Introduction to Neural Computation

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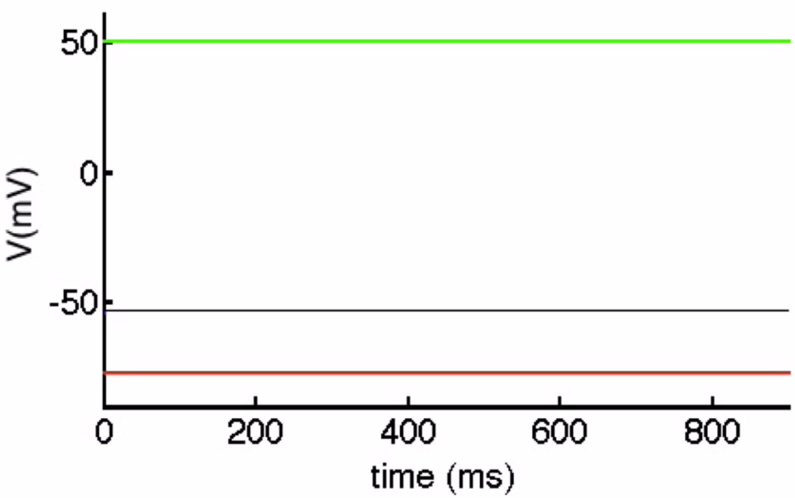
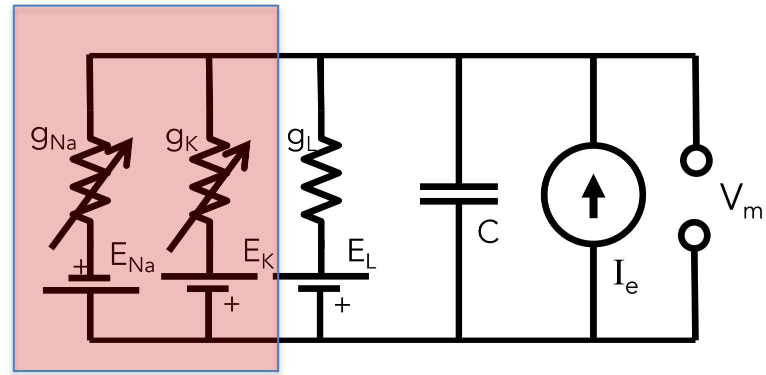
MIT BCS 9.40

Video Module on Integrate and Fire Neuron

Outline of HH model

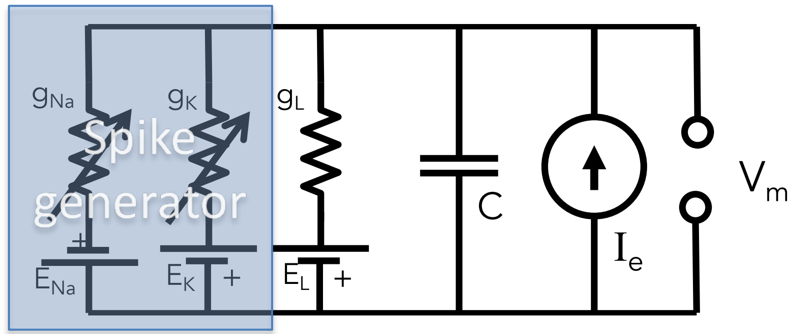
Voltage and time-dependent ion channels are the 'knobs' that control membrane potential.



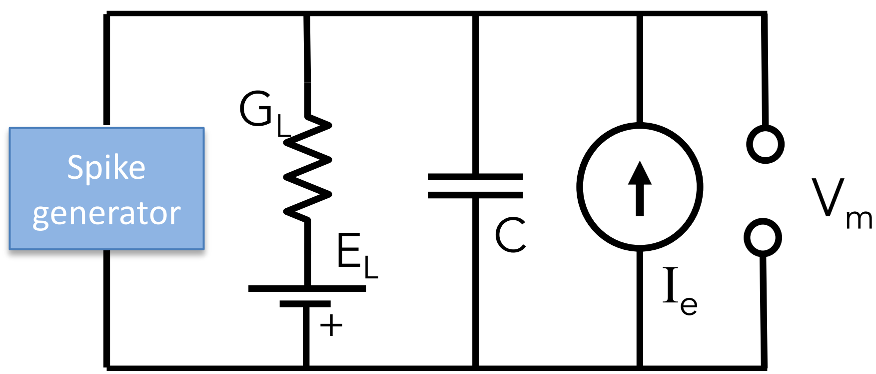


- Na^+ conductance pushes the membrane potential toward +55mV.
- K^+ conductance pushes the membrane potential toward -75mV.
- Together these conductances (and batteries) give the neuron flexible control of voltage!
- - for example to generate an action potential

Integrate and Fire model of a neuron

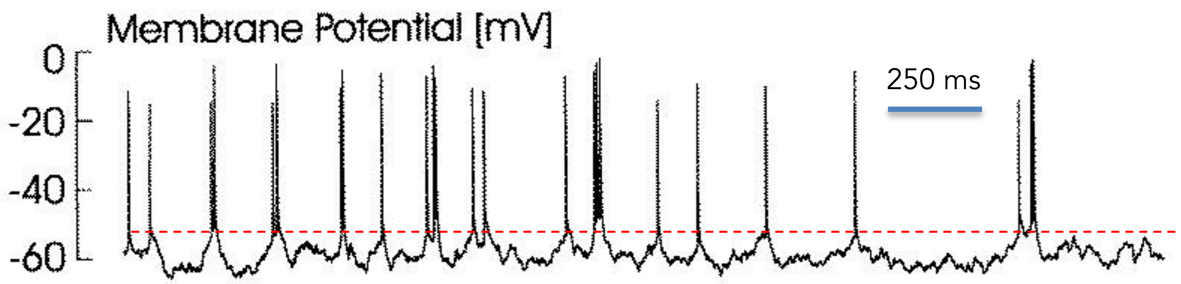


We are going to replace the fancy spike generating mechanism in a real neuron with a simplified 'spike generator'.



Louis Lapique, 1907
Knight, 1972

A simplified model of a neuron

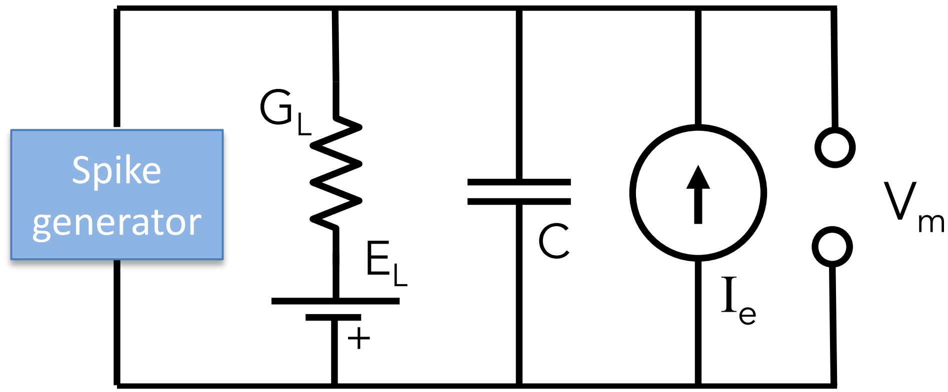


spikes as $\delta -$ *functions*

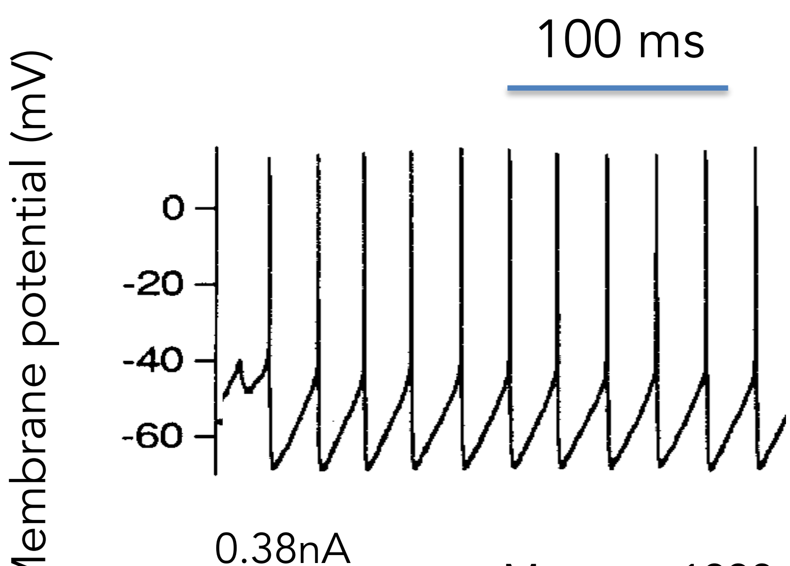
- While APs (spikes) are important, they are not what neurons spend most of their time doing. Spikes are very fast (~1ms in duration).
- This is much shorter than the typical interval between spikes (~100ms). Most of the time, a neuron is 'integrating' its inputs. (Separation of timescales)
- All spikes are the same. (No information carried in the details of action potential waveforms.)
- Spikes tend to occur when the voltage in a neuron reaches a particular membrane potential, called the spike threshold.

Integrate and Fire model of a neuron

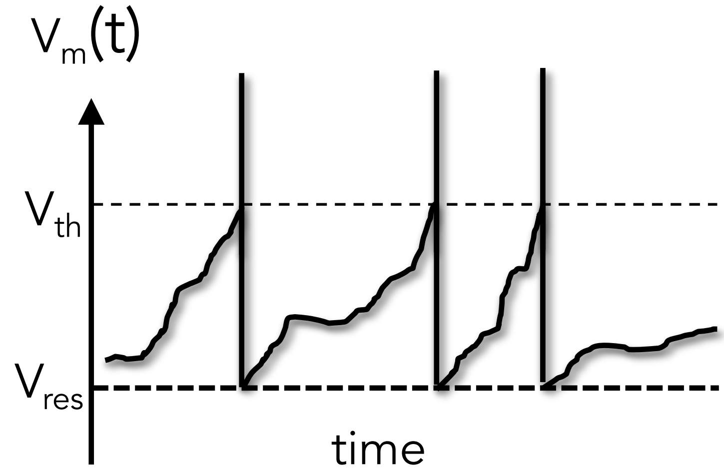
The spike generator is very simple. When the voltage reaches the threshold V_{th}, it resets the neuron to a hyper-polarized voltage V_{res}.



Louis Lapique, 1907

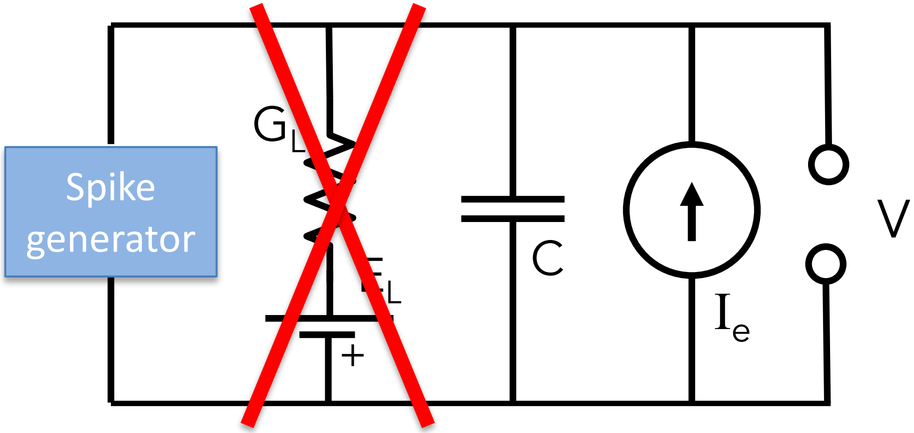


Mooney, 1992



Integrate and Fire model of a neuron

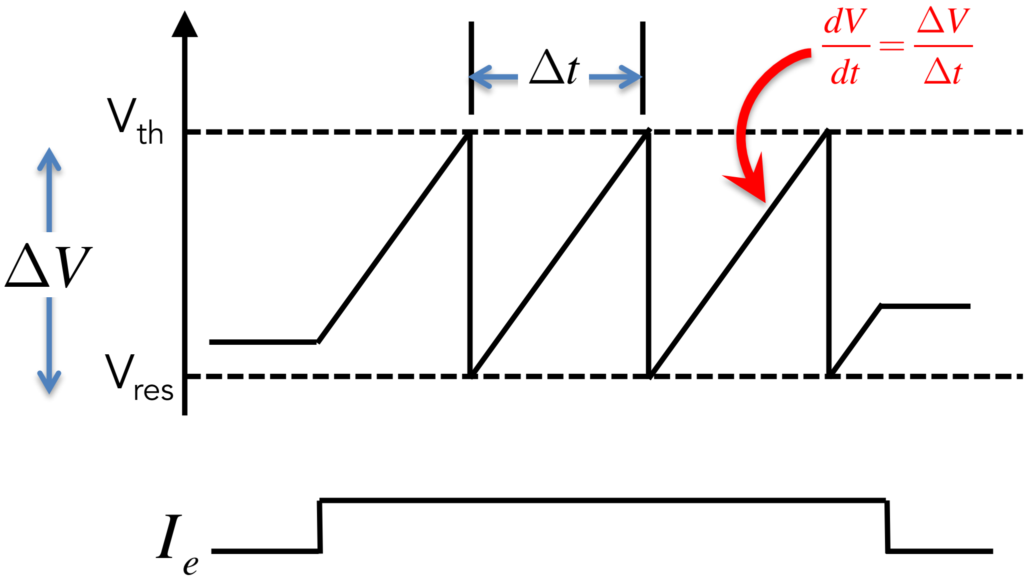
- Let's calculate the firing rate of our neuron



We'll first consider the case where there is no leak.

$$f.r. = \frac{1}{\Delta t}$$

$$\Delta V = V_{th} - V_{res}$$

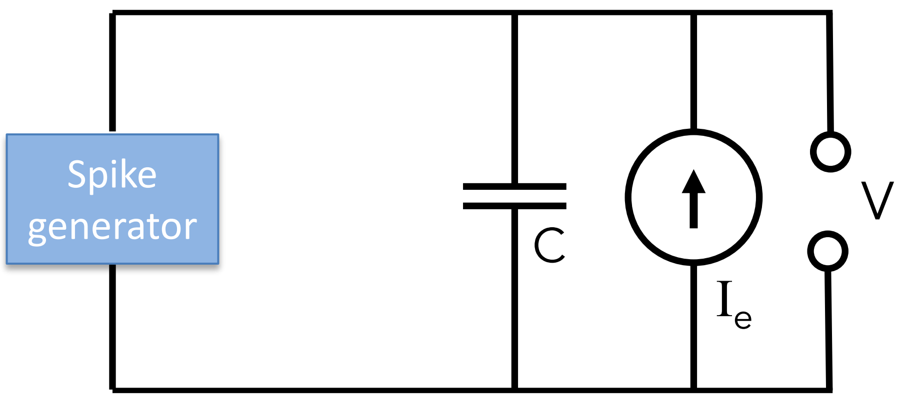


$$C\frac{dV}{dt} = I_e \qquad C\frac{\Delta V}{\Delta t} = I_e$$

$$f = \frac{1}{\Delta t} = \left(\frac{1}{C\Delta V}\right) I_e$$

Integrate and Fire model of a neuron

- Let's calculate the firing rate of our neuron



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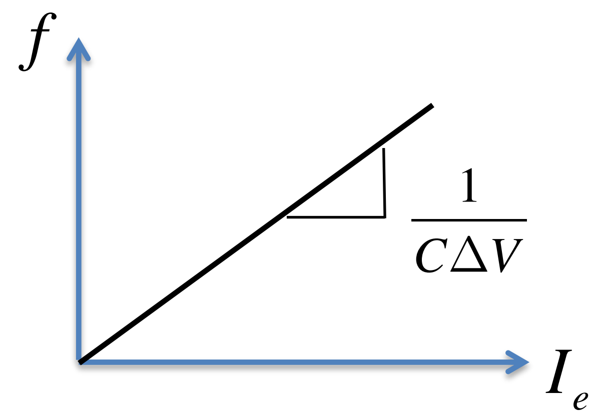
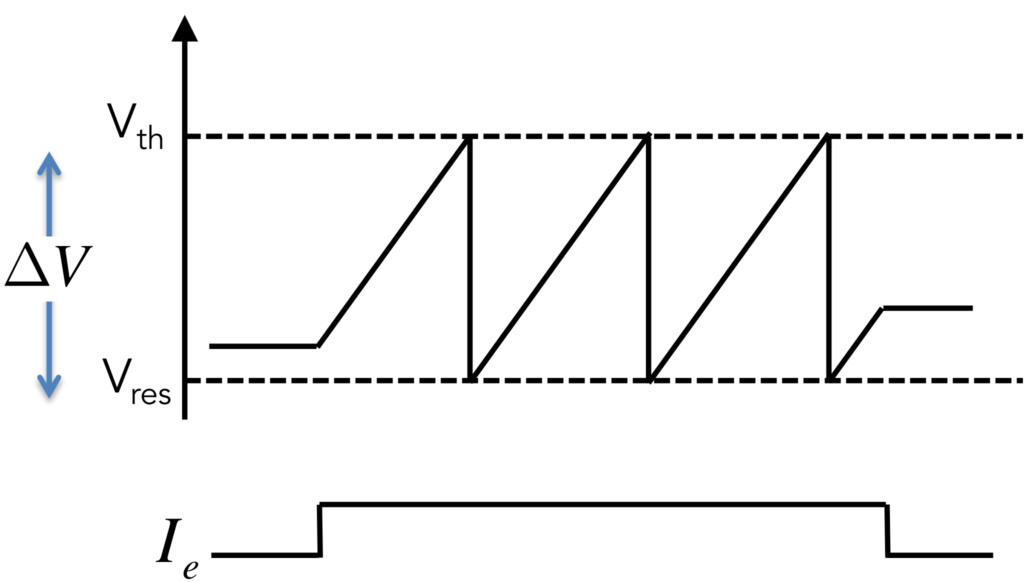
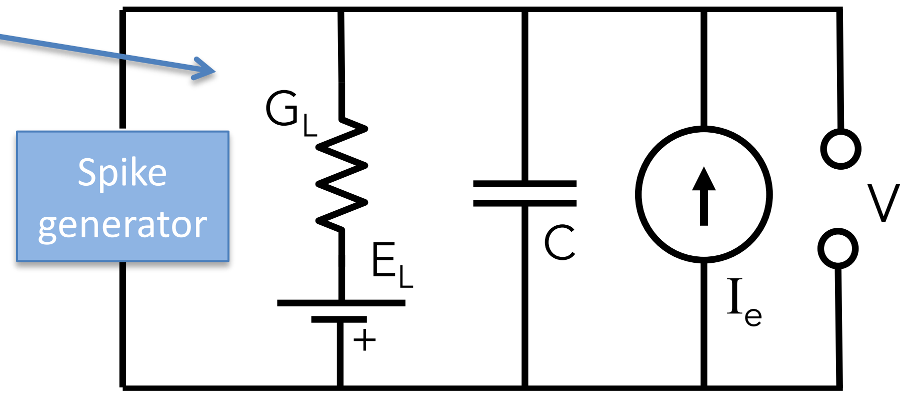




$$f = \left(\frac{1}{C\Delta V}\right) I_e$$

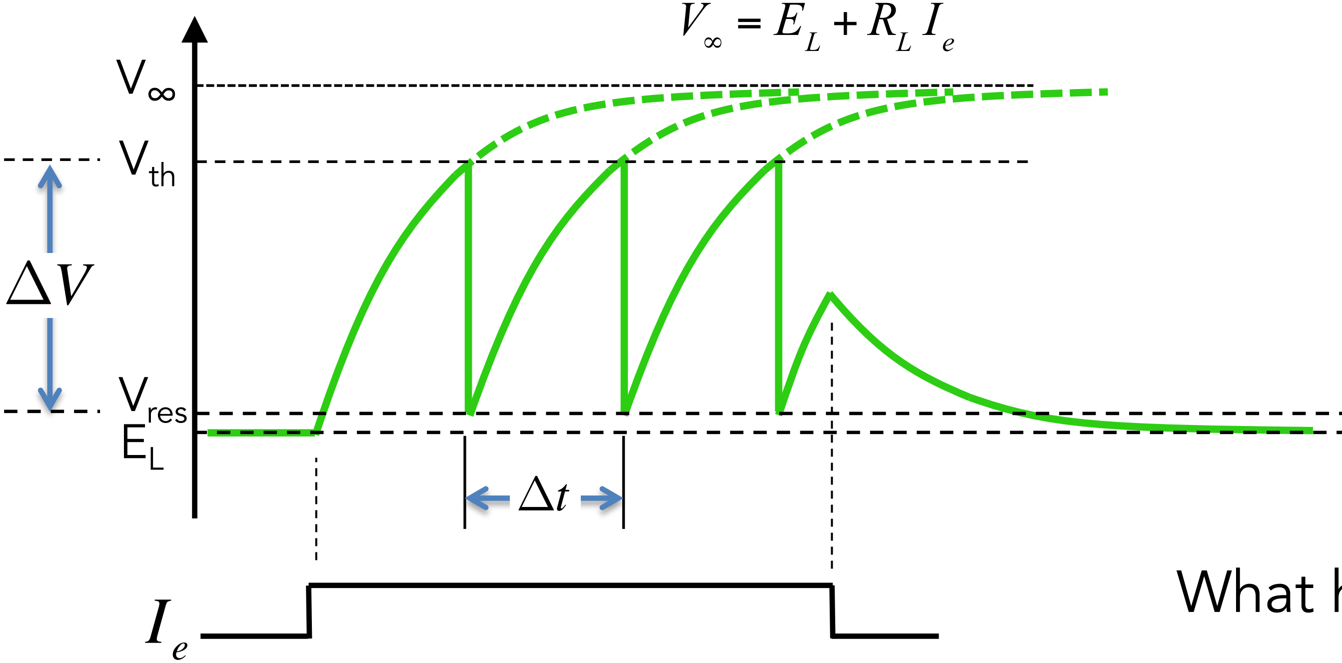
Integrate and Fire model of a neuron

Now we'll put our leak conductance back in.



Think of this G_L like a small potassium conductance that is constantly on. It has no voltage dependence and no time dependence. E_L = -75mV.

$$V_\infty = E_L + R_L I_e$$

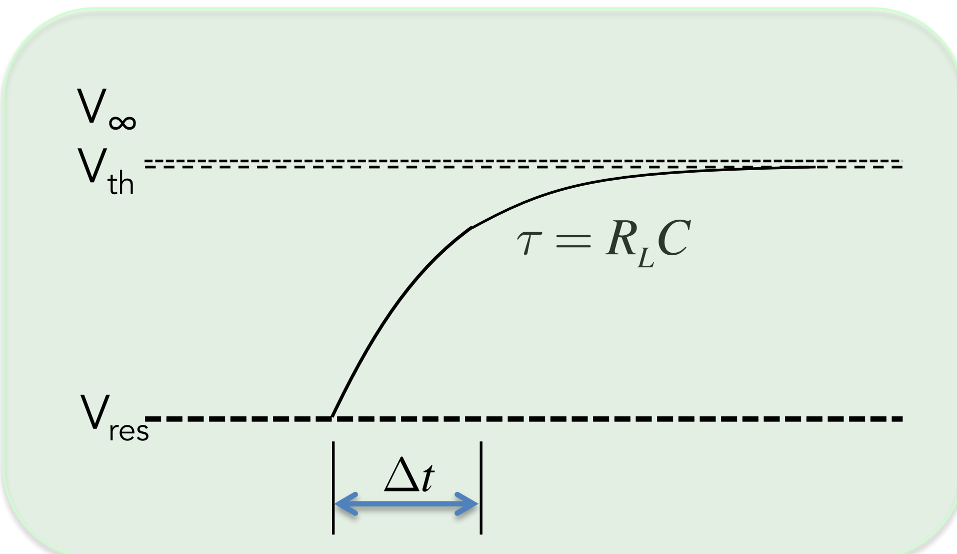


$$f.r. = \frac{1}{\Delta t}$$

What happens when $V_\infty < V_{th}$?

Integrate and fire with leak

What happens just at threshold?



The time to reach threshold (Δt) is:

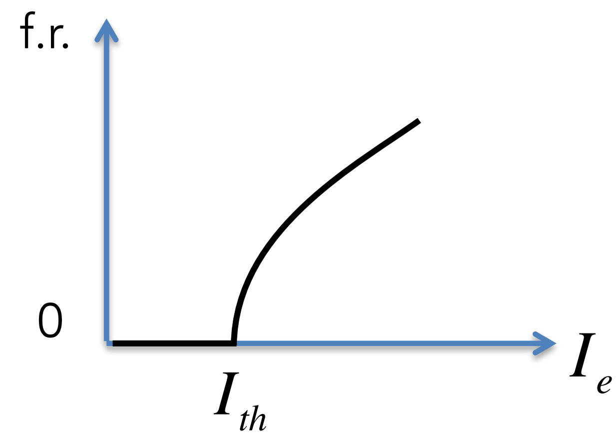
- very long
- very sensitive to injected current

Lets calculate the injected current required to reach threshold (rheobase).

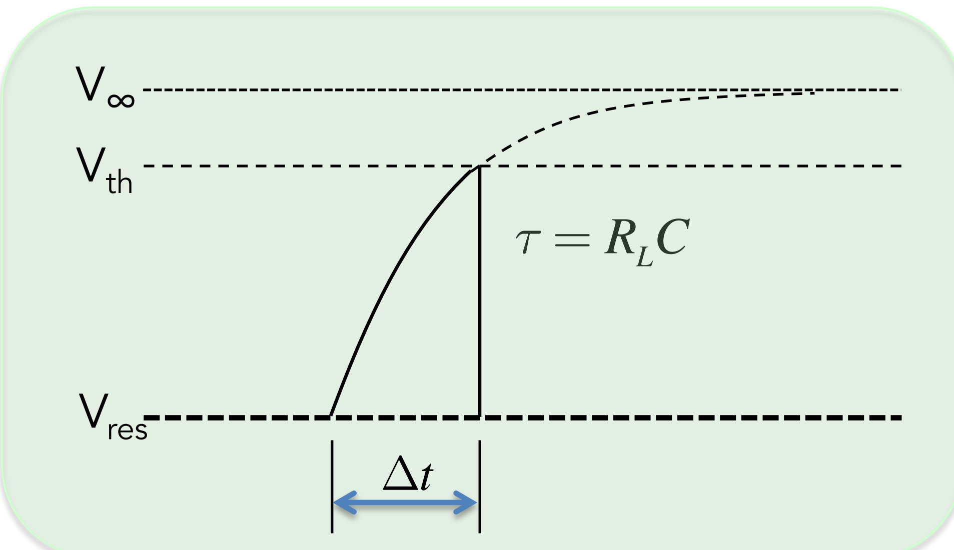
$$V_\infty = V_{th}$$

$$E_L + R_L I_e = V_{th}$$

$$I_{th} = I_e = G_L (V_{th} - E_L)$$



Integrate and fire with leak



$$e^{-\Delta t/\tau} = \frac{V_\infty - V_{th}}{V_\infty - V_{res}}$$

$$\Delta t = -\tau \ln\left(\frac{V_\infty - V_{th}}{V_\infty - V_{res}}\right)$$

$$V(t) - V_\infty = (V_0 - V_\infty)e^{-t/\tau}$$

$$V_{th} - V_\infty = (V_{res} - V_\infty)e^{-\Delta t/\tau}$$

$$f = \Delta t^{-1} = \left[\tau \ln\left(\frac{V_\infty - V_{res}}{V_\infty - V_{th}}\right)\right]^{-1}$$

Integrate and fire

At high input currents, the solution has a simple approximation

$$f = \left[\tau \ln \left(\frac{V_\infty - V_{res}}{V_\infty - V_{th}} \right) \right]^{-1}$$

$$V_\infty \gg V_{th}, V_{res}$$

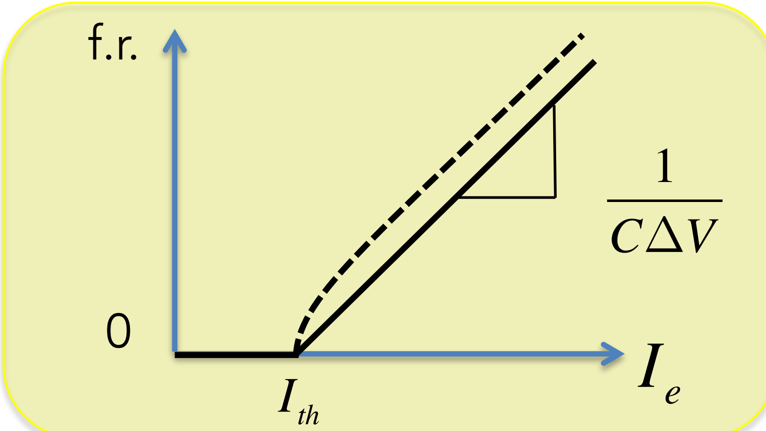
$$\ln(1+\alpha) \sim \alpha$$

$$\boxed{f = \frac{1}{C\Delta V}\left(I_e - I_{th}\right)}$$

$$I_{th} = G_L\left(V_{th} - E_L\right)$$

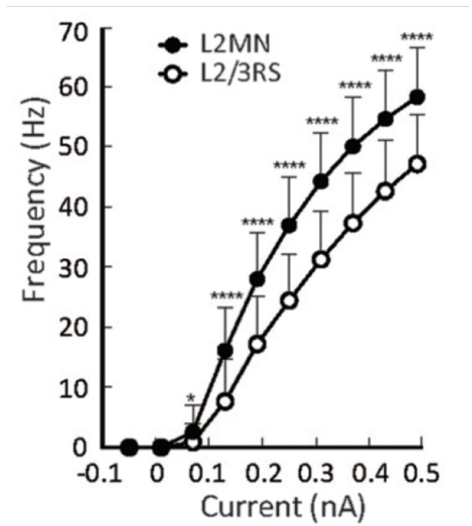
Integrate and fire

This equation is linear in injected current I_e , just like the case of no leak!



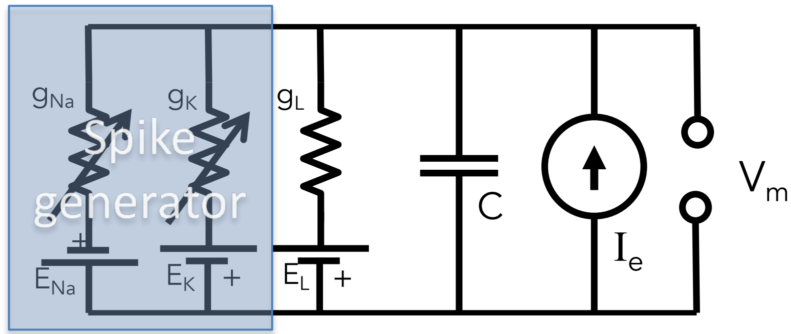
$$f = \frac{1}{C\Delta V}\left(I_e - I_{th}\right)$$

The F-I curve of many neurons look approximately like this!



Luo et al 2017

Integrate and Fire model of a neuron



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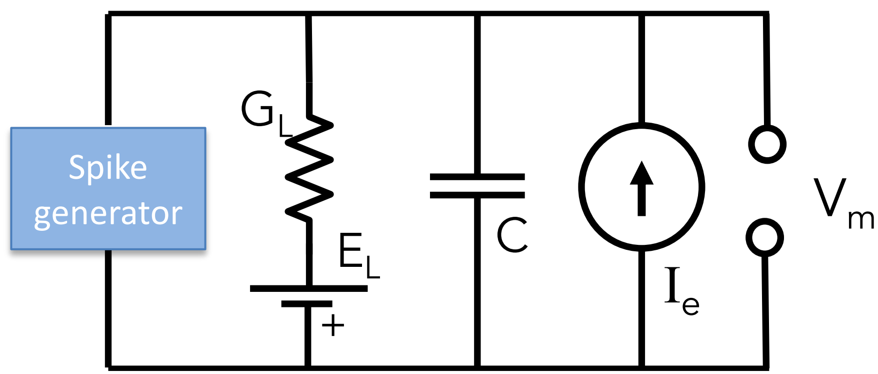

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