28-05-2020 AM5650 - END-SEM EXAM S. TARUN PRASAD

ME17B114

"The work being submitted is my own work. I have not sought the help of any person in doing this work."

(1.). (ia.) $x^3 - (2+\varepsilon)x^2 - (1-\varepsilon)x + 2 + 3\varepsilon = 0.$

For $\varepsilon = 0$.

$\Rightarrow \quad x^3 - 2x^2 - x + 2 = 0.$

$x = 1$ is a root.

$\Rightarrow \quad x^3 - 2x^2 - x + 2 = (x^2 - x - 2)(x - 1) = 0$

$\Rightarrow \quad (x-2)(x+1)(x-1) = 0$

The 3 roots are $x = -1, 1, 2$

Assuming the roots for ~~so~~ small epsilon be small perturbations away from the roots for $\varepsilon = 0$

The 3 actual roots are $x = -1 + \alpha_1, \; 1 + \alpha_2, \; 2 + \alpha_3.$

For $(-1 + \alpha_1)$:

$(-1+\alpha_1)^3 - (2+\varepsilon)(-1+\alpha_1)^2 - (1-\varepsilon)(-1+\alpha_1) + 2 + 3\varepsilon = 0.$

Considering all 2nd order and higher order terms as H.O.T.

$\Rightarrow (-1 + 3\alpha_1) - (2+\varepsilon)(+1 - 2\alpha_1) - (1-\varepsilon)(-1+\alpha_1)$

$+ 2 + 3\varepsilon + H.O.T = 0.$

$\Rightarrow -1 + 3\alpha_1 - 2 - \varepsilon + 4\alpha_1 + 1 - \varepsilon - \alpha_1 + 2 + 3\varepsilon + \cancel{H.O.T} = 0$

0

$\Rightarrow \quad 6\alpha_1 + \varepsilon = 0$

$\Rightarrow \quad \alpha_1 = -\dfrac{\varepsilon}{6}.$

$$\Rightarrow \boxed{1^{st} \text{ Root} = -1 - \frac{\varepsilon}{6}}$$

For $(1+\alpha_2)$:

$(1+\alpha_2)^3 - (2+\varepsilon)\cancel{x^2}(1+\alpha_2)^2 - (1-\varepsilon)(1+\alpha_2) + 2 + 3\varepsilon = 0$

$\Rightarrow (1+3\alpha_2) - (2+\varepsilon)(1+2\alpha_2) - (1-\varepsilon)(1+\alpha_2) + 2 + 3\varepsilon + H.O.T = 0$

$\Rightarrow 1 + 3\alpha_2 - 2 - \varepsilon - 4\alpha_2 - 1 + \varepsilon - \alpha_2 + 2 + 3\varepsilon + \underbrace{\cancel{H.O.T}}_{0} = 0.$

$\Rightarrow \quad -2\alpha_2 + 3\varepsilon = 0 \quad \Rightarrow \alpha_2 = 1.5\varepsilon.$

$\Rightarrow \boxed{2^{nd} \text{ root} = 1 + 1.5\varepsilon}$

For $(2+\alpha_3)$:

$(2+\alpha_3)^3 - (2+\varepsilon)(2+\alpha_3)^2 - (1-\varepsilon)(2+\alpha_3) + 2 + 3\varepsilon = 0$

$\Rightarrow (8 + 12\alpha_3) - (2+\varepsilon)(4+4\alpha_3) - (1-\varepsilon)(2+\alpha_3) + 2 + 3\varepsilon + H.O.T = 0.$

$\Rightarrow 8 + 12\alpha_3 - 8 - 4\varepsilon - 8\alpha_3 - 2 + 2\varepsilon - \alpha_3 + 2 + 3\varepsilon + \underbrace{\cancel{H.O.T}}_{0} = 0$

$\Rightarrow 3\alpha_3 + \varepsilon = 0 \Rightarrow \alpha_3 = -\frac{\varepsilon}{3}$ 0.

$\Rightarrow \boxed{3^{rd} \text{ root} = 2 - \frac{\varepsilon}{3}}$

(b) $x^3 - (3+\varepsilon)x - 2 + \varepsilon = 0$

For $\varepsilon = 0$

$\Rightarrow \quad x^3 - 3x - 2 = 0 \Rightarrow (x-2)(x^2+2x+1) = 0$

$\Rightarrow (x-2)(x+1)^2 = 0$

Carrying forward the same assumptions from part 'a' the roots are $(2+\alpha_1)$, $(-1+\alpha_2)$

For $(2+\alpha_1)$:

$(2+\alpha_1)^3 - (3+\varepsilon)(2+\alpha_1) - 2 + \varepsilon = 0$

$(8 + 12\alpha_1) - (3+\varepsilon)(2+\alpha_1) - 2 + \varepsilon + H.O.T = 0$

$\Rightarrow 8 + 12\alpha_1 - 6 - 2\varepsilon - 3\alpha_1 - 2 + \varepsilon + \underbrace{H.O.T}_{0} = 0$

$9\alpha_1 - \varepsilon = 0$

$\Rightarrow \alpha_1 = \frac{\varepsilon}{9}$

$$\boxed{\text{First root} = 2 + \frac{\varepsilon}{9}}$$

For $(-1+\alpha_2)$:

$(-1+\alpha_2)^3 - (3+\varepsilon)(-1+\alpha_2) - 2 + \varepsilon = 0$

$\Rightarrow (-1+3\alpha_2) + 3 + \varepsilon - 3\alpha_2 - 2 + \varepsilon + \underbrace{H.O.T}_{0} = 0$

$\Rightarrow 2\varepsilon = 0$

This leads to no different root, therefore trying along with second order terms

$$\Rightarrow (-1+\alpha_2)^3 - (3+\varepsilon)(-1+\alpha_2) - 2 + \varepsilon = 0$$

$$\Rightarrow (-1+3\alpha_2-3\alpha_2^2) + 3 + \varepsilon - 3\alpha_2 - \alpha_2\varepsilon - 2 + \varepsilon + \text{H.O.T} = 0$$

$$\Rightarrow -3\alpha_2^2 + 2\varepsilon - \alpha_2\varepsilon = 0$$

$$\Rightarrow 3\alpha_2^2 + \varepsilon\alpha_2 - 2\varepsilon = 0$$

$$\Rightarrow \alpha_2 = \frac{-\varepsilon \pm \sqrt{\varepsilon^2+24\varepsilon}}{6}$$

The remaining two roots are:

$$\boxed{-1 + \left(\frac{-\varepsilon + \sqrt{\varepsilon^2+24\varepsilon}}{6}\right)} \ \& \ \boxed{-1 + \left(\frac{-\varepsilon - \sqrt{\varepsilon^2+24\varepsilon}}{6}\right)}$$

for $\varepsilon^2 + 24\varepsilon \geq 0 \simeq \varepsilon \geq 0$

(c.) $\varepsilon(x^3+x^2) + 4x^2 - 3x - 1 = 0$

For $\varepsilon = 0$

$$\Rightarrow 4x^2 - 3x - 1 = 4x^2 - 4x + x - 1 = 0$$

$$\Rightarrow 4x(x-1) + 1(x-1) = (4x+1)(x-1) = 0.$$

$\Rightarrow x = \frac{-1}{4}, 1$ are the two roots for $\varepsilon = 0$

Carrying forward the same assumptions from part (a), the two roots are:

$$x = (1+\alpha_1) \text{ and } \left(\frac{-1}{4} + \alpha_2\right).$$

For $(\cancel{\ } 1+\alpha_1)$:

$$\Rightarrow \varepsilon\left((1+\alpha_1)^3+(1+\alpha_1)^2\right)+4(1+\alpha_1)^2-3(1+\alpha_1)-1=0.$$

$$\Rightarrow 2\varepsilon+4(1+2\alpha_1)-3(1+\alpha_1)-1+H.O.T=0$$

$$\Rightarrow 2\varepsilon+4+8\alpha_1-3-3\alpha_1-1+\cancel{H.O.T}=0$$

0

$$\Rightarrow 2\varepsilon+5\alpha_1=0.$$

$$\Rightarrow \alpha_1=\frac{-2\varepsilon}{5}.$$

$$\Rightarrow \boxed{1^{st}\text{ root}=1-0.4\varepsilon}$$

For $\left(-\frac{1}{4}+\alpha_2\right)$:

$$\Rightarrow \varepsilon\left[\left(-\frac{1}{4}+\alpha_2\right)^3+\left(-\frac{1}{4}+\alpha_2\right)^2\right)+4\left(-\frac{1}{4}+\alpha_2\right)^2-3\left(-\frac{1}{4}+\alpha_2\right)-1=0.$$

$$\Rightarrow \varepsilon\left(\frac{-1}{64}+\frac{1}{16}\right)+4\left(\frac{+1}{16}-\frac{\alpha_2}{2}\right)-3\left(-\frac{1}{4}+\alpha_2\right)-1+\cancel{H.O.T}=0$$

0

$$\Rightarrow \frac{3\varepsilon}{64}+\frac{1}{4}-2\alpha_2+\frac{3}{4}-3\alpha_2-1=0.$$

$$\Rightarrow 5\alpha_2=\frac{3\varepsilon}{64} \Rightarrow \alpha_2=\frac{3}{320}\varepsilon$$

$$\Rightarrow \boxed{2^{nd}\text{ root}=\left(-\frac{1}{4}+\frac{3\varepsilon}{320}\right)}$$