Nonlinear harmonic vibration analysis of a plate-cavity system
Term Project: Paper Review | S Tarun Prasad | ME17B114

The paper “Nonlinear harmonic vibration analysis of a plate-cavity system” by Mehran Sadri and Davood Younesian is an important paper in the realm of vibro-acoustic analysis of plate cavity systems. Understanding this paper was a very good experience for me as I was able to appreciate the amount of elegance and methodology put into solving an interesting nonlinear problem. The authors of the paper set out on a goal to analytically model and solve the nonlinear harmonic vibration of a plate-cavity system by addressing the configuration of a cuboidal air cavity-backed by an elastic plate. Importance has been given to formulate a methodological and rigorous process to identify closed-form frequency-amplitude relations and its variation with parameters. This has been done to provide a standard process to tackle vibro-acoustic analysis of plate-cavity systems in an analytical way. The Von-Karman theory models large deflections of flat thin plates with a set of nonlinear differential equations. This has been employed to formulate the governing equations of the plate deflection in the air cavity when the flexible plate is applied with an external acoustic vibration. The authors have used Galerkin’s method for obtaining plate stress and displacement functions which are then substituted in the Von-Karman equations to arrive at coupled nonlinear equations for three first mode shapes. Due diligence has been given by the authors to the fact that the coupling between modes cannot be ignored in nonlinear systems and to investigate how symmetric and unsymmetric modes enhance or cancel out each other in nonlinear systems. This can be seen in the steps where the authors arrive at the fact that external excitation contributions cancel out each other in both unsymmetrical modes due to symmetry between them while a different condition of combinational resonance conditions may exist due to coupling between symmetrical and unsymmetrical mode shapes.

Multiple scales method has been used by the authors to solve the analytical model of the system. The displacement functions of different modes are expressed as summations of different functions whose coefficients are the different non-negative integral powers of a small dimensionless parameter ‘ε’. The substitution of these summations in the coupled nonlinear equations leads to a new set of equations. For each of the primary resonance conditions, its frequency is substituted as the excitation pressure frequency and the terms with coefficients as first and third-order power of the dimensional parameters ‘ε’ are isolated to obtain independent equations. These equations are combined and the secular terms are eliminated from them. The authors come up with a beautiful substitution for the magnitude term of displacement function generated out of the isolation of terms with first-order ‘ε’ as coefficient. The real and imaginary parts are separated out as separate equations with this substitution. The steady-state condition is finally substituted to obtain the final equations. Similar procedure is followed for secondary: super-harmonic resonance condition, sub-harmonic resonance condition and combination resonance conditions. The findings of the paper are revealed in the case study constructed for a steel plate of reasonable dimensional and property values based on the closed-form frequency response. The frequency response curves of different resonance conditions for different levels of acoustic vibrations are studied. For non-symmetric primary resonance conditions, the only stable solution is the steady-state solution of 0 which is identical to linear systems while in the symmetric case non-zero stable solution exists. In the super-harmonic case, the stable zone increases with increasing amplitude whereas in subharmonic case it decreases. The amplitude of second and third modes in secondary resonance conditions vanish identically to linear systems. In the first combination-resonance condition studied the stable zone becomes smaller with an increase in excitation amplitude and the second mode amplitude is larger than the first mode. For the second combination-resonance condition the magnitude of excitation has negligible influence on the frequency curves and the first mode is larger than the third mode. The natural frequency is inversely proportional to the depth of the cavity. Some of the key applications of the paper are the conditions under which the nonlinear system behaves like a linear system. This can be of key-value when the expectation is to modify the behaviour of a given nonlinear system to that of a linear system with the flexibility of changing a few parameters to an extent. The variation of stable zones with excitation amplitudes in different resonance conditions is also of key applicational value. The methodology proposed in the paper serves as an inspiration for research work focussing on analytical approach to plate-cavity vibro-acoustic problems.

One of the main shortcomings of the paper is that it deals with only a single case study for a broad-spectrum of possible cases. The conclusions made from the paper could be of more-scalable value if unsymmetrical and conical cavity cross-sections were studied which would have lead to a more detailed and screened out conclusion set. The effect of the deviation parameter could have been more plausibly studied in the paper. The paper doesn't give sufficient reasoning as to why the information from the terms with second-order 'ε' as their coefficient, has been ignored. It could have been explored in more detail to serve as a more comprehensive methodology to future work along the same lines which is the author's intention. Another shortcoming in the paper is that the transformation of F_{mn} into $\varepsilon * f_{mn}$ is applied in the primary resonance condition but stated only in the context of secondary resonance condition. The reasoning for the same would improve the reader's understanding of the paper. The paper only briefly touches upon the distinctions between structural and acoustic resonance and the topic could have been given more value. State-plane analysis could have been performed for more resonance conditions especially to study points close to stability to instability transitions. A good research problem as a follow up to this paper would be to study and investigate the validity of the conclusions made in this paper on nonlinear harmonic vibration analysis of plate-cavity systems with unsymmetrical and conical cavity cross-sections.