Find the root of the equation $2x^3 - 2x - 5 = 0$ by regula-Falsi Method?

Sol:-

Given:

$2x^3 - 2x - 5 = 0$

$f(x) = 2x^3 - 2x - 5$

$f(x) = 2x^3 - 2x - 5$

$f(1) = -5 < 0$

$f(2) = 7 > 0$

∴ Root lies between 1 and 2.

$a = 1$ and $b = 2$

First ⇒ Approximation

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$x_1 = 1.45667 \Rightarrow f(x_1) = -2.14699 < 0$

Second Iteration

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$x_2 = 1.55359$

$f(x_2) = -0.60759 < 0$

Third Iteration

$$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$x_3 = 1.58924$

$f(x_3) = -0.15063 < 0$

n	a	b	f(a)	f(b)	x_i	$f(x_i)$
1	1	2	-5	7	1.41667	-2.15469
2	1.45667	2	-2.14699	7	1.55359	-0.6075
3	1.55359	2	-0.60759	7	1.58924	-0.15063
4	1.58924	2	-0.15063	7	1.59789	-0.0361
5	1.59789	2	-0.0361	7	1.59996	-0.00858
6	1.59996	2	-0.00858	7	1.60045	-0.00203
7	1.60045	2	-0.00203	7	1.60056	-0.00048

$\therefore$ Root is 1.600

Find the root of the equation $3x - \cos x = 1$ by Regula-Falsi Method?

Sol:- Given :-

$$3x^2 - \cos x - 1 = 0$$

$$f(x) = 3x - \cos x - 1$$

$$f(0) = -2 < 0$$

$$f(1) = 1.459697 > 0$$

$$a = 0 \; ; \; b = 1$$

By using Regula falsi Method.

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$$x = \frac{0 - 1(-2)}{1.459697 - (-2)}$$

$$x = \frac{2}{3.459697}$$

$$x = 0.578085$$

a	b	f(a)<0	f(b)>0	x	f(x)
0	1	-2	1.459697	0.578085	-0.103255<0
0.578085	1	-0.103255	1.459697	0.605958	-0.004082<0
0.605958	1	-0.004082	1.459697	0.607056	-0.000162<0
0.607056	1	-0.000162	1.459697	0.607099	-0.000006<0

∴ Root is 0.6070

1) Find a root of the equation $f(x) = x^3 - x - 1$ using Newton Raphson Method?

Soln:-

Given:-

$f(x) = x^3 - x - 1$

$f(1) = -1 < 0$

$f(2) = 5 > 0$

Root lies between 1 and 2.

$a = 1 \; ; \; b = 2$

First Iteration:

$$x_0 = \frac{1+2}{2} = 1.5$$

$f(x_0) = f(1.5) = 0.875$

$f'(x_0) = 5.75$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$x_1 = 1.34783$

Second Iteration:

$f(x_1) = 0.10068$

$f'(x_1) = 4.44991$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$x_2 = 1.3252$

3rd Iteration:

$f(x_2) = 0.00206$

$f'(x_2) = 4.26847$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$x_3 = 1.32472$

4th Iteration:

$f(x_3) = 0$

$f'(x_3) = 4.26463$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$x_4 = 1.32472$

∴ Root is 1.32472.

2) Find a root of the equation $4(x-\sin x)=1$ by Newton Raphson Method?

Soln :-

Given:-

$$4(x-\sin x)-1=0$$

$$f(x) = 4(x-\sin x)-1$$

$$f(1) = -0.365883 < 0$$

$$f(2) = 3.362810 > 0$$

$$x_0 = \frac{a+b}{2}$$

1st Iteration $\Rightarrow \frac{3}{2} = 1.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.5$$

$$f(x_1) = 1.010020$$

$$f'(x_1) = 3.929262$$

Second Iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 - \frac{1.010020}{3.929262}$$

$$x_2 = 1.242949$$

Third Iteration

$$f(x_2) = 0.184845$$

$f'(x_2) = 3.677994$

$x_3 = 1.242949 - \frac{0.184845}{3.677994}$

$x_3 = 1.192691$

4th Iteration:-

$f(x_3) = 0.053301$

$f'(x_3) = 3.070634$

$x_4 = 1.192691 - \frac{0.053301}{3.070634}$

$x_4 = 1.175332$

5th Iteration:-

$f(x_4) = 0.000056$

$f'(x_4) = 3.614763$

$x_5 = 1.175332 - \frac{0.000056}{3.614763}$

$x_5 = 1.172550$

6th Iteration:-

$f(x_5) = 0.003230 \quad f'(x_5) = 3.612197$

$x_6 = 1.172550 - \frac{0.003230}{3.612197}$

$x_6 = 1.171655$

End

7th Iteration:-

$f(x_6) = 0.001039$ | $f'(x_6) = 3.611372$

$$x_7 = 1.171655 - \frac{0.001039}{3.611372}$$

$$x_7 = 1.171367$$

8th Iteration:-

$f(x_7) = 0.000335$

$f'(x_7) = 3.611107$

$$x_8 = 1.171367 - \frac{0.000335}{3.611107}$$

$$x_8 = 1.171274$$

9th Iteration:-

$f(x_8) = 0.000108$

$f'(x_8) = 3.611021$

$$x_9 = 1.171274 - \frac{0.000108}{3.611021}$$

$$x_9 = 1.171244$$

∴ Root is 1.1712

Find the root of the equation $x^2 - x - 1 = 0$ by Iteration Method?

Soln:-

Given:

$x^2 - x - 1 = 0$

$f(x) = x^2 - x - 1$

$f(1) = -1 < 0$

$f(2) = 1 > 0$

$a = 1$; $b = 2$

Root lies between 1 and 2.

$x^2 = x + 1$

$x = \sqrt{x+1}$

$x = \phi_1(x)$

$\phi_1'(x) = \frac{1}{2\sqrt{x+1}}$

$x = x^2 - 1$

$x = \phi_2(x)$

$\phi_2'(x) = 2x$

$|\phi_1'(x)| = \left|\frac{1}{2\sqrt{x+1}}\right| < 1$ for $x \in (1,2)$

$|\phi_2'(x)| = |2x| > 1$ for $x \in [1,2]$

$\phi(x) = \sqrt{x+1}$

$x_0 = \frac{1+2}{3} = 1.5$

$x_1 = \phi(x_0) = 1.581138 = x_1$

$x_2 = \phi(x_1) = 1.606592 = x_2$

$x_3 = \phi(x_2) = 1.614494 = x_3$

$x_4 = 1.616939$

$x_5 = 1.617695$

$x_6 = 1.617929$

$x_7 = 1.618001$

$x_8 = 1.618023$

$\therefore$ Root $= 1.6180$

Find the root of the equation $x = \cos x$ by Iteration Method?

Given:-

$$x - \cos x = 0$$

$$f(x) = x - \cos x$$

$$f(0) = 0 - \cos 0 = -1 < 0$$

$$f(1) = 0.459697 > 0$$

Root lies between 0 and 1.

By Iteration Method =>

$$x = \cos x$$

$$x = \phi_1(x)$$

$$\phi_1'(x) = -\sin x$$

$$|\phi_1'(x)| = |-\sin x| < 1 \quad \text{for } x \in (0,1)$$

$$\phi(x) = \cos x$$

$$x_0 = \frac{a+b}{2}$$

$$x_0 = \frac{0+1}{2}$$

$$x_0 = 0.5$$

$$x_1 = \phi(x_0) = \cos(0.5) = 0.877582$$

$$x_2 = 0.639012$$

$$x_3 = 0.802685$$

$$x_4 = 0.694778$$

$$x_5 = 0.768195$$

Find

$x_6 = 0.719166$

$x_7 = 0.752355$

$x_8 = 0.730081$

$x_9 = 0.745120$

$x_{10} = 0.735006$

$x_{11} = 0.741826$

$x_{12} = 0.737236$

$x_{13} = 0.740329$

$x_{14} = 0.738246$

$x_{15} = 0.739650$

$x_{16} = 0.738704$

$x_{17} = 0.739341$

$x_{18} = 0.738912$

$x_{19} = 0.739201$

$x_{20} = 0.739007$

$x_{21} = 0.739137$

$x_{22} = 0.739050$

$x_{23} = 0.739108$

$x_{24} = 0.739069$

$x_{25} = 0.739096$

$\therefore$ Root is 0.7390.

Find the root of the equation $xe^x = \cos x$ by Bisection Method?

Sol":-

Given:

$xe^x = \cos x$

$xe^x - \cos x = 0$

$f(x) = xe^x - \cos x$

$f(0) = -1 < 0$

$f(1) = 2.147979 > 0$

Root lies between 0 and 1.

$a = 0$; $b = 1$

$f(a) < 0$; $f(b) > 0$

First Approximation is given by

$x_i = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$

$f(x_i) = f(0.5) = -0.053221 < 0$

$x_i = 0.5$

$f(x_i) = -0.053221 < 0$

S.No.	a	b	$f(a) < 0$	$f(b) > 0$	x_i	$f(x_i)$
1.	0	1	< 0	> 0	0.5	$-0.0432221 < 0$
2	0.5	1	< 0	> 0	0.75	$0.8564661 > 0$
3	0.5	0.75	< 0	> 0	0.625	$0.3546907 > 0$
4	0.5	0.625	< 0	> 0	0.5625	$0.1141293 > 0$
5	0.5	0.5625	< 0	> 0	0.53125	$0.0441512 > 0$
6	0.5	0.53125	< 0	> 0	0.515625	$-0.0006475 < 0$
7	0.515625	0.53125	< 0	> 0	0.523437	$0.0173607 > 0$
8	0.515625	0.523437	< 0	> 0	0.519531	$0.0054 > 0$
9	0.515625	0.519531	< 0	> 0	0.517578	$-0.000006 < 0$
10	0.517578	0.519531	< 0	> 0	0.518554	$0.0034 > 0$
11	0.517578	0.518554	< 0	> 0	0.518066	$0.00013 > 0$
12	0.517578	0.518066	< 0	> 0	0.517822	$0.00019 > 0$
13	0.517578	0.517822	< 0	> 0	0.5177	$-0.00017 < 0$
14	0.5177	0.517822	< 0	> 0	0.51774	$0.00011 > 0$

$\therefore$ Root is 0.5177

Find the root of the equation $x^2-3=0$ by bisection method?

Soln:-

Given:

$x^2-3=0$

$f(x) = x^2-3$

$f(x=1) = 1-3 = -2 < 0$

$f(x=2) = 4-3 = 1 > 0$

Root lies between 1 and 2.

$$x_0 = \frac{1+2}{2}$$

$x_0 = 1.5$

$f(x_0) = (1.5)^2 - 3 = -0.75 < 0$

If $f(x_0) < 0$ assume $a = x_0$

If $f(x_0) > 0$ assume $b = x_0$

$f(x_0)$ is negative, so a is replaced with $x_0 = 1.5$ for the next iterations.

Iterations	a	b	x_0	f(a)	f(b)	$f(x_0)$
1	1	2	1.5	-2	1	-0.75
2	1.5	2	1.75	-0.75	1	0.062
3	1.5	1.75	1.625	-0.75	0.0625	-0.359
4	1.625	1.75	1.6875	-0.3594	0.0625	-0.1523

5.	1.6875	1.75	1.7188	-0.1523	0.0625	-0.0457
6	1.7188	1.75	1.7344	-0.0457	0.0625	0.0081
7	1.7188	1.7344	1.7266	-0.0457	0.0081	-0.0189
8	1.7266	1.7344	1.7305	-0.0188	0.0081	-0.0053
9	1.7305	1.7344	1.7324	-0.0053	0.0081	0.0012
10	1.7305	1.7324	1.7314	-0.0053	0.0012	-0.0022
11	1.7314	1.7324	1.7319	-0.0022	0.0012	-0.0005
12	1.7319	1.7324	1.7321	-0.0005	0.0012	0.0007
13	1.7319	1.7321	1.7320	-0.0005	0.0007	-0.0001
14	1.7320	1.7321	1.7320	-0.0001	0.0007	-0.0000027

∴ Root is 1.7320

1) Fit a straight line to the following Data

x	1	6	11	16	20	26
y	13	16	17	23	24	31

Sol^n :- Let the straight line be $Y = a + bX$, to obtain a & b for this straight line the normal equations are

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

x	y	x^2	xy
1	13	1	13
6	16	36	96
11	17	121	187
16	23	256	368
20	24	400	480
26	31	676	806
$\Sigma x = 80$	$\Sigma y = 124$	$\Sigma x^2 = 1490$	$\Sigma xy = 1950$

$$124 = 6a + 80b \quad - (1)$$

$$1950 = 80a + 1490b \quad - (2)$$

Solving eq^{ns} (1) & (2), we get $\Rightarrow$

$$a = 11.3228$$

$$b = 0.7007$$

$\therefore$ Line of best fit is $Y = 11.3228 + 0.7007X$

Fit a straight line to the following data

x	6·3	7·4	8·6	9·7	11·5
y	5·4	4·3	3·2	2·1	1·5

Soln:-

Let $y = a + bx$ be the straight line.

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

x	y	x^2	xy
6·3	5·4	39·69	34·02
7·4	4·3	54·76	31·82
8·6	3·2	73·96	27·52
9·7	2·1	94·09	20·37
11·5	1·5	132·25	17·25
$\Sigma x = 43·5$	$\Sigma y = 16·5$	$\Sigma x^2 = 394·75$	$\Sigma xy = 130·98$

$$16·5 = 5a + b\,43·5 \quad \text{—} \quad (1)$$

$$130·98 = a\,43·5 + 394·75\,b \quad \text{—} \quad (2)$$

Solving eqn's (1) & (2), we get $\Rightarrow$

$$a = 10·0091$$

$$b = -0·7711$$

$\therefore$ Line of best fit is $y = 10·0091 + 0·7711x$.

Fit a parabola for the following data

x	1996	1997	1998	1999	2000
y	40	50	62	58	60

Soln:- Parabola equation is $y = a + bx + cx^2$ and the normal equations are

$$\Sigma y = an + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	x $x-1998$	x^2	x^3	x^4	xy	x^2y
1996	40	-2	4	-8	16	-80	160
1997	50	-1	1	-1	1	-50	50
1998	62	0	0	0	0	0	0
1999	58	1	1	1	1	58	58
2000	60	2	4	8	16	120	240
$\Sigma x = 9990$	$\Sigma y = 270$	$\Sigma x = 0$	$\Sigma x^2 = 10$	$\Sigma x^3 = 0$	$\Sigma x^4 = 34$	$\Sigma xy = 48$	$\Sigma x^2 y = 508$

$$270 = 5a + 0b + 10c \quad \text{---} (1)$$
$$48 = 0a + 10b + 0c \quad \text{---} (2)$$
$$508 = 10a + 0b + 34c \quad \text{---} (3)$$

Solving eqns (1), (2) & (3), $\Rightarrow$ we get $\Rightarrow$

$$a = 58.5714$$
$$b = 4.8$$
$$c = -2.2857$$

$\therefore$ parabola of best fit is

$$y = 58.5714 + 4.8x + (-2.2857)x^2.$$

Fit a parabola for the following data

x	0	1	2	3	4
y	1	3	4	5	6

Sol:- Let $y = a + bx + cx^2$ be the parabola.

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	xy	x^2	x^2y	x^3	x^4
0	1	0	0	0	0	0
1	3	3	1	3	1	1
2	4	8	4	16	8	16
3	5	15	9	45	27	81
4	6	24	16	96	64	256
$\Sigma x = 10$	$\Sigma y = 19$	$\Sigma xy = 50$	$\Sigma x^2 = 30$	$\Sigma x^2y = 160$	$\Sigma x^3 = 100$	$\Sigma x^4 = 354$

$$19 = 5a + 10b + 30c \quad \text{---} (1)$$

$$50 = 10a + 30b + 100c \quad \text{---} (2)$$

$$160 = 30a + 100b + 354c \quad \text{---} (3)$$

Solving eqn^{n}s (1), (2) & (3), we get $\Rightarrow$

$$a = 1.114$$

$$b = 1.7717$$

$$c = -0.1429$$

$\therefore$ Parabola of best fit is

$$y = 1.114 + 1.7717x - 0.1429x^2.$$

Fit an exponential curve for the following data.

x	2	4	6	8	10
y	1	3	6	12	24

Soln

Let the exponential curve be $y = ab^x$ and normal equations are

$$\Sigma Y = nA + B\Sigma x$$

$$\Sigma xY = A\Sigma x + B\Sigma x^2$$

where, $a = e^A$; $b = e^B$

x	y	$Y = \log_e y$	xY	x^2
2	1	0	0	4
4	3	1.0986	$3.2958 + 1.09 = 4.3944^{86}$	16
6	6	1.7917	10.7502	36
8	12	2.4849	19.8792	64
10	24	3.1780	31.780	100
$\Sigma x = 30$	$\Sigma y = 46$	$\Sigma Y = 8.5532$	$\Sigma xY = 66.8038$	$\Sigma x^2 = 220$

$$8.5532 = 5A + B\,30 \quad \text{—} \quad (1)$$

$$66.08038 = 30A + B220 \quad \text{—} \quad (2)$$

$$A = -0.5035 \; ; \quad B = 0.3690$$

$$a = 0.6044 ; \quad b = 1.4462$$

∴ Exponential curve of best fit is

$$y = (0.6044)(1.4462)^x.$$

Fit an exponential curve for the following data.

x	1	2	4
y	5	10	30

Soln :-

$y = a e^{bx}$

Normal equations are

$$\Sigma Y = nA + b\Sigma x$$

$$\Sigma xY = A\Sigma x + b\Sigma x^2$$

x	y	$Y = \log_e y$	x^2	xY
1	5	1.6094	1	1.6094
2	10	2.3025	4	4.605
4	30	3.4011	16	13.6044
$\Sigma x = 7$	$\Sigma y = 45$	$\Sigma Y = 7.313$	$\Sigma x^2 = 21$	$\Sigma xY = 19.8188$

$$7.313 = 3A + b7$$

$$19.8188 = 7A + 21b$$

$A = 1.0601$ | $a = e^A = 2.8866$

$b = 0.5903$

∴ Exponential curve of best fit is

$$y = (2.8866)\, e^{0.5903x}.$$