

Matrix Project

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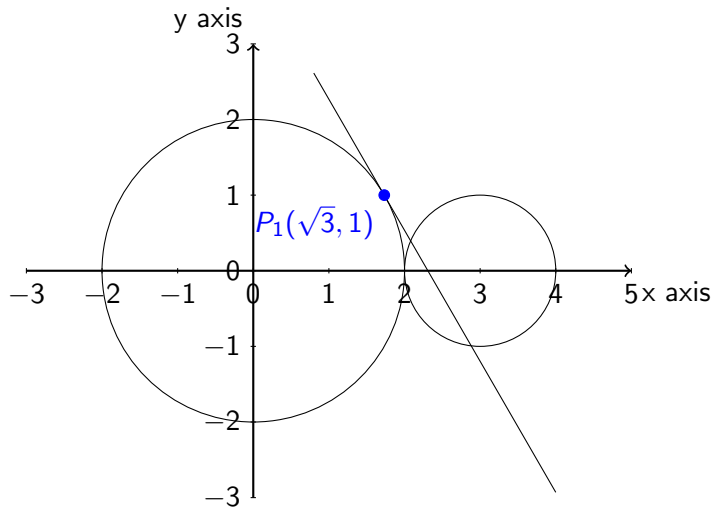
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February 14, 2019

JEE-ADV-2012-P2-Q52(Code 7)

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at point $p(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$. Find the possible equations of L.

Diagram



Matrix Transformation of Question

JEE-ADV-2012-P2-Q52(Code 7)

A tangent PT is drawn to the circle $X^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = 4$,

at a point $P^T = [\sqrt{3} \quad 1]$.

A straight line L, perpendicular to PT is a tangent to the circle

$$X^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X + 2 \begin{bmatrix} -3 & 0 \end{bmatrix} X + 8 = 0.$$

Find the possible Equations of L.

Solution

$$C1 : X^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = 4$$

$$C2 : X^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X + 2 \begin{bmatrix} -3 & 0 \end{bmatrix} X + 8 = 0$$

Comparing the given circle equations with the general equation of quadratic curve : $X^T V X + 2u^T X + F = 0$, we have -

$$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, u_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, F_1 = -4, V_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, u_2 = \begin{bmatrix} -3 \\ 0 \end{bmatrix}, F_2 = 8.$$

The Equation of tangent at $P_1 = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ to the first circle is

$$(P_1^T V_1 + u_1^T) X + P_1^T u_1 + F_1 = 0$$

Solution

$$\begin{bmatrix} \sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X + \begin{bmatrix} 0 & 0 \end{bmatrix} X - 4 = 0$$

$$\begin{bmatrix} \sqrt{3} & 1 \end{bmatrix} X = 4$$

Let n_1 is normal vector to tangent at P_1

Let n_2 is normal vector to tangent at P_2

Let m_1 is direction vector to tangent at P_1

Let m_2 is direction vector to tangent at P_2

Where P_2 is point of contact of second circle and required tangent

Solution

$$n_1 = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}, m_2 = n_1, m_2 = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$$

Now, $n_2^T m_2 = 0$

$$\text{So, } n_2 = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$$

Equation of Tangent to the second Circle is

$$(P_2^T V_2 + u_2^T)X + P_2^T u_2 + F_2 = 0, \implies (1)$$

Which is same as equation of line passing through P_2 with normal vector

$$n_2 \implies n_2^T (X - P_2) = 0 \implies (2)$$

Solution

Comparing the equations (1) and (2) with scaling factor k , we get

$$P_2^T V_2 + u_2^T = kn_2^T \implies (3)$$

$$P_2^T u_2 + F_2 = -kn_2^T P_2 \implies (4)$$

From equation (3),

$$P_2 = ((kn_2^T - u_2^T) V_2^{-1})^T$$
$$P_2 = \begin{bmatrix} k \\ -k\sqrt{3} \end{bmatrix} - \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

$$\text{Therefore, } P_2 = \begin{bmatrix} k+3 \\ -k\sqrt{3} \end{bmatrix}$$

Putting the value of P_2 in equation (4), we get

$$\begin{bmatrix} k+3 & -k\sqrt{3} \end{bmatrix} \begin{bmatrix} -3 \\ 0 \end{bmatrix} + 8 = -k \begin{bmatrix} 1 & -\sqrt{3} \end{bmatrix} \begin{bmatrix} k+3 \\ -k\sqrt{3} \end{bmatrix}$$

Solution

$$\implies -3k - 9 + 8 = -k(3 + 4k)$$

$$\implies -3k - 1 = -3k - 4k^2$$

$$\implies 4k^2 = 1$$

$$\implies k = \pm 1/2$$

Putting the two values of k in P_2 , we get two values for P_2 .

Therefore the two solutions of P_2 are $\begin{bmatrix} 7/2 \\ -\sqrt{3}/2 \end{bmatrix}$ and $\begin{bmatrix} 5/2 \\ \sqrt{3}/2 \end{bmatrix}$

Solution

The equation of a line is given by,

$n^T(X - A)$, where A is any point on the line.

We get the two possible equations of L for two values of P_2 ,

$$\text{Equation 1: } \begin{bmatrix} 1 & -\sqrt{3} \end{bmatrix} \left(X - \begin{bmatrix} 7/2 \\ -\sqrt{3}/2 \end{bmatrix} \right) = 0$$

$$\text{Equation 2: } \begin{bmatrix} 1 & -\sqrt{3} \end{bmatrix} \left(X - \begin{bmatrix} 5/2 \\ 3/2 \end{bmatrix} \right) = 0$$

Solution Diagram

