17-05-2020 ME6230: END-SEMESTER S. TARUN PRASAD

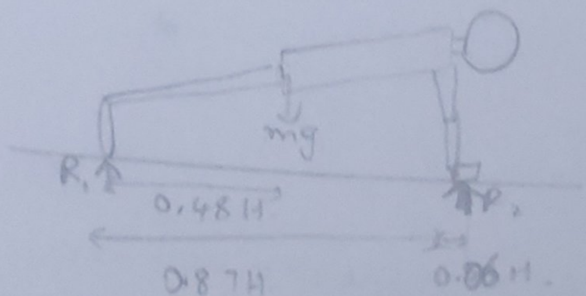
ME17B114

TAKE-HOME EXAM.

→ My Height: 170 cm.

→ My Weight: 86 kg.

(1.) (i.) Plank:

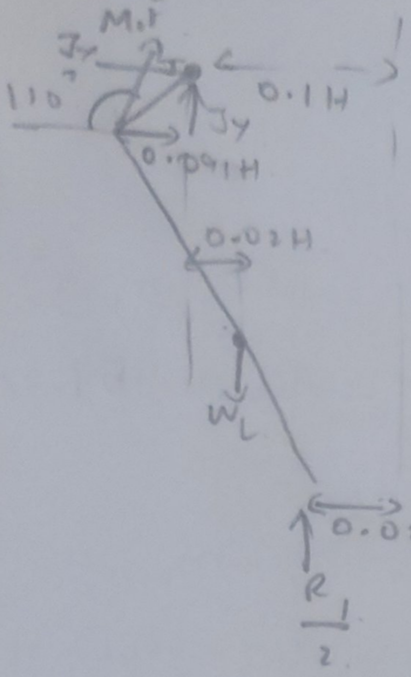


Balancing moment about foot.

$$R_2 = \frac{mg \times 0.48}{0.87 + 0.06} = 434.99 N.$$

$$\Rightarrow R_1 = mg - 434.99 N = 407.81 N$$

Hip Analysis (Plank): The muscle is gluteus medius



$\Sigma M_J = 0$.

$\Rightarrow MF \sin 70° \times 0.091H$

$= W_L \times 0.02H + \frac{R_1}{2} \times (0.191 - 0.0275)H$

$\Rightarrow M.F \times 0.07 = 334.76 \times 0.02$

$+ \frac{407.81}{2} \times 0.1635$

$\Rightarrow M.F = 571.91\,N \Rightarrow \cancel{M.R = 1048.17N}.\ \cancel{571.91N}$

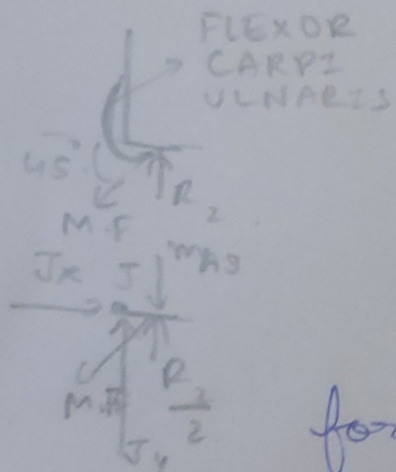
$\Sigma F_x = 0 \Rightarrow J_x = -MF\cos 70° = \cancel{-663.83N}\ -362.20N$

$\Sigma F_y = 0 \Rightarrow W_2 - \frac{R_1}{2} - MF \sin 70° = J_y$

$\Rightarrow J_y = 334.76 - \frac{407.81}{2} - \cancel{1048.17}\ 571.91 \sin 70°$.

$\Rightarrow \boxed{\cancel{J_y = -884.22N}} \Rightarrow J_y = -311.74$

Wrist Analysis:



Assuming flexor carpi ulnaris to be the sole stabilising muscle acting at the middle of the palm and the reaction force also acting at the middle of the palm. Assuming muscle force at 45° to palm.

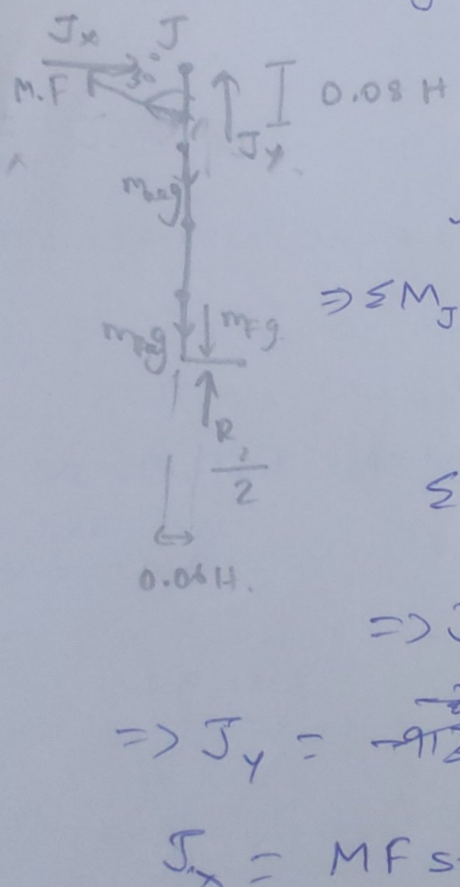
$\Sigma M_J = 0 \Rightarrow M.F \sin 45° = \frac{R_2}{2} - m_h g = \frac{R_2}{2} - 0.006mg$

$\Rightarrow M.F = \sqrt{2}\left(\frac{434.99}{2} - 0.006 \times 9.8 \times 86\right) = \cancel{608.02N}\ 300.39\,N$

$\Sigma F_y = 0 \Rightarrow J_y = 0$

$\Sigma F_x = 0 \Rightarrow J_x = MF \cos 45° = \cancel{429.93N}\ 212.44N$

Shoulder analysis



Assuming latissmus dorsi to stabilise the shoulder with parameters as shown in the diagram.

$$\Rightarrow \Sigma M_J = 0 \Rightarrow MF\sin 30^\circ \times 0.08H = 0.06H\left(\frac{R_2}{2} - m_h g\right)$$

$$\Rightarrow MF = \cancel{644.89\,N}\ \cancel{318.66\,N}\ 159.33$$

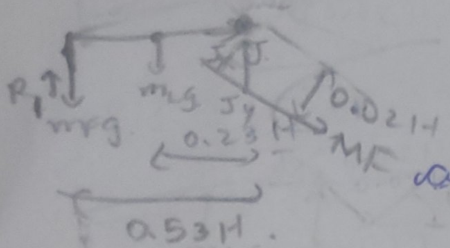
$$\Sigma F_y = 0 \Rightarrow J_y = -\left(MF\cos 30^\circ + \frac{R_2}{2}\right) + (m_u + m_F + m_h)\,g.$$

$$\Rightarrow J_y = \cancel{-912.01}\ -266.65 + (0.0271 + 0.0162 + 0.006) \times 86 \times 9.8$$

$$\Rightarrow J_y = \cancel{-912.01}\ \cancel{-266.65}\ -355.48 + 41.38 = \cancel{-870.63\,N}\ \cancel{-225.27}\ -314.09$$

$$J_x = MF\sin 30^\circ = \cancel{322.45\,N}\ \cancel{159.33\,N}\ 79.665\,N.$$

L-5 Sacral Analysis:



In the ~~plank position the core abdominal muscles are the~~ major ~~stabiliser~~

assuming the erector spinae to act.

$$M.F \times 0.02H = +R_1 \times 0.53H$$

In the plank position assuming the abdominal muscles to be the major stabilisers

$$-(m_f + \cancel{m_L} \times 0.53H + m_L \times 0.23H) \times g$$

$$\Rightarrow M.F \times 0.02H = +407.81 \times 0.53H$$

$$-\left[\left(\frac{1.37 \times 2}{100} \times 0.53\right) + \left(\frac{(14.16 + 4.33) \times 2}{100} \times 0.23\right)\right] \times 86 \times 9.8 \times H$$

$$\Rightarrow M.F \times 0.02 = -0.1 \times 86 \times 9.8 + 216.14$$

$$\Rightarrow M.F = 6593N$$

$$\Sigma F_x = 0 \Rightarrow J_x + M.F \cos 45^\circ = 0$$

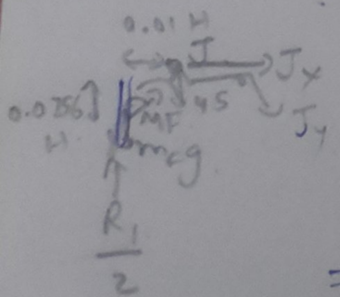
$$\Rightarrow J_x = -\frac{6593}{\sqrt{2}} = -4661.96N$$

$$\Sigma F_y = 0 \Rightarrow J_y + M.F \sin 45^\circ + R_1 = (m_L + m_F) g$$

$$\Rightarrow J_y + 4661.96 + 407.81 = \frac{(14.16 + 4.33 + 1.37) \times 2 \times 86 \times 9.8}{100}$$

$$\Rightarrow J_y = 334.76 - 401.81 - 4661.96 = -4735.01N$$

Ankle Analysis: (The muscle is soleus)



$$\Sigma M_J = 0$$

$$\Rightarrow M.F(\cos 45^\circ \times 0.0756H - \sin 45^\circ \times 0.01H)$$

$$= \left(\frac{R_1}{2} - m_F g\right) \times 0.01H$$

$$\Rightarrow M.F \times 0.046 = \left(\frac{407.81}{2} - \frac{1.37}{100} \times 86 \times 9.8\right) \times 0.01$$

$$\Rightarrow M.F = \cancel{8614.43N}\ \cancel{86.14N}\ 41.82N$$

$\Sigma F_x = 0 \Rightarrow J_x = -MF\cos 45° = \cancel{-60.91.22N} \;-29.57N$

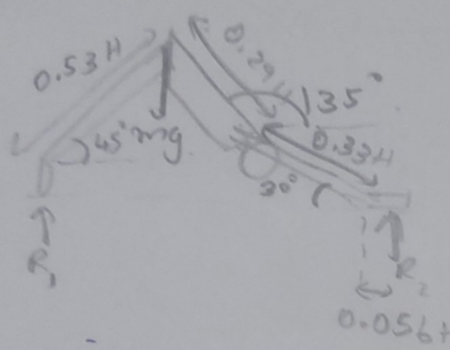
$\Sigma F_y = 0 \Rightarrow J_y = m_f g - \frac{R_1}{2} - MF\sin 45°.$

$= \frac{1.37 \times 86 \times 9.8}{100} - \frac{407.81}{2} - \cancel{\frac{86.14}{\sqrt{2}}} \; 29.57$

$= \cancel{-482.407AP} \; -221.93\,N.$

(ii) Dog-facing ground posture:

Assuming the posture to have the following values with references from slides anthropometric data & online photos



Balancing moment about foot:

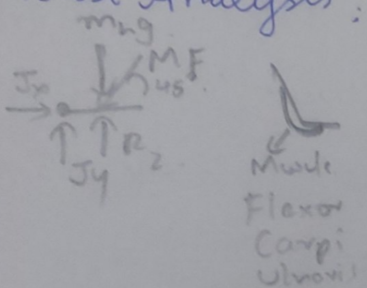
$\Rightarrow mg \times 0.53H \times \cos 45° = R_2 \times [0.53H\cos 45° + 0.29H\cos 45° + 0.33H\cos 30° + 0.056H]$

$\Rightarrow R_2 = \frac{86 \times 9.8 \times 0.53}{0.922 \times \sqrt{2}}$

$= 342.72\,N.$

$\Rightarrow R_1 = mg - R_2 = 500.08N.$

Wrist Analysis: All assumptions valid as per the wrist analysis in the plank part.



$\Sigma M_J = 0 \Rightarrow MF\sin 45° = \frac{R_2}{2} - m_h g.$

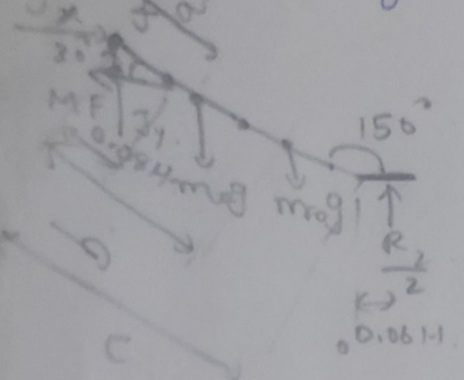
$\Rightarrow MF = \sqrt{2}(342.72/2 - 0.006 \times 9.8 \times 86)$

$\Rightarrow MF = \cancel{417.53N} \; 235.19N.$

$\Sigma F_y = 0 \Rightarrow J_y = 0.$

$\Sigma F_x = 0 \Rightarrow J_x = MF\cos 45° = \cancel{337.66N} \; 166.30N.$

Shoulder Analysis: Carrying forward the same assumptions from shoulder analysis in plank posture



$$a = \frac{57.72}{100} \times 0.1861 = 0.1074$$

$$b = 0.1861 + \frac{45.74}{100} \times 0.146H = 0.253H$$

$$c = 0.186H + 0.146H = 0.332H$$

$$\Sigma M_J = 0 \Rightarrow m_{ua}g \times a\cos 30° + m_{fa}g \times b\cos 30° + MF \sin 30° \times 0.08H$$
$$= \frac{R_2}{2}(c\cos 30° + 0.06H)$$

$$\Rightarrow \left(0.0271 \times 0.107 \times \frac{\sqrt{3}}{2} + 0.0162 \times 0.253 \times \frac{\sqrt{3}}{2}\right) \times 86 \times 9.8 + MF \times 0.04$$
$$= \frac{342.72}{2}\left(0.332 \times \frac{\sqrt{3}}{2} + 0.06\right)$$

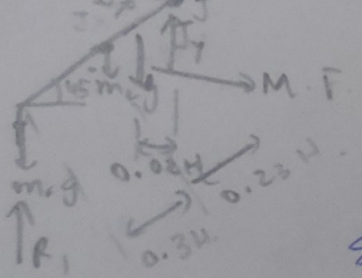
$$\Rightarrow MF = \frac{-5.11 + \cancel{119.19}\ 59.55}{0.04} = \cancel{2849.81N}\ 1361.03N$$

$$\Sigma F_x = 0 \Rightarrow J_x = MF = \cancel{2849.81N}\ 1361.03N$$

$$\Sigma F_y = 0 \Rightarrow J_y = (m_{ua} + m_{fa})g - \frac{R_2}{2}$$
$$= (0.0271 + \cancel{0.0107}\ 0.0162) \times 9.8 \times 86 - \frac{342.72}{2}$$

$$\Rightarrow J_y = \cancel{-306.23N}\ -134.87N$$

L-5 Sacral Analysis:



Going with the same assumptions for the sacral analysis in the plank part.

$$\Sigma M_J \Rightarrow MF \times 0.02H + m_L g \times 0.23 \times \cos 45° + m_f g \times 0.53 \times \cos 45°$$
$$= R_1 \times 0.53 \times \cos 45°$$

$$\Rightarrow M.F = \frac{560.08 \times 0.53 - \left(\frac{1.37 \times 2}{100} \times 0.53 + \frac{(14.16 + 4.33) \times 2 \times .23}{100}\right) \times 86 \times 9.8}{\sqrt{2} \times 0.02}$$
$$= \cancel{128.02N}\ 6403.55N$$

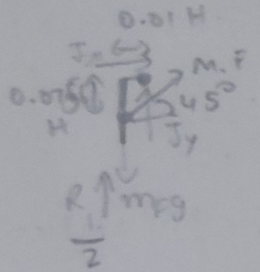
$\Sigma F_x = 0 \Rightarrow J_x = 0.$

$\Sigma F_y = 0 \Rightarrow J_y = M.F + (m_L + m_F)g - R_1$

$\Rightarrow J_y = 6403.55 + \frac{(14.16 + 4.33 + 1.37)}{100} \times 2 \times 86 \times 9.8 - 500.08$

$= 6238.23N.$

Ankle Analysis: The muscle is soleu.



$\Sigma M_J = 0.$

$M.F(\cos 45^\circ \times 0.0756 - \sin 45^\circ \times 0.01H)$

$= (\frac{R_1}{2} - m_F g) \times 0.01H$

$\Rightarrow M.F \times 0.046 = (\frac{500.08}{2} - \frac{1.37}{100} \times 86 \times 9.8) \times 0.01$

$\Rightarrow M.F = \cancel{106.2011}.\ 51.85N.$

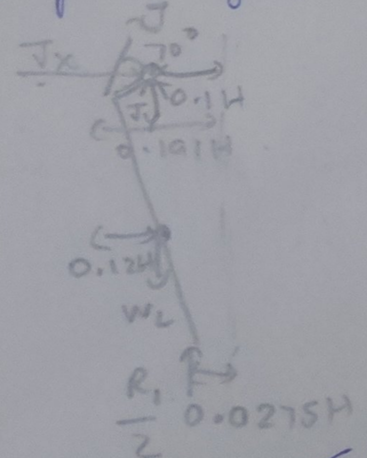
$J_x = -MF\cos 45^\circ = \cancel{-75.09N}.\ -36.66N$

$J_y = m_F g - \frac{R_1}{2} - MF\sin 45^\circ.$

$= \frac{1.37}{100} \times 86 \times 9.8 - \frac{500.08}{2} - \frac{\cancel{106.2}\ 51.85}{\sqrt{2}}.$

$\Rightarrow J_y = \cancel{-583.63N}\ -275.16N.$

Hip Analysis: The muscle is gluteus medius.



$\Sigma M_J = 0$

$\Rightarrow M.F\sin 70^\circ \times 0.091H = W_L \times 0.02H$
$+ R_1 \times (0.191 - 0.0275H)$

$\Rightarrow M.F \times 0.07 = 334.76 \times 0.02$
$+ \frac{500.08}{2}(0.16)$

$\Rightarrow M.F = \cancel{1263.69N}.\ 667.16N.$

$\Sigma F_x = 0 \Rightarrow J_x = -MF\cos 70^\circ = \cancel{-800.32N}\ -422.53N$

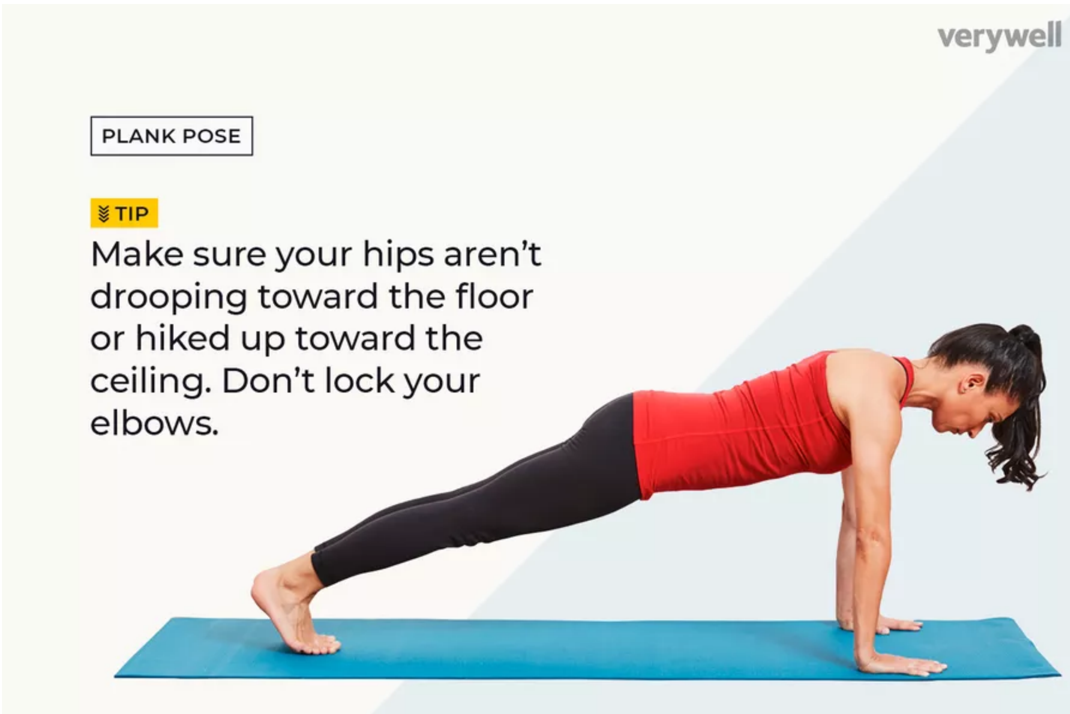
$\Sigma F_y = 0 \Rightarrow J_y = W_L - R_1/2 - MF\sin 70^\circ.$

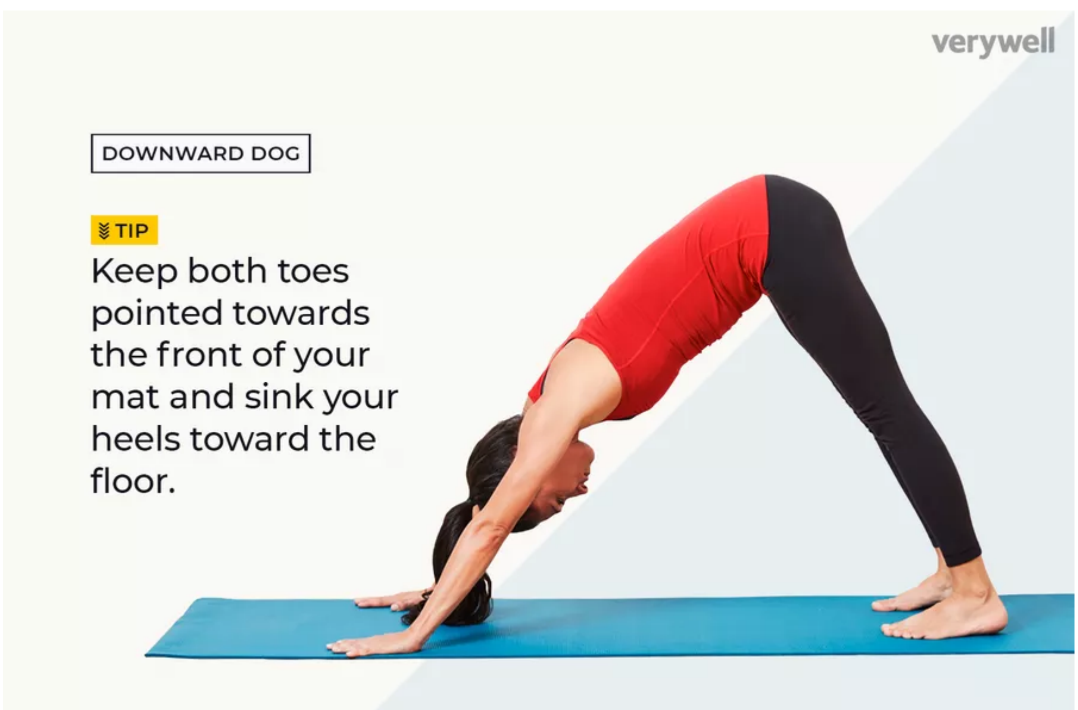
$= 334.76 - \frac{500.08}{2} - \cancel{800.31}\ \cancel{1263.69}\ 667.16N\ \sin 70$

$\Rightarrow J_y = \cancel{-1143.28N}$
$-431.59N.$

Question 1 Picture Source

Source Link:
https://www.verywellfit.com/illustrated-stepbystep-sun-salutation-3567187





Question 2

Stance Phase:

Initial Contact:

- The moment when the foot makes contact with the ground.
- Kinematics:
    - ❖ Ankle: Neither plantarflexed nor dorsiflexed.
    - ❖ Knee: It is slightly hyperextended.
    - ❖ Hip: It is flexed at around 30 degrees.
- Ground Reaction Force: A peak is observed as the heel makes impact on the ground.
- Moments induced and Power:
    - ❖ Ankle: A dorsiflexing moment would be acting opposing the GRF which will try to plantarflex the foot and hence power will be negative.
    - ❖ Knee: A flexing moment acts on the knee opposing hyperextension and the power is thus positive.
    - ❖ Hip: The GRF acts to flex the hip while the muscles try to extend it. As it is in overall motion direction the power is positive.
- Key Muscle Actions: To stop swinging of the leg, hamstrings provide eccentric action. The quadriceps cause the extension of the knee. The tibialis anterior prepares to act upon heel rocker.

Loading Response:

- This is the initial double support period when the limb is accepting the weight
- Kinematics:
    - ❖ Ankle: As heel-rocker takes place till the foot becomes flat the ankle plantarflexes.
    - ❖ Knee: The knee flexes and this provides shock absorption for heel contact.
    - ❖ Hip: Hip starts to extend from its flexed position.
- Ground Reaction Force: Along with opposition to vertical force the added friction causes the net GRF to act completely in the opposite direction of motion. The vertical GRF increases till maximum is reached.
- Moments induced and Power:

    - ❖ Ankle: A dorsiflexing moment would be acting opposing the GRF which will try to plantarflex the foot and hence power will be negative.
    - ❖ Knee: A flexing moment acts on the knee and the power is positive.
    - ❖ Hip: Hip shows extensor moment. The power is positive.
- Key Muscle Actions: The quadriceps contribute to controlled flexion of the knee. Gluteus maximus also acts in this phase. The tibialis anterior acts to pull the tibia as the person moves.

Mid Stance:

- The body advances over the supporting-limb and moves ahead of the stance-limb.
- Kinematics:
    - ❖ Ankle: The ankle rocker-motion and dorsiflexion of the foot occurs post-foot-flat position. The ankle reaches zero and moves beyond.
    - ❖ Knee: Continues flexing till it reaches 20 degrees and then extends till zero.
    - ❖ Hip: Extends till anatomical zero position and beyond.
- Ground Reaction Force: The net GRF decreases till a local minimum as foot flat is achieved. The GRF is almost vertical due to the reduction of horizontal GRF.
- Moments induced and Power:
    - ❖ Ankle: A plantarflexing moment is observed as the force now has shifted to the front and tries to dorsiflex the ankle. The power is negative.
    - ❖ Knee: A extensor moment is applied as the GRF tries to flex the knee. The power is positive.
    - ❖ Hip: The extensor moment decreases and becomes a flexion moment as the GRF is positive. The power transitions from negative to positive.
- Key Muscle Actions: The quadriceps and soleus muscles act in this phase. Soleus and Gastroc muscles act on the ankle in the later stage.

Mid Stance:

- This is the last phase of single-support and ends with opposite initial contact.
- Kinematics:
    - ❖ Ankle: Dorsiflexes till a maximum of 5-10 degrees and then plantarflexes as foot rocker takes place.
    - ❖ Knee: Stays extended and starts to flex towards the end.
    - ❖ Hip: Further extended until it reaches -20 degrees.

- Ground Reaction Force: The vertical GRF again starts increasing till it reaches a maximum as the leg prepares for push-off. The net GRF is in the direction of motion as friction acts forward accelerating the body as the body pushes the ground back.
- Moments induced and Power:
    - ❖ Ankle: The plantarflexing moment increases until a peak and then reduces slightly. Power is initially negative but then increases until a peak.
    - ❖ Knee: Turns extensor from flexor and power oscillates between positive and negative.
    - ❖ Hip: Continues to flex and absorbs power.
- Key Muscle Actions: Triceps surae acts to shift the foot forward to the forefoot rocker position.

Pre-Swing:

- The weight is shifted to the other limb as preparation for swing occurs.
- Kinematics:
    - ❖ Ankle: Due to forefoot rocker continues to plantarflex.
    - ❖ Knee: Continues flexion
    - ❖ Hip: Starts to flex.
- Ground Reaction Force: The foot starts to leave the ground after reaching a maximum and the other foot starts to accept the load. Thus the GRF decreases until lift off when it becomes zero.
- Moments induced and Power:
    - ❖ Ankle: As GRF reduces, the plantarflexion moment continues to decrease and the power generated also decreases.
    - ❖ Knee: The knee starts to flex due to hip flexion and thus an extensor moment is applied. The power is negative.
    - ❖ Hip: Still shows a flexor moment which is reducing and the power transitions from negative to positive.
- Key Muscle Actions: Iliopsoas acts to lift the leg. The quadriceps acts eccentrically to prevent flexion in the knee.

Swing Phase:

Initial-Swing:

- The first 1/3rd of the swing phase which ends with the knee being adjacent.
- Kinematics:
    - ❖ Ankle: Achieves maximum plantarflexion. Post this starts dorsiflexing to make clearance as the foot leaves the ground.
    - ❖ Knee: Flexes till a maximum of 45-60 degrees and then starts extending.
    - ❖ Hip: Flexes beyond the anatomical zero position.
- Ground Reaction Force: The GRF is zero as the foot is in the air.
- Moments induced and Power:
    - ❖ Ankle: A dorsiflexing moment is produced as the ankle starts to plantarflex due to gravity. The magnitude is negligible though.
    - ❖ Knee: A extensor moment is applied due to gravity and hip flexion. Power continues being absorbed.
    - ❖ Hip: Continues providing decreasing flexor moment. Power generation reaches maximum.
- Key Muscle Actions: The triceps surae start acting concentrically to dorsiflex the foot. The iliopsoas muscle continues to act to provide moments to lift the foot.

Mid-Swing:

- The second 1/3rd of the swing phase which ends with the tibia being vertical.
- Kinematics:
    - ❖ Ankle: Dorsiflexion continues beyond the neutral position.
    - ❖ Knee: Continues extending.
    - ❖ Hip: Flexes till maximum and stays there.
- Ground Reaction Force: The GRF is zero as the foot is in the air.
- Moments induced and Power:
    - ❖ Ankle: Continues showing negligible moment and power.
    - ❖ Knee: Shows flexor moment and power continues being negative.
    - ❖ Hip: Shows extensor moment and power is negative.
- Key Muscle Actions: The triceps surae acts in this phase. The hip flexor acts though the moment is low.

Terminal-Swing:

- The last 1/3rd of the swing phase and the gait cycle itself where the knee extends in preparation for the next initial contact.
- Kinematics:
    - ❖ Ankle: Dorsiflexion slightly and then plantarflexes in preparation for the next gait cycle.
    - ❖ Knee: Fully extends in preparation for the next gait cycle.
    - ❖ Hip: Stays at the maximum flexed position.
- Ground Reaction Force: The GRF is zero as the foot is in the air.
- Moments induced and Power:
    - ❖ Ankle: Continues showing negligible moment and power.
    - ❖ Knee: Shows flexor moment and power reaches maximum and decreases to zero.
    - ❖ Hip: Continues showing increasing extensor moment. Power is almost zero as the hip angle doesn't change much.
- Key Muscle Actions: The triceps surae contracts to keep the foot in place. Quadriceps act concentrically to extend the knee. Hamstrings provide an eccentric action.

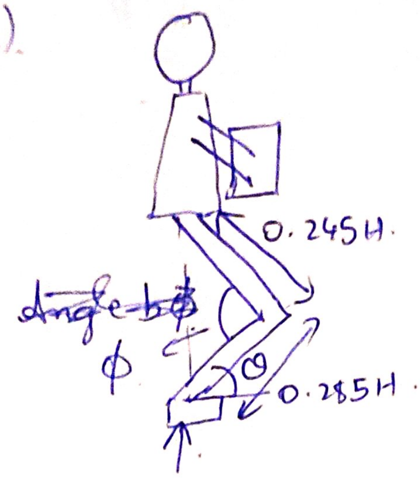
Question 3 Solution

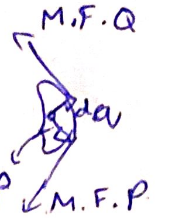
Product Name	Picture	Key Features	Disadvantages	Suggestions
Ossur Balance™ Knee		Possesses a four-bar geometric design which can be easily adjusted to optimize the required balance between stability and walking dynamics for each individual. Balance Knee facilitates easy kneeling. The design is easily adjustable without requiring disassembly. Mid-swing shortening allows the prosthesis to swing more easily through each step.	Can only support a single slow speed although it is marketed as the USP of the product. Although modelled for in-home use doesn't possess flexion resistance to assist easy sitting.	The weight of the knee is at 600 grams and this can be improved for a prosthetic knee without pneumatic or computerised assistance.
Ossur Balance™ Knee OFM1		It is a mechanical polycentric knee joint that is suitable for users with high safety requirements. The knee can be locked (e.g. when standing) to cater to new amputees and less mobile users. Ideal for first standing and walking exercises Flexion and extension can be released depending on the patient's progress More ground clearance for a safer and more confident gait Rates of flexion and extension can be adapted to the user	Is heavier than the conventional balance knee and also less mobile. Locking is the significant safety feature but knee design offers lesser flexibility and is more analogous to a rigid knee.	The servicing of the knee can be made simpler and more adjustments can be incorporated.

Ottobock 3R67™ Knee		Customised for children this knee offers easy movements, a large flexion angle and a high level of stability support. It is a hydraulic knee with powerful swing phase control and has a large flexion angle. It supports walking, running and outdoor mobility.	Supports limited body weight and is relatively bulky even for lightweight users. The maintenance and product life is again a worry with the product complexity.	As scaling down weight in hydraulic prosthetic knees is a harder goal, it can be improved to support heavier users to cater to a larger scope of users.
Plié® 3 Microprocessor Controlled Knee		This is a microprocessor polycentric knee with interchangeable battery feature. The battery cap is watertight and it offers features such as stance flexion resistance and an integrated alignment guide. Offers customised stumble recovery and also can endure slight water exposure.	Caters only to users with adequate hip strength in flexion and extension. As this is a computerised knee will involve a steep learning curve for the user and also increased servicing requirements which also can never be self tended to.	The water exposure limits can be increased and the reliance on hip strength can be reduced to cater to a wider user profile.
Ottobock C-Leg		The Cockpit app lets you operate the C-Leg directly using your smartphone – Android or iOS – and access information about the joint, such as the battery charge level. Makes precise real-time adjustments helping navigate ramps, stairs, and other rugged surfaces and even walking backwards. Offers an Intuitive Stance feature, which recognizes you have stopped moving and dampens the knee in a slightly flexed position thereby providing stance support.	Cost is very high and compatible only with custom feet which are again on the expensive side. It doesn't offer a replaceable battery feature though the battery life is around 40 hours.	A replaceable battery feature can be provided so that parallel charging of batteries can enable continuous usage.

Using anthropometric data from the slide.

(4.)





The ratios of moment arms of the quadriceps and ~~patellofemoral~~ patellar tendon are taken from an online source.

Balancing moment about patellofemoral joint.

$$\text{M.F.Q}\left(\begin{array}{c}\text{quadriceps}\\ \text{Muscle force}\end{array}\right) \times d_q = \text{M.F.P}\left(\begin{array}{c}\text{patellar tendon}\\ \text{force}\end{array}\right) \times d_p$$

$$\Rightarrow \frac{\text{M.F.Q}}{\text{M.F.P}} = \frac{d_p}{d_q} = \frac{18.4}{17.1} = 1.076$$

Assuming the leg origination point to be directly above the feet.

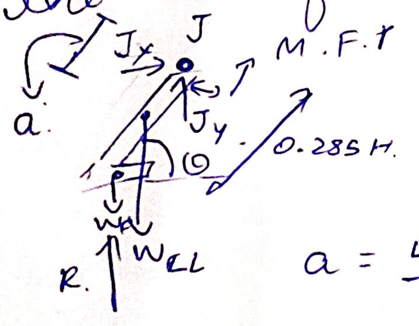
$$\Rightarrow 0.285H \times \cos\theta = 0.245\cos(\phi - \cancel{180 \pm}\ \theta).$$

$$\Rightarrow \frac{0.285}{0.245} = \frac{\cos\phi\cos\theta + \sin\phi\sin\theta}{\cos\theta} = \cos\phi + \sin\phi\tan\theta.$$

$$\Rightarrow \cancel{\tan}\ \theta = \tan^{-1}\left[\left(\frac{0.285}{0.245} - \cos\phi\right) \times \frac{1}{\sin\phi}\right].$$

Separating the ~~leg~~ portion below the knee (SHANK) about the tibiofemoral joint

~~The value of patellofemoral moment arm of patellar tendon about tibiofemoral joint from o~~



$$\Sigma M_J = 0$$

$$R \times 0.285H = W_F \times 0.285H\cos\theta + \cancel{J_F} \ d_p \times MFP + W_{LL} \times a\cos\theta$$

$$a = \frac{43.95 \times 0.285H}{100} = 0.125H.$$

$$W_{LL} = \frac{4.33 \times W}{100} = 0.0433\,W.$$

$$W_F = 0.0137\,W.$$

d_p = Value of patellar tendon force moment arm about tibiofemoral joint taken from online source = 49 mm.

Knowns: W, H, Shank Length, Thigh Length, Shank to Thigh Angle (phi), Moment Arm Ratios, Patellar Tendon Force Moment Arm, External Load

Unknowns: Patellar Tendon Force, Patellofemoral Joint Reaction, Tibiofemoral Joint Reaction, Quadriceps Force, Theta.

cos(theta)

x

$$\Rightarrow \frac{(W+10)\times 0.285H - (0.0137W)\times 0.285H\cos\theta - (0.0433W)\times \cancel{0.285}\ 0.125H\cos\theta}{0.049} = M.F.P.$$

~~J.R.~~ $\sum F_x = 0 \Rightarrow J_x = -M.F.P\cos\theta$

$\sum F_y = 0 \Rightarrow J_y = W_F + W_{LL} - R = (0.0433+0.0137)W - (W+10)$

-MFP x sin(theta)

-MFP x sin(theta)

J_x and J_y are the two perpendicular components of the tibiofemoral joint force.

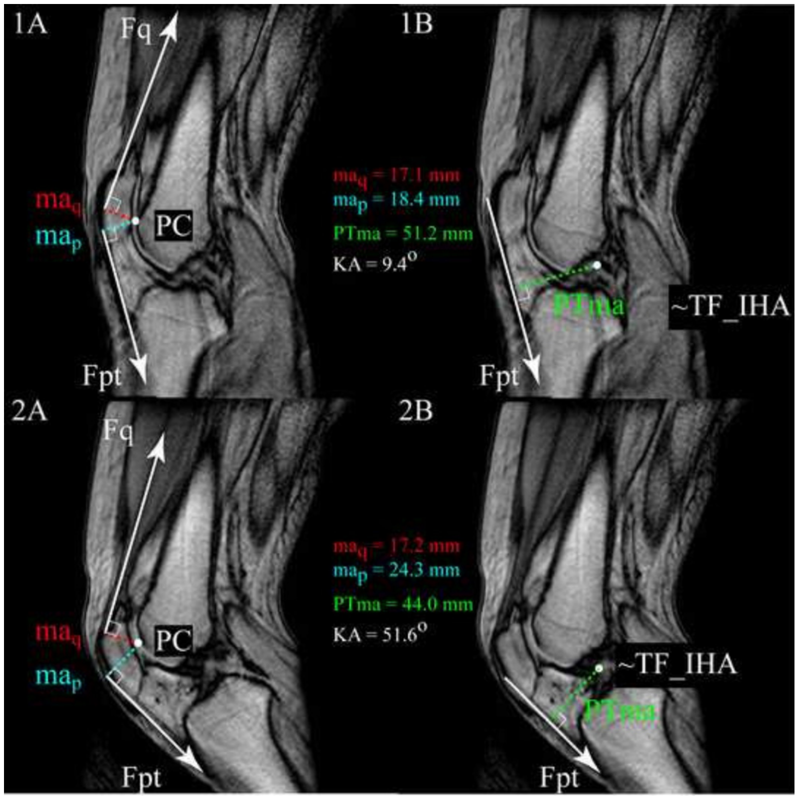
M.F.P $\rightarrow$ patellar tendon force.

Let ~~Q_f be~~ $Q_x\hat{i} + Q_y\hat{j}$ be the patellofemoral joint force then,

$$Q_x = M.F.P\cos\theta + M.F.Q\cos(\phi-\theta)$$

$$Q_y = M.F.P\sin\theta - M.F.Q\sin(\phi-\theta)$$

Link used for Moment Arm ratio of Patellar Tendon Force to Quadriceps force about the patellofemoral joint: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4662057/



Link used for finding moment arm of patellar tendon force about tibiofemoral joint: https://me.queensu.ca/People/Deluzio/JAM/files/6.26.2009_Stacey.pdf

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Table 1
Maximum patella tendon moment arm length, the angular position where it occurs and method of calculation as reported in the literature (M: male, F: female)

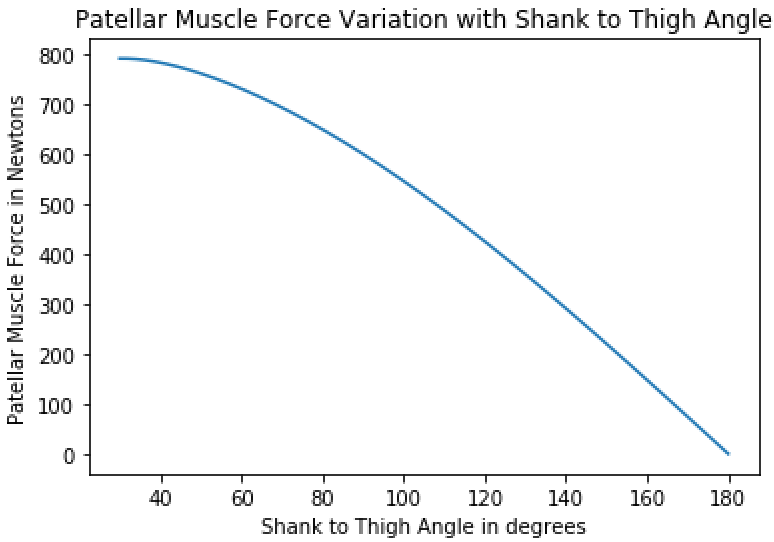
	n	Age	Height (m)	Body mass (kg)		Peak moment arm (mm)	Knee flexion angle (deg)	Method
Smidt (1973)	26	28	1.76	82	In vivo	49	30	ICR
Krevolin et al. (2004)	6	–	–	–	In vitro	51.9	45	ISA
Imran et al. (2000)	–	–	–	–	2D-model	54	30	IP
Gill and O'Connor (1996)	–	–	–	–	2D-model	41.9	~120	IP
Baltzopoulos (1995)	5	20.8 (SD 3. 9)	1.79 (SD 0.03)	79 (SD 7.2)	In vivo	39.87	45	TFCP
Kellis and Baltzopoulos (1999)	10	23 (SD 1.5)	1.74 (SD 0.04)	74 (SD 3.8)	In vivo	42.6	45	TFCP
Nisell et al. (1986) M	10	27	1.82	75	In vivo	46.2	60	TFCP
Nisell et al. (1986) F	10	23	1.67	59	In vivo	37.9	60	TFCP
Wretenberg et al. (1996) M	10	29 (SD 5)	1.81 (SD 0.06)	79 (SD 7.8)	In vivo	50.8	0	TFCP
Wretenberg et al. (1996) F	7	25 (SD 5)	1.65 (SD 0.03)	60 (SD 6.7)	In vivo	47.1	30	TFCP
Herzog and Read (1993)	5	79.2	–	–	In vitro	52.8	30	TFCP
Lindahl and Movin (1967)	15	–	–	–	In vivo	48	30	TFCP
Yamaguchi and Zazac (1989)	–	–	–	–	2D-model	43	40	TFCP
Lu and O'Connor (1996)	–	–	–	–	2D-model	47.9	40	TFCP
Buford et al. (1997)	15	55.9	–	–	In vitro	51.1	0	TE

The computations were performed for 1500 steps in a python notebook and it can be found in the link below:
https://github.com/tarunprasadoff/College/blob/master/MOHM/MOHM%20Endsem%20Question%204.ipynb

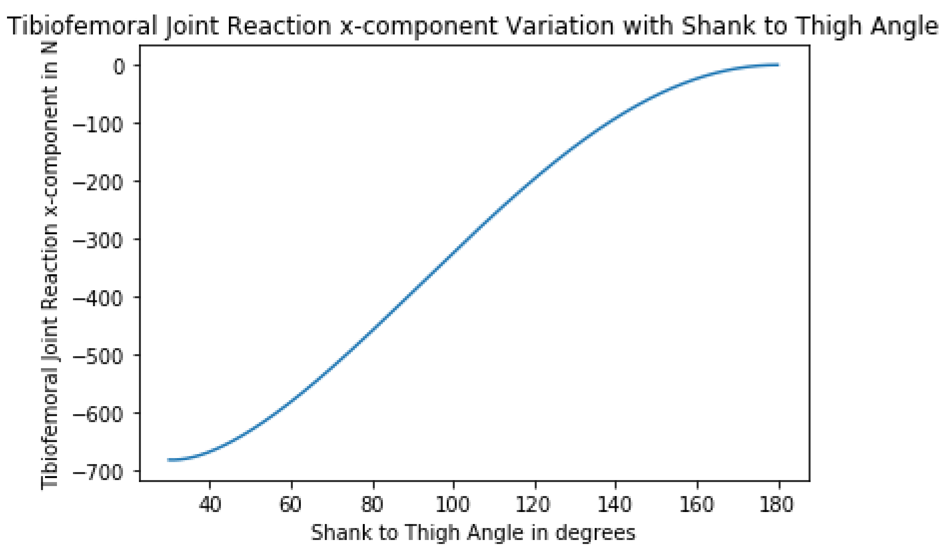
Some of the computed Values:

	phi	MFP	Jx	Jy	Jm	Qx	Qy	Qm
0	30.000000	7.920383e+02	-6.808197e+02	-495.831551	842.237714	1.533002e+03	4.156011e+02	1.588339e+03
1	30.100067	7.920551e+02	-6.808485e+02	-495.815897	842.251765	1.533067e+03	4.140670e+02	1.588001e+03
2	30.200133	7.920692e+02	-6.808729e+02	-495.802630	842.263671	1.533122e+03	4.125397e+02	1.587656e+03
3	30.300200	7.920809e+02	-6.808929e+02	-495.791728	842.273453	1.533167e+03	4.110194e+02	1.587305e+03
4	30.400267	7.920900e+02	-6.809086e+02	-495.783172	842.281129	1.533202e+03	4.095058e+02	1.586948e+03
5	30.500334	7.920967e+02	-6.809201e+02	-495.776941	842.286719	1.533228e+03	4.079990e+02	1.586585e+03
6	30.600400	7.921009e+02	-6.809273e+02	-495.773015	842.290240	1.533244e+03	4.064990e+02	1.586216e+03
7	30.700467	7.921027e+02	-6.809303e+02	-495.771375	842.291712	1.533251e+03	4.050056e+02	1.585840e+03
8	30.800534	7.921020e+02	-6.809292e+02	-495.771999	842.291151	1.533249e+03	4.035188e+02	1.585459e+03
9	30.900600	7.920989e+02	-6.809239e+02	-495.774869	842.288578	1.533237e+03	4.020387e+02	1.585071e+03
10	31.000667	7.920935e+02	-6.809145e+02	-495.779963	842.284007	1.533216e+03	4.005651e+02	1.584677e+03
11	31.100734	7.920857e+02	-6.809011e+02	-495.787264	842.277458	1.533185e+03	3.990981e+02	1.584278e+03
12	31.200801	7.920755e+02	-6.808837e+02	-495.796750	842.268947	1.533146e+03	3.976375e+02	1.583873e+03
13	31.300867	7.920631e+02	-6.808622e+02	-495.808403	842.258490	1.533098e+03	3.961834e+02	1.583462e+03
14	31.400934	7.920483e+02	-6.808369e+02	-495.822203	842.246105	1.533041e+03	3.947357e+02	1.583045e+03
15	31.501001	7.920313e+02	-6.808076e+02	-495.838131	842.231807	1.532975e+03	3.932943e+02	1.582622e+03

1485	178.599066	1.041441e+01	-1.177088e-01	-101.511741	101.511809	2.650449e-01	-7.914351e-01	8.346366e-01
1486	178.699133	9.670554e+00	-1.014945e-01	-100.768021	100.768072	2.285352e-01	-7.349454e-01	7.696578e-01
1487	178.799199	8.926694e+00	-8.648106e-02	-100.024275	100.024312	1.947295e-01	-6.784465e-01	7.058394e-01
1488	178.899266	8.182827e+00	-7.266855e-02	-99.280504	99.280531	1.636278e-01	-6.219391e-01	6.431037e-01
1489	178.999333	7.438954e+00	-6.005698e-02	-98.536712	98.536730	1.352303e-01	-5.654239e-01	5.813703e-01
1490	179.099400	6.695075e+00	-4.864639e-02	-97.792898	97.792910	1.095371e-01	-5.089017e-01	5.205567e-01
1491	179.199466	5.951191e+00	-3.843682e-02	-97.049067	97.049074	8.654822e-02	-4.523731e-01	4.605779e-01
1492	179.299533	5.207302e+00	-2.942831e-02	-96.305219	96.305223	6.626374e-02	-3.958388e-01	4.013468e-01
1493	179.399600	4.463409e+00	-2.162087e-02	-95.561357	95.561359	4.868373e-02	-3.392996e-01	3.427745e-01
1494	179.499666	3.719513e+00	-1.501454e-02	-94.817483	94.817484	3.380824e-02	-2.827562e-01	2.847702e-01
1495	179.599733	2.975614e+00	-9.609325e-03	-94.073598	94.073599	2.163732e-02	-2.262092e-01	2.272417e-01
1496	179.699800	2.231712e+00	-5.405255e-03	-93.329706	93.329706	1.217102e-02	-1.696594e-01	1.700954e-01
1497	179.799867	1.487809e+00	-2.402339e-03	-92.585807	92.585807	5.409348e-03	-1.131074e-01	1.132367e-01
1498	179.899933	7.439049e-01	-6.005851e-04	-91.841905	91.841905	1.352338e-03	-5.655406e-02	5.657023e-02
1499	180.000000	5.642105e-14	-3.454793e-30	-91.098000	91.098000	7.172230e-30	-4.289319e-15	4.289319e-15

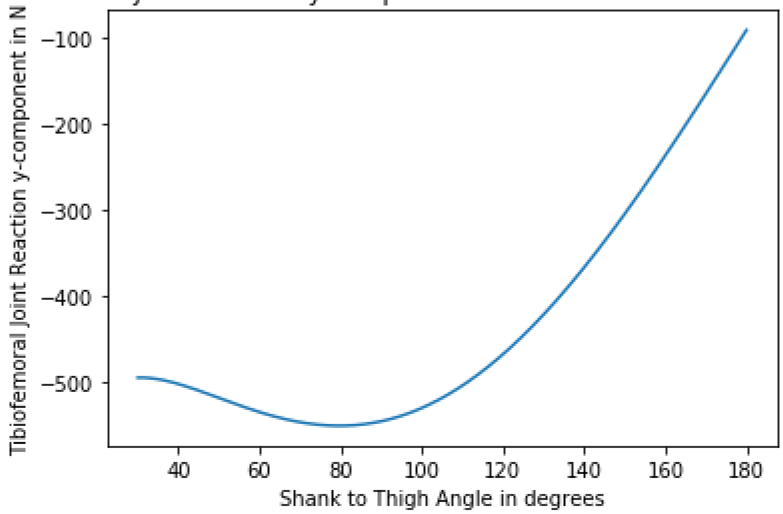


Patellar Muscle Force Variation with Shank to Thigh Angle
Patellar Muscle Force in Newtons
800
700
600
500
400
300
200
100
0
40
60
80
100
120
140
160
180
Shank to Thigh Angle in degrees

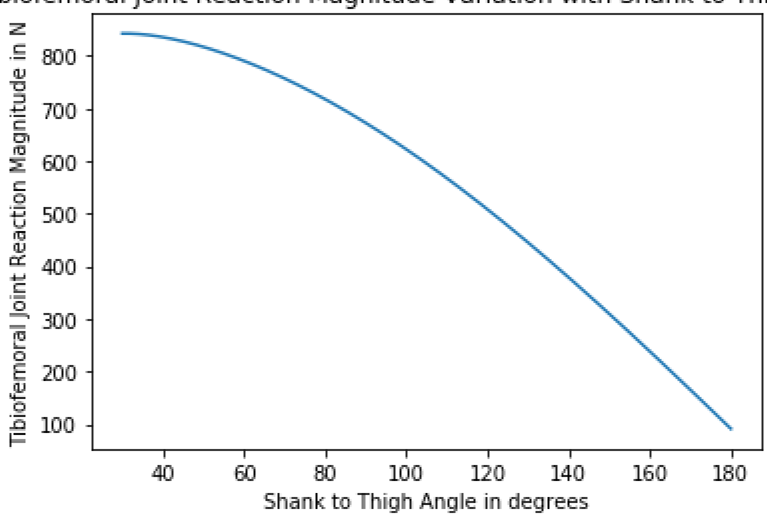


Tibiofemoral Joint Reaction x-component Variation with Shank to Thigh Angle
Tibiofemoral Joint Reaction x-component in N
0
−100
−200
−300
−400
−500
−600
−700
40
60
80
100
120
140
160
180
Shank to Thigh Angle in degrees

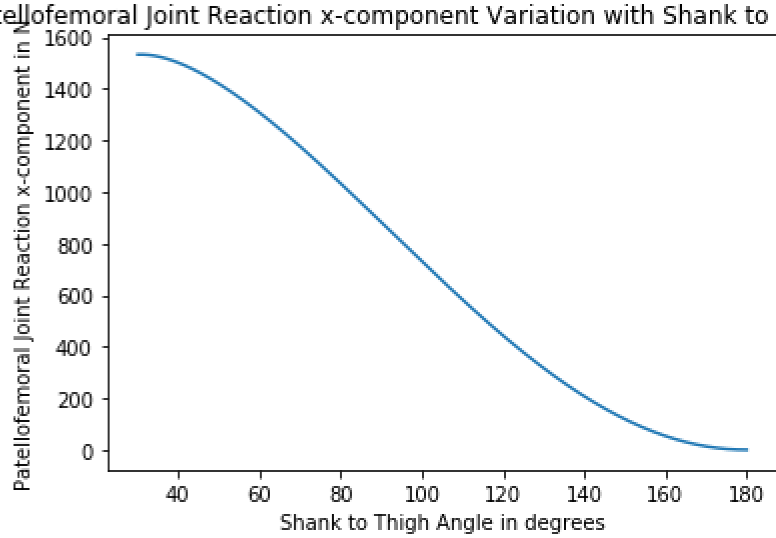
Tibiofemoral Joint Reaction y-component Variation with Shank to Thigh Angle

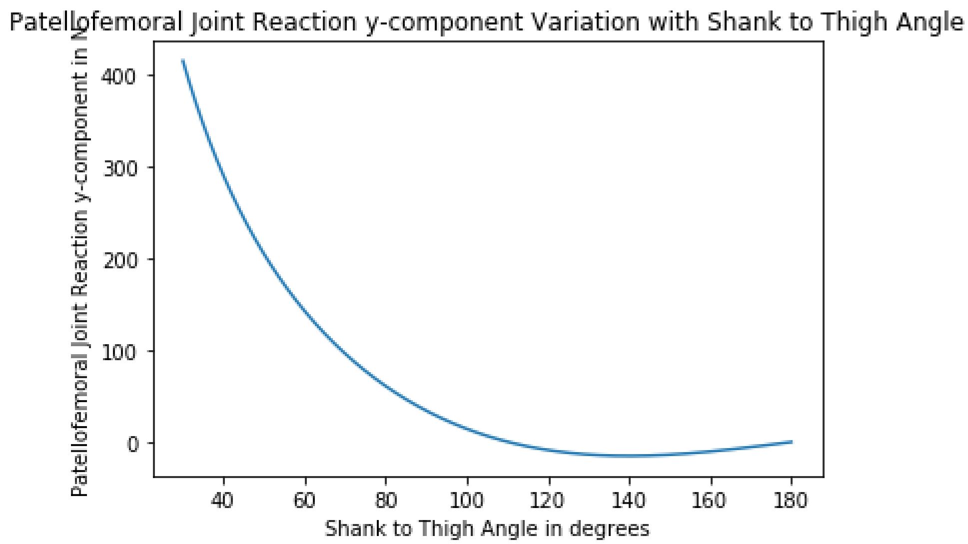


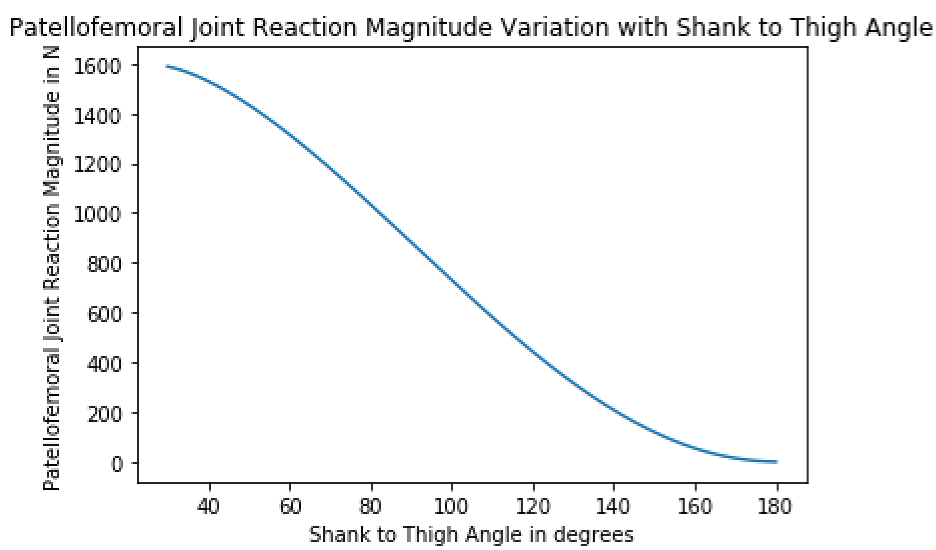
Tibiofemoral Joint Reaction Magnitude Variation with Shank to Thigh Angle



Patellofemoral Joint Reaction x-component Variation with Shank to Thigh Angle




Patellofemoral Joint Reaction y-component Variation with Shank to Thigh Angle
Patellofemoral Joint Reaction y-component in N
400
300
200
100
0
40
60
80
100
120
140
160
180
Shank to Thigh Angle in degrees


Patellofemoral Joint Reaction Magnitude Variation with Shank to Thigh Angle
Patellofemoral Joint Reaction Magnitude in N
1600
1400
1200
1000
800
600
400
200
0
40
60
80
100
120
140
160
180
Shank to Thigh Angle in degrees