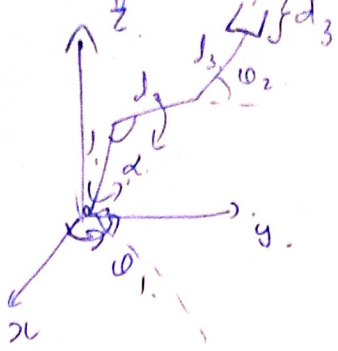
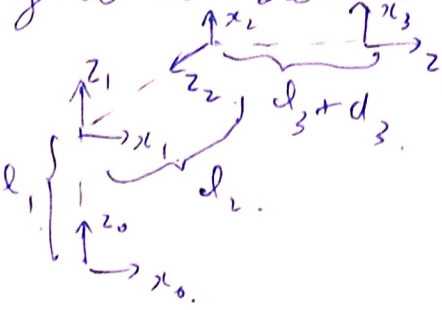
12-04-2010 ID6040: ~~Tutorial~~ Assignment (2) S Tarun Kumar MEI7B114

7. (a). Jacobian computation



At home with all the variable equal to zero we have:



DH parameters:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	$l_1 - l_2 c_2$	θ_1
2	90	$l_2 s_\alpha$	0	$\theta_2 + 90^\circ$
3.	90	0.	$l_3 + d_3$	0.

$J(q) = [j_1(q) \; j_2(q) \; j_3(q)]$.

$A_{kj} = \frac{\partial P_k(q)}{\partial q_j}$, $J(q) = \begin{bmatrix} A(q) \\ \hline B(q) \end{bmatrix}$.

Calculating the transformation matrices:

$$^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 - l_2 c_\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^0_2T = \begin{bmatrix} -c_1 s_2 & -c_1 c_2 & s_1 & l_2 s_\alpha c_1 \\ -s_1 s_2 & -s_1 c_2 & -c_1 & l_2 s_\alpha s_1 \\ c_2 & -s_2 & 0 & l_1 - l_2 c_\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0_3T = \begin{bmatrix} -c_2 s_2 & s_1 & c_1 c_2 & c_1 (d_3 c_2 + l_2 s_\alpha + l_3 c_2) \\ -s_1 s_2 & -c_1 & c_2 s_1 & s_1 (d_3 c_2 + l_2 s_\alpha + l_3 c_2) \\ c_2 & 0 & s_2 & (l_1 - l_2 c_\alpha + s_2 (d_3 + l_3)) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P_x = c_1 (d_3 c_2 + l_2 s_\alpha + l_3 c_2)$

$P_y = s_1 (d_3 c_2 + l_2 s_\alpha + l_3 c_2) \Rightarrow \therefore j_1(q)$

$P_z = l_1 - l_2 c_\alpha + s_2 (d_3 + l_3)$. $= \begin{bmatrix} -s_1 (d_3 c_2 + l_2 s_\alpha + l_3 c_2) \\ c_1 (d_3 c_2 + l_2 s_\alpha + l_3 c_2) \\ 0 \\ \hline 0 \\ 0 \\ 1 \end{bmatrix}$

$$J_2(q) = \begin{bmatrix} -C_1 S_2 (d_3 + \delta_3) \\ -S_2 S_1 (d_3 + \delta_3) \\ C_2 (d_3 + \delta_3) \\ \hline S_1 \\ -C_1 \\ 0 \end{bmatrix}, \quad J_3(q) = \begin{bmatrix} C_1 C_2 \\ C_2 S_1 \\ S_2 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow J(q) = \begin{bmatrix} -S_1 (d_3 C_2 + \delta_2 S_2 + \delta_3 C_2) & -C_1 S_2 (d_3 + \delta_3) & C_1 C_2 \\ -C_1 (d_3 C_2 + \delta_2 S_2 + \delta_3 C_2) & -S_2 S_1 (d_3 + \delta_3) & C_2 S_1 \\ 0 & C_2 (d_3 + \delta_3) & S_2 \\ \hline 0 & S_1 & 0 \\ 0 & -C_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

2b.) Trajectory planning:

(1.) $\theta_0 = 5^\circ$, $\theta_v = 15^\circ$, $\theta_g = -10^\circ$.

1st duration:

$$\theta_1(t) = a_{10} + a_{11} t + a_{12} t^2 + a_{13} t^3$$
$$\dot{\theta}_1(t) = a_{11} + 2a_{12} t + 3a_{13} t^2$$
$$\ddot{\theta}_1(t) = 2a_{12} + 6a_{13} t$$

2nd duration:

$$\theta_2(t) = a_{20} + a_{21} t + a_{22} t^2 + a_{23} t^3$$
$$\dot{\theta}_2(t) = a_{21} + 2a_{22} t + 3a_{23} t^2$$
$$\ddot{\theta}_2(t) = 2a_{22} + 6a_{23} t$$

With constraints:

$\theta_1(0) = \theta_0 \Rightarrow a_{10} = 5$ — (1)

$\theta_v = \theta_1(2) \Rightarrow 15 = 5 + 2a_{11} + 4a_{12} + 8a_{13}$ — (2)

$\dot{\theta}_1(0) = 0 \Rightarrow a_{11} = 0$ — (3)

$\theta_2(2) = \theta_g \Rightarrow -10 = a_{20} + 2a_{21} + 4a_{22} + 8a_{23}$ — (4)

$\theta_2(0) = \theta_v \Rightarrow 15 = a_{20}$ — (5)

$\dot{\theta}_2(2) = 0 \Rightarrow a_{21} + 4a_{22} + 12a_{23} = 0$ — (6)

Point velocity $\Rightarrow a_{11} + 4a_{12} + 12a_{23} = 0$ — (7)

Point acceleration $\Rightarrow 2a_{12} + 12a_{13} = 2a_{22}$ — (8)

$a_{10} = 5$, $a_{11} = 0$

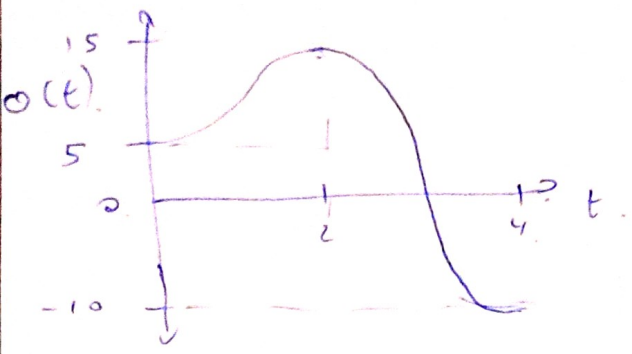
$\Rightarrow a_{12} = 10.3125$

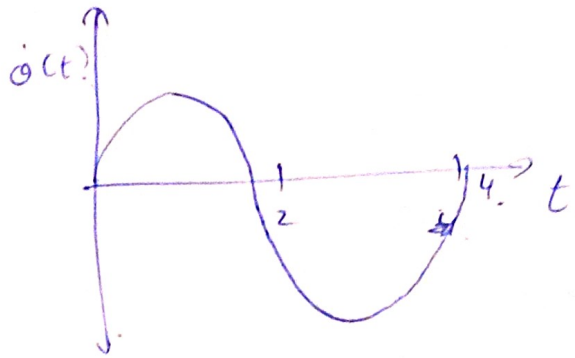
$a_{13} = -3.9063$

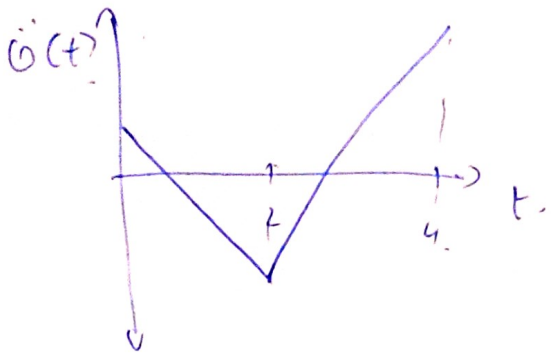
$a_{20} = 15$, $a_{21} = -5.625$

$a_{22} = -13.125$

$a_{23} = 4.846$

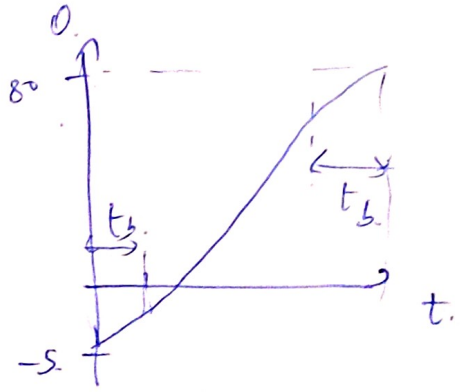






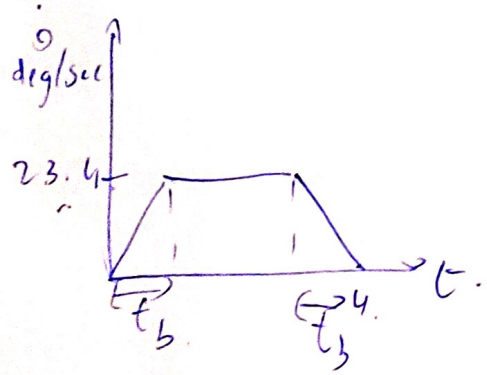
(3.) $q_1 = -5$ deg, $q_2 = 80$ deg

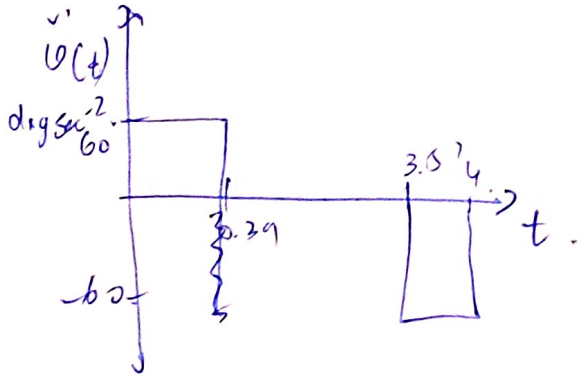
$$t_b = \frac{4}{2} - \sqrt{4 - (80 - (-5))/\ddot{\theta}} = 2 - \sqrt{4 - 85/\ddot{\theta}}$$



Choose the value of acceleration such that t_b doesn't become imaginary.

$$\therefore \dot{\theta} = \ddot{\theta}_b \times t_b = 60 \text{ deg sec}^{-2} \times 0.39 \text{ sec}$$
$$= 23.4 \text{ deg sec}^{-1}$$





(2) $q_1 = 5\,deg$ (θ_1), $q_2 = 15\,deg$ (θ_2), $q_3 = -40\,deg$ (θ_3).

$t_{d23} = t_{d12} = 2$ seconds $\quad |\ddot{\theta}_1| = |\ddot{\theta}_2| = |\ddot{\theta}_3| = 60\,deg/sec^2$

First period $(1 \to 2)$: $\ddot{\theta}_1 = SGN(\theta_2 - \theta_1)|\ddot{\theta}_1| = 60\,deg/sec^2$.

$$t_1 = t_{d12} - \sqrt{t_{d12}^2 - \frac{2(\theta_2 - \theta_1)}{\ddot{\theta}_1}} = 0.085\,sec.$$

$$\dot{\theta}_{12} = \frac{\theta_2 - \theta_1}{t_{d12} - \frac{t_1}{2}} = \frac{10}{\left(2 - \frac{0.085}{2}\right)} = 5.108\,deg/sec.$$

$$t_{12} = t_{d12} - t_1 - \frac{t_2}{2} = 1.615\,sec.$$

Final period: $\ddot{\theta}_3 = SGN(\theta_2 - \theta_3)|\ddot{\theta}_3| = 60\,deg/sec^2$

$$t_3 = t_{23} - \sqrt{t_{d23}^2 + \frac{2(\theta_3 - \theta_2)}{\ddot{\theta}_3}} = 0.528\,sec.$$

$$\dot{\theta}_{23} = \frac{\theta_3 - \theta_2}{t_{d23} - \frac{t_3}{2}} = \frac{-40 - 15}{\left(2 - \frac{0.528}{2}\right)} = -31.68\,deg/sec.$$

$$t_{23} = t_{d23} - t_3 - \frac{t_2}{2} = 1.172\,sec.$$

Intermediate segments: $\ddot{\theta}_2 = SGN(\theta_{23} - \theta_{12})|\ddot{\theta}_2| = -60\,deg/sec^2$

$$t_2 = \frac{\dot{\theta}_{23} - \dot{\theta}_{12}}{\ddot{\theta}_2} = 0.61\,sec.$$