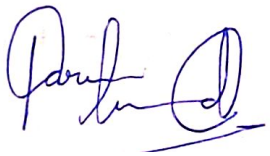
28-05-2020 AM5650 - END-SEM EXAM S. TARUN PRASAD

ME17B114

"The work being submitted is my own work. I have not sought the help of any person in doing this work."



(3). $\dot{x} = c + dx - x^3 = f(x)$.

$dx - x^3$ is the standard form for supercritical pitchfork bifurcation.

It has one solution for $d < 0$ and 3 solutions for $d > 0$ and also a minima and maxima for $d > 0$.

For 3 solutions to exist ~~in some~~ the minima and maxima should be lesser than 0 and greater than zero respectively.

For $\frac{df(x)}{dx} = 0 \Rightarrow d - 3x^2 = 0 \Rightarrow x = \pm\sqrt{\frac{d}{3}}$.

For: $\frac{d^2f(x)}{dx^2} = -6x > 0 \Rightarrow$ Minima at $x = -\sqrt{\frac{d}{3}}$.

$\frac{d^2f(x)}{dx^2} = -6x < 0 \Rightarrow$ Maxima at $x = \sqrt{\frac{d}{3}}$

Minima $\overset{=0}{\Leftarrow}$ condition

$$c + d\left(-\sqrt{\frac{d}{3}}\right) + d\left(\sqrt{\frac{d}{3}}\right)^3 \overset{=0}{\Leftarrow} \quad \leftarrow (1)$$

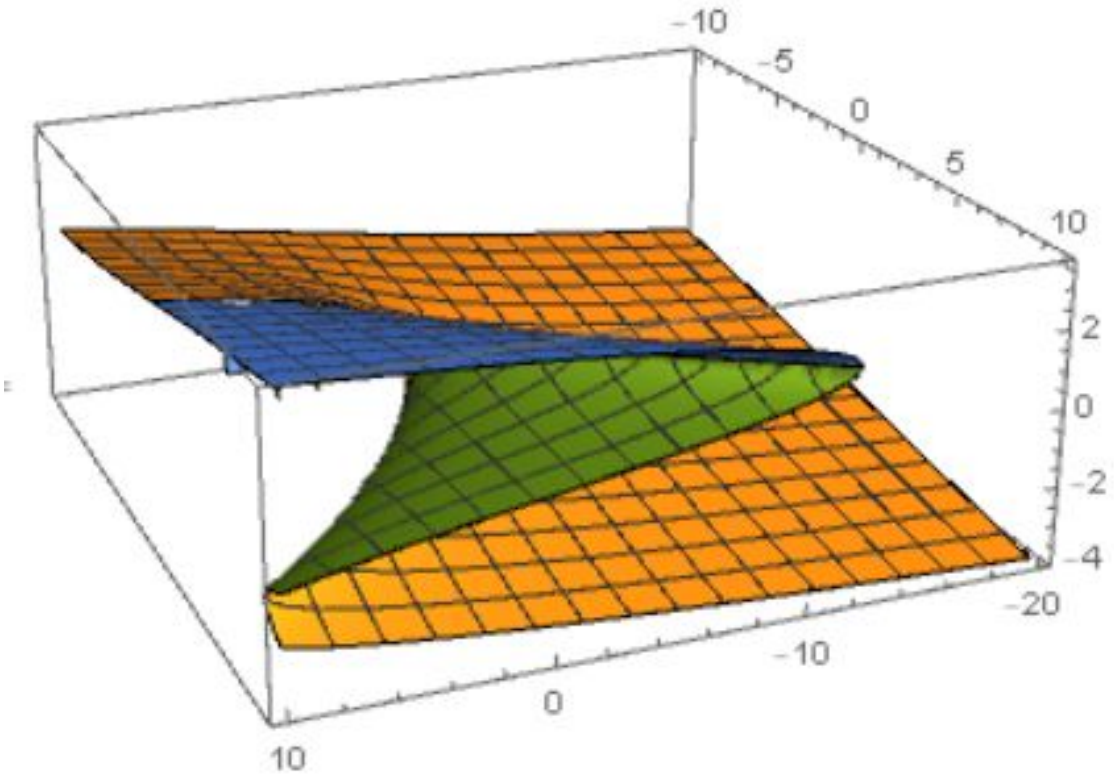
Maxima $= 0$ condition

$$c + d\left(\sqrt{\frac{d}{3}}\right) - d\left(\sqrt{\frac{d}{3}}\right)^3 = 0 \quad \text{--- (2)}$$

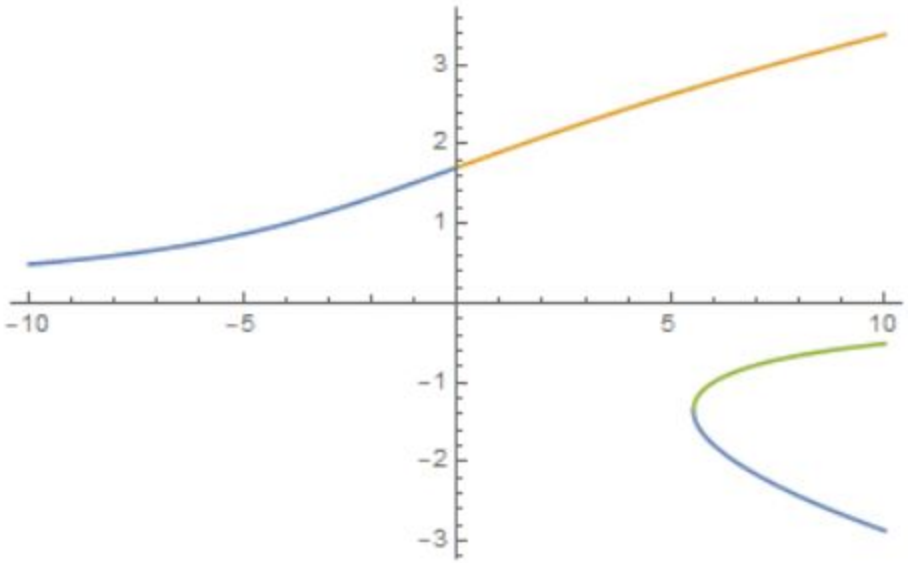
When the (c, d) domain space moves from a point where (1) & (2) is not satisfied to a point where (1) & (2) are satisfied we have a transcritical pitchfork bifurcation.

On one side of (c, d) domain space across condition (1) there is only one solution and on the other side there 2 more solutions along with the old solution. The situation is likewise for condition (2). But the old solution coincides with the two new solutions only for (1) & (2) i.e., $c = 0$ and there it forms a supercritical pitchfork bifurcation. Everywhere else where (1) or (2) is satisfied there is a pitchfork bifurcation.

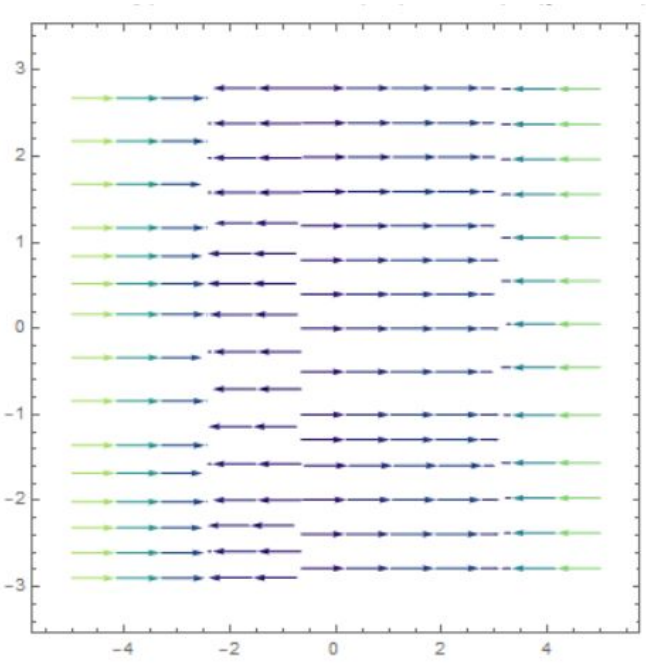
Plot of $\dot{x} = 0$ over (c, d) domain space:



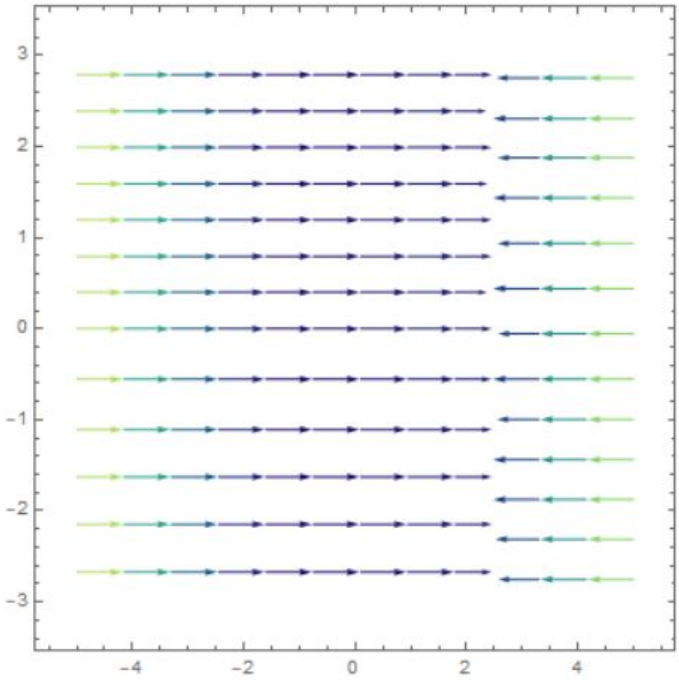
Saddle Node: Plot of $\dot{x} = 0$ versus d for c = 5 :



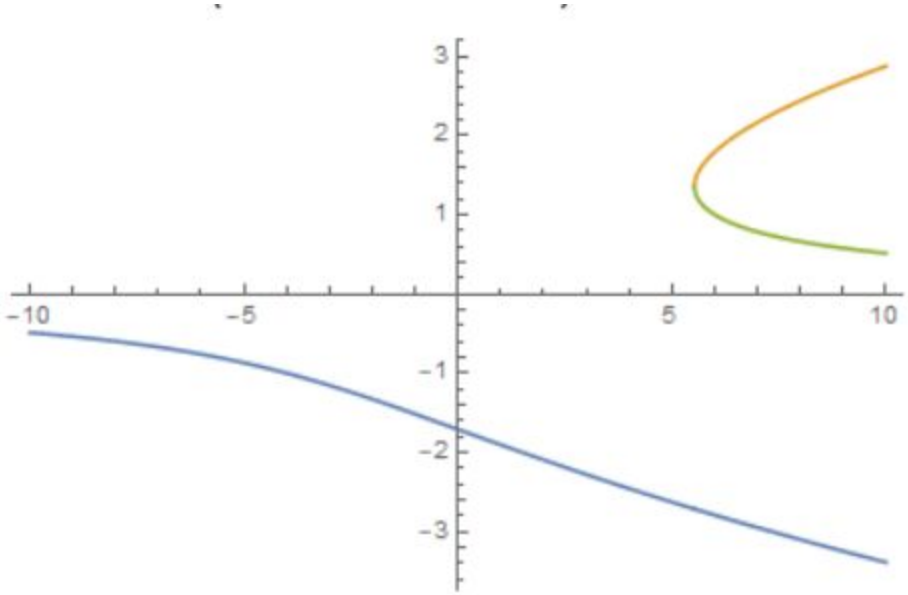
Phase Portrait for d = 8 and c = 5:



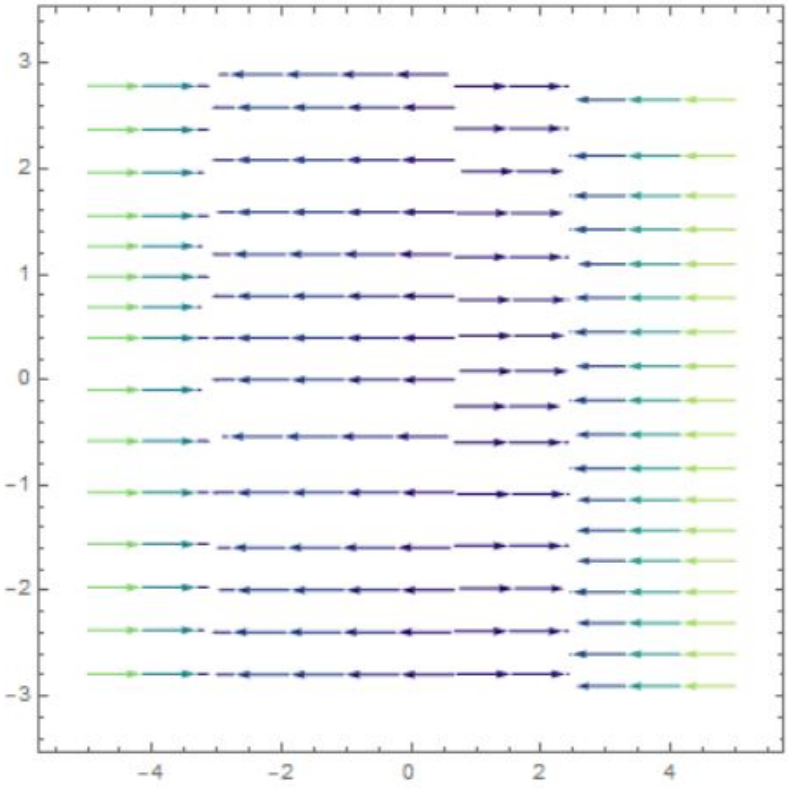
Phase Portrait for d = 4 and c = 5:



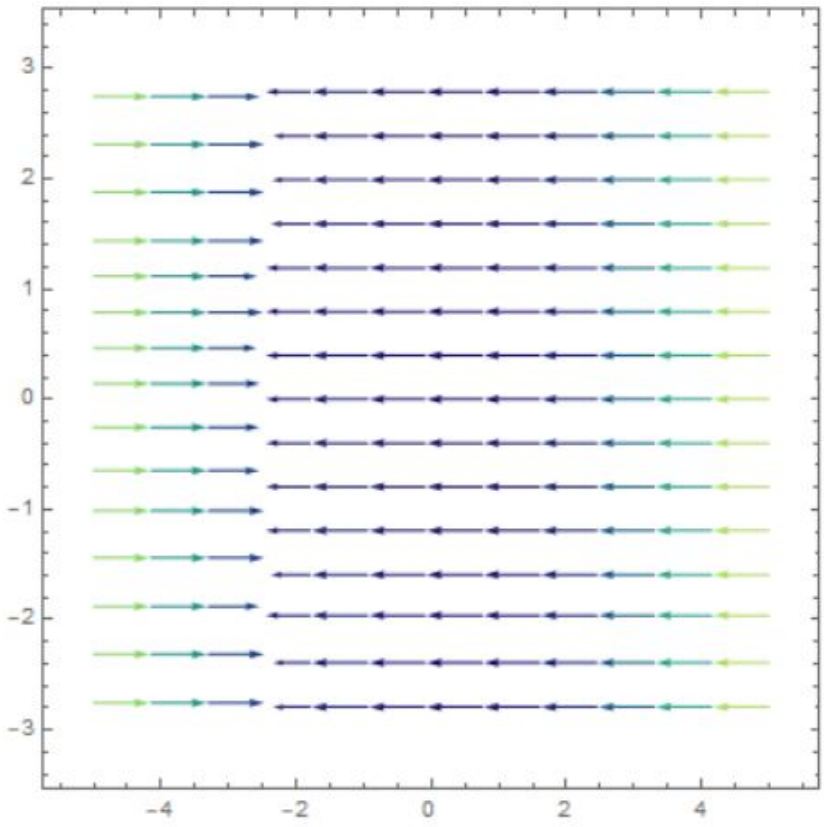
Saddle Node: Plot of $\dot{x} = 0$ versus d for c = - 5 :



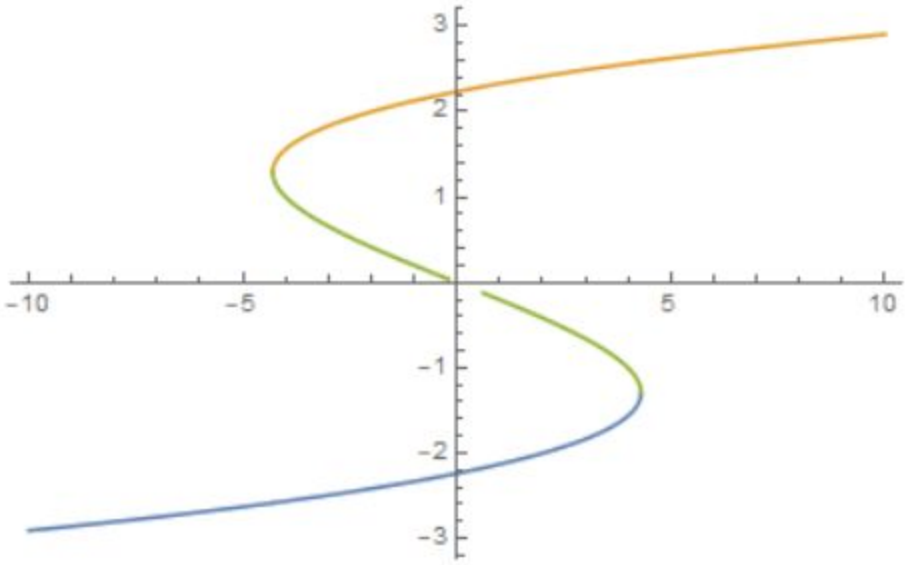
Phase Portrait for d = 8 and c = -5:



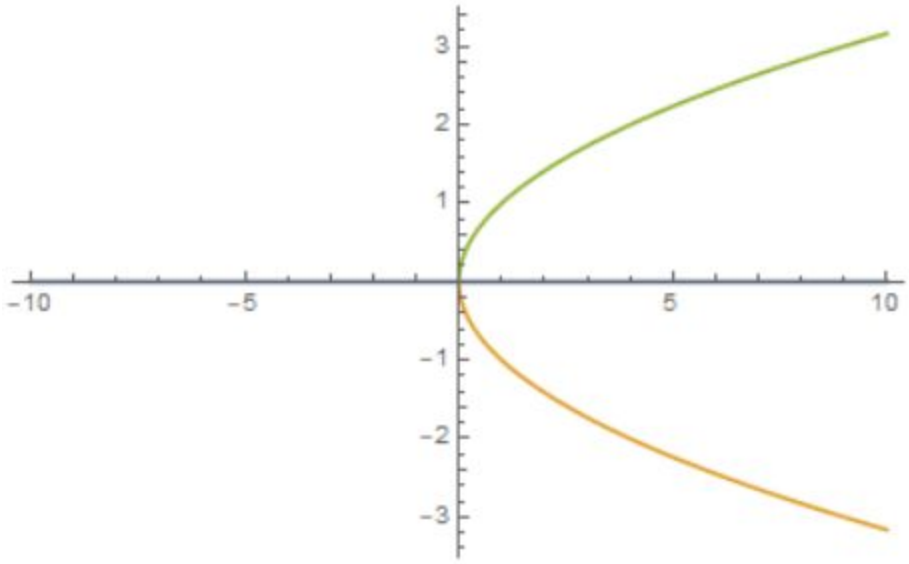
Phase Portrait for d = 4 and c = -5:



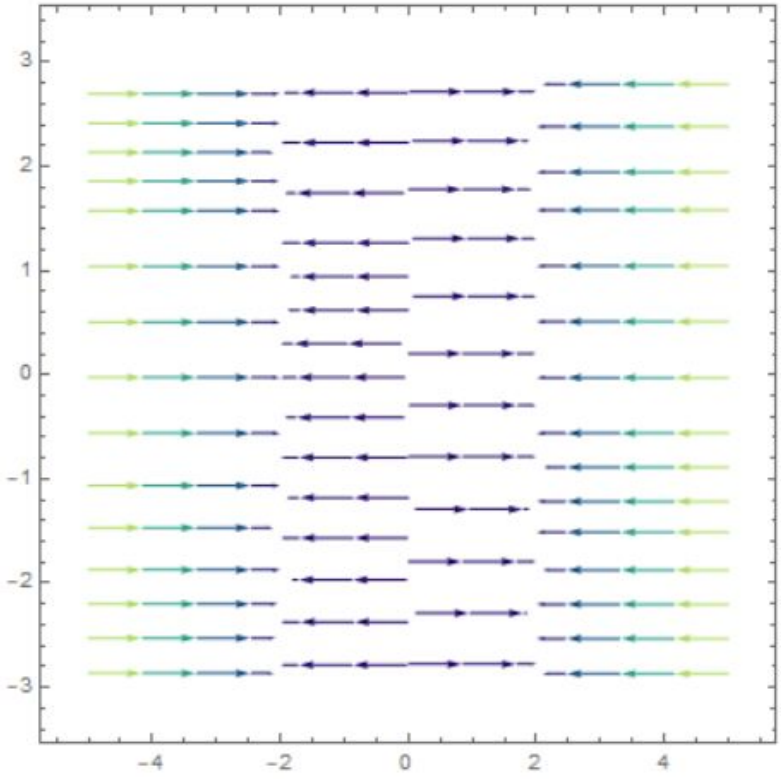
Two Saddle Nodes: Plot of $\dot{x} = 0$ versus c for d = 5 :



Supercritical Pitchfork Bifurcation: Plot of $\dot{x} = 0$ versus d for c = 0 :



Phase Portrait for d = 4 and c = 0:



Phase Portrait for d = - 4 and c = 0: