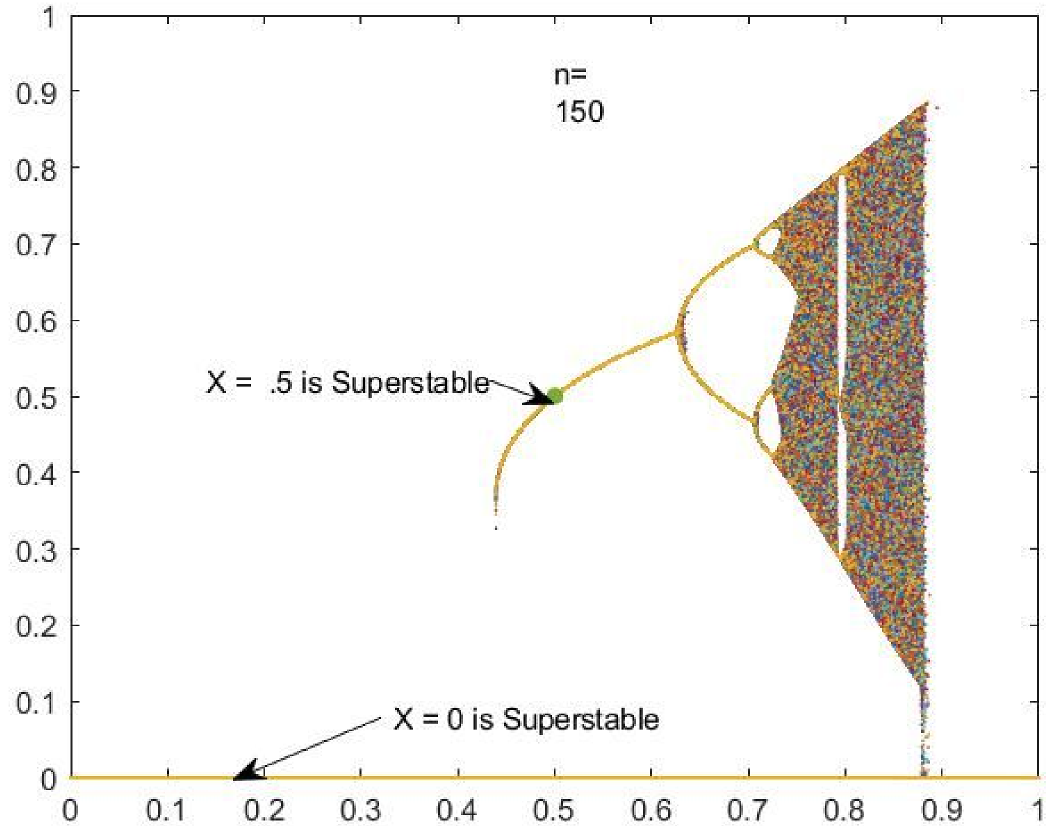
28-05-2020 AM5650 - END-SEM EXAM S. TARUN PRASAD

ME17B114

"The work being submitted is my own work. I have not sought the help of any person in doing this work."

**The Bifurcation diagram for the system is as follows:**




The superstable points are indicated in the plot.

$r \sin^2 \pi x = f(x)$.

$f'(x) = r \sin(2\pi x)$

Superstable when $f'(x) = 0$.

$\Rightarrow$ $x \in [0, 1)$, it is at $x = \boxed{0 \;\&\; 0.5}$

```
function funval = itermap(x, r)
funval = r * (sin(pi*x)).^2;
end
```

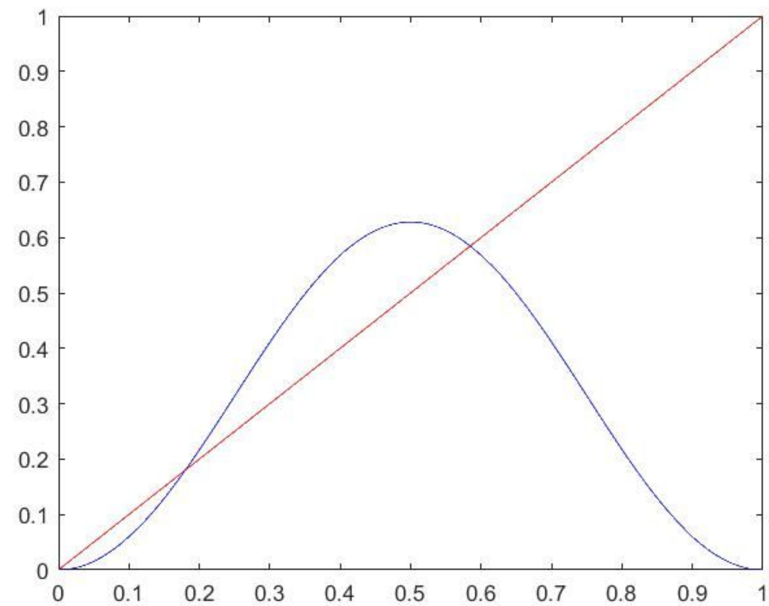
The above function returns the value of the iterated map function.

```
rset = 0.5:.001:.7;
for i=1:size(rset,2)
    r = rset(i);
    syms x;
    val = vpasolve(itermap(x,r) == x, x, 0.5);
    der = double(abs(subs(diff(itermap(x,r),x), val)));
    if(der>=1)
        r
        der
        break
    end
end
```

The above code prints the value of r at which the period one solution goes unstable numerically and the point where period two fixed point appears.

The value returned is r = 0.6280.

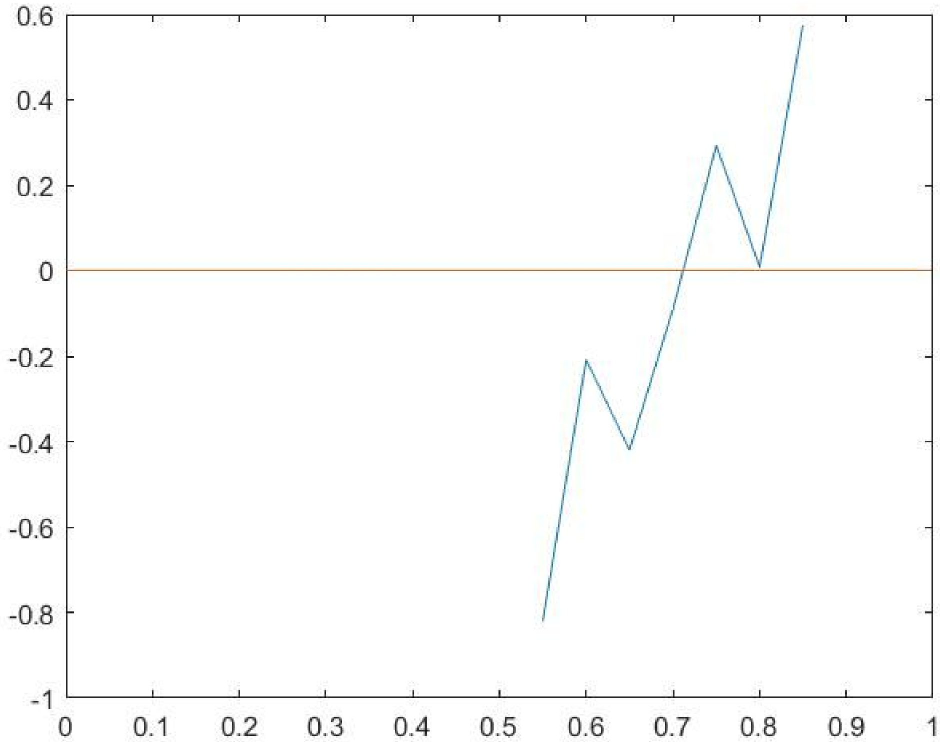
**Plotting x and f(x) at this value of r versus x:**

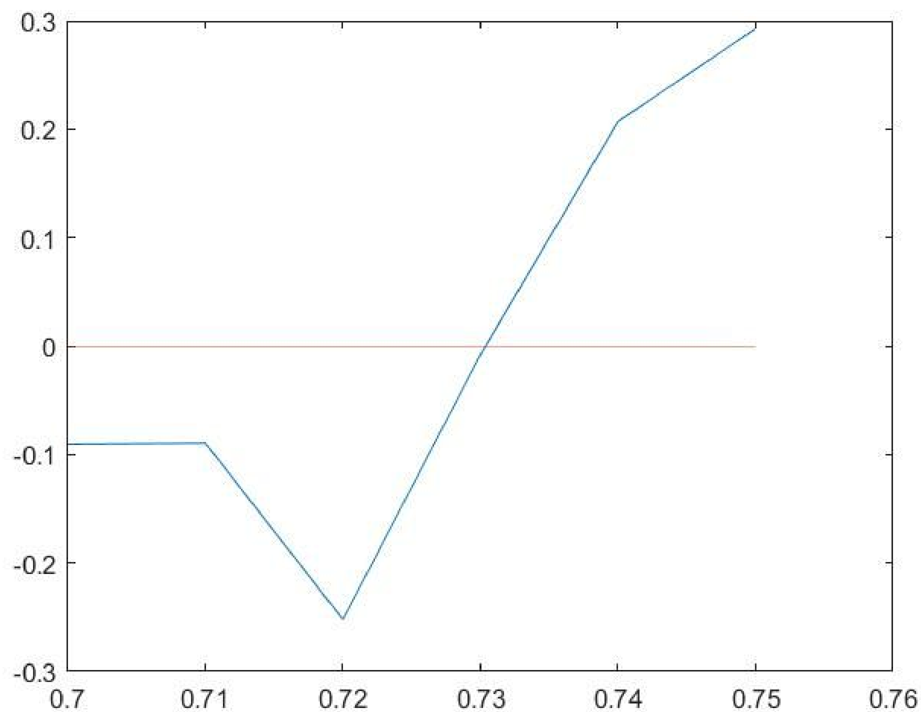



The slope of f(x) versus x at the point of intersection of f(x) and x is less than -1 as clearly evident visually from the plot. Thus the period one solution goes unstable at this value of ‘r’ and the period two solution appears.

```
rset = 0:0.05:1;
lambdaset = zeros(size(rset,2),1);
for i=1:size(rset,2)

    r = rset(i);

    n = 1000;
    x0 = .7;
    xi = x0;

    syms x;
    fdash = diff(itermap(x,r),x);
    sum = 0;

    for j=0:n-1
        sum = sum + log(double(abs(subs(fdash, x, xi))));
        xi = itermap(xi,r);
    end

    lambda = sum / n;

    lambdaset(i) = lambda;
end

plot(rset, [lambdaset, zeros(size(lambdaset))])
```

The above function computes the Lyapunov coefficient for values of r between 0 and 1 in steps of 0.05 for 1000 iterations and plots it across r. It can be seen that the Lyapunov coefficient value goes above 0 for an x0 = .7 at an ‘r’ value of approximately ‘0.732’. This can be seen from the plot between ‘r = 0 and 1’ and the zoomed plot between ‘r = 0.7 and .75’. This is where chaos will be observed.


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.7
0.71
0.72
0.73
0.74
0.75
0.76