13-05-2020 AMS650: Assignments S. Tarun Prasad

ME17B114

(1.) $u_{n+1} = \alpha u_n (1 - u_n^3) = f(u_n)$

For finding the fixed points,

$$u_n = \alpha u_n (1 - u_n^3)$$

$$\Rightarrow \quad u_n = 0 \quad \& \quad u_n = \left(1 - \frac{1}{\alpha}\right)^{1/3} \quad — (2)$$

$\hookrightarrow (1)$.

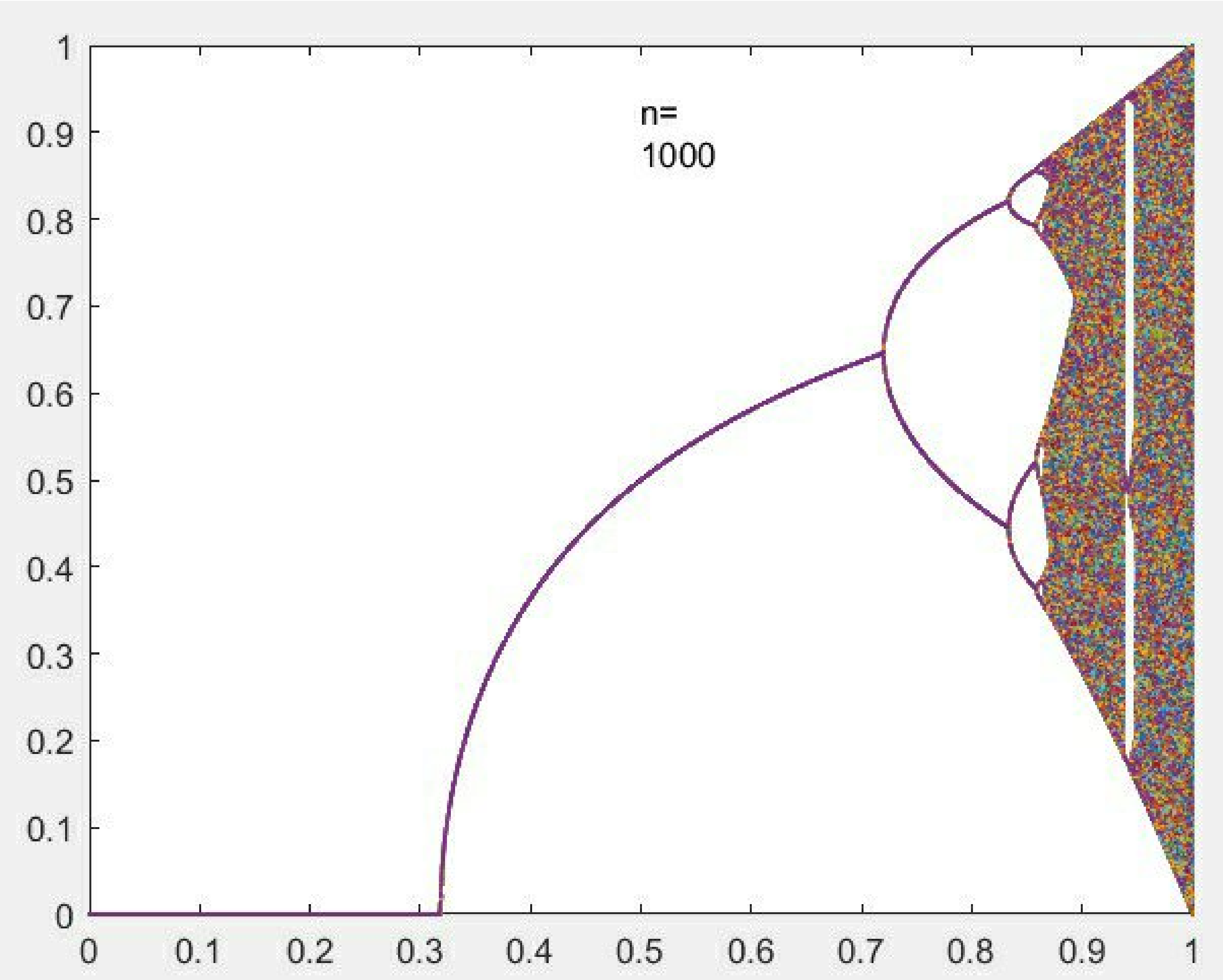
$$f'(u_n) = \cancel{B} \frac{\partial}{\partial x} (\alpha u_n - \alpha u_n^4) = \alpha - 4\alpha u_n^3$$

For (2) $\quad f'(u_n) = \alpha - 4\alpha\left(1 - \frac{1}{\alpha}\right) = \alpha - 4\alpha + 4$

$$\Rightarrow f'(u_n) > 0 \text{ for } 4 - 3\alpha > 0$$

$$\Rightarrow \boxed{\alpha < \frac{4}{3}}$$

For $\alpha < \frac{4}{3}$ non-zero fixed point becomes unstable


n=
1000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

(2.) From the graph, bifurcations for the respective periods occur at,

1st period : 0.32

2nd period : 0.73

3rd period : 0.84

4th period : 0.87

Feigenbaum constant ratio : Between 1-2 : & 2-3 :

$$\frac{0.73-0.32}{0.84-0.73} = 3.73$$

Between 2-3 & 3-4 : $\frac{0.84-0.73}{0.87-0.84} = 3.67$.

This is roughly approximate for eye-balled values.