ASSIGNMENT#1

Signals And Systems

NAME :- Vadaparthi Tarun Gangadhar

ROLL NO :-BT20EEE107

The three Linear time invariant continuous time systems (1, 2 and 3) whose equations of dynamics (in form of differential equations) are given as follows-

System 1.1: (D+2) x(t) = (3D +5) r(t)
System 1.2: D(D+2) x(t) = (D+4) r(t)
System 1.3: (D^2 +2D+1) x(t) = D r(t)

#1a

We have to obtain the step response of the above three systems using the concept of convolution theorem.

System 1.1 (D+2) x(t) = (3D +5) r(t)

(Or) Q(D) x(t) = P(D) r(t)

Here, Q(D) = D+2

P(D) = 3D+5

For zero i/p response r(t)=0 ⟹ $Q(D)x_{zi}(t)=0$

Characteristic equation Q(D)=0 ⟹ D+2=0

Characteristic roots :- D= {-2} i.e., $\lambda=-2$

$\therefore\ x_{zi}(t) = ke^{\lambda t} = ke^{-2t}$ → **equation(1)**

Order of the system n=1 , n-1 = 1-1 =0

$D^{n-1}\, x_{zi}(t)|_{t=0} = 1$ i.e., $D^{(0)}\, x_{zi}(t)|_{t=0} = 1$ ⟹ $x_{zi}(t)|_{t=0} = 1$

From equation (1),

For t=0 $x_{zi}(0) = 1 = ke^{0t} = k \quad \therefore k=1$

$\therefore$ Zero i/p response of the system is :-

$\boxed{x_{zi}(t) = e^{-2t}}$ ⟹ **ZERO INPUT RESPONSE**

For impulse response h(t)

$$\left.\begin{array}{l} Q(D) = D+2 \\ \\ P(D) = 3D+5 \end{array}\right\} b_0=3$$

∴ $\boxed{h(t) = b_0\delta(t) + [P(D)x_{zi}(t)]\, u(t)}$ $\longrightarrow$ **equation(2)**

$P(D)\, x_{zi}(t) = (3D+5)e^{-2t} = 3De^{-2t}+5e^{-2t} = -6e^{-2t}+5e^{-2t} = -e^{-2t}$

∴ Substituting the values in equation(2)

We get

$\boxed{\mathbf{h(t) = 3\delta(t) - e^{-2t}u(t)}}$ $\longrightarrow$ **UNIT IMPULSE RESPONSE**

Now, from the concept of convolution theorem,

$X_{zs}(t) = h(t) * r(t)$ (zero state response)

$= h(t) * u(t)$ [unit step response $\longrightarrow$ r(t)=u(t)]

$$= \int_0^t h(\tau)u(t-\tau)d\tau$$

$$= \int_0^t \{3\delta(\tau) - e^{-2\tau}\}.1d\tau$$

$$= \int_0^t 3\delta(\tau) - \int_0^t e^{-2\tau}d\tau$$

$$= 3 - \left[\frac{e^{-2\tau}}{-2}\right]_0^t$$

$h(\tau) = 3\delta(\tau) - e^{-2\tau}$ $\{\tau \geq 0\}$

$u(t-\tau) = 1$ $\{-\infty \leq \tau \leq t\}$

$$\boxed{\mathbf{Xzs(t) = \left(\frac{5}{2} + \frac{e^{-2t}}{2}\right) u(t)}}$$ $\longrightarrow$ **STEP RESPONSE OF THE SYSTEM**

Output= Zero input response + Zero state response

$$x(t) = \mathrm{Xzi}(t) + \mathrm{Xzs}(t)$$

$$= e^{-2t} + \left(\frac{5}{2} + \frac{e^{-2t}}{2}\right)$$

$$\boxed{\boldsymbol{x(t) = \left(\frac{5}{2} + \frac{3(e^{-2t})}{2}\right) u(t)}}$$ $\longrightarrow$ **OUTPUT**

System 1.2 $D(D+2)\,x(t) = (D+4)\,r(t)$

(Or) $Q(D)\,x(t) = P(D)\,r(t)$

Here, $Q(D) = D(D+2)$

$P(D) = D+4$

For zero i/p response $r(t)=0$ ⟹ $Q(D)x_{zi}(t)=0$

Characteristic equation $Q(D)=0$ ⟹ $D(D+2)=0$

Characteristic roots :- $D=\{0,-2\}$ i.e., $\lambda=\{0,-2\}$

$$x_{zi}(t) = k_1e^{0t} + k_2e^{-2t}$$

$$\therefore\ x_{zi}(t) = k_1 + k_2e^{-2t}$$

Order of the system n=2 $\begin{pmatrix} n-1=2-1=1 \\ n-2=2-2=0 \end{pmatrix}$

$D^{n-1}\,x_{zi}(t)|_{t=0} = 1$ and $D^{n-2}\,x_{zi}(t)|_{t=0} = 0$

$Dx_{zi}(t)|_{t=0} = -2k_2e^{-2t}$ $\qquad x_{zi}(t) = k_1 + k_2e^{-2t}$

$Dx_{zi}(0) = -2k_2e^{0}$ $\qquad x_{zi}(0) = k_1 + k_2 = 0$

⟹ $k_2 = -1/2$ $\qquad k_1 = -k_2 = 1/2$ ⟹ $k_1 = 1/2$

$\therefore$ Zero i/p response of the system is :-

$$\boxed{x_{zi}(t) = \tfrac{1}{2} - \tfrac{1}{2}e^{-2t}}$$ ⟹ **ZERO INPUT RESPONSE**

For impulse response h(t)

$Q(D) = D^2+2D$

$P(D) = D+4 = 0.D^2+D+4$

$\Big\}\ b_0=0$

$\therefore$ $\boxed{h(t) = b_0\delta(t) + [P(D)x_{zi}(t)]\,u(t)}$ ⟶ **equation(3)**

$P(D)\,x_{zi}(t) = (D+4)\left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)$

$= D\left(\frac{1}{2}\right) - D\left(\frac{1}{2}\right)e^{-2t} + 2 - 2e^{-2t}$

$= 0 + e^{-2t} + 2 - 2e^{-2t}$

$= 2 - e^{-2t}$

$\therefore$ Substituting the values in equation(3)

We get

$\boxed{h(t) = (2 - e^{-2t})\, u(t)}$ → **UNIT IMPULSE RESPONSE**

Now, from the concept of convolution theorem,

$X_{zs}(t) = h(t) * r(t)$ (zero state response)

$= h(t) * u(t)$ [unit step response → r(t)=u(t)]

$$= \int_0^t h(\tau)u(t-\tau)d\tau$$

$$= \int_0^t (2 - e^{-2\tau}).1 d\tau$$

$$= [2\tau]_0^t - \left[\frac{e^{-2\tau}}{-2}\right]_0^t$$

$$= 2t - \frac{1}{2}(1 - e^{-2t})$$

$h(\tau) = 2 - e^{-2\tau}$ $\{\tau \geq 0\}$

$u(t-\tau) = 1$ $\{-\infty \leq \tau \leq t\}$

$$\boxed{\mathbf{Xzs(t)} = \left[\mathbf{2t} - \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{1} - \mathbf{e}^{-\mathbf{2t}})\right] \boldsymbol{u(t)}}$$ → **STEP RESPONSE OF THE SYSTEM**

Output= Zero input response + Zero state response

$$x(t) = \mathrm{Xzi}(t) + \mathrm{Xzs}(t)$$

$$= [½ - ½e^{-2t}] + \left[2t - \frac{1}{2}(1 - e^{-2t})\right]$$

$$\boxed{\mathbf{x(t) = 2tu(t)}}$$ → **OUTPUT**

System 1.3 $(D^2+2D+1)\, x(t) = D\, r(t)$

(Or) $Q(D)\, x(t) = P(D)\, r(t)$

Here, $Q(D) = D^2+2D+1$

$P(D) = D$

For zero i/p response $r(t)=0$ $\Rightarrow$ $Q(D)x_{zi}(t)=0$

Characteristic equation $Q(D)=0$ $\Rightarrow$ $D^2+2D+1=0$

Characteristic roots :- $D= \{-1,-1\}$ i.e., $\lambda=\{-1,-1\}$

$x_{zi}(t) = (k_1 + k_2 t)e^{-\lambda t}$

$\therefore$ $x_{zi}(t) = k_1 + k_2 e^{-t}$

Order of the system n=2 $\begin{pmatrix} n-1=2-1=1 \\ n-2=2-2=0 \end{pmatrix}$

$D^{n-1} x_{zi}(t)|_{t=0} = 1$ and $D^{n-2} x_{zi}(t)|_{t=0} = 0$

$Dx_{zi}(t)|_{t=0} = -k_1e^{-t}+k_2e^{-t}-k_2te^{-t}$

$Dx_{zi}(0) = -k_1+k_2=1$ $x_{zi}(0) = k_1 =0$

$\Rightarrow$ $k_2= 1+k_1$ $k_2=1+0$ $\Rightarrow$ $k_2= 1$

$\therefore$ Zero i/p response of the system is :-

$\boxed{x_{zi}(t) = te^{-t}}$ $\Rightarrow$ **ZERO INPUT RESPONSE**

For impulse response h(t)

$$\left.\begin{aligned} Q(D) &= D^2+D+1 \\ P(D) &= D+4= 0.D^2+D+0 \end{aligned}\right\} b_0=0$$

$\therefore$ $\boxed{h(t) = b_0\delta(t) + [P(D)x_{zi}(t)]\, u(t)}$ $\longrightarrow$ **equation(4)**

$P(D)\, x_{zi}(t) = (D)(te^{-t})$

$= = e^{-t}(1-t)$

$\therefore$ Substituting the values in equation(4)

We get

$$\boxed{h(t) = e^{-t}(1-t)\,u(t)}$$ $\longrightarrow$ **UNIT IMPULSE RESPONSE**

Now, from the concept of convolution theorem,

$X_{zs}(t) = h(t) * r(t)$ (zero state response)

$= h(t) * u(t)$ [unit step response $\rightarrow$ r(t)=u(t)]

$$= \int_0^t h(\tau)u(t-\tau)d\tau$$

$$= \int_0^t e^{-\tau}(1-\tau).1d\tau$$

$$= [(-1)(1-\tau)e^{-\tau}]_0^t - \int_0^t(-1)(-e^{-\tau})$$

$$= 1-(1-t)e^{-t}+e^{-t}-1$$

$h(\tau) = e^{-\tau}(1-\tau) \quad \{\tau \geq 0\}$

$u(t-\tau) = 1 \quad \{-\infty \leq \tau \leq t\}$

$$\boxed{\mathbf{Xzs(t)} = [\mathbf{t}e^{-t}]\boldsymbol{u(t)}}$$ $\longrightarrow$ **STEP RESPONSE OF THE SYSTEM**

Output= Zero input response + Zero state response

$$x(t) = \mathrm{Xzi}(\mathrm{t}) + \mathrm{Xzs}(\mathrm{t})$$

$$= [te^{-t}] + [te^{-t}]$$

$$\boxed{\mathbf{x(t)} = \mathbf{2}\boldsymbol{te^{-t}}\mathbf{u(t)}}$$ $\longrightarrow$ **output**

#1b

This **GENERALISED CODE** is written in python language.

The best recommended IDE to run my code is Anaconda (Jupiter Notebook) since in it all the libraries are already available and its user friendly.

I am pasting the script here as well as I will be uploading the python (.py) file as well.

This is a flexible code in which on entering the coefficients of differential equations of Q(D) and P(D)

the user will be getting the equations of zero input response, unit impulse response , step response and the output. Also, the graphs of them are showed there itself.

This code is limited to differential equations of 1 order and 2 order .

A very limited number of assumptions and in-built functions have been used.

The graph which is plotted is taken from the library

sympy and different colors have been used for all the three different graphs.

CODE:-

```
 #SnS Assignment   BT20EEE107 Vadaparthi Tarun Gangadhar
import numpy as np
from matplotlib import pyplot as plt
import sympy as smp
import warnings
warnings.filterwarnings('ignore')
import sys


t = smp.symbols('t', real = True)
m,n = smp.symbols('m,n', real = True)


print("This is a Generalized code to obtain the impulse response, and the output for a given input for any LTIC whose dynamics in \nterms of differential equations(maximum order 2 and minimum order 1) is known to us.\n")
print("Assume the dynamics as Q(D)*x(t)=P(D)*R(t) where Q(D)=aD^2+bD+c and P(D)=dD^2+eD+f and condition P(D) order <= Q(D) order \nmust be taken for practical noise considerations \n")



def solver(a1,b1,c1):
   if a1==0:
      m = 1
```

```
        eqt = m*smp.exp(-c1*t/b1)
        return eqt
    else:
        x1 = (-b1 + (b1**2 - 4*a1*c1)**0.5)/2*a1
        x2 = (-b1 - (b1**2 - 4*a1*c1)**0.5)/2*a1
        if x1==x2:
            m = 0
            n = 1
            eqt = (m + n*t)* smp.exp(x1*t)
            return eqt
        elif x1!=x2:
            m = -1/(x2-x1)
            n = 1/(x2-x1)
            eqt = m*smp.exp(x1*t) + n*smp.exp(x2*t)
            return eqt


def unit_impulse_response(a2,b2,c2,flag):
    if flag==True:
        unit = smp.symbols('u(t)', real = True)
        delta = smp.symbols('δ(t)', real = True)
    elif flag == False:
        unit,delta = 1,0
    if a1==0:
```

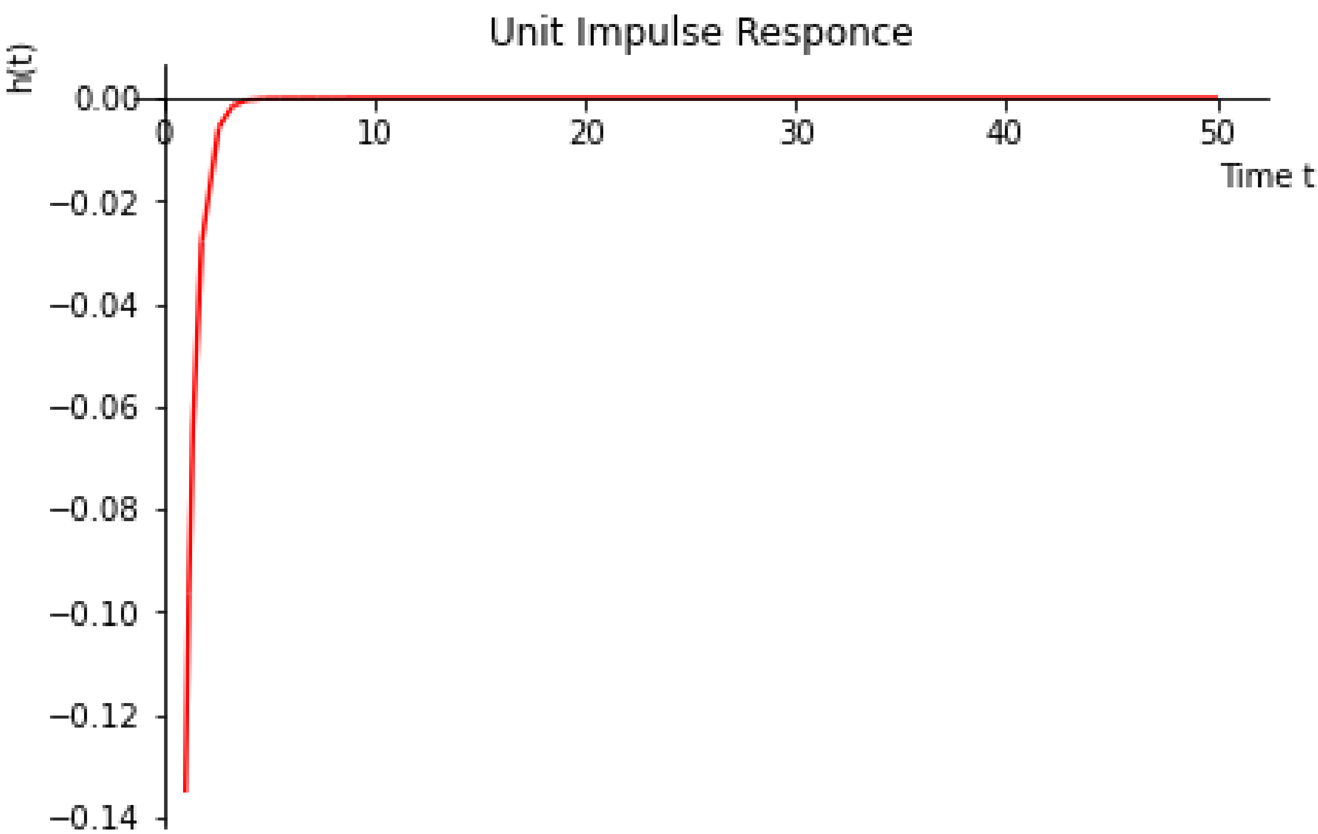
```
        return b2*delta + (a2*(smp.diff(solver(a1,b1,c1),t,t)) +
b2*smp.diff(solver(a1,b1,c1)) + c2*solver(a1,b1,c1))*unit
        #return ans
    else:
        return a2*delta + (a2*smp.diff(smp.diff(solver(a1,b1,c1))) +
b2*smp.diff(solver(a1,b1,c1)) + c2*solver(a1,b1,c1))*unit
        #return ans
#unit_impulse(0,1,2,0,3,5)


def zero_state_response(a1,b1,c1,a2,b2,c2):
    if a1==0:
        return b2 + smp.integrate((a2*(smp.diff(solver(a1,b1,c1),t,t)) +
b2*smp.diff(solver(a1,b1,c1),t) + c2*solver(a1,b1,c1)),(t,0,t))
    else:
        return a2 + smp.integrate((a2*(smp.diff(solver(a1,b1,c1),t,t)) +
b2*smp.diff(solver(a1,b1,c1),t) + c2*solver(a1,b1,c1)),(t,0,t))


# write the equations of the left side
a1 = float(input("enter a value i.e coeffecient of x^2 in Q(D) "))
b1 = float(input("enter b value i.e coeffecient of x in Q(D) "))
c1 = float(input("enter c value i.e constant term in Q(D) "))
# write the equation on the right side
a2 = float(input("enter d value i.e coeffecient of D^2 in P(D) "))
b2 = float(input("enter e value i.e coeffient of D in P(D) "))
c2 = float(input("enter f value i.e constant term in P(D) "))
```

```
if(a1==0 and a2!=0):
    sys.exit("In practical noise considerations,we require P(D) order <= Q(D) order. Please re enter values accordingly")
elif(a1==b1==0 or a2==b2==0):
    sys.exit("Please check the orders. Minimim and maximum orders of Q(D) and P(D) are 1 and 2 respectively")

else:
    flag = True
    Xi = solver(a1,b1,c1)
    h = unit_impulse_response(a2,b2,c2,flag)
    print('\nZero input response = ',Xi)
    print('\nUnit Impulse response = ',h)
    Xs = zero_state_response(a1,b1,c1,a2,b2,c2)
    print('\nStep response = [',Xs,']*u(t)')
    Xo = Xi+Xs
    print('\nOutput = [',Xo,']*u(t)')

#Xo = 2.5 + 1.5*smp.exp(-2.0*t)
graph1 = smp.plot(Xo,(t,0,50),show = False,title = 'Output',xlabel = 'Time t',ylabel = 'x(t)',line_color='b')
graph1.show()

def h1():
```

```
    if a1==0:
        return b2
    else:
        return a2
#(h1(),(t,-0.1,0.1)),
# At t=0
h2 = unit_impulse_response(a2,b2,c2,False)
graph2 = smp.plot((h2,(t,1,50)),show = False,title = 'Unit Impulse Responce',xlabel = 'Time t',ylabel = 'h(t)',line_color='r')
#graph2.extend(smp.plot((h1(),(t,0,0.01)),show = False,legend=True,line_color='b'))
graph2.show()


graph3 = smp.plot(Xs,(t,0,50),show = False,title = 'Step Response',xlabel = 'Time t',ylabel = 'Xzs(t)',line_color='g')
graph3.show()
```

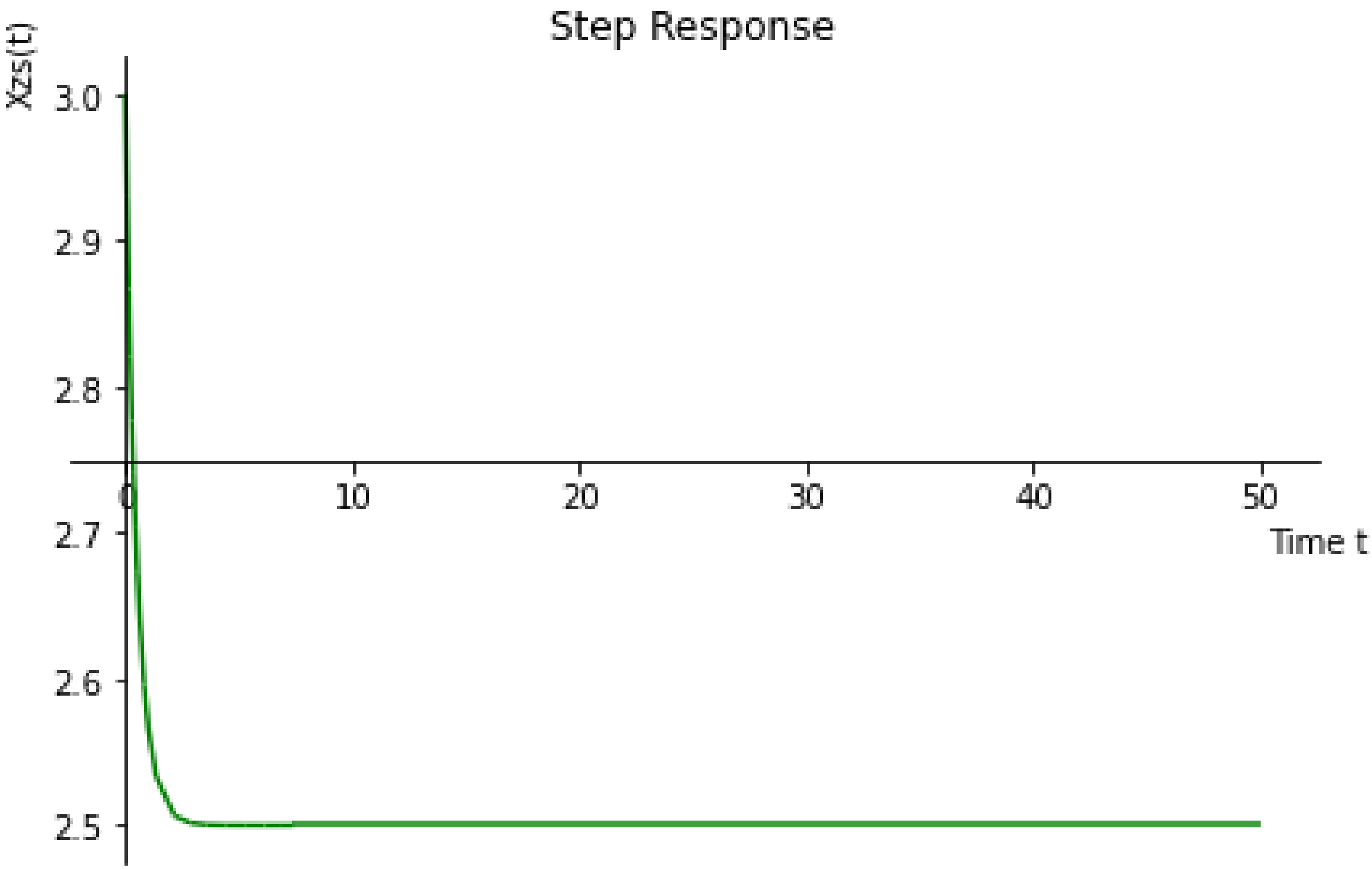
#1c (**Graphs of impulse response step response and output from the code developed**)

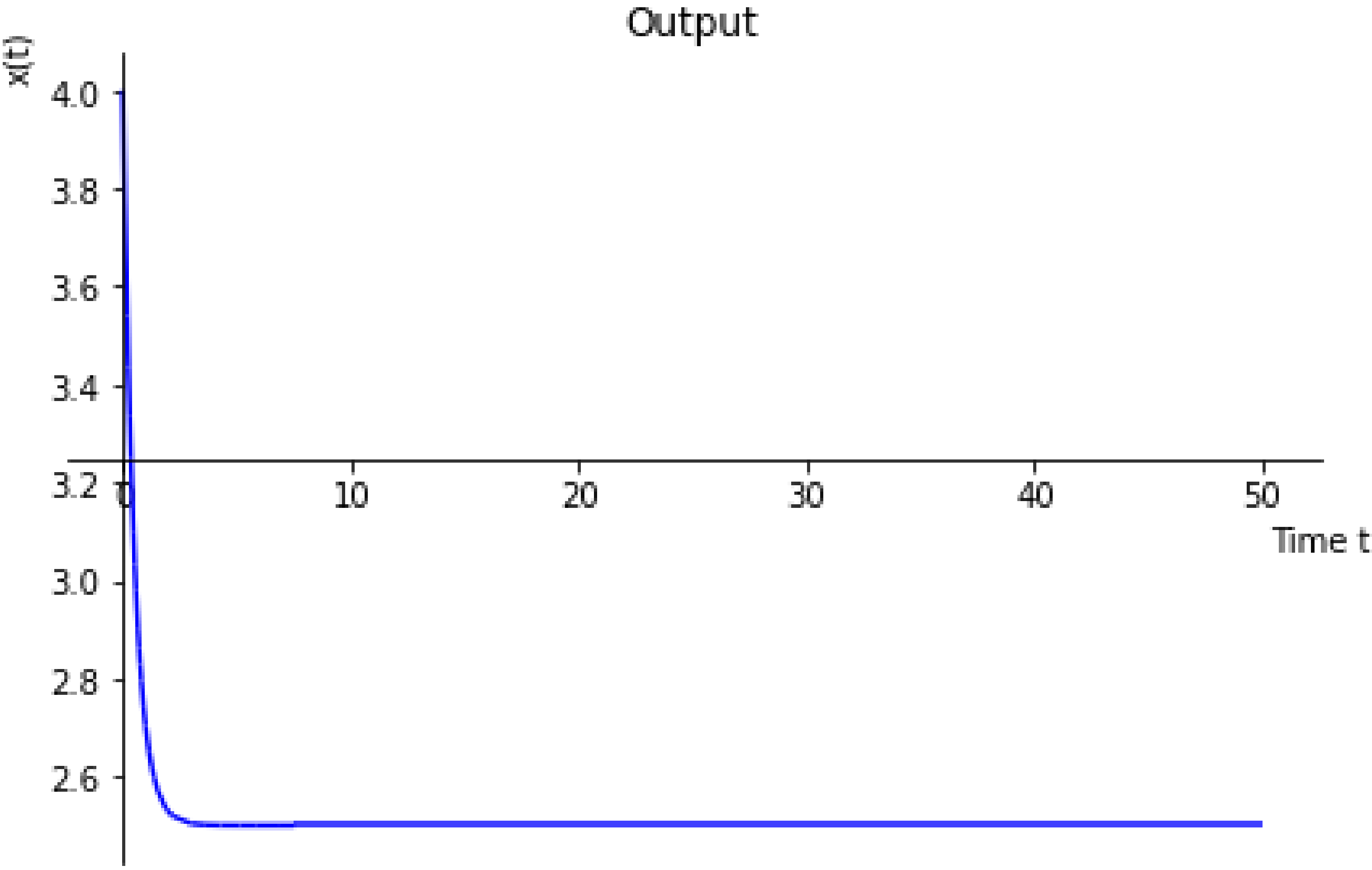
For system 1.1 ((D+2) x(t) = (3D +5) r(t))



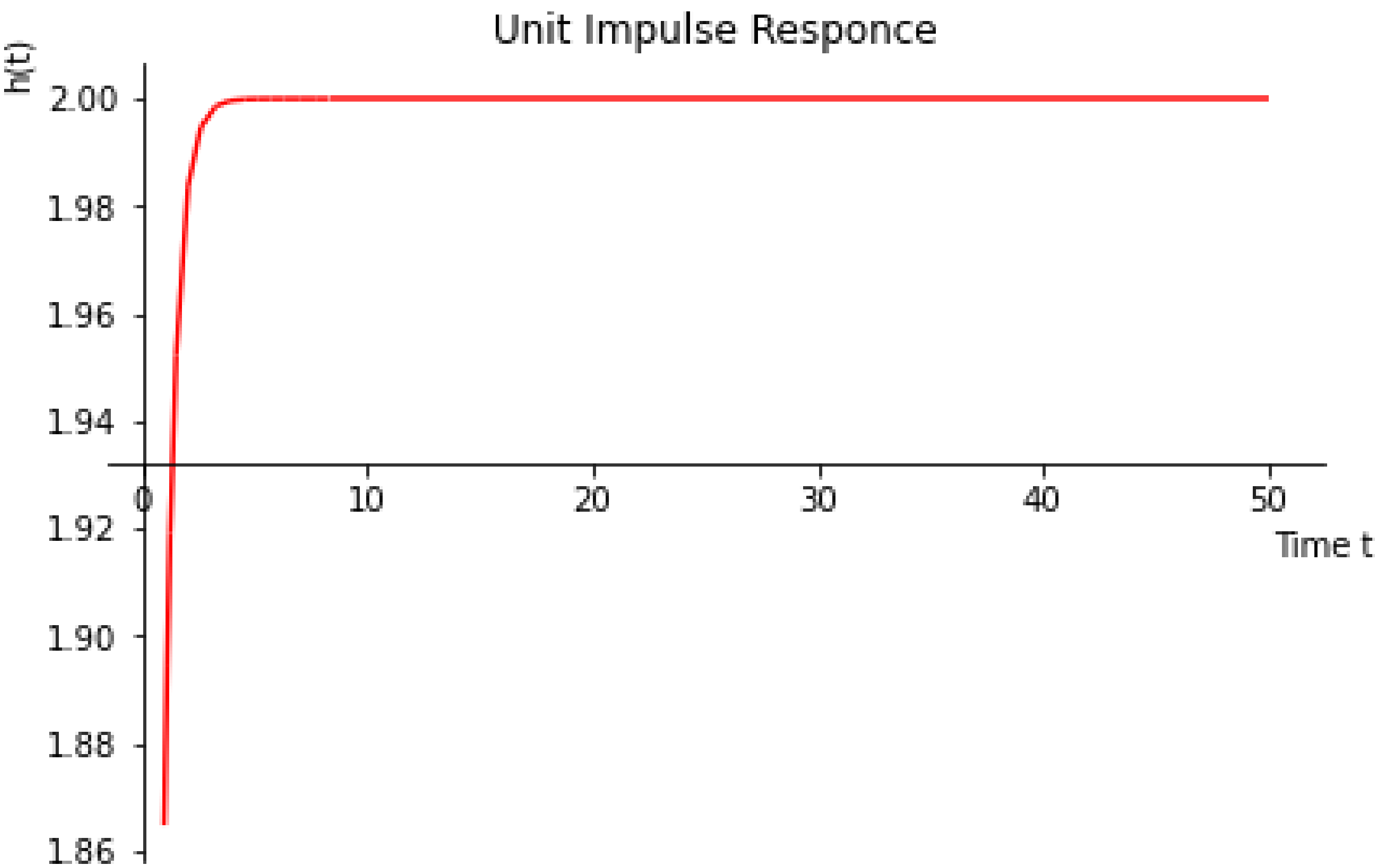
`Value of Impulse response at t=0 is 3.0`

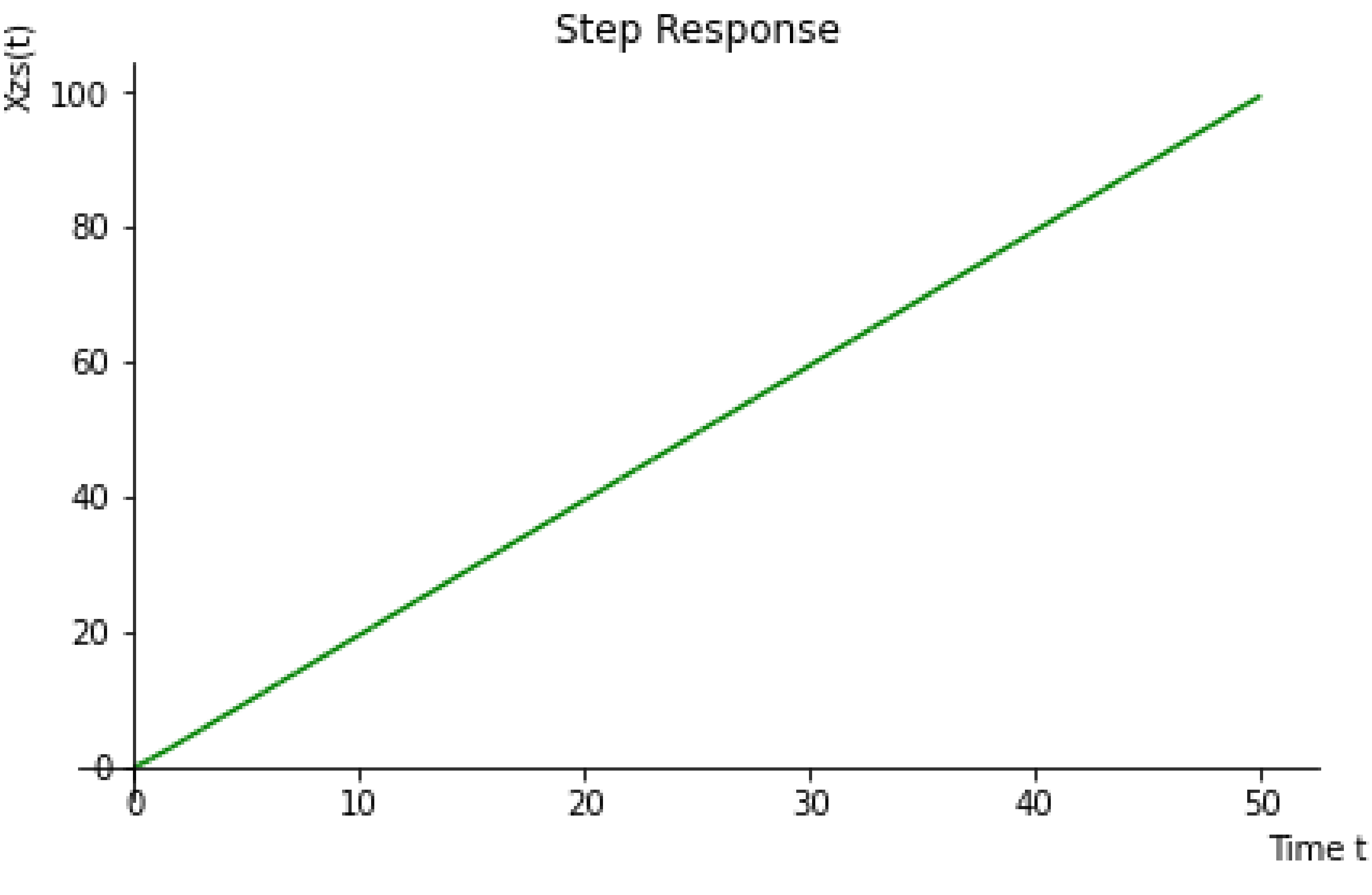
`(since` δ(t) is 1 at t=0)


Step Response
Xzs(t)
3.0
2.9
2.8
2.7
2.6
2.5
0
10
20
30
40
50
Time t

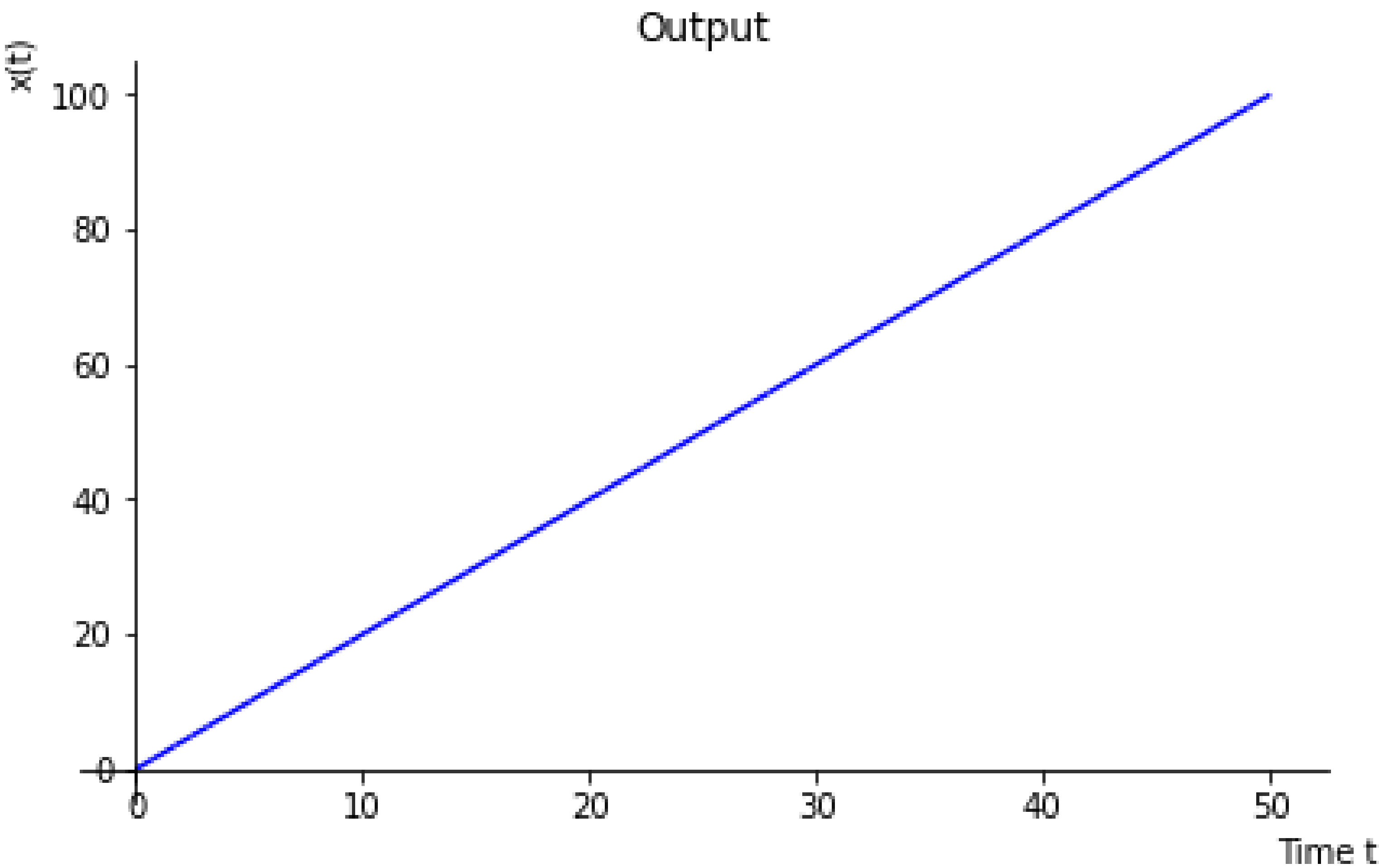

Output
x(t)
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
0
10
20
30
40
50
Time t

For system 1.2 $(D(D+2)\,x(t) = (D+4)\,r(t))$

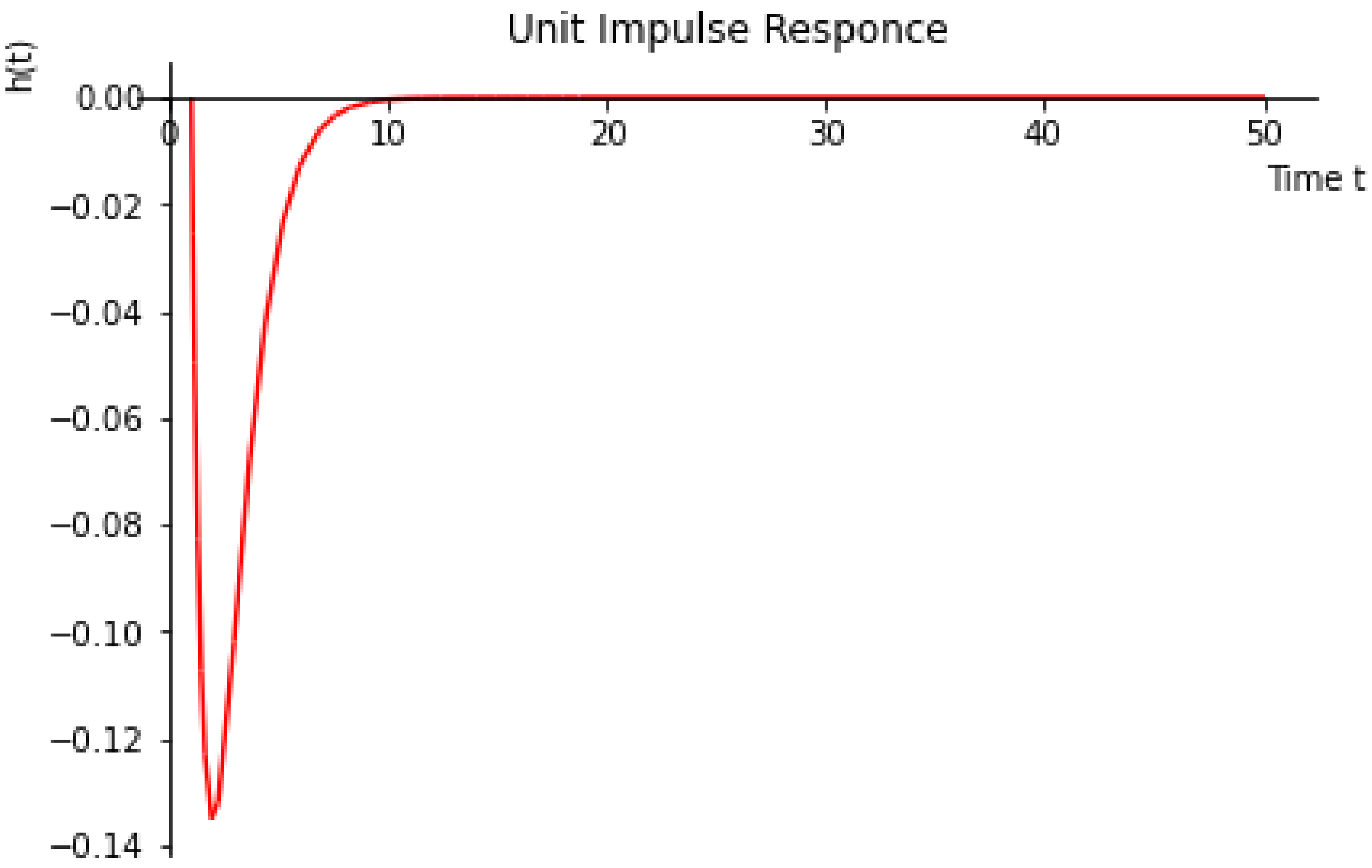


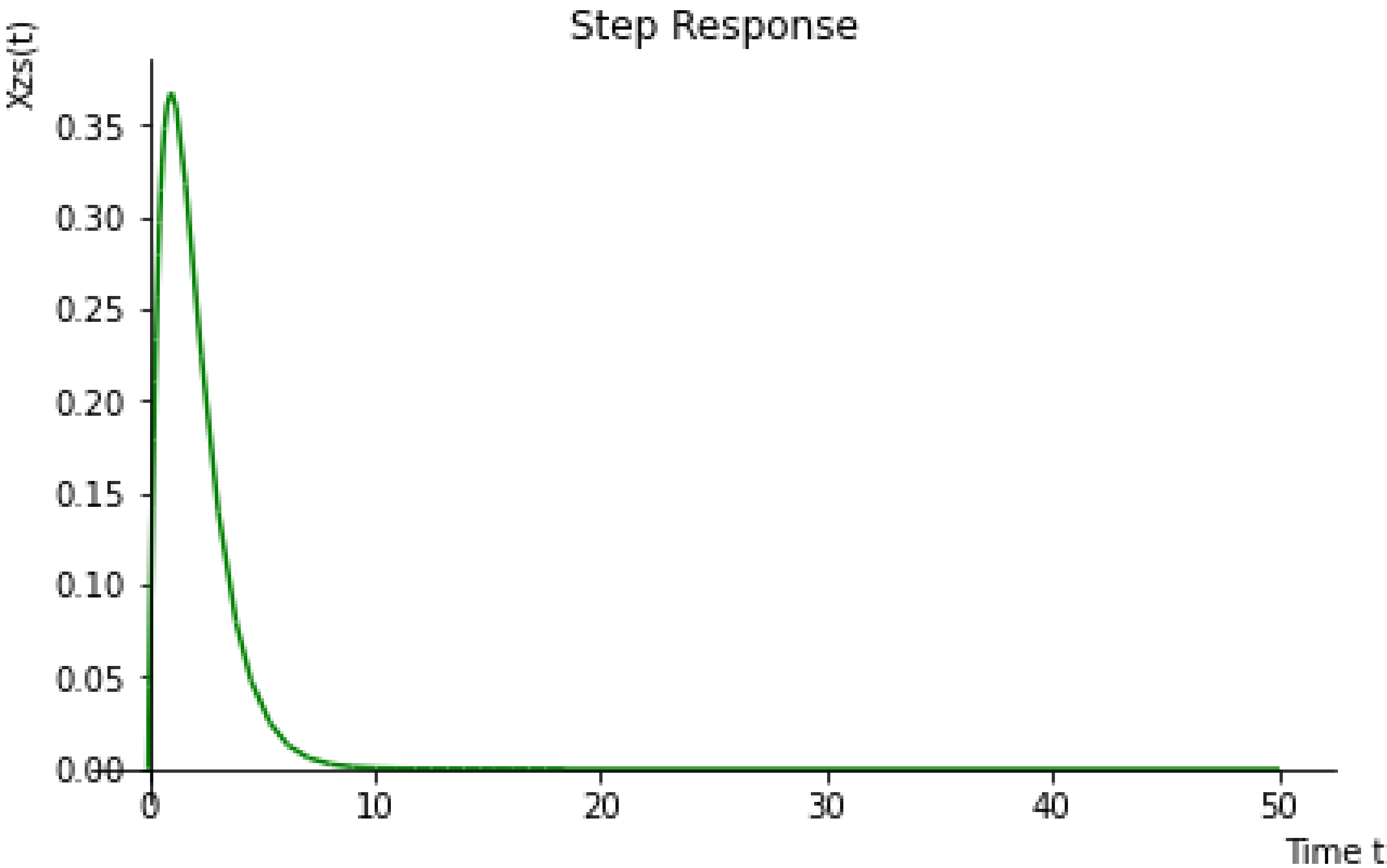


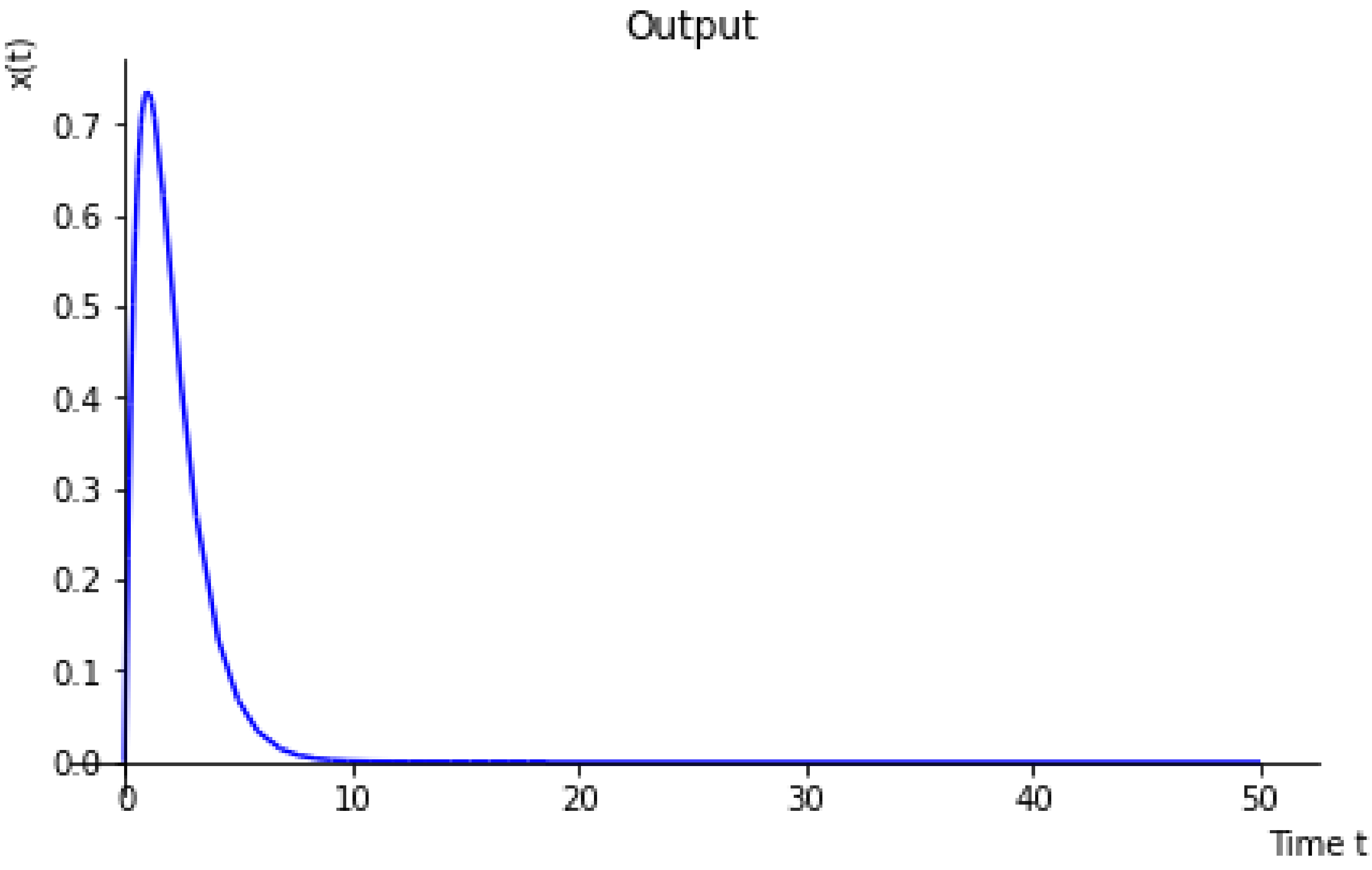
Step Response
Xzs(t)
100
80
60
40
20
0
0
10
20
30
40
50
Time t


Output
x(t)
100
80
60
40
20
0
0
10
20
30
40
50
Time t

For system 1.3 ((D^2 +2D+1) x(t) = D r(t))




Step Response
Xzs(t)
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
10
20
30
40
50
Time t


Output
x(t)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
10
20
30
40
50
Time t

I sincerely thank **Ritesh Kr. Keshri** Sir for acknowledging me and guiding us which led to completion of this assignment successfully……….

THE

END