(7) (a) Lets consider two infinitesimal rotation $\vec{\delta\theta_1}$ & $\vec{\delta\theta_2}$ about axes $\vec{k_1}$ & $\vec{k_2}$ respectively. Lets study the net equivalent rotation matrices for both the different orders of rotation.

$$R_{\vec{k_1}}(\vec{\delta\theta_1}) = I_3 + K_1 \sin\vec{\delta\theta_1} + K_1^2(1-\cos\vec{\delta\theta_1})$$

$$R_{\vec{k_2}}(\vec{\delta\theta_2}) = I_3 + K_2 \sin\vec{\delta\theta_2} + K_2^2(1-\cos\vec{\delta\theta_2}).$$

where K_1 & K_2 are $\vec{k_1}^\vee$ and $\vec{k_2}^\vee$ respectively.

$$\Rightarrow R_{\vec{k_1}}(\vec{\delta\theta_1}) = I_3 + K_1\vec{\delta\theta_1} \quad \text{as} \quad \sin\delta\theta \approx \delta\theta$$

$$\& \; R_{\vec{k_2}}(\vec{\delta\theta_2}) = I_3 + K_2\vec{\delta\theta_2} \qquad \& \; \cos\delta\theta \approx 1.$$

$$\Rightarrow R_{\vec{k_1}}(\vec{\delta\theta_1}).R_{\vec{k_2}}(\vec{\delta\theta_2}) = I_3 I_3 + K_1\vec{\delta\theta_1} + K_2\vec{\delta\theta_2} + K_1K_2\vec{\delta\theta_1}\vec{\delta\theta_2}$$

$$= I_3 + K_1\vec{\delta\theta_1} + K_2\vec{\delta\theta_2}$$

as $\vec{\delta\theta_1}\vec{\delta\theta_2} \approx 0$.

$$R_{\vec{k_2}}(\vec{\delta\theta_2}).R_{k_1}(\vec{\delta\theta_1}) = I_3 I_3 + K_2\vec{\delta\theta_2} + K_1\vec{\delta\theta_1} + K_1K_2\vec{\delta\theta_1}\vec{\delta\theta_2}$$

$$= I_3 + K_2\vec{\delta\theta_2} + K_1\vec{\delta\theta_1}$$

$$= I_3 + K_1\vec{\delta\theta_1} + K_2\vec{\delta\theta_2}$$

$$\Rightarrow R_{\vec{k_1}}(\vec{\delta\theta_1}).(R_{\vec{k_2}}(\vec{\delta\theta_2})) = R_{\vec{k_2}}(\vec{\delta\theta_2}) . R_{\vec{k_1}}(\vec{\delta\theta_1})$$

Hence two generic infinitesimal rotations are ~~commut~~ commutative (b.). For two generic rotation matrices to commute two orthogonal matrices of determinant 1 should commute. This happens only when the eigenvectors for both the rotation matrices are same.

(c.) As the eigenvectors need to be the same for two generic rotation matrices to commute, we have already shown that one of the eigenvectors is the real axis of rotation and hence the axis of rotation needs to be same for two generic rotation matrices to commute.