Theoretical minimum for the exam on Analysis 1 for Engineers

To be able to formulate and apply the following definitions, statements, laws, rules, methods, and to give examples and counterexamples.

1. Laws of propositional logic. Negation of statements. Method of mathematical induction.
2. Laws of algebra of sets. Bounded and unbounded sets. Infimum and supremum of a set and their properties.
3. Properties of real and complex numbers, operations with real and complex numbers. Archimedean property. Polar and exponential form of a complex number.
4. Function and related notions (domain, codomain, image, range, etc.). Injective, surjective, bijective functions.
5. Properties of the functions (monotonicity, symmetry, boundedness). Composite functions, inverse functions.
6. Basic elementary functions and their graphs and properties.
7. Sequences of real numbers and their properties (monotonicity, boundedness and unboundedness, convergence and divergence). Infimum and supremum of a sequence.
8. Limit of a sequence and its arithmetic properties. Theorem on uniqueness of a limit of a sequence. Common limits.
9. Limit passage in inequalities. Sandwich Theorem for sequences and its corollaries.
10. Theorem on boundedness of convergent sequences.
11. Weierstrass theorem for monotonic sequences and its corollaries.
12. Subsequences and their properties. Subsequential limit. Limit inferior and limit superior.
13. Bolzano–Weierstrass theorem and its corollaries.
14. Cauchy sequence. Cauchy convergence criterium for sequences.
15. Infinite series and their properties. Sum of a series. Convergent and divergent series. Common series.
16. Necessary condition for convergence of a series.
17. Alternating series. Leibniz convergence test.
18. Absolutely and conditionally convergent series and their properties.
19. Convergence tests (Comparison convergence test, D'Alembert's ratio test, Cauchy's root test, Dirichlet's test, Abel's test, Integral test).
20. Limit of a function and its arithmetic properties. Special limits. L'Hôpital's Rule.
21. Limit passage in inequalities. Sandwich Theorem for functions.
22. One-sided limits. Theorem on existence of a limit of a function.
23. Continuity, one-sided continuity, discontinuity of a function at the point.
24. Continuous functions and their properties.

25. Theorems about continuity of inverse functions, composite of continuous functions and about limits of continuous functions.
26. Types of discontinuity. Asymptotes.
27. Bolzano–Cauchy Intermediate value theorem and its corollaries.
28. Derivative at a point, derivative as a function. One-sided derivatives.
29. Geometrical and physical meaning of a derivative.
30. Differentiable functions and their properties.
31. Necessary condition of differentiability.
32. Calculation of derivatives: differentiation rules; chain rule; derivative of inverse function; implicit differentiation; derivative formulas of elementary functions.
33. Higher order derivatives and their properties. Differentiability classes.
34. Mean values theorems (Rolle's Lemma, Lagrange's Mean value theorem and corollaries, Cauchy's Mean value theorem).
35. Extremum points of a function. Local and global extrema. Weierstrass Extreme value theorem. Fermat's local extremum theorem. First- and second derivative tests for local extrema.
36. Theorem on monotonicity of a function.
37. Concave and convex functions. Second derivative test for concavity.
38. Points of inflection. Second derivative criteria for points of inflection.
39. Graph sketching.
40. Power series. Radius and interval of convergence.
41. Convergence theorem for power series and its corollary.
42. Operations on power series. Differentiation of power series.
43. Taylor and Maclaurin series. Taylor / Maclaurin series of common functions. Taylor's formula.
44. Antiderivative. Indefinite integral. Calculation of antiderivatives and integrals: basic integration rules and formulas; substitution rule; integration by parts.
45. Riemann integrability of a function. Definite integral and its properties. Substitution and integration by parts in definite integrals. Definite integrals of symmetric functions.
46. Calculation of the area between curves.
47. Theorem about boundedness of integrable functions. Sufficient integrability conditions.
48. Mean value theorem for definite integrals.
49. Fundamental theorem of Calculus, Parts 1 and 2.
50. Improper integrals. Comparison tests for improper integrals.

Possible bonus questions:

To be able to prove the following statements.

1. Sandwich Theorem for sequences.
2. Necessary condition for convergence of a series.
3. D'Alembert's ratio test.
4. Cauchy's root test.
5. Lagrange's Mean value theorem.
6. L'Hôpital's Rule.
7. Fermat's local extremum theorem.
8. Substitution rule for indefinite integrals.
9. Theorem on integration by parts for indefinite integrals.
10. Fundamental theorem of Calculus.

Literature:

- Thomas' Calculus: Early Transcendentals. – Thirteenth edition / as revised by M. D. Weir, J. Hass.
- M. Spivak. Calculus.
- Lecture slides, https://moodle.aau.at/course/view.php?id=35302.