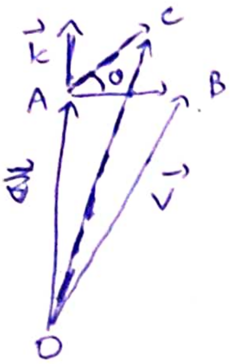
(3.) For a vector $\vec{v}$ in $\mathbb{R}^3$ transformed by a rotation θ about an axis with unit vector $\vec{k}$, the new vector $\vec{v}_{rot}$ obtained is given by the following formula known as the Rodrigues formula,

$$\vec{v}_{rot} = \vec{v}\cos\theta + (\vec{k}\times\vec{v})\sin\theta + \vec{k}(\vec{k}\cdot\vec{v})\cdot(1-\cos\theta)$$

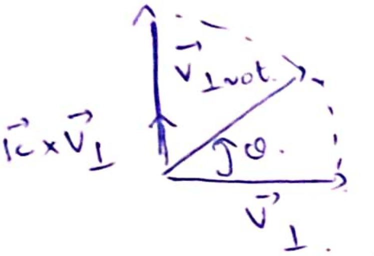


$$\boxed{\begin{aligned}\vec{OB} &= \vec{v}\\ \vec{OC} &= \vec{v}_{rot}\\ \vec{AB} &= \vec{v}_\perp\\ \vec{OA} &= \vec{v}_{//}\\ \vec{AC} &= \vec{v}_{\perp rot}\end{aligned}}$$

$\vec{v}$ has two components one parallel to $\vec{k}$: $\vec{v}_{//} = (\vec{v}\cdot\vec{k})\,\vec{k}$ and another perpendicular to $\vec{k}$: $\vec{v}_\perp = \vec{v} - \vec{v}_{//}$

$$= \vec{v} - (\vec{v}\cdot\vec{k})\,\vec{k}.$$

Observing the plane of rotation.



$$\Rightarrow \vec{v}_{\perp rot} = \vec{v}_\perp\cos\theta + (\vec{k}\times\vec{v}_\perp)\sin\theta.$$

The new vector $\vec{v}_{rot}$ will have two components $\vec{v}_{\perp rot}$ and $\vec{v}_{//}$, which remains invariant from before rotation.

Hence $\vec{v}_{rot} = \vec{v}_{\perp rot} + \vec{v}_{//}$

$$\Rightarrow \vec{v}_{rot} = (\vec{k}\cdot\vec{v})\vec{k} + \left(\vec{v} - (\vec{v}\cdot\vec{k})\,\vec{k}\right)\cos\theta + \left(\vec{k}\times\left(\vec{v} - (\vec{v}\cdot\vec{k})\vec{k}\right)\right)\sin\theta.$$

$$\Rightarrow \vec{v}_{rot} = \vec{v}\cos\theta + \vec{k}(\vec{k}\cdot\vec{v})(1-\cos\theta) + (\vec{k}\times\vec{v})\sin\theta.$$

<u>H.P</u>