PSET 3

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Problem 1: Somatic Inhibition

1. So we first have to write the equations for the current in each of the dendritic and somatic compartments. However, by Kirchhoff's Current Law, we know that the current going into a junction must be equal to the current going out of a junction. Thus we can write:

$$
\begin{aligned}
&\text{Dendritic} : G^* * (V_s - V_d) = G_e * (V_d - E_e) + G_d * V_d \\
&\text{Somatic} : G_i * (-V_s) + G_s * V_s = (V_s - V_d) * G^*
\end{aligned}
$$

2. We can simplify the above equations by solving for V_d in each of them and then setting them equal to each other:

$$
\begin{aligned}
&\text{Dendritic} : \\
&G^* * (V_s - V_d) = G_e * (V_d - E_e) + G_d * V_d \\
&= G^* * V_s - G^* * V_d = G_e * V_d - G_e * E_e + G_d * V_d \\
&= G_e * V_d + G_d * V_d + G^* * V_d = G^* * V_s + G_e * E_e \\
&= V_d = \frac{\left(G^* * V_s + G_e * E_e\right)}{G_e + G_d + G^*}
\end{aligned}
$$

We can do the same thing for the somatic compartment

$$
\begin{aligned}
&-G_i * V_s - G_s * V_s = (V_s - V_d) * G^* \\
&= G^* * V_d = \left(G^* + G_i + G_s\right) * V_s \\
&= V_d = \frac{\left(G^* + G_i + G_s\right)}{G^*} * V_s
\end{aligned}
$$

We can think of the expression for the somatic compartment as $V_d = a * V_s$. We can think of the expression for the dendritic compartment as $V_d = b * V_s + c$. Thus if we equate these, then

$V_s = \frac{c}{a - b}$ with $a = \frac{\left(G^* + G_i + G_s\right)}{G^*}$, $b = \frac{(G^*)}{G_e + G_d + G^*}$, and $c = \frac{(G_e * E_e)}{G_e + G_d + G^*}$. Thus we can do:

$$V_s = \frac{\left(\frac{(G_e * E_e)}{G_e + G_d + G^*}\right)}{\frac{(G^* + G_i + G_s)}{G^*} - \frac{G^*}{G_e + G_d + G^*}}$$

$$= \frac{(G_e * E_e)}{\frac{((G_e + G_d + G^*) * (G^* + G_i + G_s))}{G^*} - G^*}$$

$$= \frac{(G_e * E_e * G^*)}{G_e * G^* + G_d * G^* + {G^*}^2 + G_e * G_i + G_d * G_i + G_i * G^* + G_s * G_e + G_s * G_d - {G^*}^2}$$

$$= \frac{(G_e * E_e * G^*)}{G^* * G_d + G_s * G_d + G_s * G^* + G_e * (G^* + G_i + G_s) + G_i * (G_d + G^*)}$$

We have now arrived at the equation that we were hoping to arrive at.

3.So the Equation can be simplified as such:

$$V_S = \frac{(G_e * G^* * E_e)}{G^* * G_d + G_s * G_d + G_s * G^* + G_e * (G_i + G_s * G^*) + G_i * (G_d + G^*)}$$

now put in that $G_d = G_s$

$$= \frac{(G_e * G^* * E_e)}{G^* * G_d + G_d^2 + G_d * G^* + G_e * (G^* + G_i + G_d) + G_i * (G_d + G^*)}$$

$$= \frac{(G_e * E_e * G^*)}{G_d^2 + 2 * G^* * G_d + G_e * G_i + (G^* + G_d) * (G_e + G_i)}$$

now put in that $G_s = 1 = G_d$

$$= \frac{(G_e * E_e * G^*)}{1 + 2 * G^* + G_e * G_i + (1 + G^*) * (G_e + G_i)} \text{ now put in that } G^* = \frac{1}{9}$$

$$= \frac{(G_e * E_e)}{9 * G_e * G_i + 10 * (G_e + G_i) + 11}$$

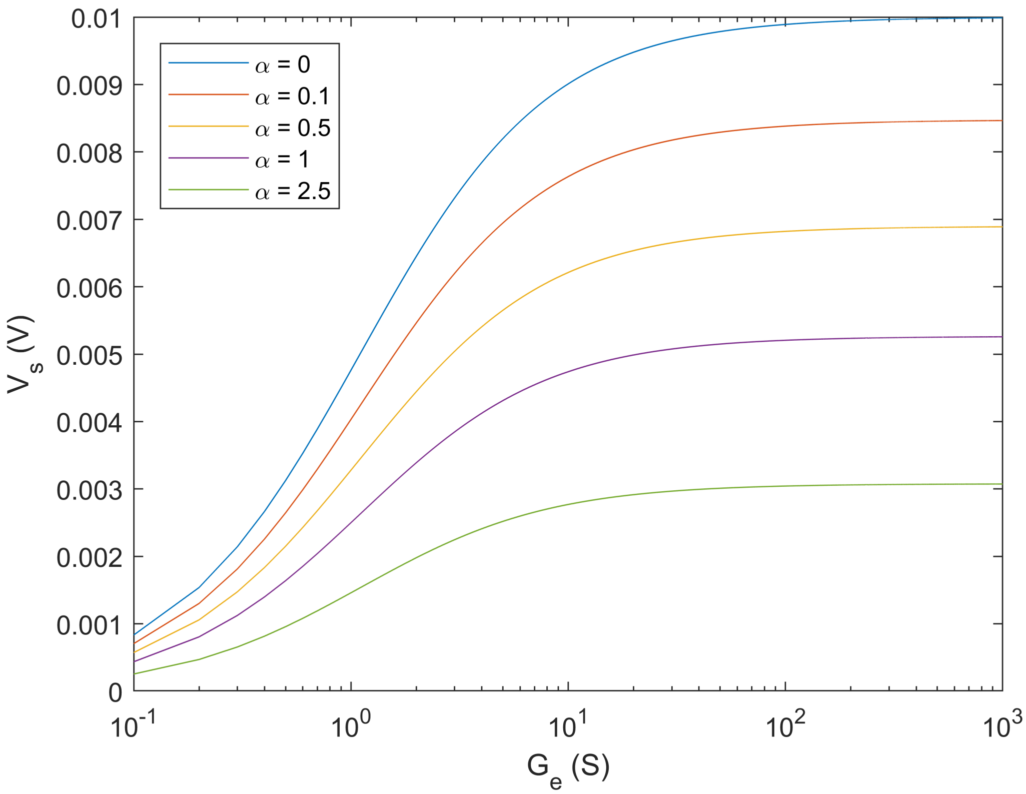
now $G_i = \alpha * G_d = \alpha$ and $E_e = 0.1\ V$

$$= \frac{(0.1 * G_e)}{9 * G_e * G_i + 10 * (G_e + G_i) + 11}$$

4. We can go on to graph this using the code shown below, where the G_i will be either {0, 0.2, 0.5, 1, 2.5}

```
G_i = [0, 0.2, 0.5, 1, 2.5]; %Set the values of G_i in an array
V_s = [];
xval = [];
currentval = 0;
for y = 1:5 %Iterating through each of the G_i values
    indexer = 0; %Reset the index of the matrix when we have a new value of G_i (a new curve)
    for x = 0.1:0.1:1000 %Iterating through all of G_e in 0.1 S steps
        indexer = indexer + 1;
        xval(indexer) = x; %For plotting the voltage values on the scale of 0.1 S steps
```

```
        currentval = (0.1*x)/(9*x*G_i(y) + 10*(x+G_i(y)) + 11); %This is the above equation,
        %                                                        we solve for it at each step
        V_s(y, indexer) = currentval; % Set the appropriate
        %                               part in the array equal to the equation value
    end
end
semilogx(xval,V_s);
legend ('\alpha = 0', '\alpha = 0.1', '\alpha = 0.5', '\alpha = 1', '\alpha = 2.5',...
'Location', 'northwest');
xlabel('G_e (S)');
ylabel('V_s (V)');
```



5. We can mathematically look at this as such:

$$V_s = \frac{0.1 * G_e}{9 * G_e * G_i + 10 * (G_e + G_i) + 11}$$

$$= \frac{0.1 * G_e}{G_e * (9 * G_i + 10) + (10 * G_i + 11)} \text{ thus we are doing :}$$

$$\lim_{G_e \to \infty} \frac{0.1 * G_e}{G_e * (9 * G_i + 10) + (10 * G_i + 11)}$$

so as $G_e \to \infty$, the second part of the denominator $\to 0$ so we have :

$$\to \frac{0.1 * G_e}{(9 * G_i + 10) * G_e} = \underline{\frac{0.1}{9 * G_i + 10}}$$

Thus V_s, as the G_e approaches infinity, is dependent on the level of inhibition. Thus this is a proximal inhibition circuit, which is logical since the plot shows that as we increase the value of G_i, which is the level of inhibition, the voltage will tend to decrease, and this can not be corrected by simply increasing the G_e value. It is not that at larger values of G_e the V_s all tends to the same value - for all values of G_e, the curves which have a higher value of G_i show lower values of V_s.

Problem 2: Dendritic Inhibition

1. So this can be thought of in the same way as the problem 1 in the earlier section. The current input to a junction must be equal to the current leaving the junction, thus we can write the currents in the dendritic and somatic compartments as such:

$$\begin{aligned}
&\text{Dendritic Compartment :}\\
&G^* * (V_s - V_d) = G_e * (V_d - E_e) + G_d * V_d + G_i * V_d\\
&\text{Somatic Compartment :}\\
&G_s * (-V_s) = G^* * (V_s - V_d)
\end{aligned}$$

2. So now we can solve for V_d in each compartment and then set these equal to one another

Dendritic:

$$\begin{aligned}
&G^* * (V_s - V_d) = G_e * (V_d - E_e) + G_d * V_d + G_i * V_d\\
&= G^* * V_s + G^* * V_d = G_e * V_d - G_e * E_e + G_d * V_d + G_i * V_d\\
&= G_e * V_d + G_i * V_d + G_d * V_d + G^* * V_d = G^* * V_s + G_e * V_e\\
&= V_d = \frac{\left(G^* * V_s + G_e * V_e\right)}{G_e + G_i + G_d + G^*}
\end{aligned}$$

Somatic:

$$\begin{aligned}
&G^* * V_s + G_s * V_s = G^* * V_d\\
&= V_d = \frac{\left(G^* * V_s + G_s * V_s\right)}{G^*}\\
&= V_d = \frac{\left(G^* + G_s\right)}{G^*} * V_s
\end{aligned}$$

Now we can combine the two expressions, in a similar manner as we had stated in Problem 1. We will say that the dendritic equation resembles an equation where $V_d = b * V_s + c$ and hte somatic compartment resembles an equation where $V_d = a * V_s$. Thus, setting these expressions equal to one another gives us that, $V_s = \frac{c}{a - b}$ and we have that $a = \frac{\left(G^* + G_s\right)}{G^*}$, $b = \frac{\left(G^*\right)}{G_e + G_i + G_d + G^*}$ and $c = \frac{\left(G_e * V_e\right)}{G_e + G_i + G_d + G^*}$. So we can say:

$$V_s = \frac{\left(\frac{(G_e * V_e)}{G_e + G_i + G_d + G^*}\right)}{\frac{(G^* + G_s)}{G^*} - \frac{G^*}{G_e + G_i + G_d + G^*}}$$

$$= \frac{(G_e * V_e)}{\frac{((G^* + G_s) * (G_e + G_i + G_d + G^*))}{G^*} + G^*}$$

$$= \frac{(G_e * V_e * G^*)}{G_e * G^* + G_i * G^* + G_d * G^* + G^{*2} + G_s * G_e + G_s * G_i + G_s * G_d + G_s * G^* - G^{*2}}$$

$$= \frac{(G_e * V_e * G^*)}{G^* * G_d + G_d * G_s + G^* * G_s + G_e * (G^* + G_s) + G_i * (G^* + G_s)}$$

Thus we arrived at the equation that we expect to reach.

3. So the equation can be simplified as such:

$$V_s = \frac{(G_e * G^* * E_e)}{G^* * G_d + G_d * G_s + G^* * G_s + G_e * (G^* + G_s) + G_i * (G^* + G_s)}$$

and then when $G_d = G_s$ we find

$$V_s = \frac{(G_e * G^* * E_e)}{G_d^2 + 2 * G^* * G_d + (G_e + G_i) * (G^* + G_d)}$$

and then $G^* = \frac{1}{9} * G_s = \frac{1}{9} * G_d$

$$V_s = \frac{\left(G_e * \frac{1}{9} * G_d * E_e\right)}{G_d^2 + \frac{2}{9} * G_d^2 + (G_e + G_i) * \left(\frac{10}{9} * G_d\right)}$$

but we also know that $G_s = 1 = G_d$ so we substitute this in

$$V_s = \frac{\left(\frac{1}{9} * G_e * E_e\right)}{1 + \frac{2}{9} + \frac{10}{9} * (G_e + G_i)}$$

$$= \frac{(G_e * E_e)}{11 + 10 * (G_e + G_i)}$$

we can say that $E_e = 0.1\ V$ and substitute this in:

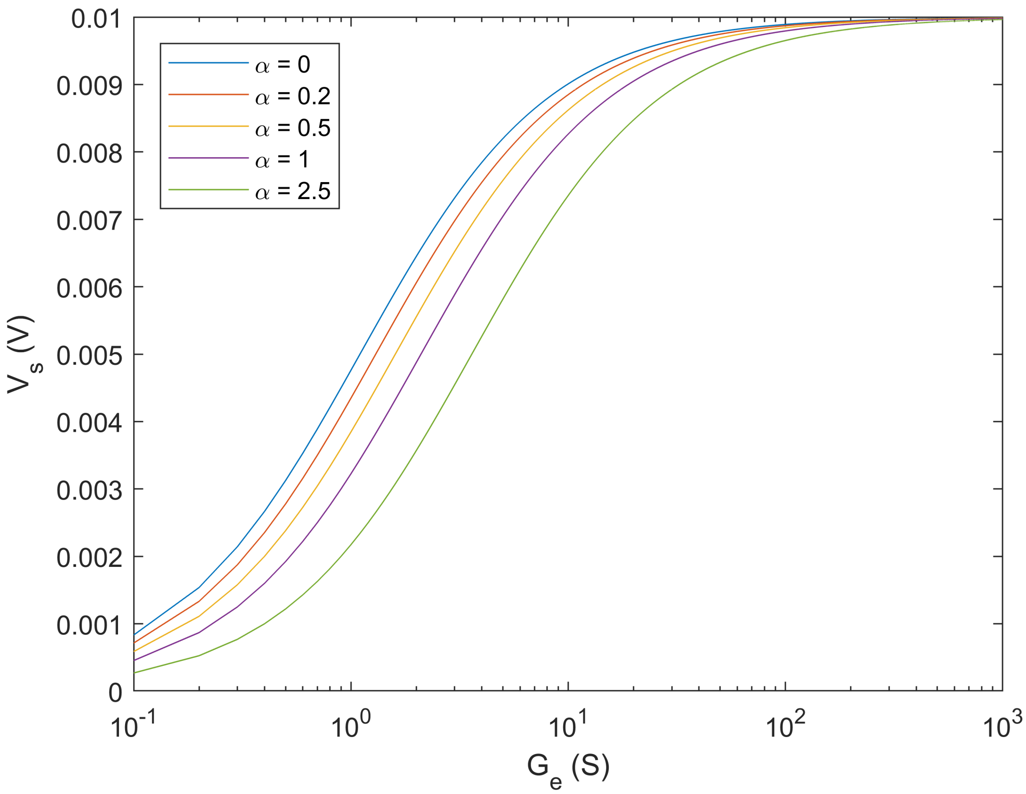
$$V_s = \frac{(0.1 * G_e)}{11 + 10 * (G_e + G_i)}$$

$$\text{Finally, } \alpha = \frac{G_i}{G_d} \text{ so } G_i = \alpha * G_d = \alpha \text{ since } G_d = 1, \text{thus :}$$

$$V_s = \frac{(0.1 * G_e)}{11 + 10 * (G_e + \alpha)}$$

4. We can graph the above equation as such:

```
alpha2 = [0, 0.2, 0.5, 1, 2.5]; %Set the values of G_i in an array
V_s2 = [];
xval2 = [];
currentval2 = 0;
for n = 1:5 %Iterating through each of the G_i values
    indexer2 = 0; %Reset the index of the matrix when we have a new value of G_i (a new curve)
    for m = 0.1:0.1:1000 %Iterating through all of G_e in 0.1 S steps
        indexer2 = indexer2 + 1;
        xval2(indexer2) = m; %For plotting the voltage values on the scale of 0.1 S steps
        currentval2 = (0.1*m)/(11+10*(m+alpha2(n))); %This is the above equation,
        %                                             we solve for it at each step
        V_s2(n,indexer2) = currentval2; % Set the appropriate
        %                                  part in the array equal to the equation
    end
end
semilogx(xval2,V_s2);
legend('\alpha = 0', '\alpha = 0.2', '\alpha = 0.5', '\alpha = 1', '\alpha = 2.5', ...
'Location', 'northwest');
xlabel('G_e (S)');
ylabel('V_s (V)');
```



5. We can mathematically look at this as such:

$$\lim_{G_e \to \infty} \frac{(0.1 * G_e)}{11 + 10 * (G_e + \alpha)} = \frac{(0.1 * G_e)}{10 * (G_e)}$$

the constant term in the denominator (11) and the alpha both are relatively small as G_e approaches infinity. this simplifies to:

$$V_s = \frac{(0.1)}{10} = .01\ V$$

In this expression, V_s is independent of inhibition as G_e approaches infinity (it will always approach 0.01 V), which is what we would expect from looking at our graph. We see on our graph that at high values of G_e, the curves are all roughly the same value, independent of their alpha value (amount of inhibition), despite being slightly different values of V_s at lower values of G_e. This is dendritic, distal inhibition.