Matrix Project

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Geometry Question

JEE-ADV-2012-P2-Q52(Code 7)

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at point $p(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$. Find the possible equations of L.

Matrix Transformation of Question

JEE-ADV-2012-P2-Q52(Code 7)

A tangent PT is drawn to the circle

$$X^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = 4,$$

at a point $P^T = \begin{bmatrix} \sqrt{3} & 1 \end{bmatrix}$.
A straight line L, perpendicular to PT is a tangent to the circle
$X^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X + 2 \begin{bmatrix} -3 & 0 \end{bmatrix} X + 8 = 0.$
Find the possible Equations of L.

Solution

$$C1 : X^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = 4$$

$$C2 : X^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X + 2 \begin{bmatrix} -3 & 0 \end{bmatrix} X + 8 = 0$$

Comparing the given circle equations with the general equation of quadratic curve : $X^T VX + 2u^T X + F = 0$, we have -

$$V_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, u_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, F_1 = -4, V_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, u_2 = \begin{bmatrix} -3 \\ 0 \end{bmatrix}, F_2 = 8.$$

The Equation of tangent at $P_1 = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ to the first circle is

$$(P_1^T V_1 + u_1^T)X + P_1^T u_1 + F_1 = 0$$

Solution

$$\begin{bmatrix}\sqrt{3} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\mathrm{X} + \begin{bmatrix}0 & 0\end{bmatrix}\mathrm{X} - 4 = 0$$

$$\begin{bmatrix}\sqrt{3} & 1\end{bmatrix}\mathrm{X} = 4$$

Let n_1 *is normal vector to tangent at* P_1

Let n_2 *is normal vector to tangent at* P_2

Let m_1 *is direction vector to tangent at* P_1

Let m_2 *is direction vector to tangent at* P_2

Where P_2 *is point of contact of second circle and required tangent*

Solution

$$n_1 = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}, m_2 = n_1, m_2 = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$$

Now, $\mathrm{n}_2^T m_2 = 0$

So , $\mathrm{n}_2 = \begin{bmatrix} 1 \\ -\sqrt{3} \end{bmatrix}$

Equation of Tangent to the second Circle is

$$(\mathrm{P}_2^T V_2 + u_2^T)X + P_2^T u_2 + F_2 = 0, \implies (1)$$

Which is same as equation of line passing through

P_2 *with normal vector* $n_2 \implies n_2^T(X - P_2) = 0 \implies (2)$

Solution

Comparing the equations (1) and (2) with scaling factor k, we get

$\mathrm{P}_2^T V_2 + u_2^T = kn_2^T \implies (3)$

$\mathrm{P}_2^T u_2 + F_2 = -kn_2^T P_2 \implies (4)$

From equation (3),

$$P_2 = ((kn_2^T - u_2^T)V_2^{-1})^T$$

$$P_2 = \begin{bmatrix} k \\ -k\sqrt{3} \end{bmatrix} - \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

Therefore, $\mathrm{P}_2 = \begin{bmatrix} k+3 \\ -k\sqrt{3} \end{bmatrix}$

Putting the value of $\mathrm{P}_2 in\ \ equation\ \ (4), we\ \ get$

$\begin{bmatrix} k+3 & -k\sqrt{3} \end{bmatrix} \begin{bmatrix} -3 \\ 0 \end{bmatrix} + 8$=-k$\begin{bmatrix} 1 & -\sqrt{3} \end{bmatrix} \begin{bmatrix} k+3 \\ -k\sqrt{3} \end{bmatrix}$

Solution

$$\implies -3k - 9 + 8 = -k(3 + 4k)$$

$$\implies -3k - 1 = -3k - 4k^2$$

$$\implies 4k^2 = 1$$

$$\implies k = \pm 1/2$$

Putting the two values of k in $\mathrm{P}_2, we\ \ get\ \ two\ \ values\ \ for\ \ P_2.$

Therefore the two solutions of $\mathrm{P}_2\ \ are \begin{bmatrix} 7/2 \\ -\sqrt{3}/2 \end{bmatrix} and \begin{bmatrix} 5/2 \\ \sqrt{3}/2 \end{bmatrix}$

Solution

The equation of a line is given by,

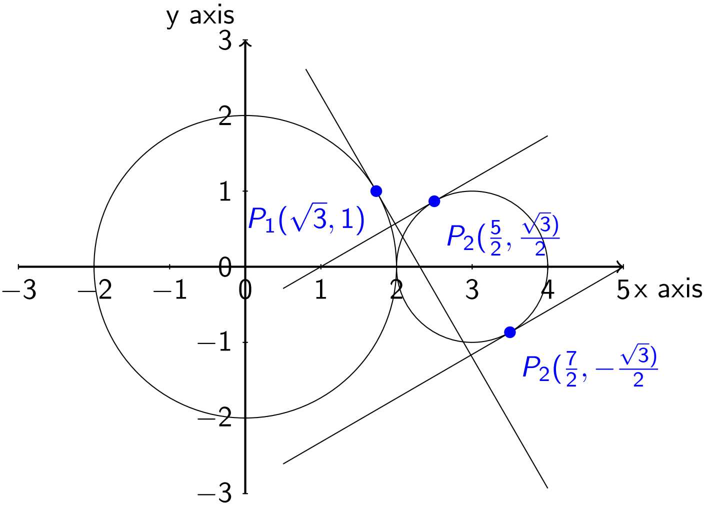
$\mathrm{n}^T(X-A), where\ \ A\ \ is\ \ any\ \ point\ \ on\ \ the\ \ line.$

We get the two possible equations of L for two values of P_2,

Equation 1: $\begin{bmatrix}1 & -\sqrt{3}\end{bmatrix}$ (X-$\begin{bmatrix}7/2\\-\sqrt{3}/2\end{bmatrix}$)=0

Equation 2: $\begin{bmatrix}1 & -\sqrt{3}\end{bmatrix}$ (X-$\begin{bmatrix}5/2\\3/2\end{bmatrix}$)=0

Diagram



y axis
x axis
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
4
5
$P_1(\sqrt{3}, 1)$
$P_2(\frac{5}{2}, \frac{\sqrt{3})}{2}$
$P_2(\frac{7}{2}, -\frac{\sqrt{3})}{2}$