JCD 891 – MINOR PROJECT Development of Fully Homomorphic Encryption Scheme

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Problem Statement

Problem - For a third party to work on any encrypted data, it needs to decrypt the data first. This creates a privacy issue for the user, as the third party service providers like cloud storage, file sharing, collaboration, servers can keep physically identifying elements of users long after the user has ended the relationship with the services.

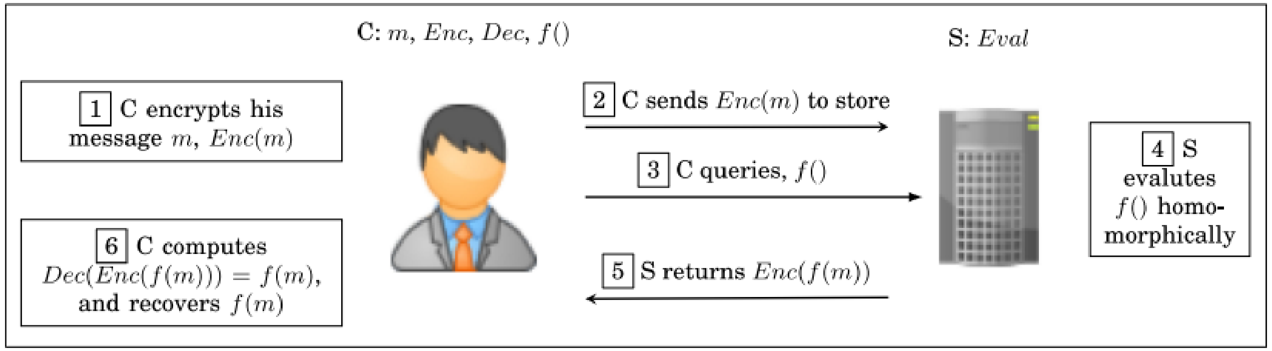
Solution – We need to develop a scheme that can operate on encrypted data. This scheme is referred to as Homomorphic Encryption scheme.

Homomorphic Encryption

- Homomorphic encryption is a form of encryption that allows computations to be performed on encrypted data without first having to decrypt it.
- An encryption scheme is called homomorphic over an operation " * ", if it supports the following equation:

$$E(m1) * E(m2) = E(m1 * m2), \forall m1,m2 \in M$$

- An HE scheme is primarily characterized by four operations: KeyGen, Enc, Dec, and Eval.



Why Homomorphic Encryption?

- Confidentiality Problem
- Ability to compute over ciphertext instead of plaintext
- One could use information without knowing the content of that information
- Privacy guaranteed

Homomorphic Encryption is of three types:

1. Partially Homomorphic Encryption(PHE)
2. Somewhat Homomorphic Encryption(SWHE)
3. Fully Homomorphic Encryption(FHE)

Partially Homomorphic Encryption Schemes

1. RSA

Security Assumption - Hardness of Factoring Problem

Homomorphic Property:

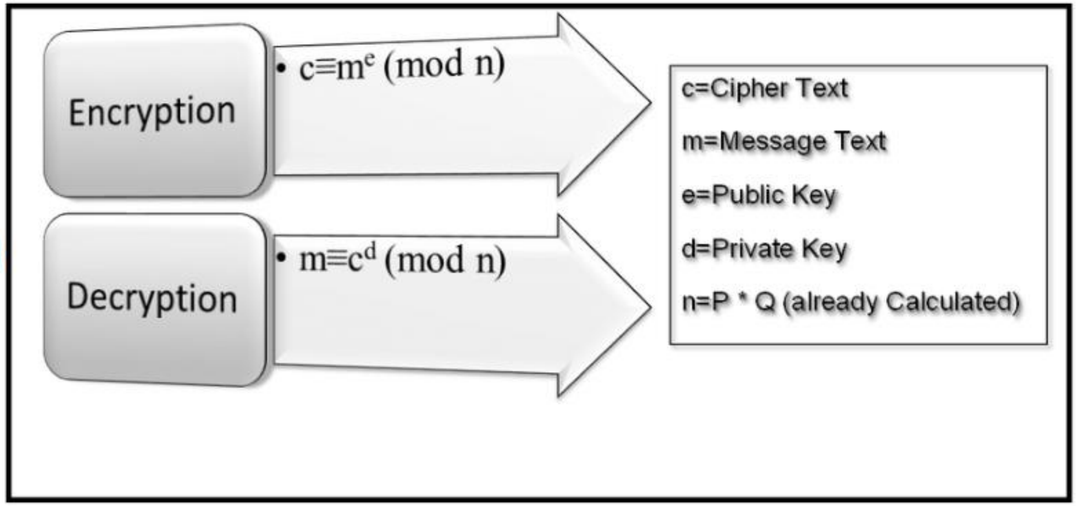
$m1, m2 \in M$

$E(m1) * E(m2) = (m1^e \pmod n) * (m2^e \pmod n)$

$= (m1 * m2)^e \pmod n$

$= E(m1 * m2)$

Homomorphic over multiplication operation.



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2. GM Cryptosystem

Security Assumption – Hardness of Quadratic Residuosity Problem

KeyGen : n = pq, x - a quadratic non-residue modulo n

Public key = (x,n) ; Private key = (p,q)

Encryption : message is encrypted bit by bit, y_i is a quadratic non-residue value

$$c_i = E(m_i) = y_i^2 x^{m_i} \pmod{n}, \quad \forall m_i = \{0, 1\},$$

Decryption: To decrypt the ciphertext c_i , one decides whether c_i is a quadratic residue modulo n or not; if so, m_i returns 0, or else m_i returns 1.

Homomorphic Property:

$$E(m_1) * E(m_2) = (y_1^2 x^{m_1} \pmod{n}) * (y_2^2 x^{m_2} \pmod{n})$$
$$= (y_1 * y_2)^2 x^{m_1+m_2} \pmod{n} = E(m_1 + m_2).$$

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3. Paillier Cryptosystem

Security Assumption :Composite Residuosity Problem

KeyGen : public key =(n,g) private key = (p,q)

Encryption: For each m choose a random r,

$$c = E(m) = g^m r^n \pmod{n^2}.$$

Decryption :

$$D(c) = \frac{L(c^\lambda \pmod{n^2})}{L(g^\lambda \pmod{n^2})} \mod n = m,$$

Homomorphic Property :

$$\begin{aligned} E(m_1) * E(m_2) &= (g^{m_1} {r_1}^n \pmod{n^2}) \quad * (g^{m_2} {r_2}^n \pmod{n^2}) \\ &= g^{m_1+m_2} (r_1 * r_2)^n \pmod{n^2} = E(m_1 + m_2). \end{aligned}$$

Homomorphic over addition operation.

Results of Paillier

```python
# Homomorphic Properties
m1 = int(input("First message : "))
m2 = int(input("Second message : "))
c1,m1= encrypt(m1,public_key)
c2,m2 = encrypt(m2,public_key)

# (i)Homomorphic over addition

print("Paillier cipher is homomorphic over addition ")
ct = gmpy2.mul(c1,c2)
msg = decrypt(ct,private_key,public_key)

if msg == (m1+m2)%n:
    print("True")
    print(msg)
else:
    print("False")
```

```
tarun@tarunlm10:~/Documents$ python3 paillier_cipher.py
3467675791
7013790659134944590
First message : 54
Second message : 6
Paillier cipher is homomorphic over addition
True
60
Raising an encrypted message to the power of a second message results in the multiplication of plaintext messages
True
324
```

Somewhat Homomorphic Encryption Schemes

BGN Cryptosystem

Security Assumption : Subgroup Decision Problem

KeyGen : public key = (n,G,G1,e,g,h) , here n =q1*q2 and e : G × G → G1

Private key = (q1)

Encryption: choose a random r

$$c = E(m) = g^m h^r \mod n.$$

Decryption: first compute $c' = c^{q_1} = (g^m h^r)^{q_1}$ and let $g' = g^{q_1}$

$$m = D(c) = \log_{g'} c'.$$

<u>Homomorphic Property:</u>

Addition - Homomorphic addition of plaintexts m1 and m2 using ciphertexts E(m1) = c1 and E(m2) = c2

$$c = c_1 c_2 h^r = (g^{m_1} h^{r_1})(g^{m_2} h^{r_2}) h^r = g^{m_1+m_2} h^{r'}$$

where r = r1 + r2 + r and it can be seen that m1 + m2 can be easily recovered from the resulting ciphertext c.

Multiplication - The homomorphic multiplication of messages m1 and m2 using the ciphertexts c1 = E(m1) and c2 = E(m2) are computed as follows:

$$c = e(c_1, c_2) {h_1}^r = e(g^{m_1} h^{r_1}, g^{m_2} h^{r_2}) {h_1}^r$$
$$= {g_1}^{m_1 m_2} {h_1}^{m_1 r_2 + r_2 m_1 + \alpha q_2 r_1 r_2 + r} = {g_1}^{m_1 m_2} {h_1}^{r'}.$$

It is seen that r is uniformly distributed like r and so m1m2 can be correctly recovered from resulting ciphertext c.

Limitations

Computational complexity

Performance is often a disadvantage.

Do not provide verifiable computing.

Number of homomorphic functions and the number of times they can be applied is limited.

Thank You!