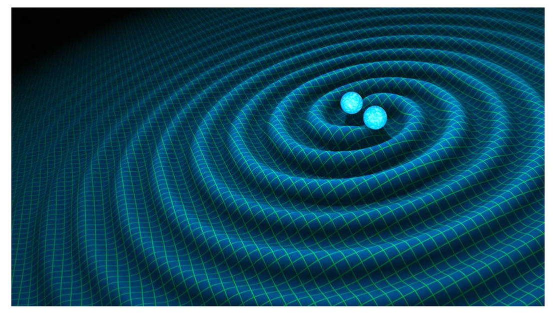
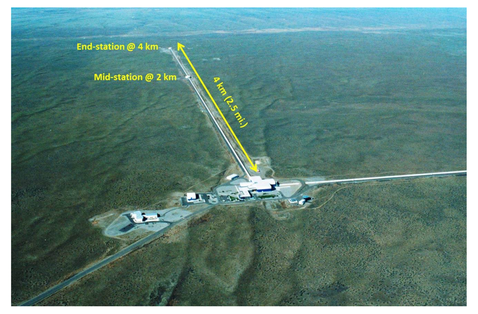
Collisions that Shake the Universe: Systems of Black Holes and Neutron Stars

Tarunyaa Sivakumar and Shivani Nellore

Introduction

Gravitational waves are disturbances or 'ripples' in space-time. They originate from massive accelerating masses such as binary black holes or neutron stars and travel at the speed of light. The first Gravitational waves were detected by LIGO in 2015. This detection was a result of a *binary black hole merger*, an event dubbed GW150914. As time went on, two other types of binary mergers were detected. The first *binary neutron star merger*, GW170817, was detected by LIGO in 2017, and the first *binary black hole - neutron star* merger, GW200105, was detected by LIGO in 2020





This Project attempts to Model the Motion of Different Gravitational Waves Collision events and compare the Timescales of Collisions.

Background

The Theory of General Relativity (GR): Describes gravity as a geometric property of space and time. It states that the curvature of spacetime depends on the energy and momentum of the matter present. It was predicted by Albert Einstein in 1916 and then experimentally confirmed in 1974 with the first discovery of the binary pulsar.

Black Holes: Bodies of intense gravity from which no particles or electromagnetic radiation can escape. Most of the time, they are a result of a large amount of mass being concentrated into tiny space after a star (> 25 M☉) collapse. As a result, following the predictions made by GR, they compress spacetime to such an extent that all paths taken by particles or waves that are past the event horizon, bend back towards its center.

Neutron Stars: Bodies that are the smallest and densest known class of stellar objects, consisting of almost entirely neutrons. Most of the time, they're formed when a star (10 - 25 M☉) collapses. The gravitational field at the neutron star's surface is about 2×10^{11} (200 billion) times that of Earth's gravitational field, hence compressing spacetime to similar extents as a Black Hole.

Binary Black Holes or Neutron Stars: Systems consisting of 2 Black Holes or Neutron Stars in close orbit around each other.

LIGO: Large-scale physics observatory designed to detect gravitational waves. They use mirrors spaced 4 km apart and a set-up of lasers and beam splitters to detect distortions in spacetime that are less than one-thousandth the charge diameter of a proton. This is because, by the time the Gravitational Waves reach the Earth, they have dampened and only compress spacetime by tiny amounts.

Schwarzschild radius: The radius below which the gravitational attraction between the particles of a body must cause it to undergo irreversible gravitational collapse.

Methods

Assumptions: The relationship between Gravitational waves and their sources are analogous to that between Electromagnetic Waves and their sources, i.e. as charges accelerate, they produce Electromagnetic waves and as massive masses accelerate, they produce Gravitational waves. Although GR is a tensor theory and E&M is a vector theory, this assumption is reasonable until the binary objects collide. After this point, this assumption breaks down.

Orbital Angular Frequency

$$\omega^2 = G\frac{m_a + m_b}{r^3} = \frac{GM}{r^3}$$

Schwarzschild radius

$$r_S = \frac{2GM}{c^2}$$

Gravitational Wave Strain Signal: Signal registered by the interferometer in LIGO. Derived from the relative movements of parts of apparatus

$$h(t) = \frac{\Delta L_x(t) - \Delta L_y(t)}{L} = \frac{\eta (GM)^{5/3}\omega^{2/3}}{2c^4 R}\sin\beta\cos\beta\sin 2\theta_i\cos(2\omega t).$$

Radius of Separation (differential equation): Derived from our assumption, the time derivative of Keplar's Third Law and the formula for Schwarschild radius. Solved for r using both Verlet method and 4th Order Runge-Kutta Method.

$$\dot{r} = \frac{-\eta N c}{4}\left(\frac{r_S}{r}\right)^3$$

Radius of Separation (function of time): Analytical solution to differential equation

$$r^4(t) = r_i^4 - N\eta r_S^3 c\,(t - t_i) = N\eta r_S^3 c\,(t_c - t).$$

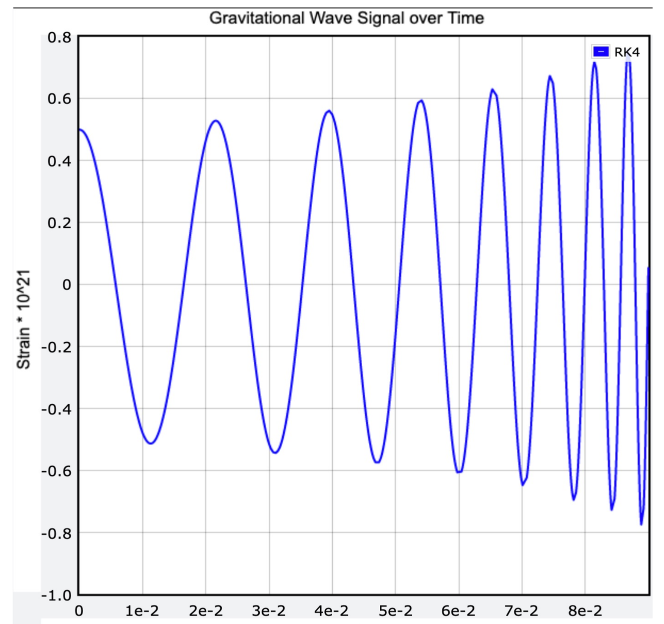
Rate of Energy Loss (derivative): Derived from our assumption. Solved for Energy as a function of time using Trapezoidal Rule

$$\frac{dE_G}{dt} = N\frac{G(\eta M)^2 r^4 \omega^6}{c^5}$$

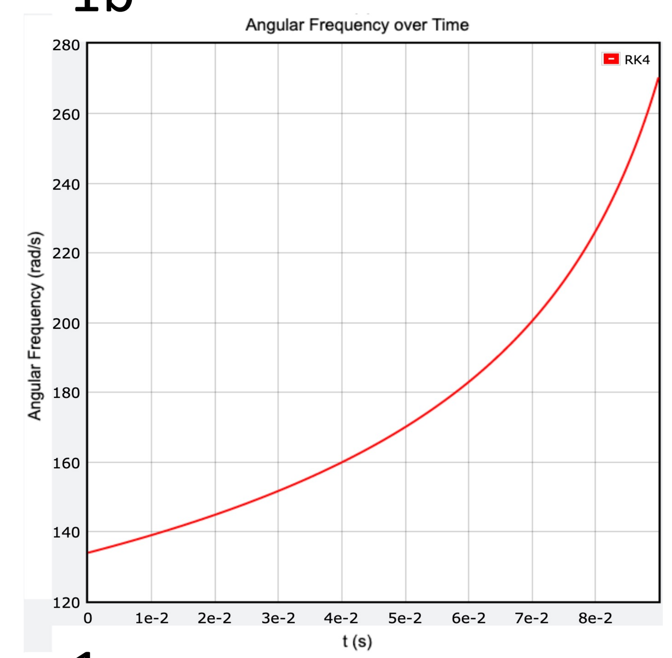
Results

1 -- GW150914 -- Binary Black Hole System

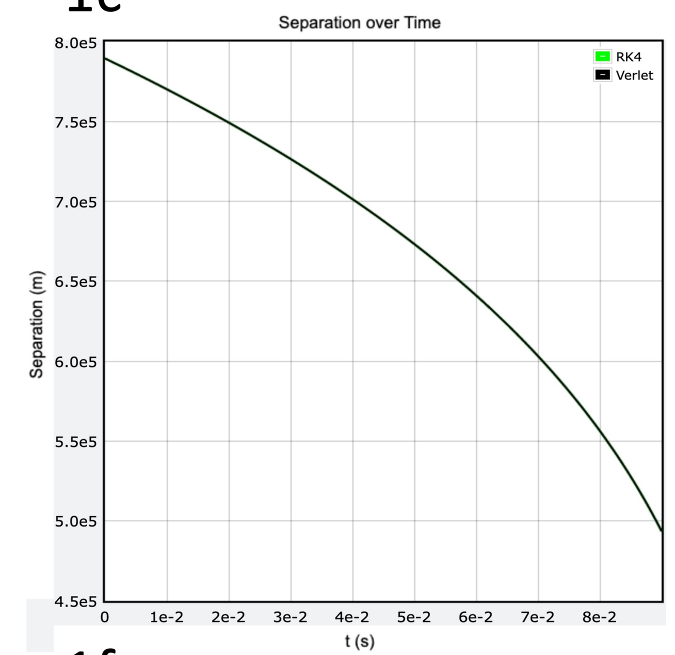
1a



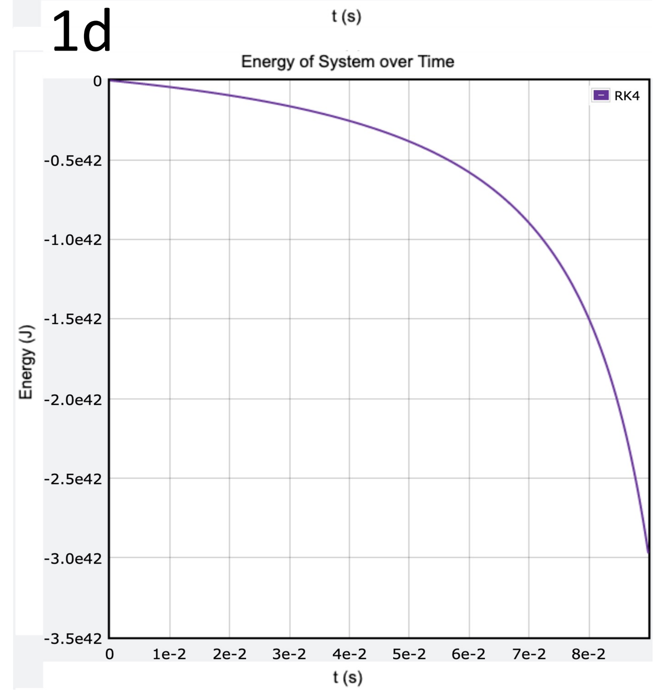
1b



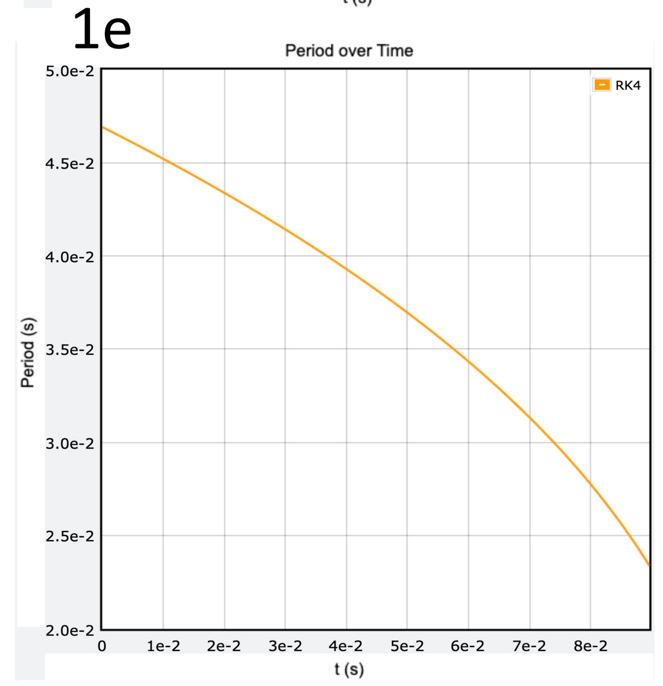
1c



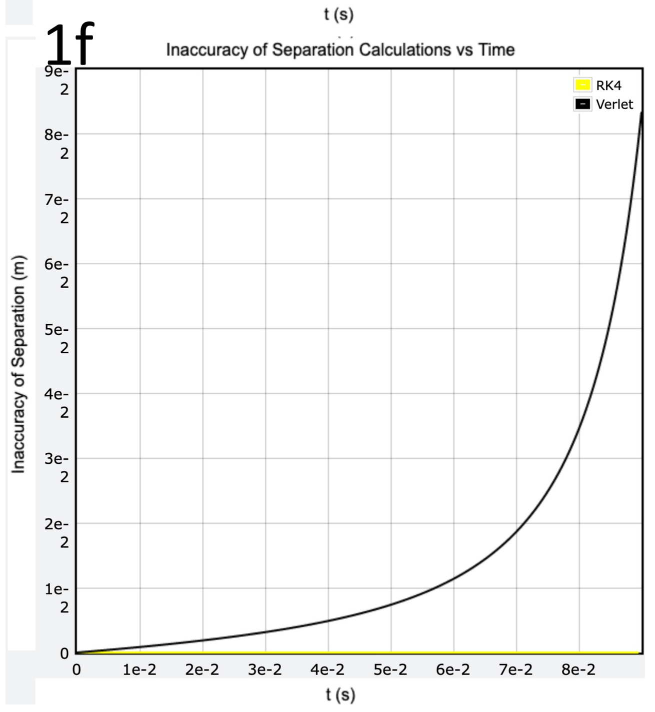
1d



1e

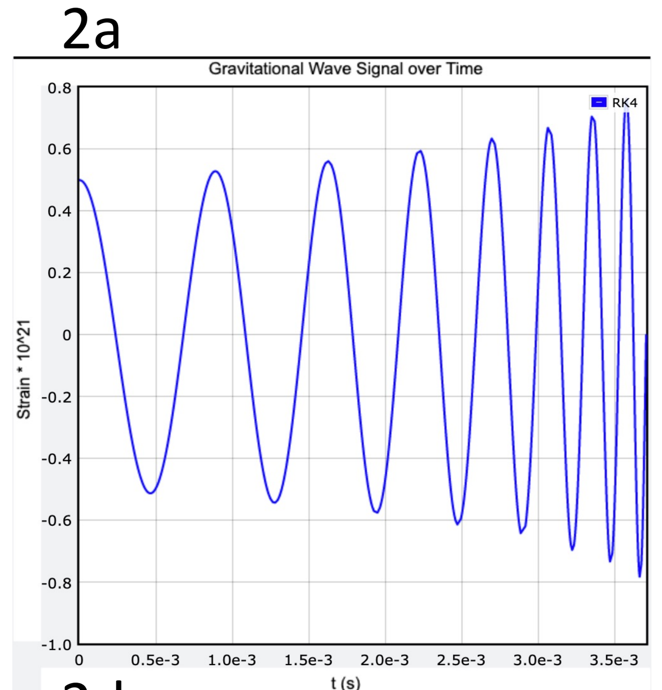


1f

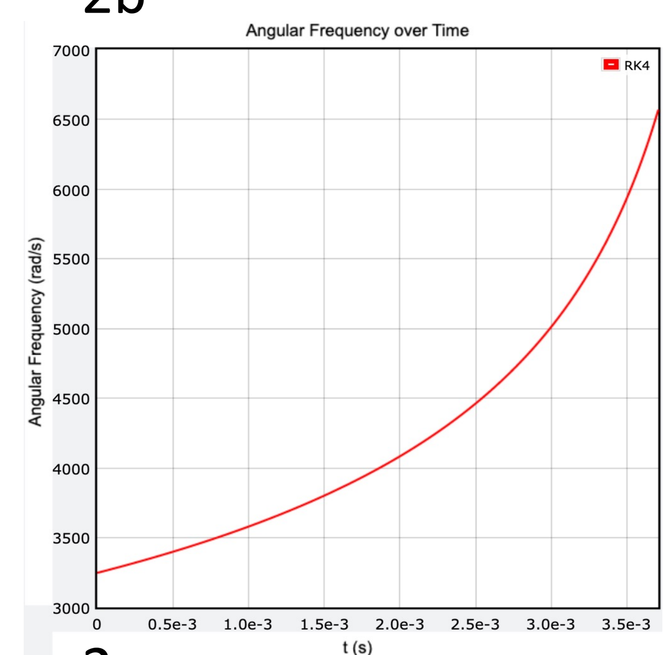


2 -- GW170817 -- Binary Neutron Star System

2a



2b



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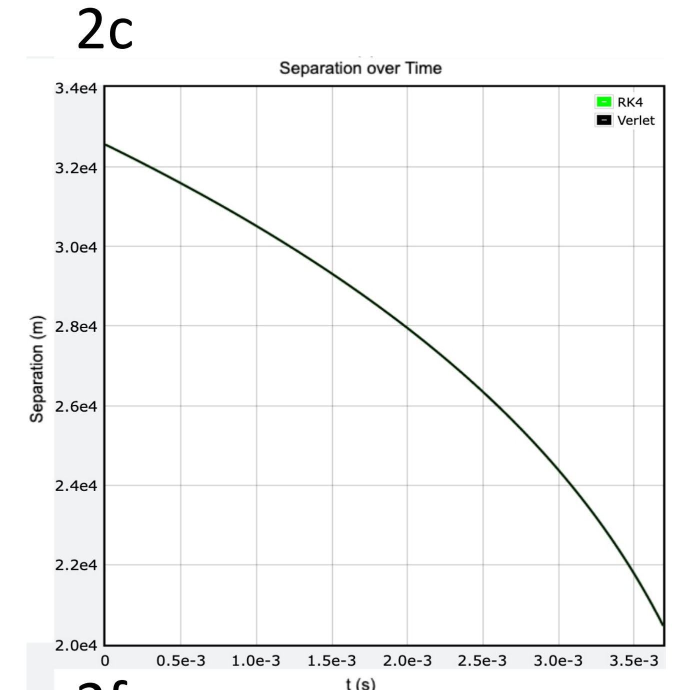
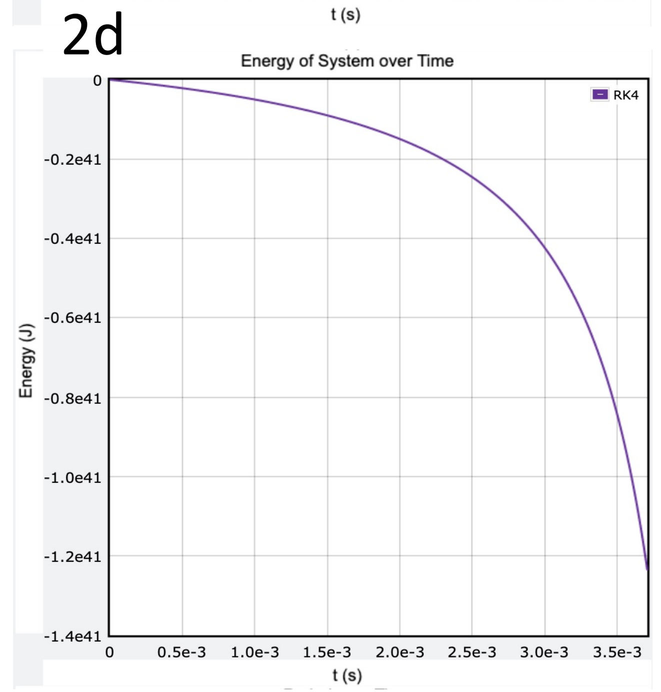


2d



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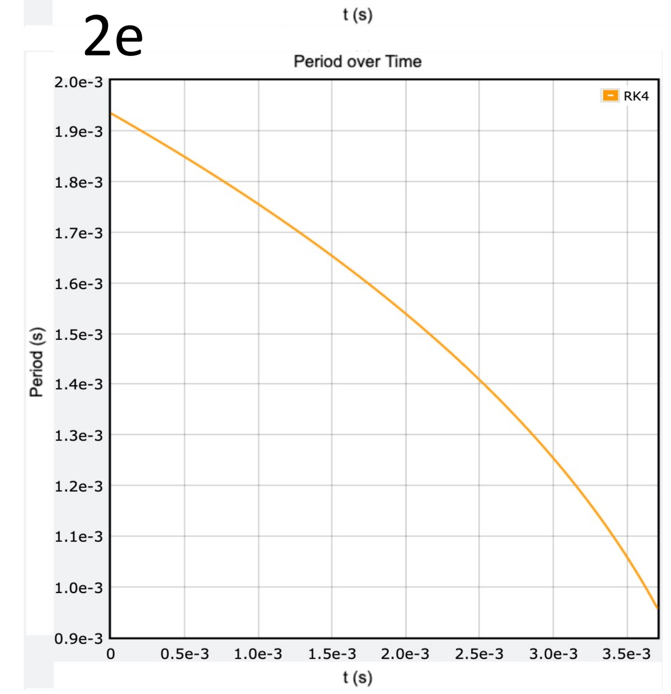
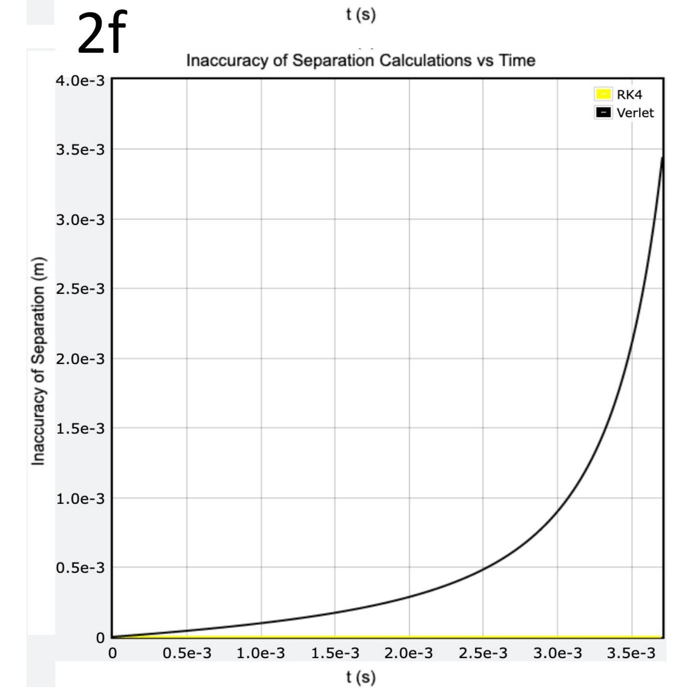
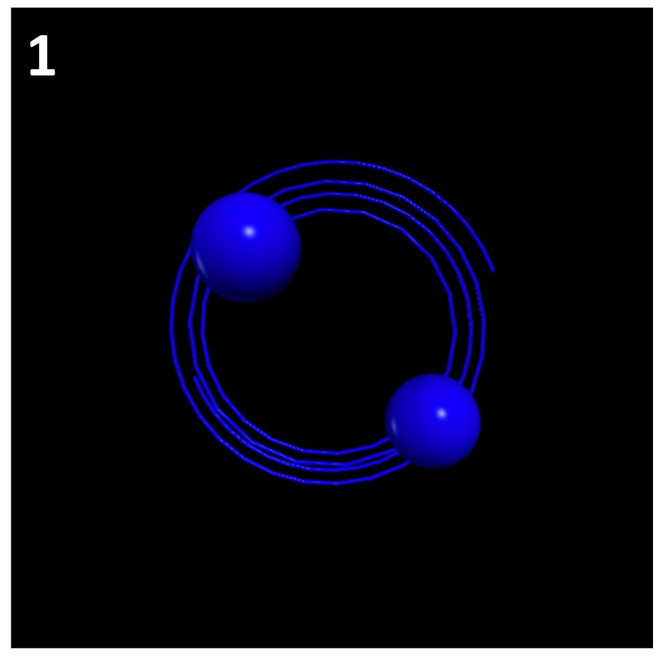
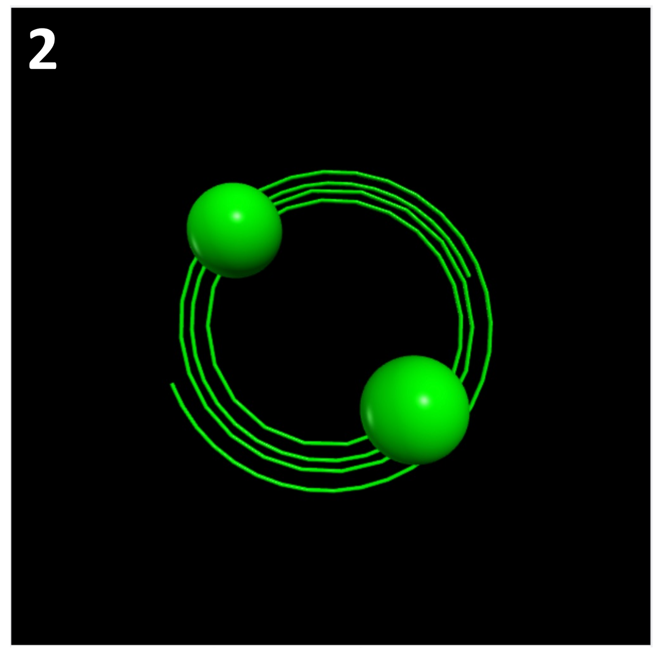
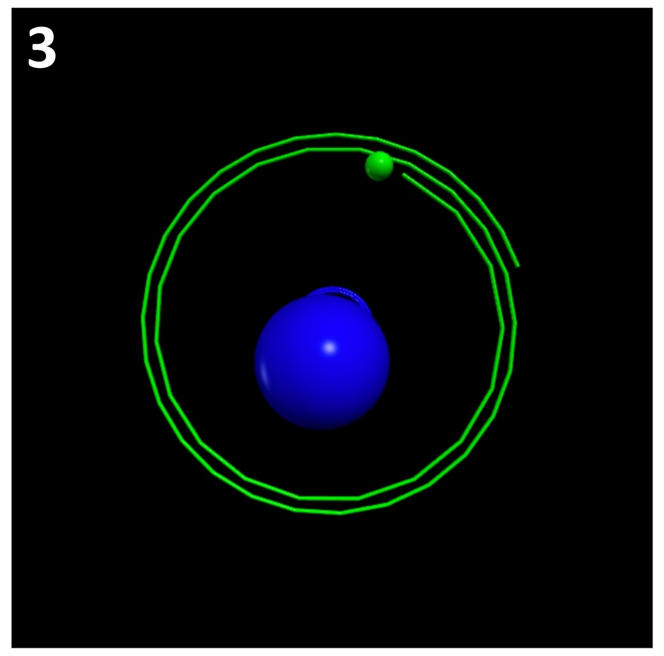


2f



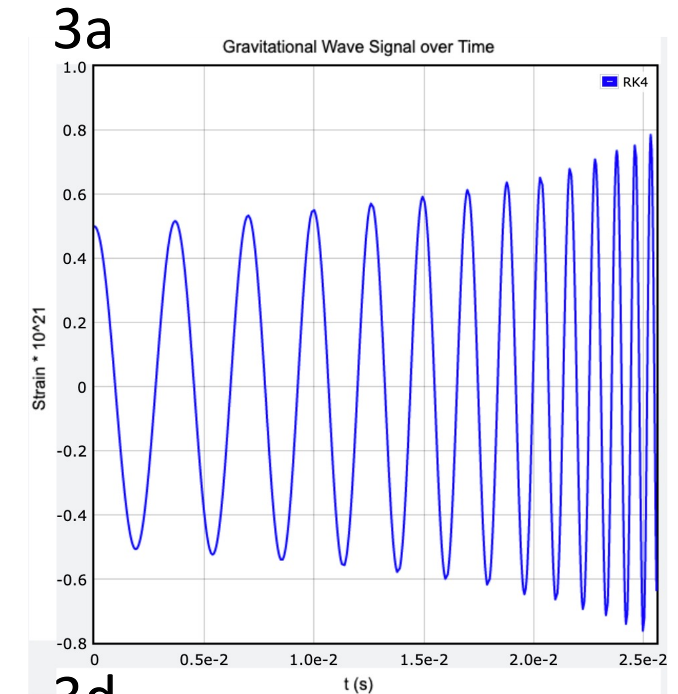




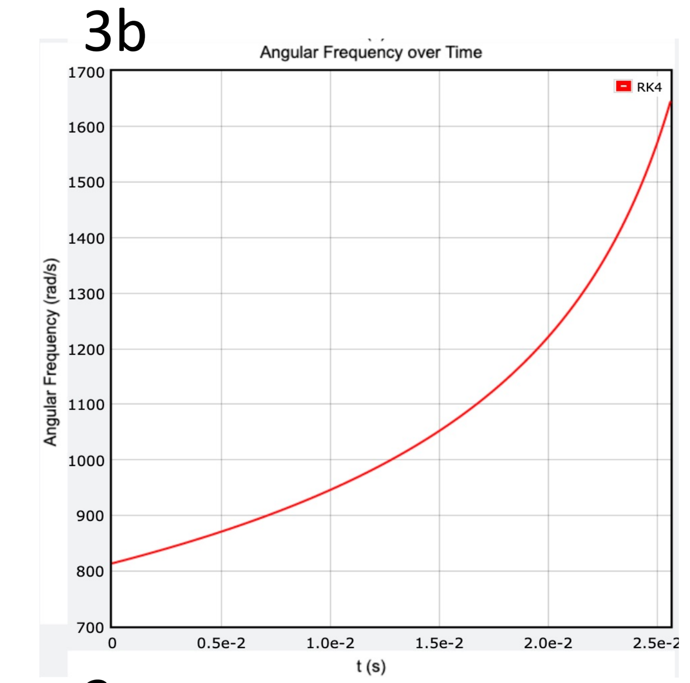


3 -- GW200105 -- Black Hole and Neutron Star collision

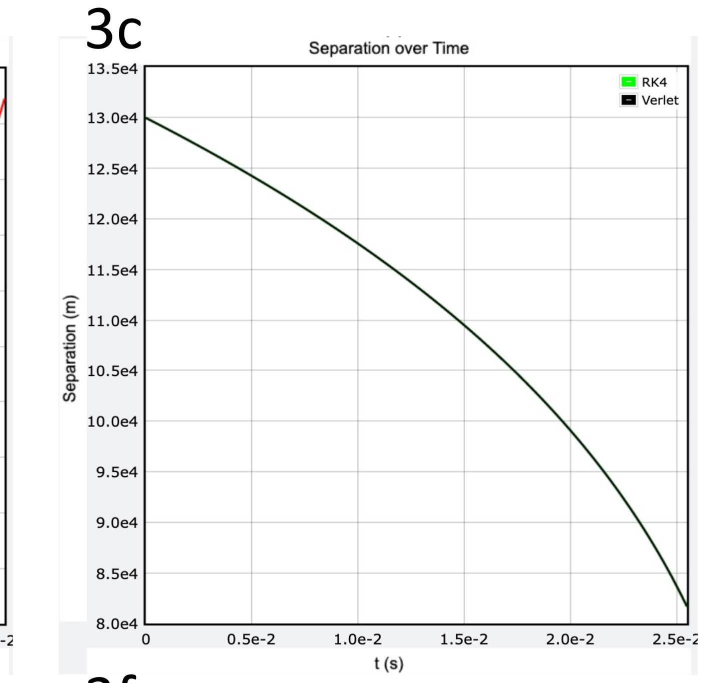
3a



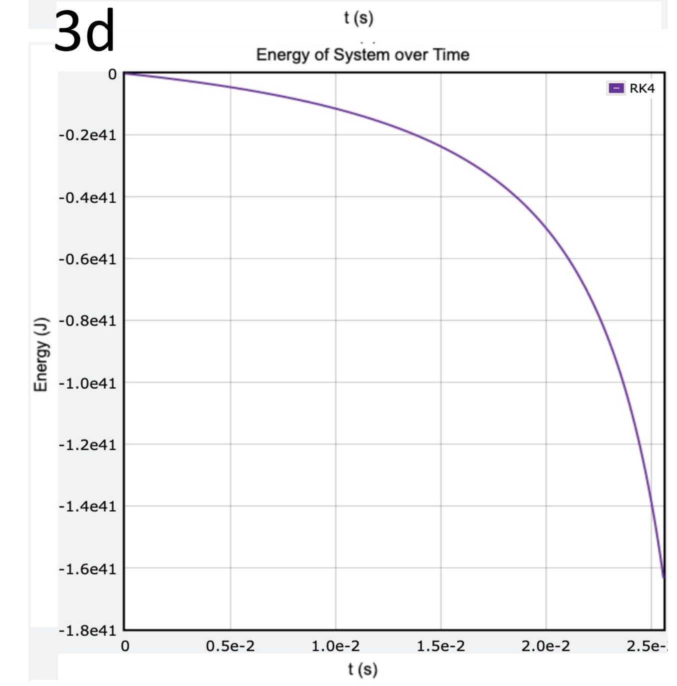
3b



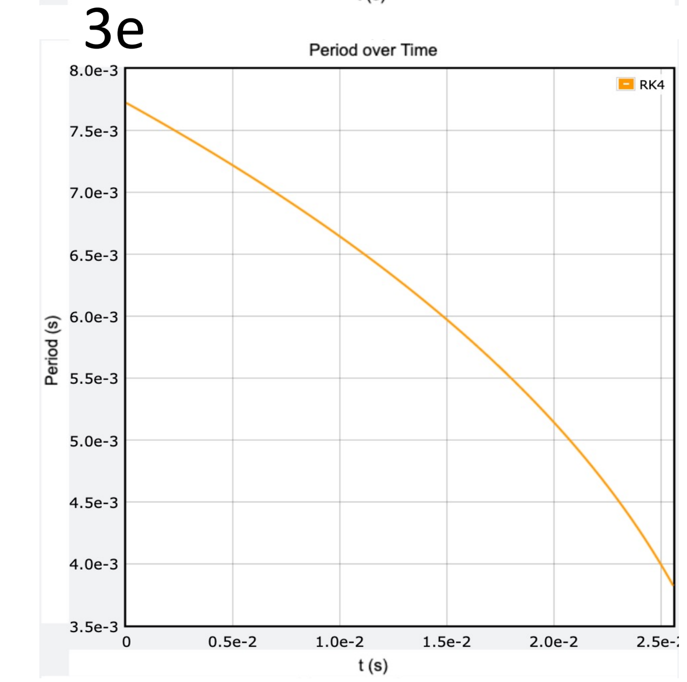
3c



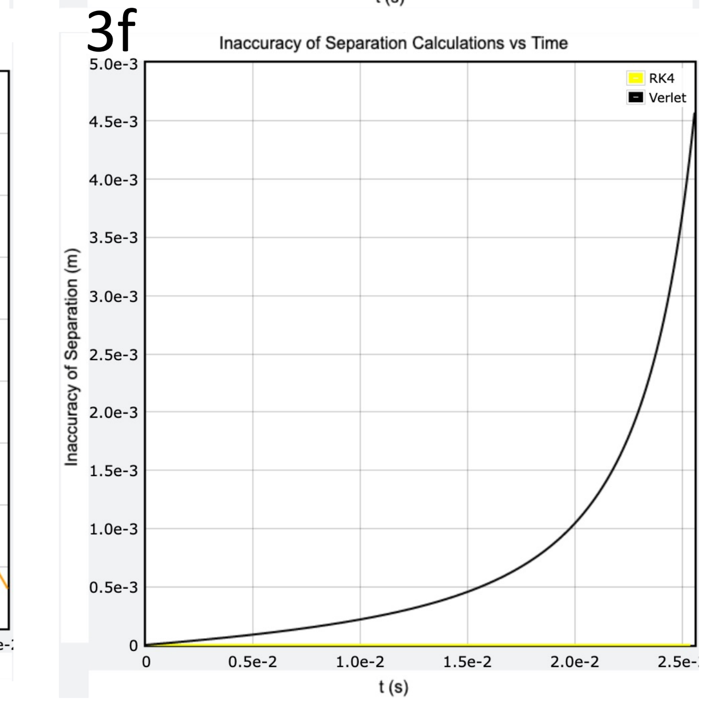
3d



3e



3f



Conclusions

- The separation of the binary objects decreases the fastest for binary neutron stars and slowest for binary black holes (graphs 1c, 2c, and 3c)
- Note that all these graphs only model the motion of the binary objects at the timestamp after which they start emitting gravitational waves that are at a frequency high enough for the LIGO to detect (> 15 Hz).
- Hence, binary black holes emit higher frequencies for a longer time before they collide than a binary neutron star - black hole or binary neutron stars. This is in line with the fact that binary systems with higher total masses will disrupt spacetime to a greater extent.
- This is in fact a burgeoning area of research - a black hole that's too massive or slow, can swallow neutron stars before they collide to produce heavy elements, or the neutron star is can be disrupted by the tidal field of the black hole, leading to mass ejection and the formation of an accretion torus around the black hole. The former case is what happened in GW200105, as is noticeable from the very small timescale in the order of -2. Understanding what is actually happening in these mergers based on timescale data is imperative in learning more about nucleosynthesis, and the properties of the binary objects, such as mass and spin.
- Compared to LIGO data, we don't get the solutions after the binary objects collide.
- Using the Runge Kutta 4th Order Method for solving the differential equation to calculate separation of the objects was more accurate than using the Verlet method (shown by graphs 1f, 2f, and 3f). This is very likely due to the Verlet method only being accurate a smaller time steps, but at the same large time step of the RK4 method, the accuracy of the Verlet method drops

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