Introduction to Neural Computation

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MIT BCS 9.40 — 2018

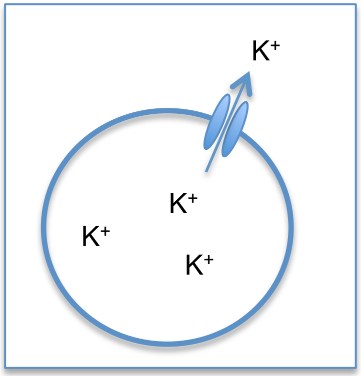
Video Module on Nernst Potential

Part 2

The Nernst potential for potassium

Intracellular and extracellular concentrations of ionic species, and the Nernst potential

Ion	Cytoplasm (mM)	Extracellular (mM)	Nernst (mV)
K^+	400	20	-75

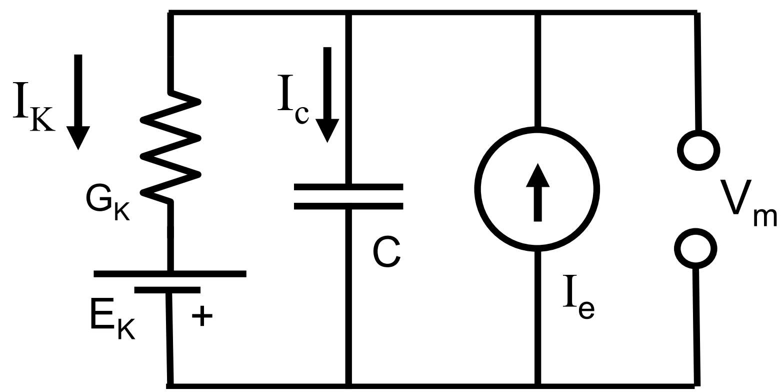


$$\Delta V = \frac{kT}{q}\ln\left(\frac{[K]_{out}}{[K]_{in}}\right)$$

$\frac{kT}{q}$ = 25mV at 300K (room temp) for monovalent ion

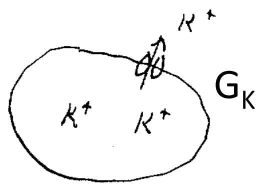
$$E_K = 25mV(-3.00) = -75mV$$

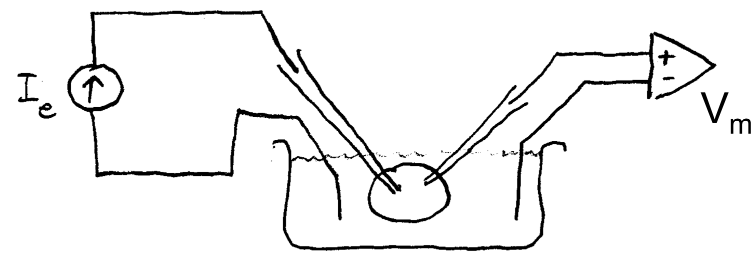
How to implement an ion specific conductance as a battery in our model neuron



Potassium I-V relation

One of the best ways to study the function of an ion channel is to plot the current-voltage relation (I-V curve). This can be measured as the current required to hold the neuron at a given voltage.

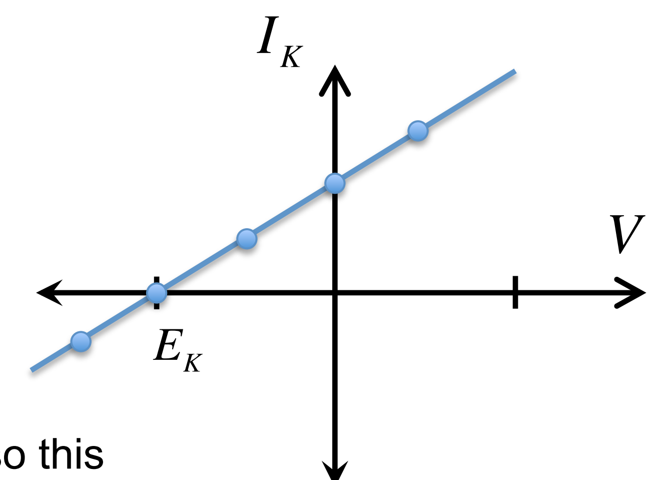




For a potassium conductance

- If you hold the voltage above the equilibrium potential, K current will flow out through the membrane (positive current)

- If you hold the cell below E_K, then the current will flow into the cell.

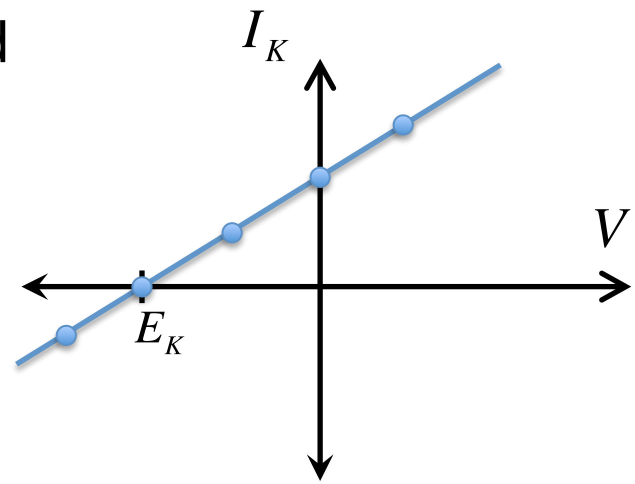


Note that the current reverses at the equilibrium potential, so this is often referred to as the 'reversal potential'

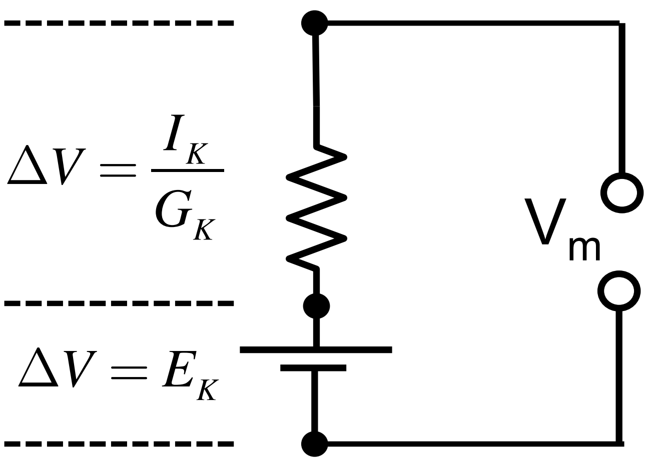
I-V relation

This relation turns out to be monotonic and roughly linear for ion channels in the open state. So we can write:

$$I_K = G_K(V - E_K)\ , \qquad G_K = R_K^{-1}$$

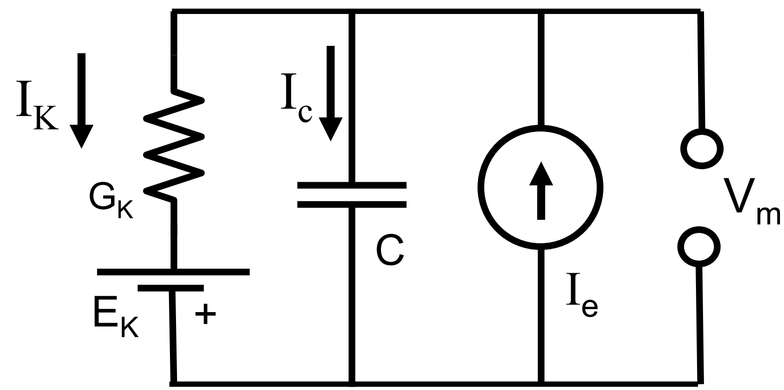


We can model this as a battery in series with a resistor! Why?



$$V_m = E_K + \frac{I_K}{G_K} \Rightarrow I_K = G_K(\underbrace{V - E_K}_{\text{driving potential}})$$

Our equation is now:



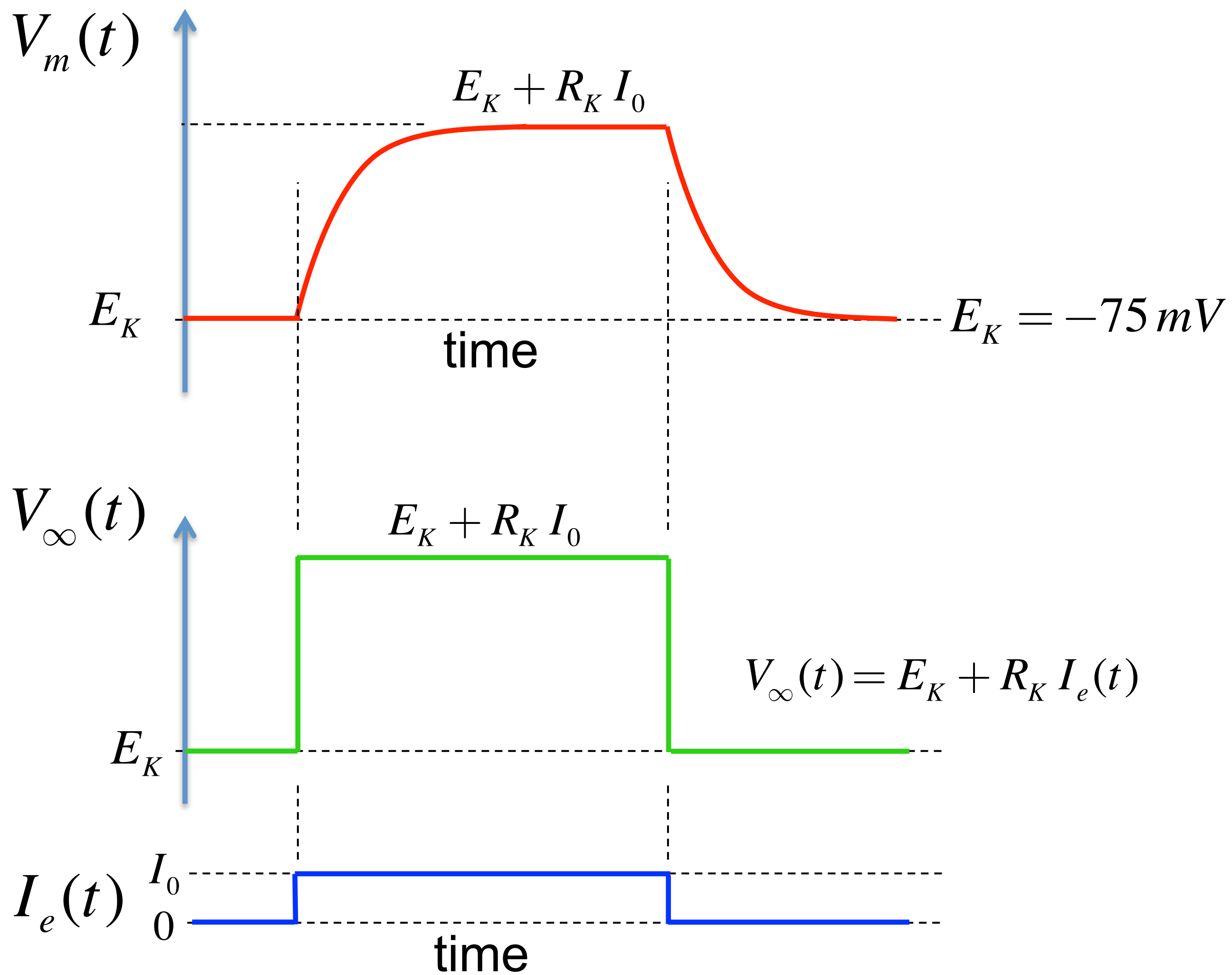
$$I_K + C\frac{dV}{dt} = I_e$$

$$G_K(V - E_K) + C\frac{dV}{dt} = I_e \quad , \qquad R_K = G_k^{-1} \;\; , \;\; \tau = R_K C$$

$$V + \tau\frac{dV}{dt} = \underbrace{E_K + R_K I_e}_{V_\infty}$$

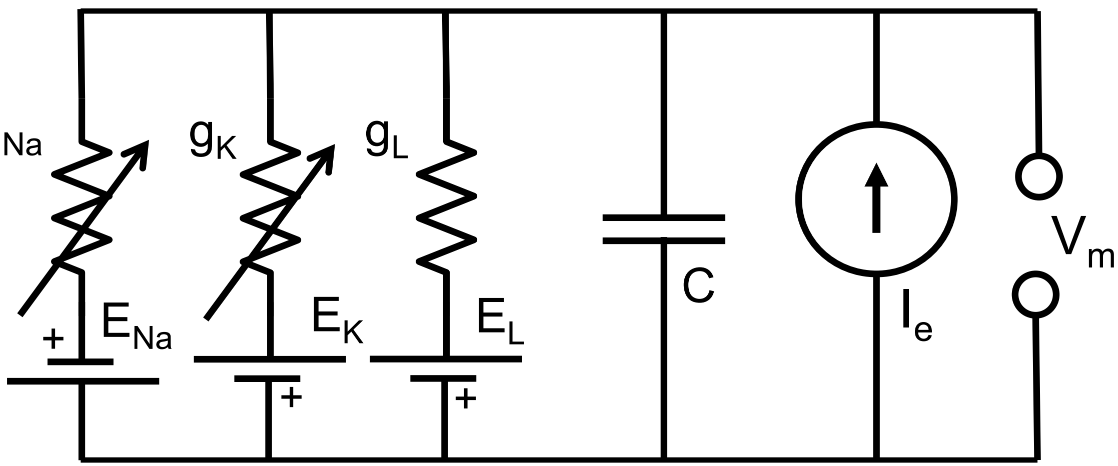
$$V + \tau\frac{dV}{dt} = V_\infty , \qquad V_\infty = E_K + R_K I_e$$

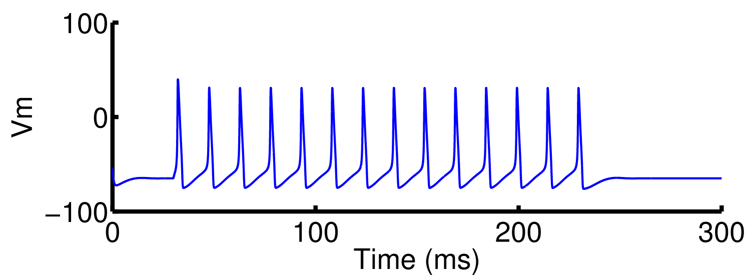
Response to current injection



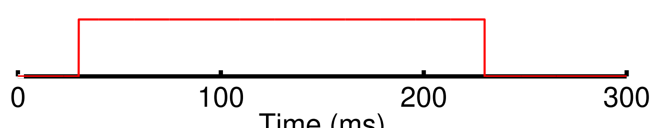
A mathematical model of a neuron

- Equivalent circuit model





Alan Hodgkin
Andrew Huxley, 1952

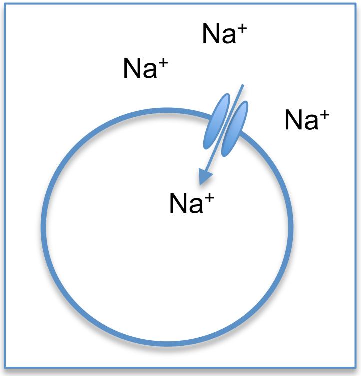


The Nernst Potential is different for different ions

Intracellular and extracellular concentrations of ionic species, and the Nernst potential

Ion	Cytoplasm (mM)	Extracellular (mM)	Nernst (mV)
K^+	400	20	-75
Na^+	50	440	

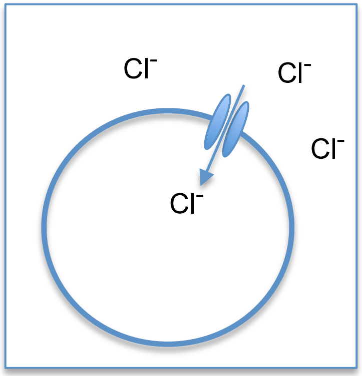
$$E_{Na} = 25mV \ln\left(\frac{440}{50}\right) = 25mV(2.17) = 54.3mV$$



The Nernst Potential is different for different ions

Intracellular and extracellular concentrations of ionic species, and the Nernst potential

Ion	Cytoplasm (mM)	Extracellular (mM)	Nernst (mV)
K^+	400	20	-75
Na^+	50	440	+54
Cl^-	52	560	



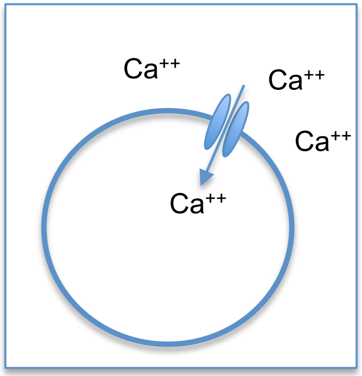
$$E_{Cl} = -25mV \ln\left(\frac{560}{52}\right) = -25mV(2.38) = -59.4\,mV$$

The negative here comes from the negative charge of the Cl^- ion (q=-e)

The Nernst Potential is different for different ions

Intracellular and extracellular concentrations of ionic species, and the Nernst potential

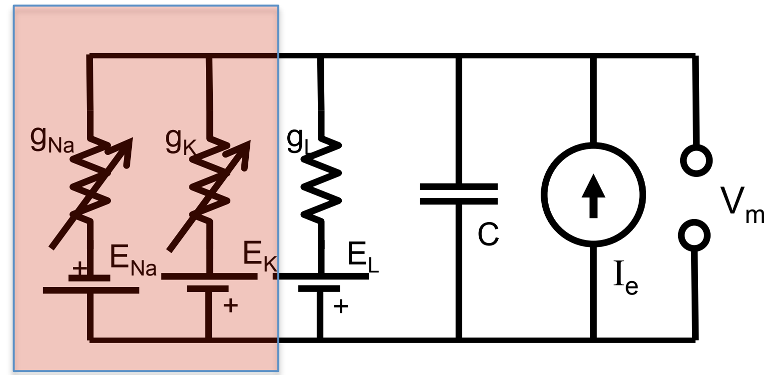
Ion	Cytoplasm (mM)	Extracellular (mM)	Nernst (mV)
K^+	400	20	-75
Na^+	50	440	+55
Cl^-	52	560	-59
Ca^{++}	10^{-4}	2	+124



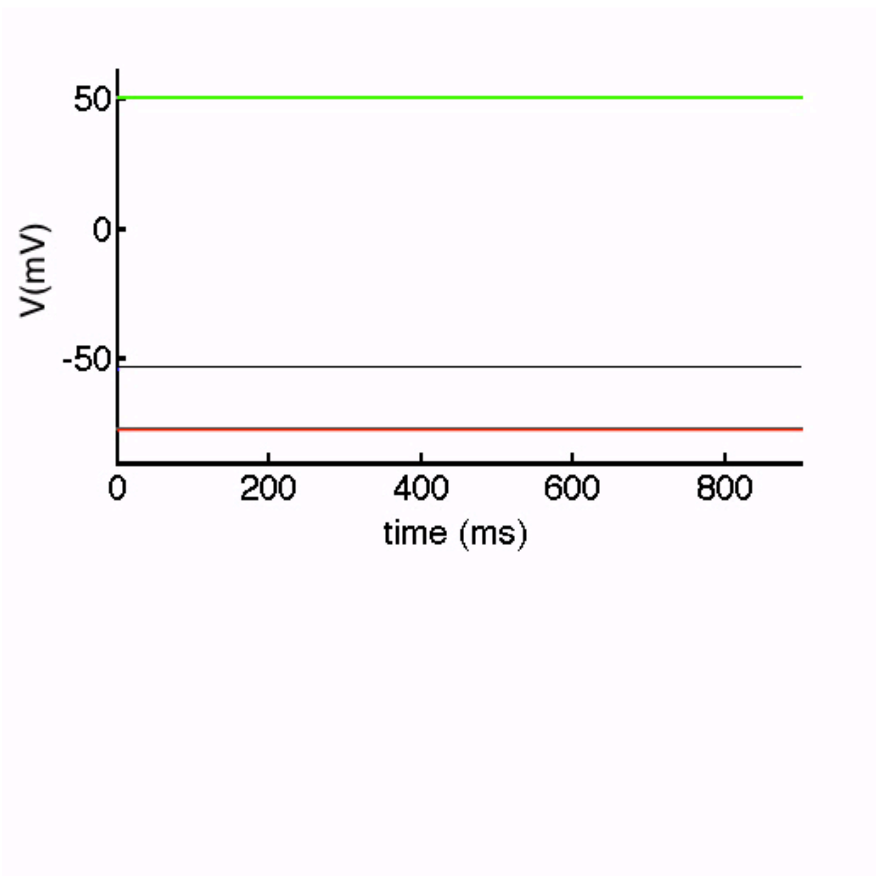
$$E_{Ca} = 12.5 mV \ln\left(\frac{2}{.0001}\right) = 124 mV$$

Why is this 12.5mV?

Outline of HH model



Voltage and time-dependent ion channels are the ‘knobs’ that control membrane potential.



- Na^+channels push the membrane potential toward +50mV.
- K^+ channels push the membrane potential toward -80mV.
- Together these channels give the neural machinery flexible control of voltage!
- - for example to generate an action potential