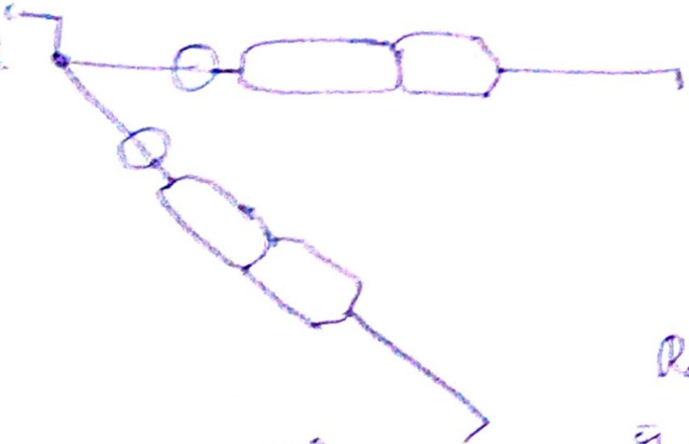
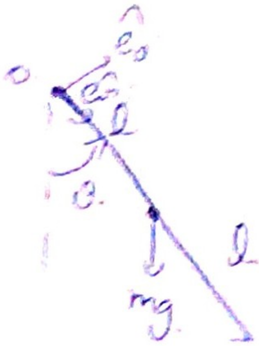
Wrist Pin Joint





Rod length → $2l$, mass → m.

$\bar{r}_{cm} = l\hat{e}_r$, $\bar{v}_{cm} = l\dot{\theta}\hat{e}_\theta$

$\bar{a}_{cm} = l\ddot{\theta}\hat{e}_\theta - l\dot{\theta}^2\hat{e}_r$

$dm = \frac{m}{2l}dr$

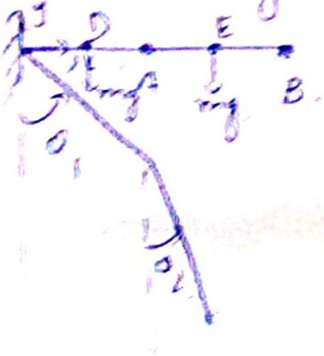
$$\bar{H}_o = \sum r_i \times m\bar{v}_i = \int_0^{2l} r\hat{e}_r \times r\dot{\theta}\hat{e}_\theta \times \frac{m}{2l}dr$$

$$= \frac{m}{2l}\dot{\theta}\int_0^{2l} r^2 dr = \frac{4}{3}ml^2\dot{\theta}.$$

$$\sum M_o = \frac{dH_o}{dt} = \frac{4}{3}ml^2\ddot{\theta} = I_o\alpha$$

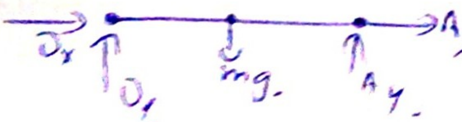
$I_c = \frac{ml^2}{3} \Rightarrow I_o = \frac{4}{3}ml^2$ (// axis theorem.

2-link system:



Released from rest $\Rightarrow \dot{\theta}_1 = \dot{\theta}_2 = 0$.

$\bar{a}_D = \alpha_1\frac{l}{2}\hat{j}$, $\bar{a}_E = \bar{a}_A + \bar{a}_{E/A} = \alpha_1 l\hat{j} + \alpha_2\frac{l}{2}\hat{j}$.



$O_x + A_x = -m\omega_1^2\frac{l}{2} = 0.$

$O_y - mg + A_y = m\alpha_1\frac{l}{2}.$

$\sum M_o = I_o\alpha_1$

$\sum M_E = I_E\alpha_2$ $\quad\Rightarrow -mg\frac{l}{2} + A_y \times l = \frac{ml^2}{3}\alpha_1$.

$$\Rightarrow \frac{A_y l}{2} = \frac{ml^2}{12}\alpha_2 \Rightarrow \boxed{A_y = \frac{ml}{6}\alpha_2}$$

$A_x = 0$ ($\omega_2 = 0$).

$$-A_y - mg = m \times a_E = m\left(\alpha_2\frac{l}{2} + \alpha_1 l_1\right).$$

$$\Rightarrow -mg\frac{l}{2} + \frac{ml^2\alpha_2}{6} = \frac{ml^2}{3}\alpha_1 \Rightarrow -3g + l\alpha_2 = 2l\alpha_1,$$

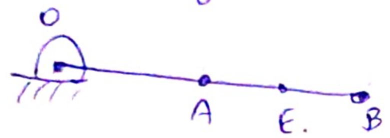
$$\Rightarrow \boxed{\alpha_2 - 2\alpha_1 = \frac{3g}{l}} \quad \text{—(1).}$$

$$-A_y - mg = m\left(\alpha_2 \frac{l}{2} + \alpha_1 l\right) \Rightarrow -\frac{ml}{6}\alpha_2 - mg = m\left(\alpha_1 \frac{l}{2} + \alpha_1 l\right)$$

$$\Rightarrow \boxed{2\alpha_2 + 3\alpha_1 = -\frac{3g}{l}} \quad -(2)$$

From (1) & (2) $\therefore$ $\boxed{\alpha_1 = -\frac{9}{7}\frac{g}{l}}$, $\boxed{\alpha_2 = \frac{3g}{7l}}$

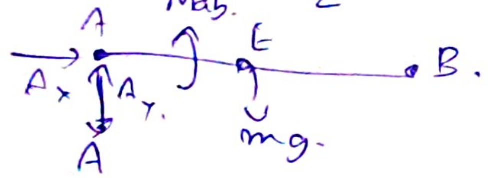
Rigid straight body from the beginning:-



$$\Sigma M_o = I_o \alpha.$$

$$-2mgl = \frac{2m \times 4l^2 \alpha}{3}. \Rightarrow \alpha = \frac{-3g}{4L}.$$

$$a_E = \alpha \times \frac{3l}{2} = -\frac{9}{8} g. \Rightarrow$$ For this the abdominal muscle needs to exert counter moment.



$$\Sigma M_A = I_E \alpha + \bar{r}_{E/A} \times m\bar{a}_{EA}$$

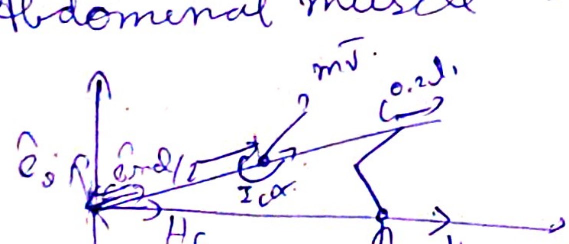
$$= \frac{ml^2}{12}\left(-\frac{3g}{4l}\right)(\hat{k}). + \left(\frac{l}{2}\right)\left(-\frac{9}{8}mg\right)\hat{k}$$

$$= -\frac{5}{8} mgl\hat{k}.$$

$$\Sigma M_A = -mgl + M_{ab} = -\frac{5}{8} mgl.$$

$$\Rightarrow M_{ab} = -\frac{1}{8} mgl.$$

Abdominal muscle $\rightarrow$ agonist, Back muscles $\rightarrow$ antagonist



$H_h \rightarrow 0$ (Assumption) Arms $\rightarrow$ weightless.

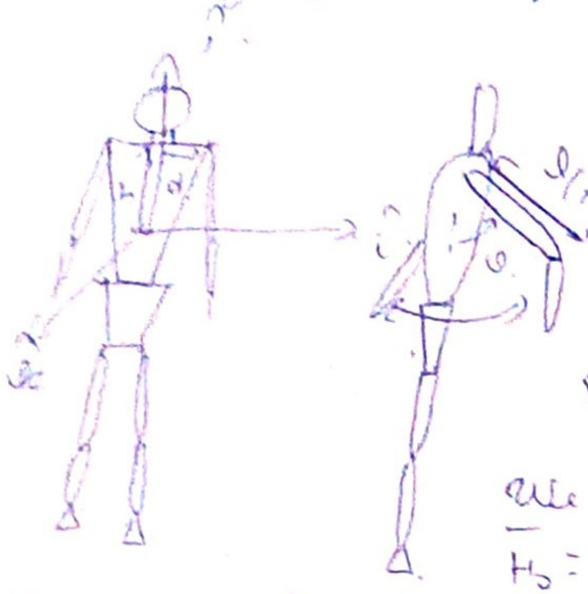
$$\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta, \quad \dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r$$

$$\bar{r}_c = \frac{l}{2}\hat{e}_r, \quad \bar{v}_c = \frac{l\dot{\theta}}{2}\hat{e}_\theta, \quad \bar{a}_c = \frac{l\ddot{\theta}}{2}\hat{e}_\theta - \frac{l}{2}\dot{\theta}^2\hat{e}_r$$

$$\Sigma M_o = I_c\ddot{\theta}\hat{k} + \frac{l}{2}\hat{e}_r \times m\left(\frac{l}{2}\ddot{\theta}\hat{e}_\theta - \frac{l}{2}\dot{\theta}^2\hat{e}_r\right) = I_o\ddot{\theta}\hat{k} = \frac{ml^2}{3}\ddot{\theta}\hat{k}.$$

$$-mg\frac{l}{2}\cos\theta + V_h(0.8l\cos\theta) = \frac{ml^2}{3}\ddot{\theta}.$$

Arm swinging during walking :



l - arm length

Arm 1 : $\bar{r}_1 = h\hat{j} + d\hat{i} - \frac{l}{2}\cos\theta\hat{j} - \frac{l}{2}\sin\theta\hat{k}$

Arm 2 : $\bar{r}_2 = h\hat{j} - d\hat{i} - \frac{l}{2}\cos\theta\hat{j} + \frac{l}{2}\sin\theta\hat{k}$

$\bar{v}_1 = \frac{l}{2}\sin\theta\dot{\theta}\hat{j} - \frac{l}{2}\cos\theta\dot{\theta}\hat{k}$

$\bar{v}_2 = \frac{l}{2}\sin\theta\dot{\theta}\hat{j} + \frac{l}{2}\cos\theta\dot{\theta}\hat{k}$

We have lumped the arm at COM.

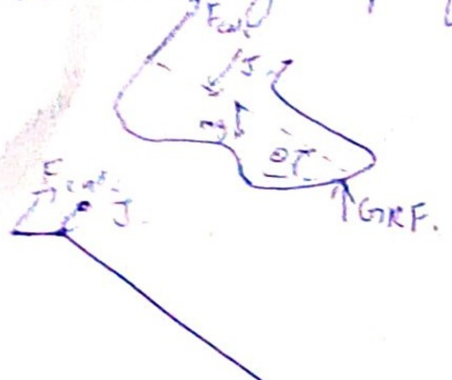
$\bar{H}_0 = \bar{r}_1 \times m_a\bar{v}_1 + \bar{r}_2 \times m_a\bar{v}_2 = 2dl_m\dot{\theta}\cos\theta\hat{j}$

When we stand and move our arms, the trunk also moves to compensate for the angular momentum.

Walking : $\bar{H}_{0\,legs} = 2dl_2 m_a\dot{\theta}_d\cos\theta_e\hat{j}$

$\bar{H}_0$ arms & $\bar{H}_0$ legs counteract and to maintain the same frequency arms have more amplitude to compensate for the increased mass and length of legs.

Vertical Jumping :



$F_{calc} \to$ achilles , $m = 75\,kg$.

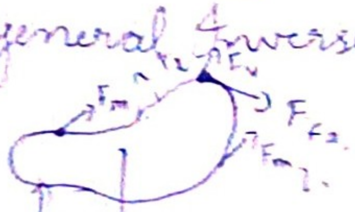
Length of foot $= 27\,cm$, $\omega = 15\,rad/s$.

Length of moment arm $= 4\,cm$, $\alpha = -150\,rad/s$.

$2F_G - mg = ma_y$, $a_y = 12\,m/s^2$

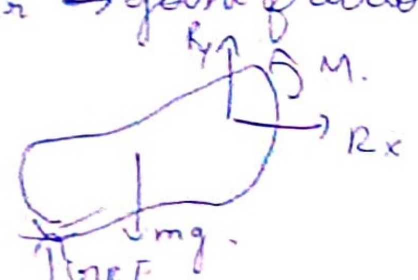
$F_G = 825N$, mass of foot $= 1.5\,kg$

General Inverse Dynamic analysis procedure :

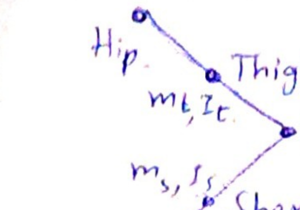


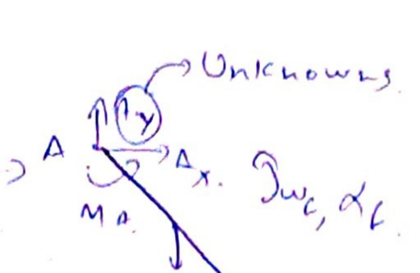
$F_l \to$ ligamentous force . F_j = Joint reaction.

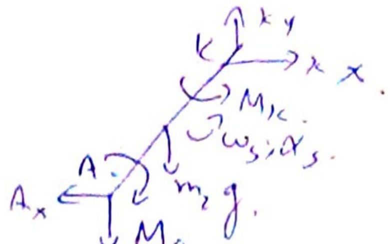
$F_{fr} \Rightarrow$ joint friction F_m = Muscle force.

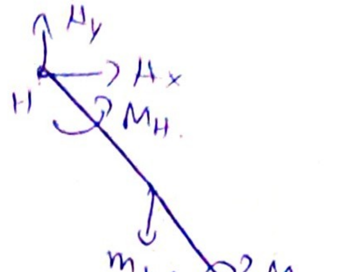


All forces reduced to 3 unknowns.









For each segment,

$\Sigma F_x = m a_x$, $\Sigma F_y = m a_y$.

$\Sigma M_{COM} = I_{COM}\ddot{\theta}$.