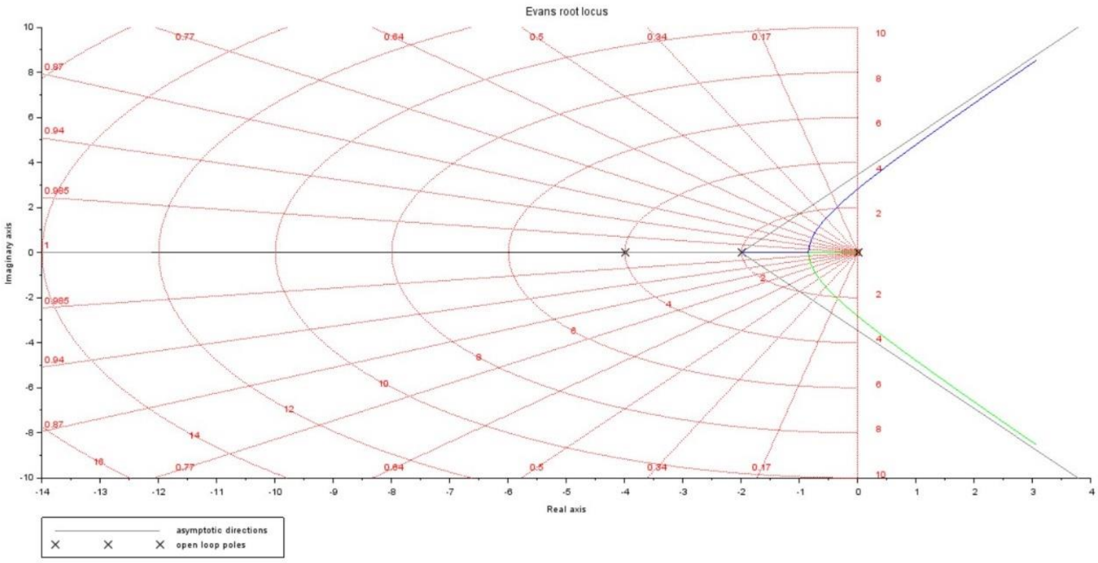

Figure 3: Loci of Closed-loop poles

2c

The code from part b was used here as well, so we obtain that the value of K < 4.8 are stable, as all poles in LHP. The plot with the critical point marked is shown below:

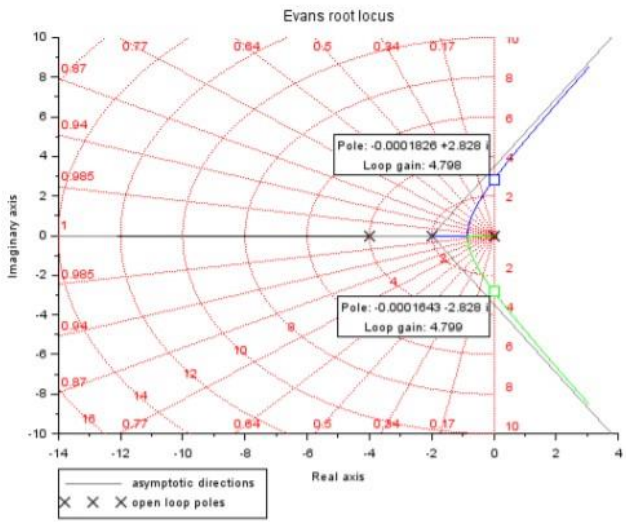

Figure 4: Critical points marked

2d

To confirm our answer, we display the number of sign changes for Kc - 0.1, Kc , Kc + 0.1. As expected, we see no sign change in H(s, K - 1 and a sign change in and . The routh tables c) H(s, Kc) H(s, Kc + 0.1) obtained for each of them are shown below:

Scilab Code:

```
s = poly(0, 's');
G = 10/(s*(s+2)*(s+4));
K = 4.8;
Tf = (K*G)/(1+K*G);
[r,num] = routh_t(Tf.den);
disp(num);
```

```
r  =

   1.           8.
   6.           47.
   0.1666667    0.
   47.          0.
```

Figure 5: Routh Table for Kc – 0.1

```
r  =

   1.           8.
   6.           48.
 -8.882D-15     0.
   48.          0.
```

Figure 6: Routh Table for Kc

```
r  =

   1.           8.
   6.           49.
 -0.1666667     0.
   49.          0.
```

Figure 7: Routh Table for Kc+0.1

Question 3

3a We have the system with transfer function:

P(s) = s^5+ 3s^4 + 5s^3 + 4s^2 + s + 3

1.	5.	1.
3.	4.	3.
3.6666667	0.	0.
4.	3.	0.
-2.75	0.	0.
3.	0.	0.

Figure 8: Routh Table

3b

P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20

$\frac{1}{1}$	$\frac{6}{1}$	$\frac{8}{1}$
$\frac{eps}{1}$	$\frac{5}{1}$	$\frac{20}{1}$
$\frac{-5 + 6eps}{eps}$	$\frac{-20 + 8eps}{eps}$	$\frac{0}{1}$
$\frac{-25 + 50eps - 8eps^2}{-5 + 6eps}$	$\frac{20}{1}$	$\frac{0}{1}$
$\frac{-2.274D-13 - 160eps - 64eps^2}{-25 + 50eps - 8eps^2}$	$\frac{0}{1}$	$\frac{0}{1}$
$\frac{20}{1}$	$\frac{0}{1}$	$\frac{0}{1}$

Figure 9: Routh Table

3c

$$P(s) = s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4$$

1.	3.	2.
-2.	-6.	-4.
-8.	-12.	0.
-3.	-4.	0.
-1.3333333	0.	0.
-4.	0.	0.

3d

$$P(s) = s^6 + s^5 - 6s^4 + s^2 + s - 6$$

$$\begin{matrix} \frac{1}{1} & \frac{-6}{1} & \frac{1}{1} & \frac{-6}{1} \\ \frac{1}{1} & \frac{0}{1} & \frac{1}{1} & \frac{0}{1} \\ \frac{-6}{1} & \frac{0}{1} & \frac{-6}{1} & \frac{0}{1} \\ \frac{-24}{1} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} \\ \frac{eps}{1} & \frac{-6}{1} & \frac{0}{1} & \frac{0}{1} \\ \frac{-144}{eps} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} \\ \frac{864}{-144} & \frac{0}{1} & \frac{0}{1} & \frac{0}{1} \end{matrix}$$

Figure 10: Routh Table

Scilab Code:

```
// Part a
s = poly(0, 's');
G = s^5 + 3*s^4 + 5*s^3 + 4*s^2 + s + 3;
[r,num] = routh_t(G);
disp(r);
//Part b
clear;
s = poly(0, 's');
G = s^5 + 6*s^3 + 5*s^2 + 8*s + 20;
[r,num] = routh_t(G); // r is the routh table
disp(r); // num is the number of sign changes
//Part c
clear;
s = poly(0, 's');
G = s^5 - 2*s^4 + 3*s^3 - 6*s^2 + 2*s - 4;
[r,num] = routh_t(G); // r is the routh table
disp(r); // num is the number of sign changes
//Part d
clear;
s = poly(0, 's');
G = s^6 + s^5 - 6*s^4 + s^2 + s - 6;
[r,num] = routh_t(G); // r is the routh table
disp(r); // num is the number of sign changes
```

Question 4

a)

Construct a degree 6 polynomial whose R-H table has its entire row corresponding to s to be zero:

S^6	1	4	7	4
S^5	2	6	8	0
S^4	1	3	4	0
S^3	0	0	0	0

P(s) = s^6+2s^5+4s^4+6s^3+7s^2+8s+4

b)

Repeat Part (a) with a polynomial of degree 8 and having the entire row corresponding to s to be zero.

S^8	1	7	17	19	4
S^7	1	6	13	12	0
S^6	1	4	7	4	0
S^5	2	6	8	0	0
S^4	1	3	4	0	0
S^3	0	0	0	0	0

P(s) = s^8+s^7+7s^6+6s^5+17s^4+13s^3+19s^2+12s+4

c)

Construct a degree 6 polynomial whose R-H table has the first entry in its row corresponding to s^3 to be zero.

S^6	1	4	15/2	4
S^5	2	6	9	0
S^4	1	3	4	0
S^3	0	1	0	0