

## **Reliability Analysis of Pricing of CAT Bonds**

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### **Abstract**

Catastrophe (CAT) bonds are one of most recent financial derivatives and recently the most talked about instrument to be traded on the world markets. In this work, we try to present a stochastic model for evaluating the price of a catastrophe bond. The price of the catastrophe bond is contingent on occurrence of natural disaster and losses incurred due to it. Understanding the losses incurred due to the disaster is cardinal to the bond valuation. We model the losses incurred due to the catastrophe events . In this report we are modelling a specific type of CAT bond with trigger event as modelled losses. The bond loses its value if the losses incurred due to the catastrophe events exceed the threshold. Reliability analysis provides the with the probability of failure of bond i.e. the probability that the bond will lose its value. The objective is to equip investors with efficient valuation model in order to develop better hedging techniques.

**Keywords:** Stochastic Model, Insurance linked securities, Reinsurance, Reliability Analysis.

## 1. INTRODUCTION

CAT bonds deal with transferring the risks posed by catastrophe events, from the insurers and reinsurers to the capital market. A market in catastrophe securities emerged in the mid 1990's in order which facilitated the direct transfer of risk of reinsurance posed by catastrophe events from the insurers, reinsurers and corporations to the capital market investors. The most prominent instrument designed for this type of risk transfer is the CAT bonds. CAT bonds are more specifically referred to as insurance-linked securities (ILS). The feature that differentiates these bonds from the other bonds available in the marketed is that the ultimate repayment of principal depends on the outcome of an insured event.

The creation of CAT bonds was motivated by the events of catastrophes leading to massive losses to the reinsurers and insurers which threatened their solvency and has also forced a few companies to go bankrupt. CAT bonds are the recent of the such insurance linked securities which basically provides insurance to the insurance companies. At the same time, they offer investors a unique opportunity to enhance their portfolios with an asset that provides an attractive return that is uncorrelated to the market.

### 1.1. STRUCTURE

The basic CAT bond structure can be summarized as follows (Lane, 2004; Sigma, 2009):

1. The sponsor establishes a special purpose vehicle (SPV) as an issuer of bonds and as a source of reinsurance protection.
2. The issuer sells bonds to investors. The proceeds from the sale are invested in a collateral account.
3. The sponsor pays a premium to the issuer; this and the investment of bond proceeds are a source of interest paid to investors.
4. If the specified catastrophic risk is triggered, the funds are withdrawn from the collateral account and paid to the sponsor; at maturity, the remaining principal – or if there is no event, 100% of principal – is paid to investors.

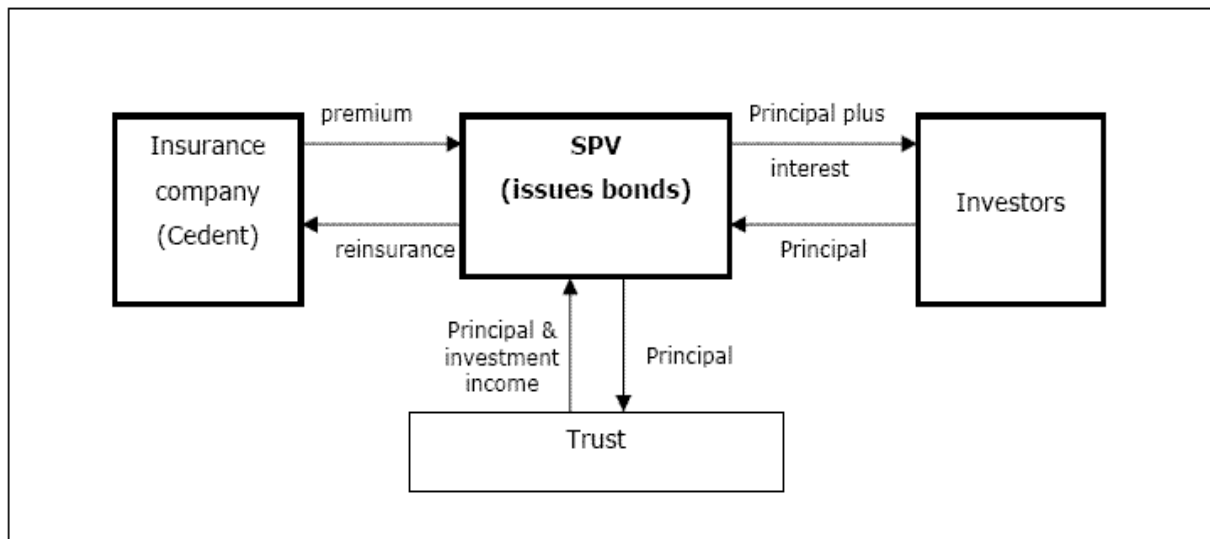


Diagram showing the structure of the Catastrophe bonds.

## 2. PRICING OF CAT BONDS

Pricing of CAT Bonds is similar to that of Default bonds. Defaultable bonds, by definition, must contain within their pricing model a mechanism that accounts for the potential (partial or complete) loss of their principal value. Considering the modelled loss trigger, we have designed the pricing model of CAT bond that accounts the potential loss of their value. We model the loss associated with the catastrophe events. Defaultable bonds yield higher returns, in part, because of this potential defaultability. Similarly, CAT bonds are offered at high yields because of the unpredictable nature of the catastrophe process.

The CAT bond we are interested in is described by specifying the region, type of events, type of insured properties, etc. More abstractly, it is described by the loss process  $L_s$  and by the threshold loss  $D$ . We are making following assumptions:

- We are considering the losses due to two catastrophes i.e. Earthquakes and Floods. Let  $L_e$  represent the aggregate losses due to Earthquake and  $L_f$  represents the losses due to Floods in the pre decided region in  $n$  years.
- The losses incurred by Earthquake and Floods respectively are assumed to be independent random variables  $X_e$  and  $X_f$  with distribution functions as  $F_{X_e}(x_e) = P(X_e < x_e)$  &  $F_{X_f}(x_f) = P(X_f < x_f)$  respectively.
- CAT bond is triggered when the cumulative losses due to Earthquake and Flood exceed the Threshold Loss ( $D$ ) i.e.  $X_e + X_f > D$ .
- Let  $r$  represent the average discount rate per annum. So the present value of the 1 USD paid after  $n$  years is given by  $\frac{1}{(1+r)^n}$
- Investment is considered failure if the CAT bond is triggered. Let  $P_f$  be the probability of failure and it is given by:  $P_f = P(X_e + X_f > D)$
- The arbitrage price of the zero coupon CAT bond associated with threshold  $D$ , incurred Losses  $L_e$  &  $L_f$  and paying  $Z$  at maturity is given by:

$$V = \frac{Z}{(1+r)^n} \times (1 - P(X_e + X_f > D))$$

In the example presented in this report, catastrophe with following contract details have been considered:

Maturity Time: 1 year

Trigger Type: Modelled Losses

Perils: Earthquake and Floods

$X_e$  - Losses due to earthquake

$X_f$  - Losses due to floods

$\Theta$  - Threshold for the losses incurred

Data for the losses has been taken from the EM-DATA agency and it covers the losses incurred in North and South America

### 3. Modelling the Losses Distribution

Consider CAT bond described by the events of Earthquake and Flood holding maturity period of 1 year i.e.  $n=1$  covering North and South America. Therefore  $L_e$  represents the aggregate losses due to Earthquake and  $L_f$  represents the losses due to Floods in 1 year. The challenge is to find appropriate distribution functions.

#### 3.1. Losses due to Earthquake

Data concerning aggregate damage due to earthquakes in a particular was collected from EM-DATA. The following figure represents the aggregate loss due to earthquakes in North and South America in different years.

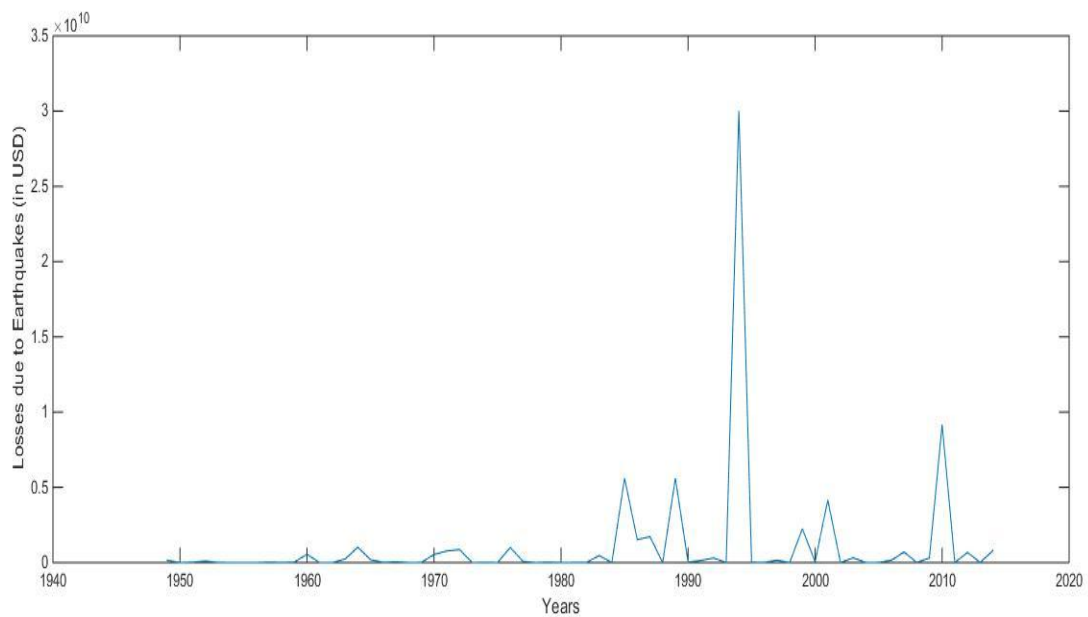


Figure 1: Figure showing the timeline of Losses due to earthquake

##### 3.1.1. Loss Distribution:

In order to estimate the best loss distribution, we first plotted histogram of Losses due to Earthquake ( $X_e$ ) and then plotted the histogram of natural log of Losses due to Earthquake.

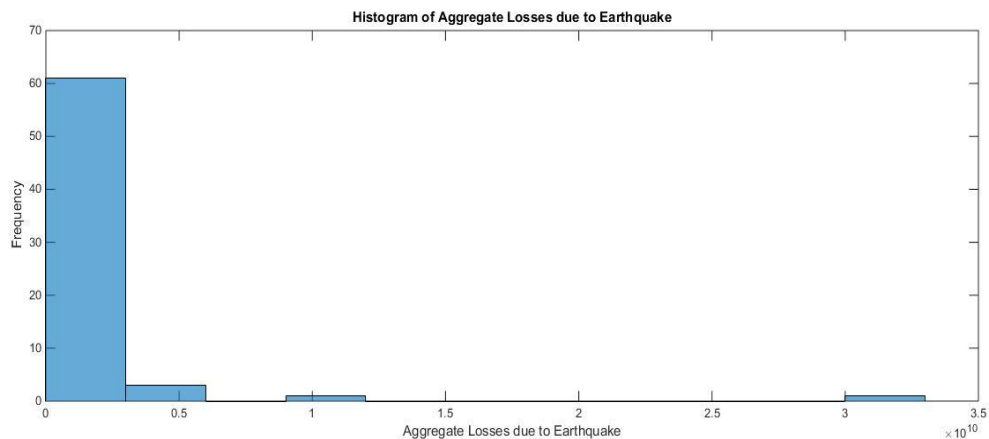


Figure 2: Histogram of Losses due to earthquake

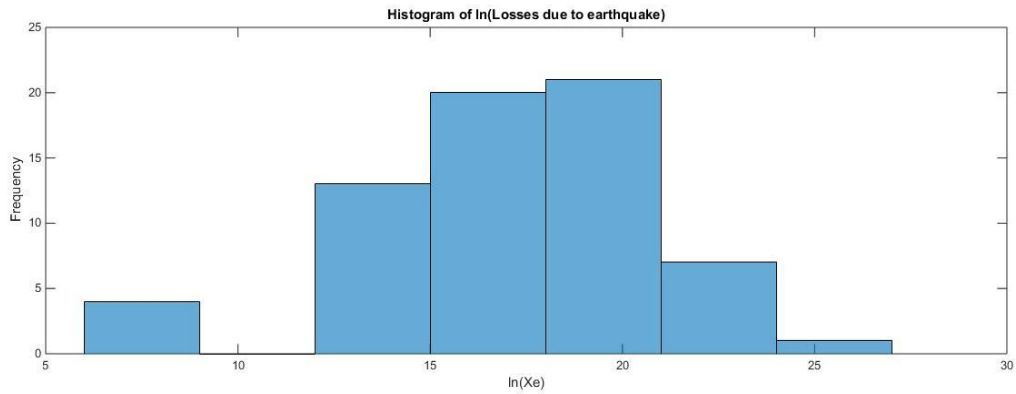


Figure 3: Histogram of log(Losses due to earthquake)

### 3.1.2. Shapiro Wilk Test for Normality:

In order to test whether our assumption that Losses due to Earthquake can be modelled with Lognormal distribution, we conduct the Shapiro Wilk Test on the data. For  $\alpha=0.05$ , we can say the data follows Lognormal distribution and hence we fail to reject our hypothesis.

Distribution:

$X_e$  represents the aggregate losses due to earthquakes

$$X_e \sim \ln N(\lambda_e, \xi_e)$$

$$f_{X_e}(x_e) = \frac{1}{\sqrt{(2\pi)(x_e \xi_e)}} \exp\left(-\frac{(\ln(x_e) - \lambda_e)^2}{2\xi_e^2}\right)$$

$$F_{X_e}(x_e) = \Phi\left(\frac{\ln(x_e) - \lambda_e}{\xi_e}\right)$$

where  $\lambda_e$  and  $\xi_e$  can be called location parameter and scale parameter respectively.

### 3.1.3 Estimation of Parameters

In order to estimate the parameters of Log normal distribution, we use the maximum likelihood approach.

$$L(x, \lambda_e, \xi_e) = \prod_{i=1}^n N(\ln x; \lambda_e; \xi_e) \frac{1}{x_i}$$

$$l_L(\lambda_e, \xi_e | x_1, x_2, x_3, \dots, x_n) = -\sum_{i=1}^n \ln(x_i) - \frac{n}{2} \ln(2\pi) - \ln(\xi_e^2) \frac{n}{2} - \frac{1}{2\xi_e^2} \sum_{j=1}^n (\ln(x_j) - \lambda_e)^2$$

$$\frac{\partial l}{\partial \lambda_e} = 0 \Rightarrow \widehat{\lambda_e} = \sum_{i=1}^n \frac{1}{n} \ln(x_i)$$

$$\frac{\partial l}{\partial \xi_e^2} = 0 \Rightarrow \widehat{\xi_e^2} = \frac{1}{n} \sum_{j=1}^n (\ln(x_j) - \widehat{\lambda_e})^2$$

Based on the data gathered, the estimates are as follows:

$$\widehat{\lambda_e} = 17.0389$$

$$\widehat{\xi_e^2} = 13.209$$

### 3.2. Losses due to Floods

Data concerning aggregate damage due to earthquakes in a particular was collected from EM-DATA. The following figure represents the aggregate loss due to earthquakes in North and South America in different years.

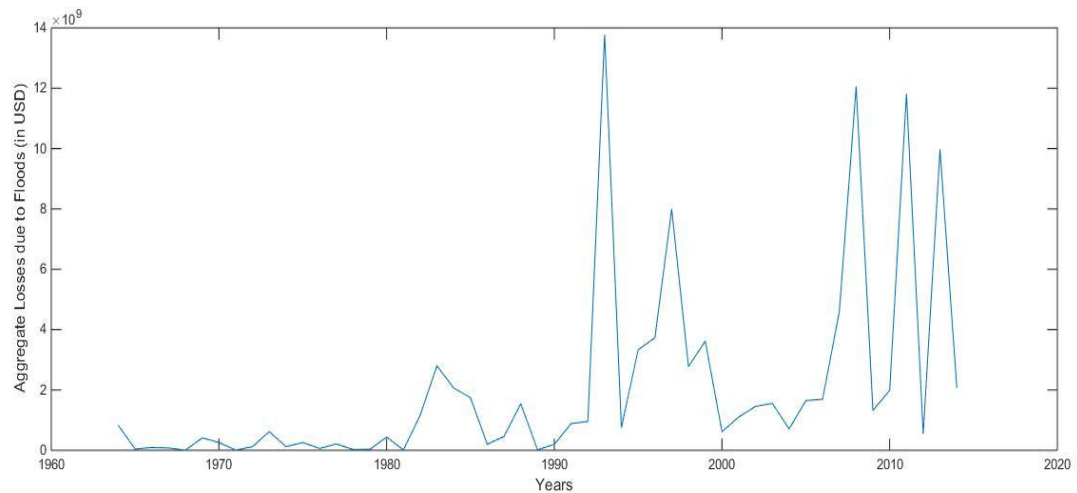


Figure 4: Plot showing timeline of Losses due to Floods

#### 3.2.1. Loss Distribution:

In order to estimate the best loss distribution, we first plotted histogram of Losses due to Earthquake ( $X_e$ ) and then plotted the histogram of natural log of Losses due to Earthquake.

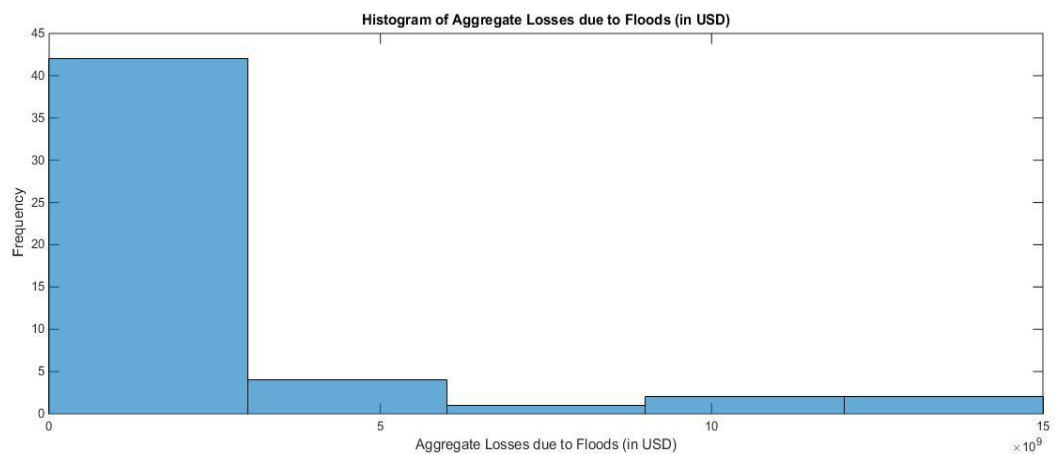


Figure 5: Histogram of Losses incurred due to Floods (in USD)

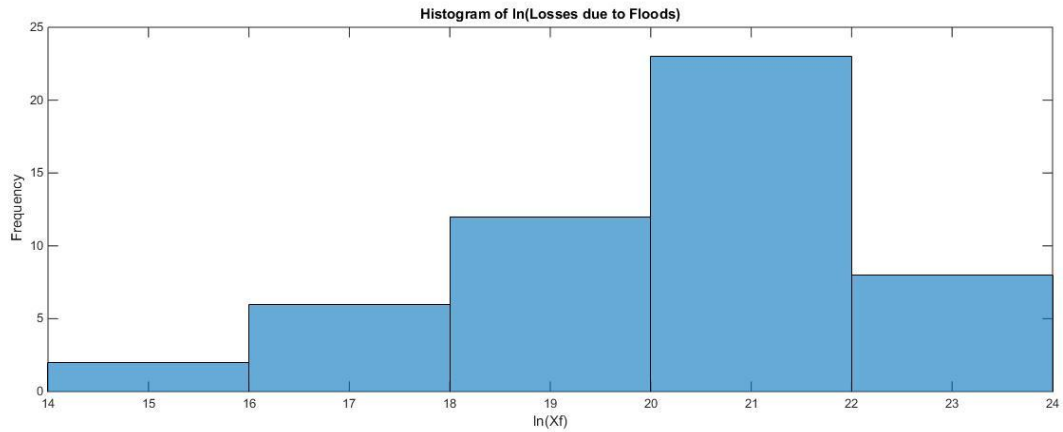


Figure 6: Histogram of log(Losses due to Floods)

### 3.2.2. Shapiro Wilk Test for Normality:

In order to test whether our assumption that Losses due to Floods can be modelled with Lognormal distribution, we conduct the Shapiro Wilk Test on the data. For  $\alpha=0.05$ , we can say the data follows Lognormal distribution and hence we fail to reject our hypothesis.

*Distribution:*

$X_e$  represents the aggregate losses due to earthquakes

$$X_f \sim \ln N(\lambda_f, \xi_f)$$

$$f_{Xf}(x_f) = \frac{1}{\sqrt{(2\pi)(x_f\xi_f)}} \exp\left(-\frac{(\ln(x_f) - \lambda_f)^2}{2\xi_f^2}\right)$$

$$F_{Xf}(x_f) = \Phi\left(\frac{\ln(x_f) - \lambda_f}{\xi_f}\right)$$

where  $\lambda_f$  and  $\xi_f$  can be called location parameter and scale parameter respectively.

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In order to estimate the parameters of Log normal distribution, we use the maximum likelihood approach.

$$L(x, \lambda_f, \xi_f) = \prod_{i=1}^n N(\ln x; \lambda_f; \xi_f) \frac{1}{x_i}$$

$$l_L(\lambda_f, \xi_f | x_1, x_2, x_3, \dots, x_n) = -\sum_{i=1}^n \ln(x_i) - \frac{n}{2} \ln(2\pi) - \ln(\xi_f^2) \frac{n}{2} - \frac{1}{2\xi_f^2} \sum_{j=1}^n (\ln(x_j) - \lambda_f)^2$$

$$\frac{\partial l}{\partial \lambda_f} = 0 \Rightarrow \hat{\lambda}_f = \sum_{i=1}^n \frac{1}{n} \ln(x_i)$$

$$\frac{\partial l}{\partial \xi_f^2} = 0 \Rightarrow \widehat{\xi_f^2} = \frac{1}{n} \sum_{j=1}^n (\ln(x_j) - \hat{\lambda}_f)^2$$

Based on the data for the Losses incurred due to Floods

$$\hat{\lambda}_f = 20.15279$$

$$\widehat{\xi_f^2} = 3.8458$$

#### 4. First Order Reliability Method ANALYSIS

##### Defining the limit state function:

$$g(\underline{X}) = \Theta - X_e - X_f$$

where,

$\Theta$  = Threshold Loss incurred

$X_e$  = Aggregate Loss due to Earthquakes

$X_f$  = Aggregate Loss due to Floods

##### Loss Distributions

$X_e$  follows log normal distribution with parameters  $\lambda_e$  and  $\xi_e$ :

$$X_e \sim \ln N(\lambda_e, \xi_e)$$

$X_f$  follows log normal distribution with parameters  $\lambda_f$  and  $\xi_f$ :

$$X_f \sim \ln N(\lambda_f, \xi_f)$$

##### Transformation:

$$u_e = \Phi^{-1}[F_{X_e}(x_e)]$$

$$u_f = \Phi^{-1}[F_{X_f}(x_f)]$$

##### Let's find the inverse relation

$$\Phi(u_e) = F_{X_e}(x_e)$$

$$\Rightarrow \Phi(u_e) = \Phi\left(\frac{\ln(x_e) - \lambda_e}{\xi_e}\right)$$

$$\Rightarrow u_e = \frac{\ln(x_e) - \lambda_e}{\xi_e}$$

$$\ln(x_e) = \xi_e u_e + \lambda_e$$

$$\Rightarrow x_e = \exp(\xi_e u_e + \lambda_e)$$

The same derivation for  $u_f$ :

$$\Phi(u_f) = F_{X_f}(x_f)$$

$$\Rightarrow \Phi(u_f) = \Phi\left(\frac{\ln(x_f) - \lambda_f}{\xi_f}\right)$$

$$\Rightarrow u_f = \frac{\ln(x_f) - \lambda_f}{\xi_f}$$

$$\ln(x_f) = \xi_f u_f + \lambda_f$$

$$\Rightarrow x_f = \exp(\xi_f u_f + \lambda_f)$$



Now we need to compute  $J_{u,x} = \text{diag} [du/dx] = \text{diag} [f(x_i) / \phi(u_i)]$  and

$$J_{x,u} = J_{u,x}^{-1} = \text{diag} [\phi(u_i) / f(x_i)],$$

where

$$f_{X_e}(x_e) = \frac{1}{\sqrt{(2\pi)(x_e\xi_e)}} \exp\left(-\frac{(\ln(x_e) - \lambda_e)^2}{2\xi_e^2}\right)$$

$$f_{X_f}(x_f) = \frac{1}{\sqrt{(2\pi)(x_f\xi_f)}} \exp\left(-\frac{(\ln(x_f) - \lambda_f)^2}{2\xi_f^2}\right)$$

Let's calculate  $\nabla g(x)$ :

$$g(\underline{X}) = \Theta - X_e - X_f$$

$$\frac{dg}{dx_e} = -1$$

$$\frac{dg}{dx_f} = -1$$

$$\nabla g(x) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

According to the following implementation of the HL-RF algorithm:

$$u^{*T} = [1.431 \quad 0.1303]$$

$$x^{*T} = [4.2701 \quad 0.7299] * 10^9$$

$$\alpha^T = [0.9958 \quad 0.0918]$$

$$\beta = 1.4191$$

$$P_f = \Phi(-\beta) = \Phi(-1.491) = 0.0779$$

Value of the bond

$$V = \frac{Z}{(1+r)^n} \times (1 - P(X_e + X_f > D))$$

$$V = \frac{Z}{(1+r)^1} \times (1 - 0.0779) = \frac{Z}{(1+r)^1} \times 0.9221$$

Value of the bond can be used to make the investment decision. If the bond is priced at higher price than the provided value then the investor would be hesitant to invest in such CAT bonds on the other side, if the bond is priced at lower level then there would be more demand for such CAT bonds.

The probability of failure depends upon various parameters. Changing such parameter would have an impact on probability of failure. We have done analysis of sensitivity of the beta and probability of failure in the sixth section. It incorporates how changing parameters like threshold affects the probability of failure and hence affects the value of the bond.

#### 4.1. VISUAL REPRESENTATION OF FORM ANALYSIS

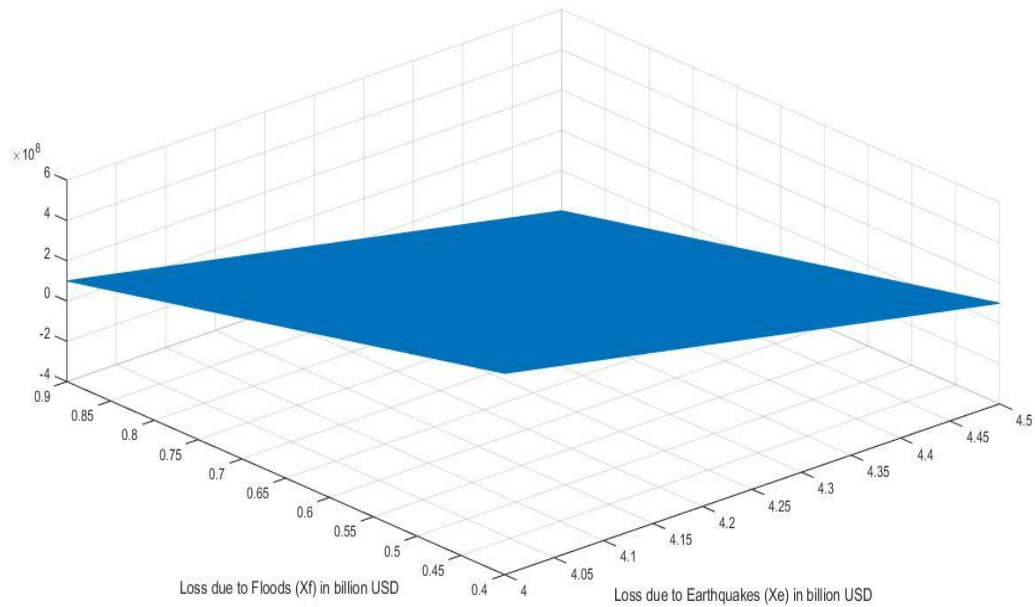


Figure7: Surface of plot of the function against Losses due to Earthquakes and Losses due to Floods in billion dollars in original space

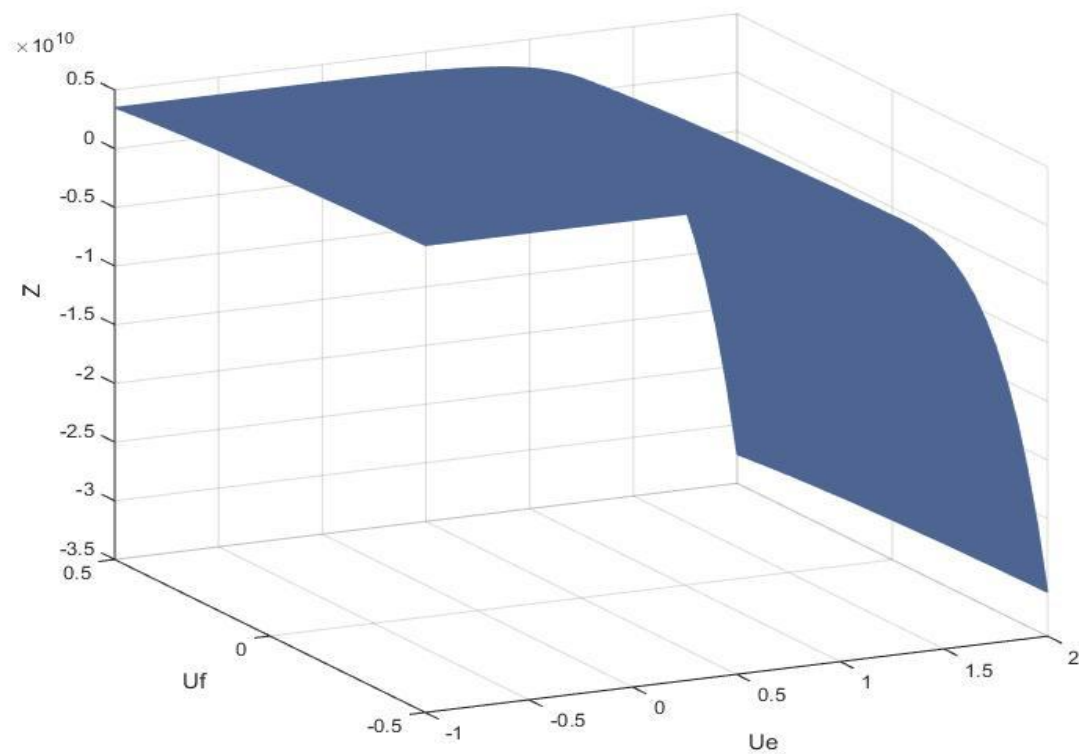


Figure8: Surface plot of the function in standard normal space

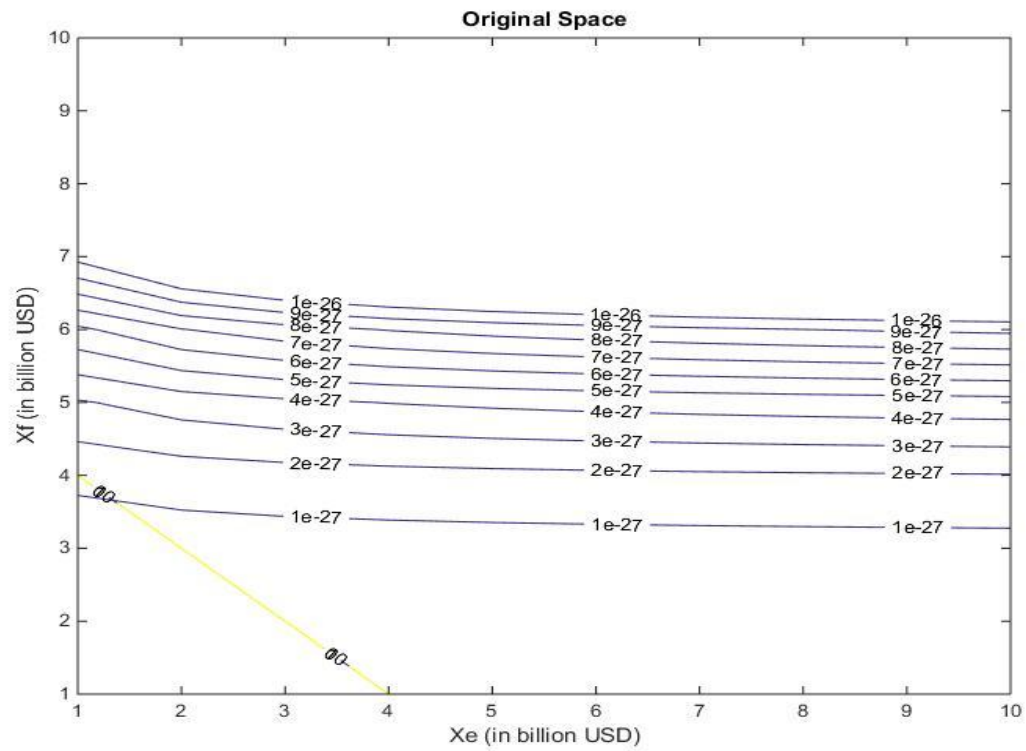


Figure 9: Contours of probability distribution function and limit state function in original space

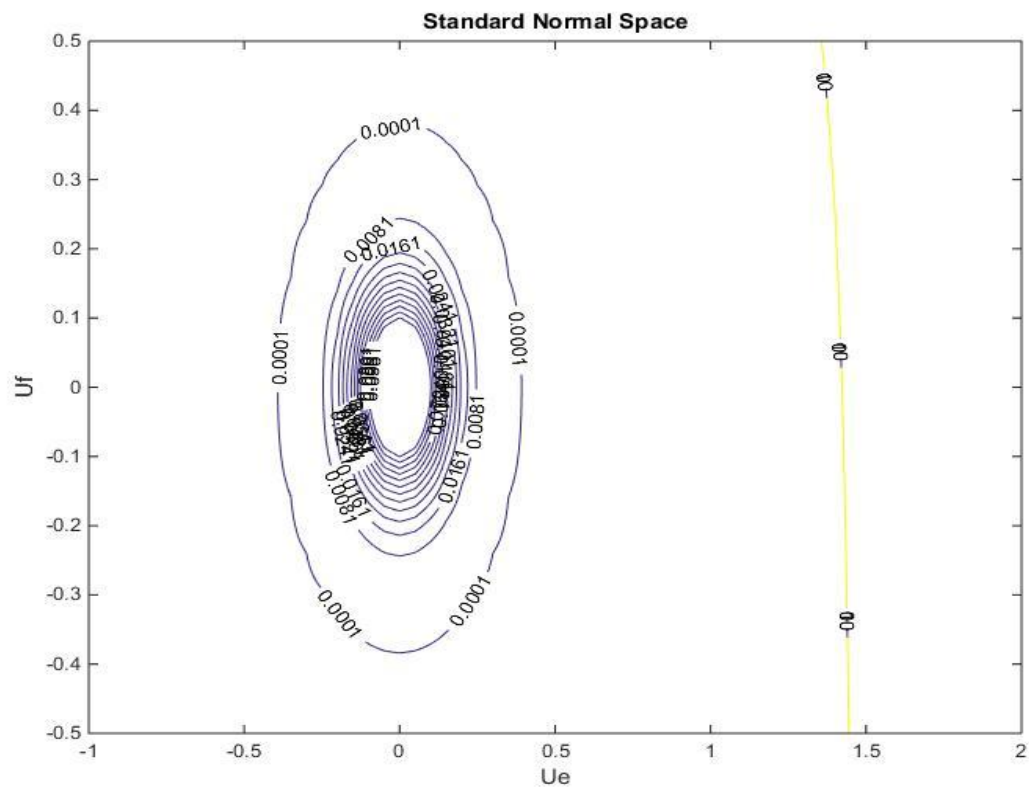


Figure 10: Contours of probability distribution function and limit state function in standard normal space

## 5. Second Order Reliability Method

Curvature fitting SORM:

From the FORM analysis, we know the following:

$$\mathbf{u}^{*T} = [1.431 \quad 0.1303]$$

$$\mathbf{x}^{*T} = [4.2701 \quad 0.7299] * 10^9$$

$$\boldsymbol{\alpha}^T = [0.9958 \quad 0.0918]$$

$$\beta = 1.4191$$

Let  $\mathbf{x}_e' = \mathbf{x}_e^* + \boldsymbol{\varepsilon}$ . Since we are on  $\mathbf{g}(\mathbf{x}_e, \mathbf{x}_f) = \mathbf{0}$  we can get  $\mathbf{x}_f' = \boldsymbol{\Theta} - \mathbf{x}_e'$ . Let  $\varepsilon = 0.015 * 10^9$

$$\mathbf{x}'^T = [4.2851 \quad 0.7149] * 10^9$$

$$\mathbf{u}'^T = [1.4141 \quad 0.1198]$$

$$\boldsymbol{\alpha}'^T = [0.9964 \quad 0.0844]$$

$$\kappa \approx \frac{\sqrt{2(1 - \boldsymbol{\alpha}^T \boldsymbol{\alpha}')}}{\|\mathbf{u}^* - \mathbf{u}'\|} = 0.3050$$

Let us calculate the second order probability approximation by Breitung's formula as

$$P_{f2} \approx \Phi(-\beta) \prod_{i=1}^n \frac{1}{\sqrt{1 + \beta \kappa_i}}$$

$$P_{f2} \approx \Phi(-1.4191) \frac{1}{\sqrt{1 + (1.4191)(0.3050)}} = 0.0651$$

$$\beta_{\text{SORM}} = \Phi^{-1}(1 - P_{f2}) = 1.5133$$

As expected the  $\beta_{\text{SORM}}$  is greater than  $\beta_{\text{FORM}}$  as the  $\kappa$  is positive

Value of the bond

$$V = \frac{Z}{(1+r)^n} \times (1 - P(X_e + X_f > D))$$

$$V = \frac{Z}{(1+r)^1} \times (1 - 0.0651) = \frac{Z}{(1+r)^1} \times 0.9349$$

Therefore the Value of the bond is close to its value at maturity with the decrease in probability of failure.

## 6. SENSITIVITY ANALYSIS

$$\Theta_g = [\Theta]$$

$$\Theta_f = [\lambda_e \quad \xi_e \quad \lambda_f \quad \xi_f]^T$$

$$\Theta_f = [\mu_e \quad \sigma_e \quad \mu_f \quad \sigma_f]^T$$

Lets first consider the first order sensitivity of  $\beta$  and  $P_f$  with respect to  $\Theta_g$ :

$$g(\mathbf{X}) = \Theta - X_e - X_f$$

$$\nabla_{\Theta} g(x^*, \theta) = 1$$

$$\nabla_{\Theta} \beta = \frac{\nabla_{\Theta} g(x^*, \theta)}{\|\nabla G\|}$$

$$\text{where } \|\nabla G\| = 1.5586 \cdot 10^{10}, \quad \Theta = 5 \cdot 10^9 \quad \& \quad \nabla_{\Theta} g(x^*, \theta) = 1$$

$$\nabla_{\Theta} \beta = 0.642 \cdot 10^{-10}$$

Accordingly,

$$\nabla_{\Theta} P_f = -\varphi(\beta) \nabla_{\Theta} \beta = -0.14575 \cdot 0.642 \cdot 10^{-10} = -0.09351 \cdot 10^{-10}$$

Lets first consider the first order sensitivity of  $\beta$  and  $P_f$  with respect to  $\Theta_f$

$$\Theta_f = [\lambda_e \quad \xi_e \quad \lambda_f \quad \xi_f]^T$$

$$\nabla_{\Theta_f} \beta = \alpha^T J_{u^*, \Theta_f}$$

$$u_e = \frac{\ln(x_e) - \lambda_e}{\xi_e}$$

$$u_f = \frac{\ln(x_f) - \lambda_f}{\xi_f}$$

$$\frac{\partial u_e}{\partial \lambda_e} = \frac{-1}{\xi_e} = -0.27514$$

$$\frac{\partial u_e}{\partial \xi_e} = \frac{-(\ln(x_e) - \lambda_e)}{\xi_e^2} = \frac{-\ln(4.2701 \cdot 10^9) + 17.03893}{3.6345^2} = -0.38881$$

$$\frac{\partial u_e}{\partial \lambda_f} = 0$$

$$\frac{\partial u_e}{\partial \xi_f} = 0$$

$$\frac{\partial u_f}{\partial \lambda_e} = 0$$

$$\frac{\partial u_f}{\partial \xi_e} = 0$$

$$\frac{\partial u_f}{\partial \lambda_f} = \frac{-1}{\xi_f} = -0.50992$$

$$\frac{\partial u_f}{\partial \xi_f} = \frac{-\ln(x_f) + \lambda_f}{\xi_f^2} = \frac{-\ln(0.7299 \cdot 10^9) + 20.15729}{1.96108^2} = -0.06530$$

$$J_{u^*, \Theta_f} = \begin{bmatrix} -0.27514 & -0.3881 & 0 & 0 \\ 0 & 0 & -0.50992 & -0.06530 \end{bmatrix}$$

$$\alpha = [0.9958 \quad 0.0918]^T$$

$$\nabla_{\Theta_f} \beta^T = \alpha^T J_{u^*, \Theta_f} = [-0.2740 \quad -0.3865 \quad -0.0468 \quad -0.0060]$$

$$\nabla_{\Theta_f} P_f^T = -\varphi(\beta) \nabla_{\Theta_f} \beta = [-0.03994 \quad -0.05633 \quad -0.00682 \quad -0.00087]$$

Let's consider the first order sensitivity of  $\beta$  and  $P_f$  with respect to  $\Theta_f$

$$\Theta_F = [\mu_e \ \sigma_e \ \mu_f \ \sigma_f]^T$$

$$\nabla_{\Theta_F} \beta^T = \nabla_{\Theta_F} \beta^T J_{\Theta_F, \Theta}$$

where

$$\Theta_F = [\lambda_e \ \xi_e \ \lambda_f \ \xi_f]^T ; \lambda_e = \ln(\mu_e) - 0.5 \ln[1 + (\sigma_e/\mu_e)^2] \text{ and } \xi_e = \sqrt{\ln \left[ 1 + \left( \frac{\sigma_e}{\mu_e} \right)^2 \right]}$$

$$\lambda_f = \ln(\mu_f) - 0.5 \ln[1 + (\sigma_f/\mu_f)^2] \text{ and } \xi_f = \sqrt{\ln \left[ 1 + \left( \frac{\sigma_f}{\mu_f} \right)^2 \right]}$$

$$\frac{\partial \lambda_e}{\partial \mu_e} = \frac{1}{\mu_e} \left( 1 + \frac{\sigma_e^2}{\sigma_e^2 + \mu_e^2} \right) = 1.0782 * 10^{-10} \quad \frac{\partial \lambda_e}{\partial \sigma_e} = 0$$

$$\frac{\partial \lambda_e}{\partial \sigma_e} = \frac{-\sigma_e}{\sigma_e^2 + \mu_e^2} = -7.2982 * 10^{-14} \quad \frac{\partial \lambda_e}{\partial \sigma_f} = 0$$

$$\frac{\partial \xi_e}{\partial \mu_e} = \frac{-\sigma_e^2}{(\sigma_e^2 + \mu_e^2)\mu_e \xi_e} = -1.14832 * 10^{-10} \quad \frac{\partial \xi_e}{\partial \mu_f} = 0$$

$$\frac{\partial \xi_e}{\partial \sigma_e} = \frac{\sigma_e}{(\sigma_e^2 + \mu_e^2)\xi_e} = 2.0080 * 10^{-14} \quad \frac{\partial \xi_e}{\partial \sigma_f} = 0$$

$$\frac{\partial \lambda_f}{\partial \mu_e} = 0 \quad \frac{\partial \lambda_f}{\partial \mu_f} = \frac{1}{\mu_f} \left( 1 + \frac{\sigma_f^2}{\sigma_f^2 + \mu_f^2} \right) = 5.1168 * 10^{-10}$$

$$\frac{\partial \lambda_f}{\partial \sigma_e} = 0 \quad \frac{\partial \lambda_f}{\partial \sigma_f} = \frac{-\sigma_f}{\sigma_f^2 + \mu_f^2} = -0.37397 * 10^{-10}$$

$$\frac{\partial \xi_f}{\partial \mu_e} = 0 \quad \frac{\partial \xi_f}{\partial \mu_f} = \frac{-\sigma_f^2}{(\sigma_f^2 + \mu_f^2)\mu_f \xi_f} = -1.2905 * 10^{-10}$$

$$\frac{\partial \xi_f}{\partial \sigma_e} = 0 \quad \frac{\partial \xi_f}{\partial \sigma_f} = \frac{\sigma_f}{(\sigma_f^2 + \mu_f^2)\xi_f} = 0.190696 * 10^{-10}$$

$$J_{u^*, \Theta_F} = \begin{bmatrix} 1.07820 & -0.00073 & 0 & 0 \\ -0.14832 & 0.000200 & 0 & 0 \\ 0 & 0 & 5.1168 & -0.37397 \\ 0 & 0 & -1.2905 & 0.19067 \end{bmatrix} * 10^{-10}$$

$$\nabla_{\Theta_F} \beta^T = \nabla_{\Theta_F} \beta^T J_{u^*, \Theta_F} = [-0.2381 \quad 0.0001 \quad -0.2317 \quad 0.0164] * 10^{-10}$$

$$\nabla_{\Theta} P_f^T = -\varphi(\beta) \nabla_{\Theta} \beta^T = [0.347 \quad -0.000147 \quad 0.338 \quad -0.0239] * 10^{-11}$$

	$\Theta$	$\lambda_e$	$\xi_e$	$\lambda_f$	$\xi_f$	$\mu_e$	$\sigma_e$	$\mu_f$	$\sigma_f$
$\frac{\partial \beta}{\partial \Theta}$	$0.642 * 10^{-10}$	-0.274	-0.386	-0.047	-0.006	$-0.238 * 10^{-10}$	$0.001 * 10^{-11}$	$-0.232 * 10^{-10}$	$0.0164 * 10^{-10}$
$\frac{\partial P_f}{\partial \Theta}$	$-0.936 * 10^{-11}$	0.03994	0.0563	0.0068	0.00087	$0.347 * 10^{-11}$	$-0.147 * 10^{-14}$	$0.338 * 10^{-11}$	$-0.024 * 10^{-11}$

Table1: Table illustrates the changes in beta and probability of failure with respect to changes in parameters

## **7. CONCLUSION**

The model provided in the report can be used to derive the value of the catastrophe bond with maturity of one year. The reliability analysis of the model derived the probability of failure and hence the value of the bond. Sensitivity analysis can equip the investors with better tools to assess the risk involved with investing in catastrophe bonds. As can be seen from the sensitivity analysis, the probability of failure can be decreased by increasing the threshold of the losses incurred. Such analysis can help investors to make more informed decisions.

The model given the section 2 is valid only models the losses incurred in a specific time period. A better model for defining the value can be developed which may incorporate the modelling of the natural catastrophe events and losses incurred due to those events.

Catastrophe bonds has attracted billions of dollars of investments in last few years and better modelling of loss function and natural catastrophe events will lead to more investment as investors there would be lesser uncertainty in the returns.

## REFERENCES

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