MScFE 620 DERIVATIVE PRICING

Group Work Project # 1

See grading rubric here.

Scenario

Suppose you are on a team of newly hired quants on the derivatives desk. As your first responsibility, you will be tasked with validating and verifying the prices of vanilla options and calculating and confirming their sensitivities. This is a very important and highly visible role. Perform it correctly and your group will be promoted to pricing exotic options. Perform it incorrectly and…well, let's not do that.

The key is to ensure you are calculating the prices correctly using Binomial and Trinomial trees. You can double-check the prices of the call and put options using put-call parity. **Be sure to do this — you will get 0 points if you fail to satisfy (within sensible rounding) put-call parity where applicable!**

Once you have priced European options, you will move on to price American options. Since American options have more optionality, they should be worth at least as much as European options. Ensuring that American options cost at least as much as European options is a good check to make. Finally, we will compute a few Greeks to see what the risks of our option positions are.

Tasks

Step 1

As a group, answer these questions on Put-Call Parity in the context of binomial tree model:

1. Does put-call parity apply for European options? Why or why not?
2. Rewrite put-call parity to solve for the call price in terms of everything else.
3. Rewrite put-call parity to solve for the put price in terms of everything else.
4. Does put-call parity apply for American options? Why or why not?

NOTE: In a group of two students, you will omit Team Member **C**'s responsibilities.

For the next set of questions, assume the following values and parameters:

$$S_0 = 100;\ r = 5\%;\ \sigma = 20\%;\ T = 3\,months$$

Team Member A will work with European calls and puts using a <u>binomial tree</u>:

5. Price an ATM European call and put using a **binomial tree**:
    a. Choose the number of steps in the tree you see convenient to achieve reliable estimates.
    b. Briefly describe the overall process, as well as a reason why you choose that number of steps in the tree.
6. Compute the Greek Delta for the European call and European put at time 0:
    a. How do they compare?
    b. Comment briefly on the differences and signs of Delta for both options. What does delta proxy for? Why does it make sense to obtain a positive/negative delta for each option?
7. Delta measures one sensitivity of the option price. But there are other important sensitivities we will look at throughout the course. An important one is the sensitivity of the option price to the underlying volatility (vega)..
    a. Compute the sensitivity of previous put and call option prices to a 5% increase in volatility (from 20% to 25%). How do prices change with respect to the change in volatility?
    b. Comment on the potential differential impact of this change for call and put options.

Team Member B will work with American calls and puts using a binomial tree:

8. Repeat Q5, but this time consider options (call and put) of American style. (Answer sections a and b of Q5 as well)
9. Repeat Q6, but considering American-style options. (Answer/comment on sections a and b of Q6 as well).
10. Repeat Q7, but considering American-style options. (Answer/comment on sections a and b of Q7 as well).

Team Member C will produce graphs and confirmations:

11. If the team answered Q1 as "Yes" (i.e. that put-call parity holds), then show that the European call and put satisfy put-call parity. Comment on the reasons why/why not the parity holds, as well as potential motives.
12. If the team answered Q4 as "Yes" (i.e. that put-call parity holds), then show that the American call and put satisfy put-call parity. Comment on the reasons why/why not the parity holds, as well as potential motives.
13. Confirm that the European call is less than or equal to the American call. Show the difference if any and comment on the reasons for this difference, would this always be the case?
14. Confirm that the European put is less than or equal to the American put. Show the difference if any and comment on the reasons for this difference. For example, would this always be the case?

Step 2

Unless stated otherwise, keep the same input parameters as for Step 1.

Team Member B will now work pricing European options using a trinomial tree:

15. Select 5 strike prices so that Call options are: Deep OTM, OTM, ATM, ITM, and Deep ITM. (E.g., you can do this by selecting moneyness of 90%, 95%, ATM, 105%, 110%; where moneyness is measured as K/S0):
    a. Using the trinomial tree, price the Call option corresponding to the 5 different strikes selected. (Unless stated otherwise, consider input data given in Step 1).
    b. Comment on the trend you observe (e.g., increasing/decreasing in moneyness) in option prices and whether it makes sense.
16. Repeat Q15 for 5 different strikes for Put options. (Make sure you also answer sections a and b of Q15).

Team Member A will now work with American options using a trinomial tree:

17. Repeat Q15, but this time consider Call options of American style. (Answer sections a and b of Q15 as well)
18. Repeat Q16, but this time consider Put options of American style. (Answer sections a and b of Q15 as well)

Team Member C will continue to provide graphs and confirmations:

19. Graph #1. Graph **European** call prices and put prices versus stock prices.
20. Graph #2. Graph **American** call prices and put prices versus stock prices.
21. Graph #3. Graph European and American **call** prices versus strike.
22. Graph #4. Graph European and American **put** prices versus strike.
23. For the 5 strikes that your group member computed in Q15 and Q16, check whether put-call parity holds (within sensible rounding). Briefly comment on the reasons why/why not this is the case.
24. For the 5 strikes that your group member computed in Q17 and Q18, check whether put-call parity holds (within sensible rounding). Briefly comment on the reasons why/why not this is the case.

Step 3

As a group, work on the following real-world questions:

25. **Dynamic Delta Hedging**. Use the following data: S0=180, r =2%, sigma=25%, T=6 months, K = 182:
    a. Price a European Put option with the previous characteristics using a 3-step binomial tree (you do not need code for this).
    b. Pick one path in the tree.
        i. Describe the Delta hedging process (how many units of the underlying you buy/sell, …) of that path throughout each step if you act as the seller of the Put option.
        ii. Make sure you include a table with how your cash account varies at each step (you can follow the format in the slides from Lesson 3 in Module 1). Also, assume you can buy fractions of the underlying asset shares.

26. **Using the same data from Q25, price an American Put option**. Still, assume you are acting as the seller of this put. Consider now 25 steps in the tree (do this via python code).
    a. Compute the delta hedging needed at each node in each step.
    b. Show the evolution of the cash-account throughout the different steps for one path of your choice.

c. Comment on the Delta hedging process as compared to the European option case.

27. **Finally, repeat Q26 considering now an Asian ATM Put option.** Comment on your results as compared to the regular American Put option case of Q25.

Reminder: In a group of two students, you will omit Team Member **C**'s responsibilities.

Submission Requirements and Format

One team member submits the following on behalf of the entire group:

1. **1 zipped folder** including:
    a. An executable Jupyter notebook* that includes the code, its output, and the answer to each question along with the solution
        i. Option prices MUST be rounded to the nearest cent: for example, **39.51, not 39.50941851234124151**
        ii. Option prices should be neatly organized in a table and **NOT presented individually**
    b. A duplicate version of the Jupyter notebook above in PDF or HTML format. In order to include the output of the code, you must RUN the code before downloading the PDF.
2. **1 PDF document** with the question number and the written answers to the questions only (the document will have the same content as the ones above but without the code). Note: this document is run against Turnitin, and must be included as part of the submission.
    a. Use the available Report Template and fill out the required information in the first page.

***Use Google Colab or GitHub to collaborate** in completing the executable Python program.*

*The PDF file with the answers to the questions must be uploaded **separately** from the zipped folder that includes any other types of files. This allows Turnitin to generate a similarity report.*

Rubric

Your instructor will evaluate your group submission for GPW1 using the following rubric:

Quantitative Analysis (Open-Ended Questions) 40 Points	Technical and Non-Technical Reports 30 Points	Writing and Formatting 20 Points
The group is able to apply results, formulas, and their knowledge of theory to real-life finance scenarios by doing the following: ● Providing all the necessary information to support their arguments. ● Presenting arguments that reflect group discussion and research. ● Using authoritative references to support a position and provide updated information ● Concluding with practical takeaways for more insightful financial decision-making.	Technical Reports contain 3 parts: 1) **code** for each question (be sure to explicitly state the question number), 2) the corresponding output of that **code**, and 3) **interpretations** and/or **recommended courses of action** that reasonably follow from those results. **Note**: Technical reports will include the technicalities of models, such as names, methods of estimation, parameter values, etc., and exclude generalities about the work done. It should NOT include names of Python code that were used.	A submission that looks professional should: ● Include the axes, labels, and scales in graphs. ● Be free of significant grammatical errors or typos. ● Be an organized, well-structured, and easy-to-read document. ● Include proper citations and a bibliography in MLA format.
	Non-Technical Reports contain 3 parts: 1) clear explanation of results; 2) the recommended course of action that follows; and 3) the identification of factors that impact each portfolio. **Note**: AVOID all references to model names, algorithms, and unnecessary details. Instead, focus on the investment decision.	

Revised: April 21, 2025