26-04-2020 Assignment 5 : AM5650 S. Tarun Prasad
ME17B114

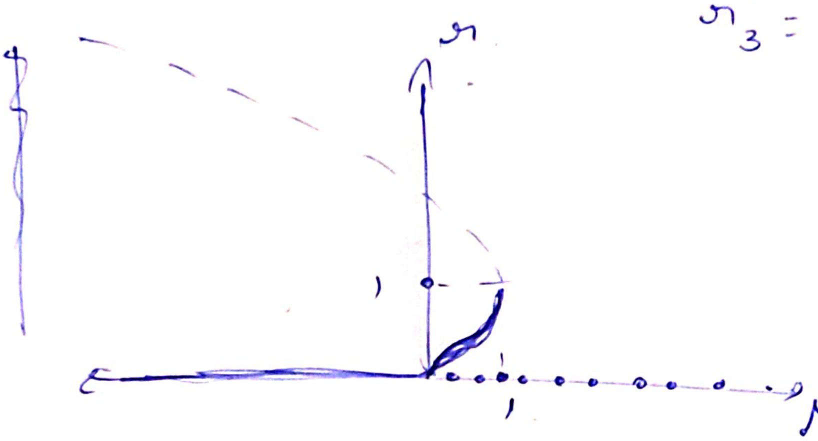
(1.) Equating $\dot{r} = (r^4 - 2r^2 + \mu) r = 0$.

$\Rightarrow r_1 = 0 \rightarrow$ sol. 1.

$\mu \le 1$

$(r^2)^2 - 2(r^2) + \mu = 0 \Rightarrow r_2^2 = \dfrac{2 + \sqrt{4 - 4\mu}}{2} = 1 \pm \sqrt{1-\mu}$

$r_3^2 = 1 - \sqrt{1-\mu} \rightarrow 0 \le \mu \le 1$.



For r_1

$\dot{r}_1' = (r_1')(r_1'^4 - 2r_1'^2 + \mu)$
$= \mu r_1' + HOT$ Stable when $\mu < 0$.

For r_2, $\dot{r}_2' = \cancel{(r_2'^4 - 2r_2'^2 + 2\mu}$

$[(r_2 + r_2')^4 - 2(r_2 + r_2')^2 + \mu](r_2 + r_2')$

$= (4r_2' r_2^3 + r_2^4 + HOT - 2r_2^2 - 4r_2 r_2' + HOT + \mu)(r_2 + r_2') + HOT$

$= r_2'(4r_2^4 - 4r_2^2) + r_1'(r_2^4 - 2r_2^2 + \mu) + HOT$

$= \cancel{-2\mu r_2' + HOT}$ ~~$\rightarrow$ stable when $\mu > 0$, unstable when $\mu < 0$~~ $\rightarrow$ same for r_3

$= 4r_2'((r_2^2)^2 - (r_2)^2) \rightarrow$ Stable $\rightarrow r_2^2 < 1$

Unstable $\rightarrow r_2^2 > 1$

~~For r_3~~ It is a supercritical Hopf bifurcation

(2.) $\dot r = -\mu r + r^2 - r^3 = 0$

$r(-\mu + r - r^2) = 0 \Rightarrow r_1 = 0$

$\Rightarrow r^2 - r + \mu = 0 \Rightarrow r_2 = \dfrac{1+\sqrt{1-4\mu}}{2}, \rightarrow \mu \le \dfrac{1}{4}$

$r_3 = \dfrac{1-\sqrt{1-4\mu}}{2} \rightarrow 0 \le \mu \le \dfrac{1}{4}$

For r_1, $\dot r_1' = (-(r_1+r_1')^2 + (r_1+r_1') - \mu)(r_1+r_1')$

$= (-r_1'^2 + r_1' - \mu) * r_1' = -\mu r_1' + O.T$

$\Rightarrow r_1 \rightarrow$ Stable : $r \ge 0$, Unstable $r \le 0$.

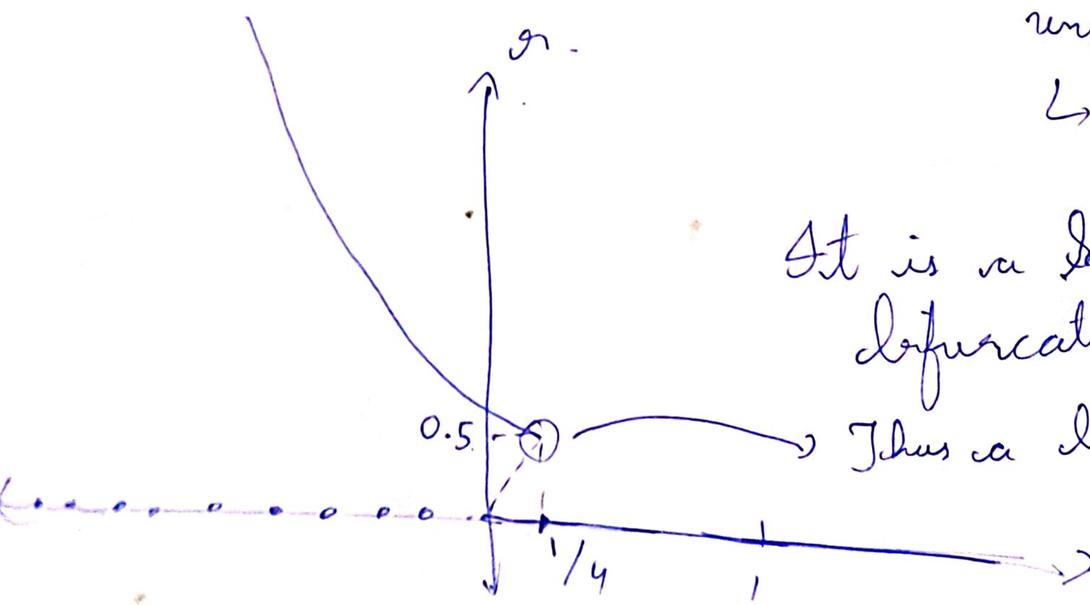
For r_2, $\dot r_2' = (-2r_2 r_2' - r_2^2 + r_2 + r_2' - \mu)(r_2 + r_2')$

$= r_2'(-2r_2^2 + r_2) + \underbrace{(-r_2^2 + r_2 - \mu)}_{0}$

$= r_2'(r_2 - 2r_2^2) \rightarrow$ stable : $r_2 \ge 0.5$.

unstable : $r_2 \le 0.5$

↳ same for r_3



It is a Subcritical Hopf bifurcation.

Thus a limit cycle of radius 0.5 appears at $\mu = 1/4$ as we move from $\mu = 1$ to $\mu = 0$.

(3) From the phase potraits plotted on mathematica,

(a) Supercritical Hopf Bifurcation

(b) Supercritical Hopf Bifurcation

(c) Subcritical Hopf Bifurcation

4) (a) : $f(x,y) = \mu x \quad , \quad g(x,y) = \mu y - x^2 y$

$\omega = z(-1)$

$$a = \frac{1}{16}\left(0 + \cancel{0+} (-2) + 0 - [0 - 0 - 0 + 0]\right)$$

$$= \frac{-2}{16} \rightarrow \text{Supercritical}$$

(b) $f(x,y) = \mu x - x^3, \quad g(x,y) = \mu y - 2y^3, \quad \omega = -1$

$$a = \frac{1}{16}\left(-6 + 0 + 0 - 12 - (0 - 0 + 0 + 0)\right)$$

$$= -\frac{18}{16} \rightarrow \text{Supercritical}$$

(c) $F(x,y) = \mu x - x^2 \quad , \quad g(x,y) = \mu y - 2x^2, \quad \omega = -1$

$$a = \frac{1}{16}\left(0 + 0 + 0 \ 0 - [0 - 0 \mp 8 + 0]\right)$$

$$= \frac{1}{2} \rightarrow \text{Subcritical}$$