Introduction to Neural Computation

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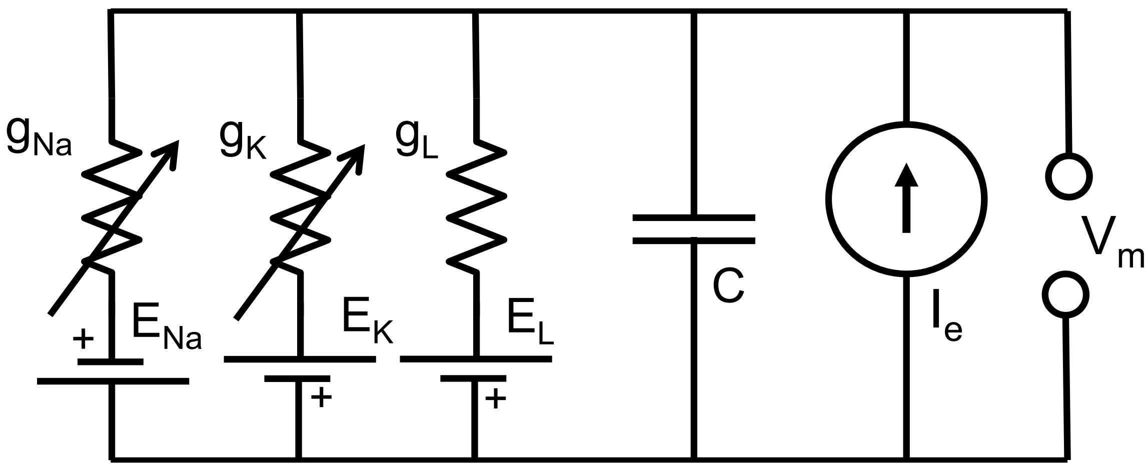
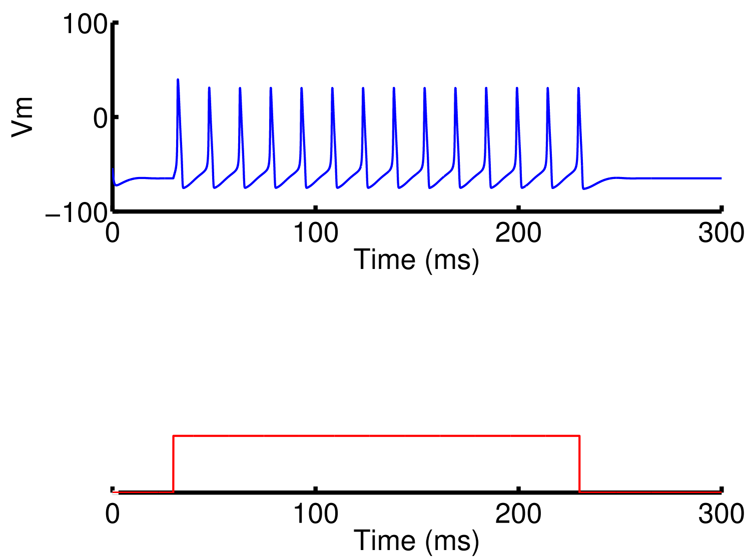
MIT BCS 9.40 — 2018
Video Module on Nernst Potential
Part 1

A mathematical model of a neuron

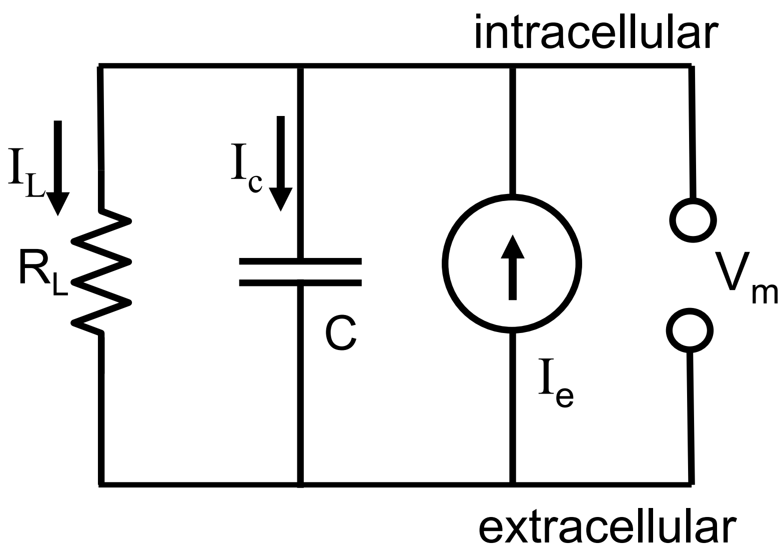
- Equivalent circuit model





Alan Hodgkin
Andrew Huxley, 1952

A neuron is a leaky capacitor



I_c = membrane capacitive current

I_L = membrane ionic current

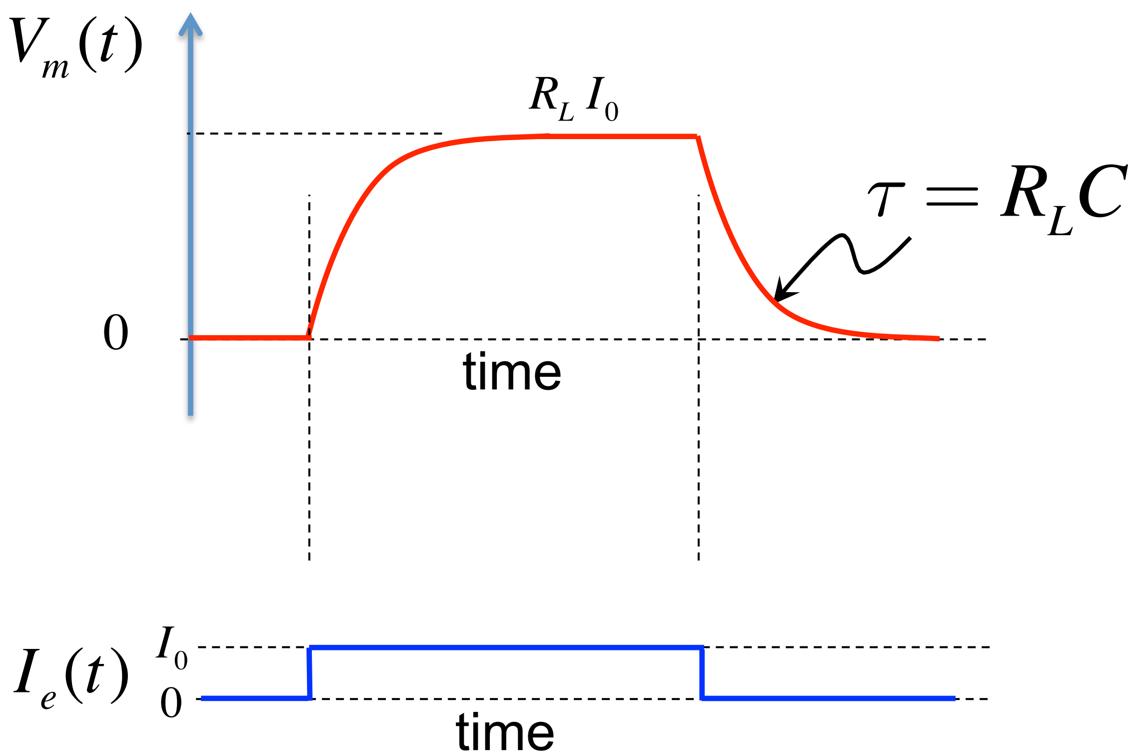
$$V_m + \tau \frac{dV_m}{dt} = V_\infty$$

where $\tau = R_L C$

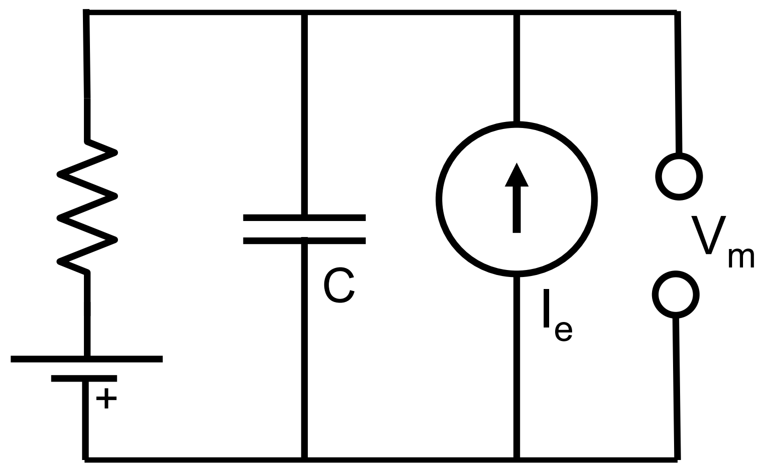
$$V_\infty(t) = R_L\, I_e(t)$$

Response to current injection

Let's see what happens when we inject current into our model neuron with a leak conductance.

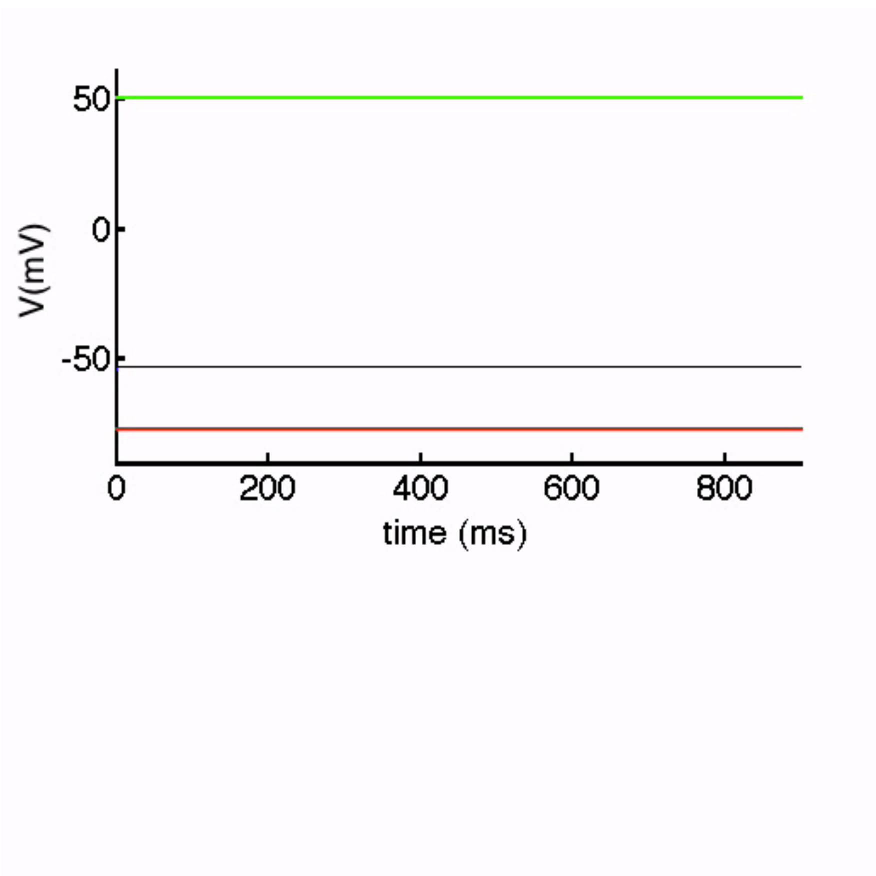


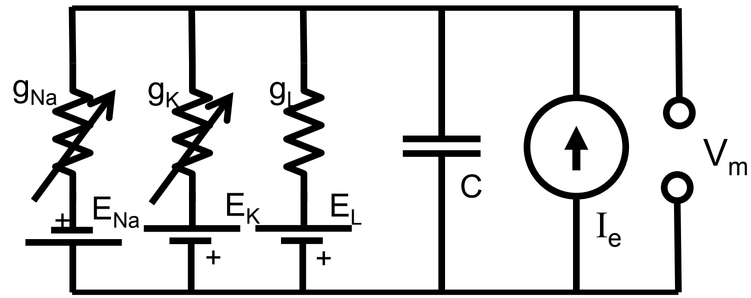
A neuron is a leaky capacitor



Outline of HH model

Voltage and time-dependent ion channels are the ‘knobs’ that control membrane potential.



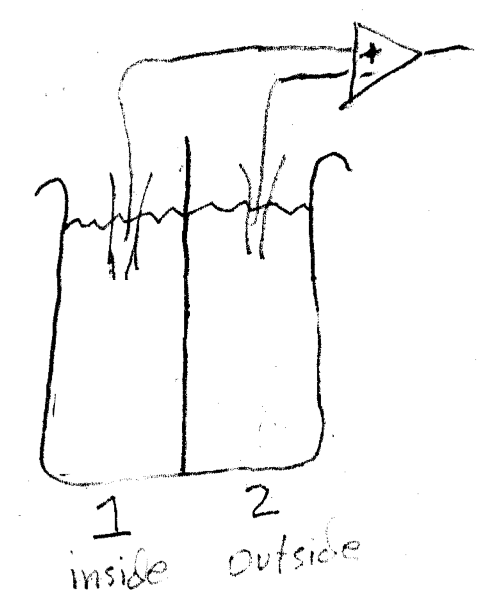


- Some ion channels push the membrane potential positive.
- Other ion channels push the membrane potential negative.
- Together these channels give the neural machinery flexible control of voltage!

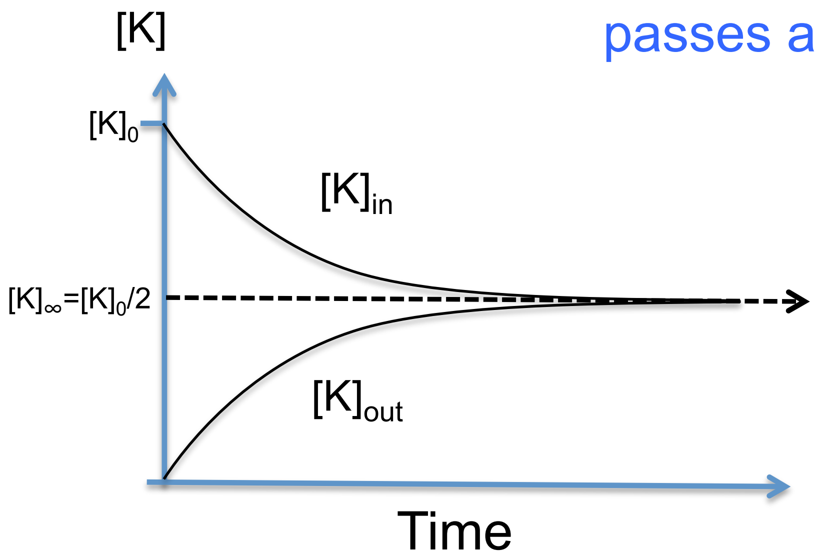
Where do the batteries of a neuron come from?

1) Ion concentration gradients

2) Ion-selective permeability of ion channels

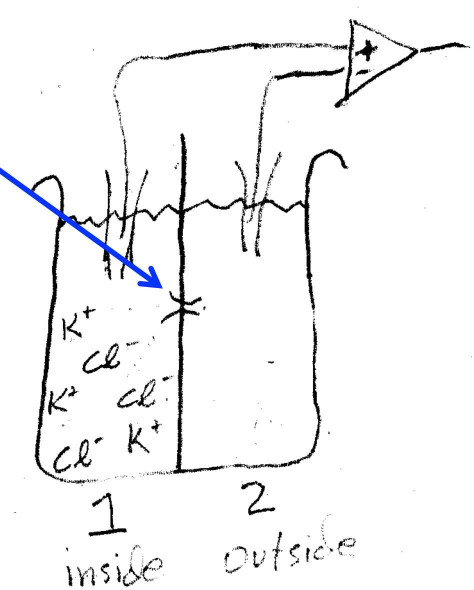


Neurons have batteries



(a long time)

'Non-selective' pore
passes all ions



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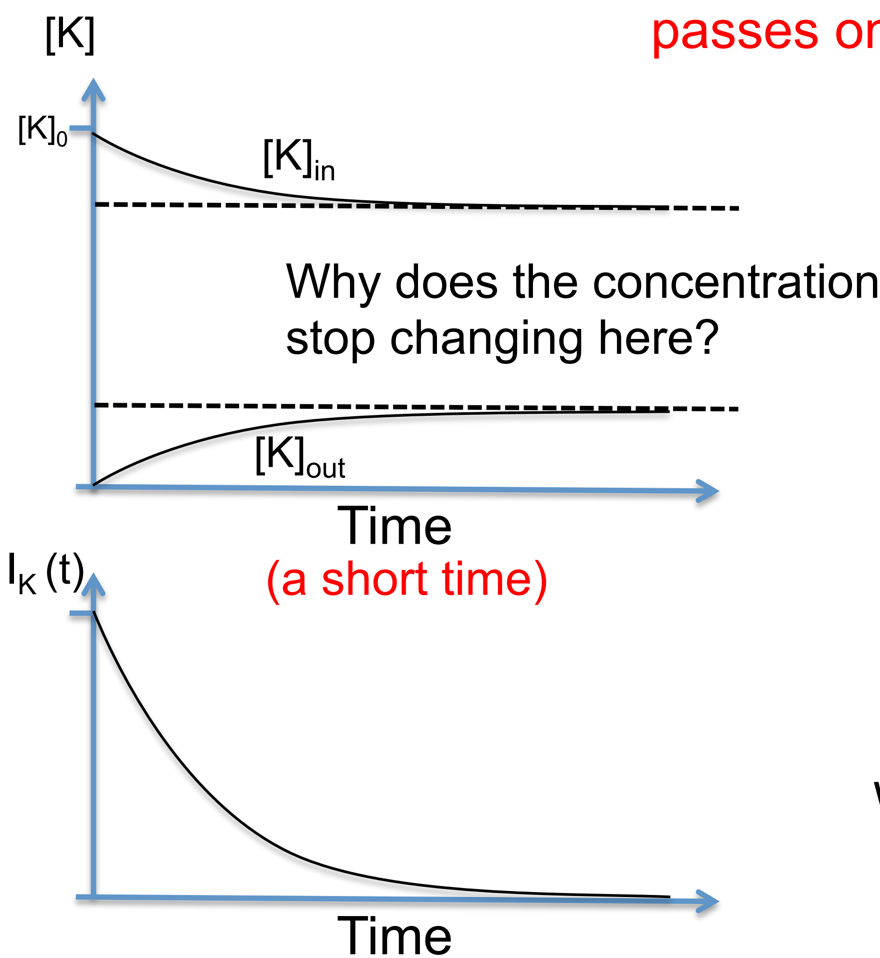
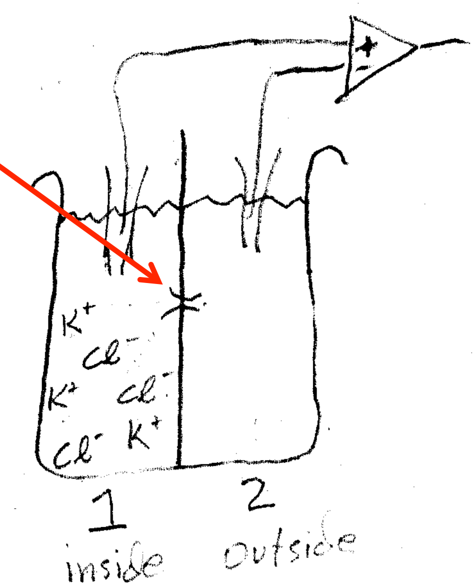
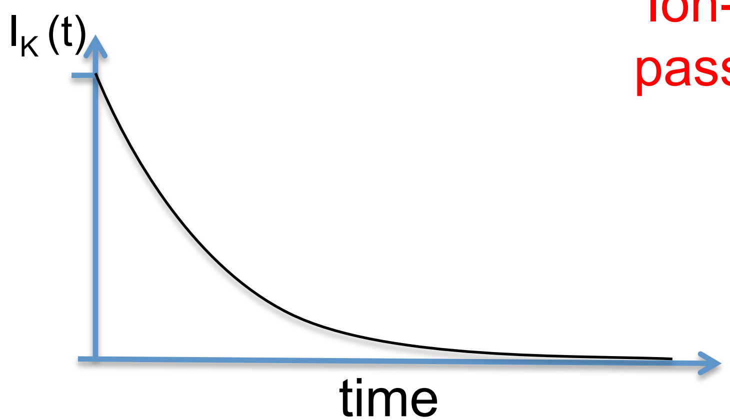


'Ion-selective' pore passes only K^+ ions



Why do the ions stop flowing from side 1 to side 2?

Neurons have batteries



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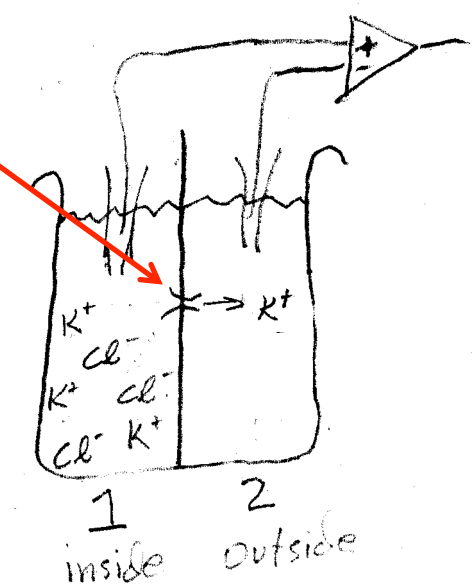
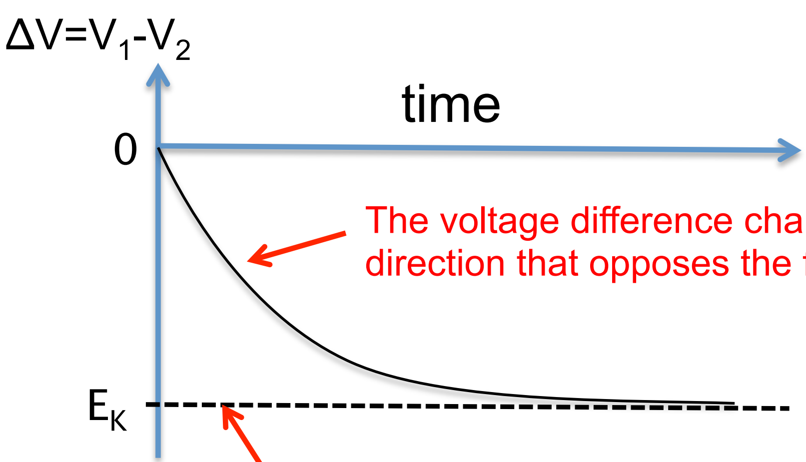


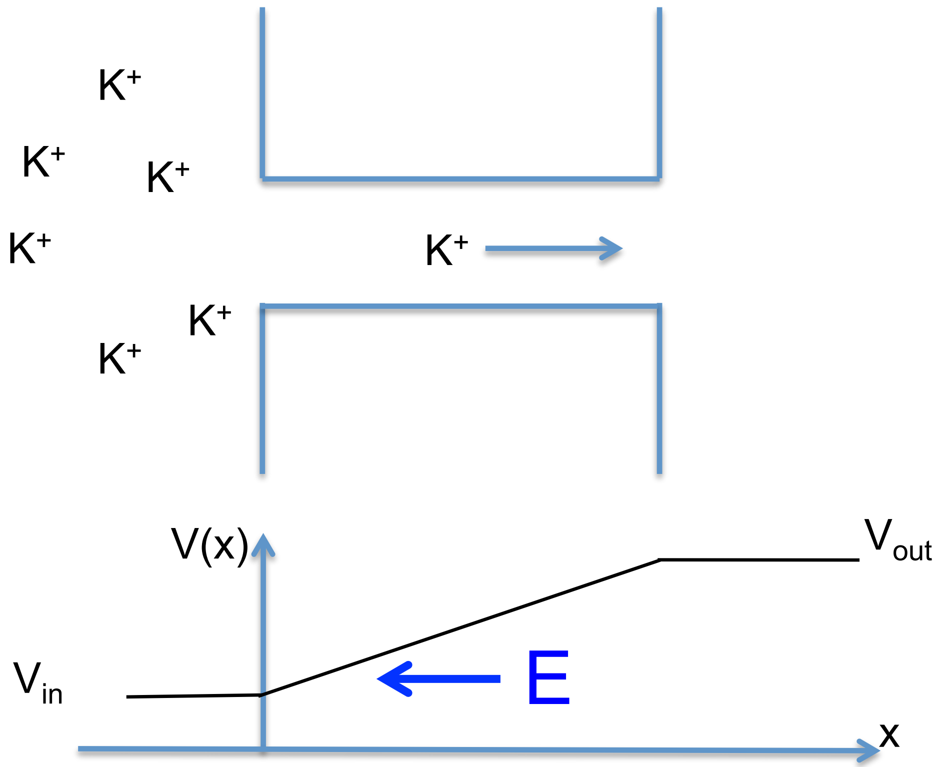


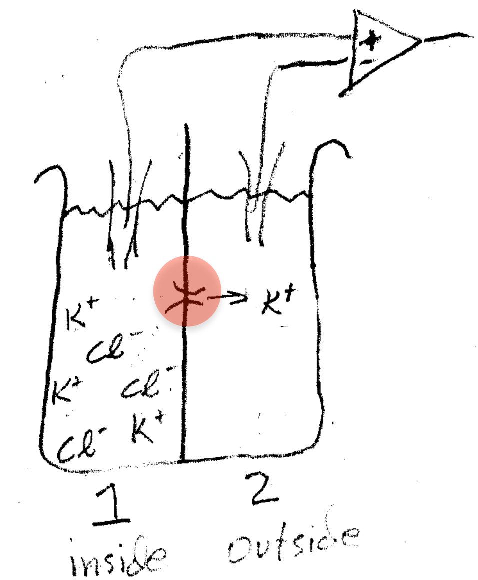
The voltage difference changes in a direction that opposes the flow of ions.

It reaches an ‘equilibrium potential’ at a value that gives zero net flow of ionic current.

This voltage difference is a battery for our model neuron!!

Neurons have batteries





There will be some electric field strength such that the ‘drift’ will exactly balance the diffusion produced by the concentration gradient…

Nernst Potential

Neurons have batteries

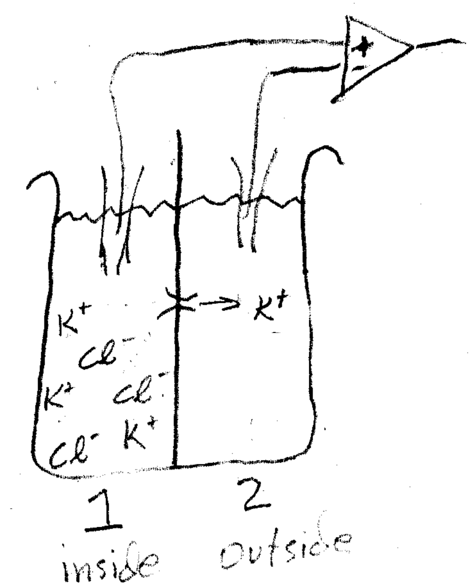
- Where do the ‘batteries’ of a neuron come from?

  1) Ion concentration gradients

  2) Ion-selective pores (channels)

- How big is the battery (how many volts?)

  This is determined by a balance between diffusion down a concentration gradient balanced by ‘drift’ in the opposing electric field.



Electrodiffusion and the Nernst Potential

One can use Ohm's law and Fick's first law to derive the Nernst potential

— At this voltage, the drift current in the electric field exactly balances current due to diffusion

$$I_{Tot} = I_{Drift} + I_{Diffusion} = 0$$

Ohm's Law

$$I_{Drift} = \frac{Aq^2\varphi(x)D}{kT}\frac{\Delta V}{L}$$

Fick's First Law

$$I_{Diffusion} = -AqD\frac{\partial\varphi}{\partial x}$$

$$\Delta V = \frac{kT}{q}\ln\left(\frac{\varphi_{out}}{\varphi_{in}}\right)$$ at equilibrium

Derive Nernst potential using the Boltzmann equation

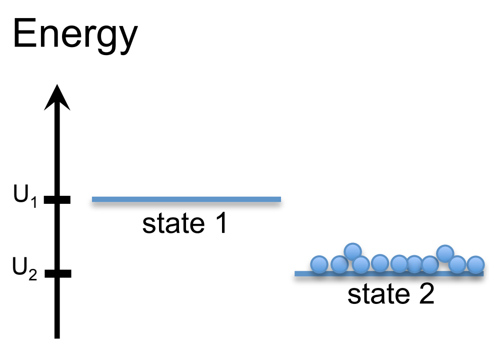
The Boltzmann equation describes the ratio of probabilities of a particle being in any two states, at thermal equilibrium:

$$\frac{P_{state1}}{P_{state2}} = e^{-\left(\frac{U_1 - U_2}{kT}\right)}$$

$k =$ Boltzmann constant (J/K)

$T =$ temperature (K) = 273 + T_C

$kT =$ thermal energy (J)



$kT = 0$

$$\frac{P_{state1}}{P_{state2}} = 0$$

Derive Nernst potential using the Boltzmann equation

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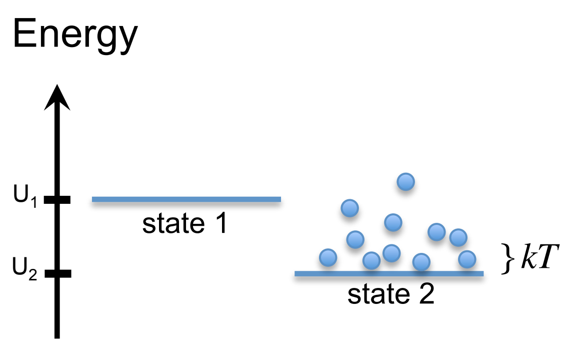

$kT > 0$

$$\frac{P_{state1}}{P_{state2}} > 0$$

Derive Nernst potential using the Boltzmann equation

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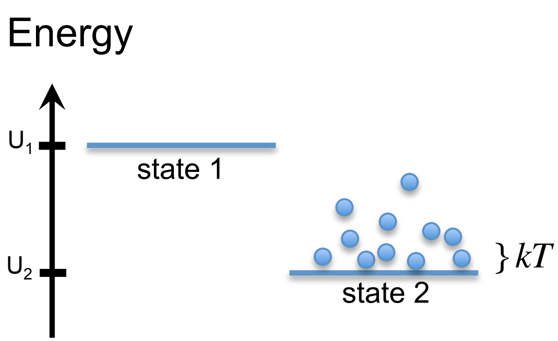
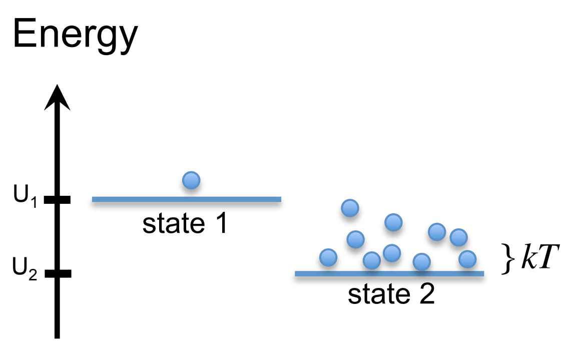


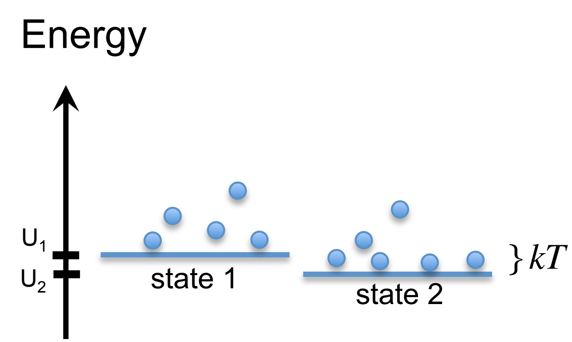
$\Delta U >> kT$

$$\frac{P_{state1}}{P_{state2}} \approx 0$$



$\Delta U = 2kT$

$$\frac{P_{state1}}{P_{state2}} = e^{-2}$$



$\Delta U < kT$

$$\frac{P_{state1}}{P_{state2}} \approx 1.0$$

Nernst Potential

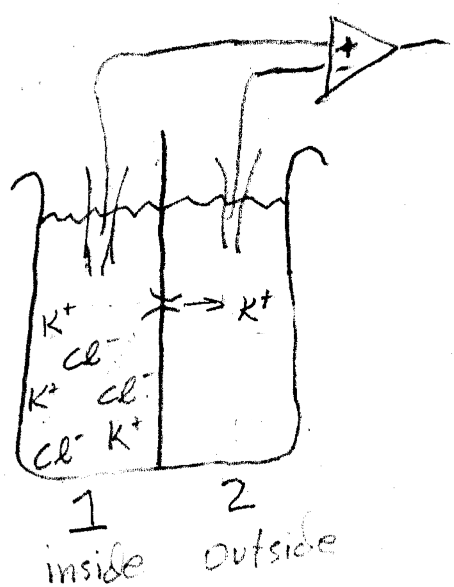
We can compute the equilibrium potential using the Boltzmann equation:

$$\frac{P_{in}}{P_{out}} = e^{-\frac{U_{in}-U_{out}}{kT}} = e^{-\frac{q(V_{in}-V_{out})}{kT}}$$

$U = qV =$ electrical potential (J)

$q =$ charge of ion

$q =$ 1.6x10^{-19}C for monovalent ion



Nernst Potential

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$U = qV =$ electrical potential (J)

$q =$ charge of ion

$q = 1.6\text{x}10^{-19}\text{C}$ for monovalent ion

$$V_{in} - V_{out} = -\frac{kT}{q}\ln\left(\frac{P_{in}}{P_{out}}\right)$$

$\frac{kT}{q} = 25mV$ for monovalent ion

$$\Delta V = V_{in} - V_{out} = 25mV\ \ln\left(\frac{P_{out}}{P_{in}}\right)$$

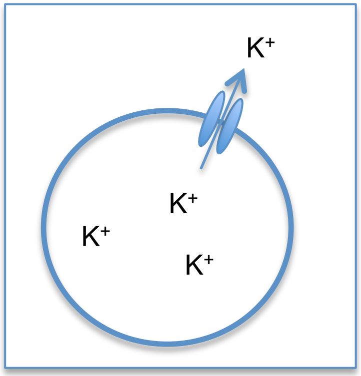
$$\Delta V = 25mV\ln\left(\frac{[K]_{out}}{[K]_{in}}\right) = E_K$$

Don't get confused by this notation. E_K is the equilibrium potential (voltage) for the K ion. 'E' here does not refer to an electric field.

The Nernst potential for potassium

Intracellular and extracellular concentrations of ionic species, and the Nernst potential

Ion	Cytoplasm (mM)	Extracellular (mM)	Nernst (mV)
K^+	400	20	-75



$$E_k = \frac{kT}{q} \ln\left(\frac{20}{400}\right)$$

$\frac{kT}{q}$ = 25mV at 300K (room temp) for monovalent ion

$$E_K = 25mV(-3.00) = -75mV$$

How to implement an ion specific conductance as a battery in our model neuron