Amdahl's Law

The performance gain that can be obtained by improving some portion of a computer can be calculated using Amdahl's law. Amdahl's law states that the performance improvement to be gained from using some faster mode of execution is limited by the fraction of the time the faster mode can be used.

Amdahl's law defines the *speedup* that can be gained by using a particular feature. What is speedup? Suppose that we can make an enhancement to a computer that will improve performance when it is used. Speedup is the ratio:

$$\text{Speedup} = \frac{\text{Performance for entire task using the enhancement when possible}}{\text{Performance for entire task without using the enhancement}}$$

Alternatively,

$$\text{Speedup} = \frac{\text{Execution time for entire task without using the enhancement}}{\text{Execution time for entire task using the enhancement when possible}}$$

Speedup tells us how much faster a task will run using the computer with the enhancement as opposed to the original computer.

Amdahl's law gives us a quick way to find the speedup from some enhancement, which depends on two factors:

1. *The fraction of the computation time in the original computer that can be converted to take advantage of the enhancement*—For example, if 20 seconds of the execution time of a program that takes 60 seconds in total can use an enhancement, the fraction is 20/60. This value, which we will call $\text{Fraction}_{\text{enhanced}}$, is always less than or equal to 1.

2. *The improvement gained by the enhanced execution mode, that is, how much faster the task would run if the enhanced mode were used for the entire program*—This value is the time of the original mode over the time of the enhanced mode. If the enhanced mode takes, say, 2 seconds for a portion of the program, while it is 5 seconds in the original mode, the improvement is 5/2. We will call this value, which is always greater than 1, $\text{Speedup}_{\text{enhanced}}$.

The execution time using the original computer with the enhanced mode will be the time spent using the unenhanced portion of the computer plus the time spent using the enhancement:

$$\text{Execution time}_{\text{new}} = \text{Execution time}_{\text{old}} \times \left((1 - \text{Fraction}_{\text{enhanced}}) + \frac{\text{Fraction}_{\text{enhanced}}}{\text{Speedup}_{\text{enhanced}}} \right)$$

The overall speedup is the ratio of the execution times:

$$\text{Speedup}_{\text{overall}} = \frac{\text{Execution time}_{\text{old}}}{\text{Execution time}_{\text{new}}} = \frac{1}{(1 - \text{Fraction}_{\text{enhanced}}) + \frac{\text{Fraction}_{\text{enhanced}}}{\text{Speedup}_{\text{enhanced}}}}$$

The Processor Performance Equation

Essentially all computers are constructed using a clock running at a constant rate. These discrete time events are called *ticks*, *clock ticks*, *clock periods*, *clocks*, *cycles*, or *clock cycles*. Computer designers refer to the time of a clock period by its duration (e.g., 1 ns) or by its rate (e.g., 1 GHz). CPU time for a program can then be expressed two ways:

$$\text{CPU time} = \text{CPU clock cycles for a program} \times \text{Clock cycle time}$$

or

$$\text{CPU time} = \frac{\text{CPU clock cycles for a program}}{\text{Clock rate}}$$

In addition to the number of clock cycles needed to execute a program, we can also count the number of instructions executed—the *instruction path length* or *instruction count* (IC). If we know the number of clock cycles and the instruction count, we can calculate the average number of *clock cycles per instruction* (CPI). Because it is easier to work with, and because we will deal with simple processors in this chapter, we use CPI. Designers sometimes also use *instructions per clock* (IPC), which is the inverse of CPI.

CPI is computed as

$$\text{CPI} = \frac{\text{CPU clock cycles for a program}}{\text{Instruction count}}$$

This processor figure of merit provides insight into different styles of instruction sets and implementations, and we will use it extensively in the next four chapters.

By transposing the instruction count in the above formula, clock cycles can be defined as IC × CPI. This allows us to use CPI in the execution time formula:

$$\text{CPU time} = \text{Instruction count} \times \text{Cycles per instruction} \times \text{Clock cycle time}$$

Expanding the first formula into the units of measurement shows how the pieces fit together:

$$\frac{\text{Instructions}}{\text{Program}} \times \frac{\text{Clock cycles}}{\text{Instruction}} \times \frac{\text{Seconds}}{\text{Clock cycle}} = \frac{\text{Seconds}}{\text{Program}} = \text{CPU time}$$

As this formula demonstrates, processor performance is dependent upon three characteristics: clock cycle (or rate), clock cycles per instruction, and instruction count. Furthermore, CPU time is *equally* dependent on these three characteristics; for example, a 10% improvement in any one of them leads to a 10% improvement in CPU time.

Unfortunately, it is difficult to change one parameter in complete isolation from others because the basic technologies involved in changing each characteristic are interdependent:

- *Clock cycle time*—Hardware technology and organization
- *CPI*—Organization and instruction set architecture
- *Instruction count*—Instruction set architecture and compiler technology

Luckily, many potential performance improvement techniques primarily improve one component of processor performance with small or predictable impacts on the other two.

Sometimes it is useful in designing the processor to calculate the number of total processor clock cycles as

$$\text{CPU clock cycles} = \sum_{i=1}^{n} \text{IC}_i \times \text{CPI}_i$$

where IC_i represents the number of times instruction i is executed in a program and CPI_i represents the average number of clocks per instruction for instruction i. This form can be used to express CPU time as

$$\text{CPU time} = \left(\sum_{i=1}^{n} \text{IC}_i \times \text{CPI}_i \right) \times \text{Clock cycle time}$$

and overall CPI as

$$\text{CPI} = \frac{\sum_{i=1}^{n} \text{IC}_i \times \text{CPI}_i}{\text{Instruction count}} = \sum_{i=1}^{n} \frac{\text{IC}_i}{\text{Instruction count}} \times \text{CPI}_i$$

The latter form of the CPI calculation uses each individual CPI_i and the fraction of occurrences of that instruction in a program (i.e., $IC_i \div$ Instruction count). CPI_i should be measured and not just calculated from a table in the back of a reference manual since it must include pipeline effects, cache misses, and any other memory system inefficiencies.

Consider our performance example on page 47, here modified to use measurements of the frequency of the instructions and of the instruction CPI values, which, in practice, are obtained by simulation or by hardware instrumentation.