ID6040 - Inverse Kinematics S. Tarun Prasad
ME17B114

Inverse kinematics: Given P & R find q.

$$\left[\begin{array}{ccc|c} n_x & s_x & a_x & P_x \\ n_y & s_y & a_y & P_y \\ n_z & s_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$\rightarrow R(q)$

$\rightarrow p(q)$

$\rightarrow$ 12 equations,

n unknowns.

$\hookrightarrow$ n - state variables (n - joints)

→ All aren't independent.

R ⇒ only 3 independent equations.

P → only 3 independent equations

All are non-linear → difficult to solve

Existence of solution:

If all joint variables can be found given end-effector location

Conditions:

→ Tool within workspace

→ Tool orientation shouldn't violate limitation

Solutions: → Closed form (analytic)

* Only some robots have it.

* Needs to satisfy sufficiency conditions

→ Numerical form (iterative)

* Time - consuming

Tricks for solving:

→ Combine 2 equation and eliminate a variable

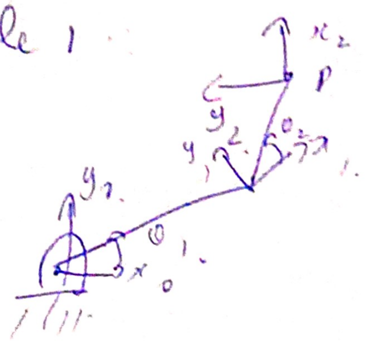
→ Use $u = \tan \theta/2$.

$$\cos\theta = \frac{1-u^2}{1+u^2}$$

$$\sin\theta = \frac{2u}{1+u^2}$$

→ $A\tan2(\sin\theta_i, \cos\theta_i)$.

Example 1



$$c_2 = \frac{P_x^2 + P_y^2 - (l_1^2 + l_2^2)}{2 l_1 l_2}$$

$$s_2 = \sqrt{1 - c_2^2}$$

$$\theta_2 = a\tan2(s_2, c_2)$$

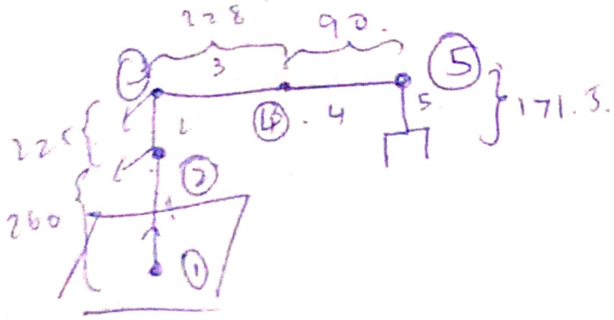
DH parameters

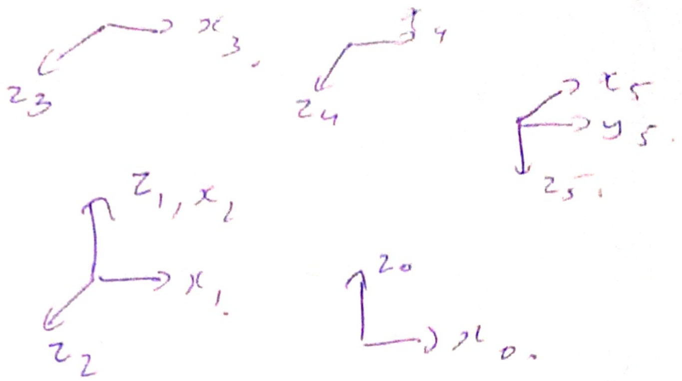
a	d	α	θ
l_1	0	0	θ_1
l_2	0	0	θ_2
0	d_3	0	θ_3

→ for solving inverse problem first proceed with forward kinetics problem.

$$\begin{bmatrix} c_{123} & -s_{123} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{123} & c_{123} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example problems:





DH parameter table:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	260	θ_1
2	90°	0	0	θ_2
3	0	228	0	θ_3
4	0	228	0	θ_4
5.	90°	90	171	θ_5

$$ {}^{0}_{5}T = \begin{bmatrix} S_1S_5 + C_1C_5C_{234} & C_5S_1 - C_1S_5C_{234} & C_1S_{234} & 3C_1\,76C_{23} + 76C_2 + 3\sqrt{461}\sin(\tan^{-1}(10/19) + t_2 + t_3 + t_4) \\ C_5S_1C_{234} - C_1S_5 & -C_1C_5 - S_1S_5C_{234} & S_1S_{234} & 3S_1\,76C_{23} + 76C_2 + 3\sqrt{461}\sin(\tan^{-1}(10/19) + t_2 + t_3 + t_4) \\ C_5S_{234} & -S_5S_{234} & -C_{234} & 228S_{23} + 228S_2 - 9\sqrt{461}\cos(\tan^{-1}(10/19) + t_2 + t_3 + t_4 + 260) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} n_x & s_x & a_x & P_x \\ n_y & s_y & a_y & P_y \\ n_z & s_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Differential Relations & Statics

$x = w(q)$ → joint variables

tool configuration ↳ tool-configuration function

differential relationship

$\dot{x} = J(q)\dot{q}$

$$J_{kj}(q) = \frac{\partial w_k(q)}{\partial q_j}$$

$X = wq$ (k → no. of joint velocities) $(1 \le k \le 6, 1 \le j \le n)$

(j → no. of DOF)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\times 1} = \begin{bmatrix} \partial w_1/\partial q_1 & \frac{\partial w_1}{\partial q_2} & \cdots & \frac{\partial w_1}{\partial q_n} \\ \partial w_2/\partial q_1 & & & \\ \vdots & & \cdots & \\ \partial w_6/\partial q_1 & \cdots & \cdots & \frac{\partial w_6}{\partial q_n} \end{bmatrix}_{6\times n} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}_{n\times 1}$$

For a rotatory manipulator,

$$\dot{x} = J(\theta)\,\dot{\theta} \Rightarrow \dot{\theta} = [J(\theta)]^{-1}\dot{x}.$$

The point at which the Jacobian loses rank is called Joint Space Singularity.

Note: The Jacobian matrix $J(q)$ is full rank as long as q is not a joint space singularity.