Portfolios Management and Analysis in the Fixed Income Market

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Abstract

This study delves into the intricate dynamics of portfolio management within the fixed-income market, focusing primarily on the comparative analysis of fixed income Exchange-Traded Funds (ETFs) and homogeneous bond laddering strategies. Through rigorous data analysis and a unique asset selection methodology, we assessed these strategies for performance, risk, and returns. Our findings are based on advanced statistical analyses, including covariance estimation and principal component analysis, which helped us uncover key risk factors that influence portfolio outcomes.

The comparison between the global minimum variance portfolio and tangency portfolio approaches underlines the importance of strategic asset allocation in minimizing risk and maximizing returns. Additionally, the study's findings on our ETF portfolio contribute to a deeper understanding of what ETFs will perform well in varying market conditions and how the fixed-income market differs from other markets such as the equity market.

Our research also goes over homogeneous bond laddering as a strategy for stable returns and risk management, providing a compelling argument for its incorporation into diversified investment portfolios by utilizing models such as bootsrapping and Nelson-Siegel. Comparative analysis between the two investment strategies explains their distinct advantages and limitations, offering valuable guidance to investors learning about the complexities of the fixed-income market.

This study offers substantial contributions to the field of portfolio management, providing empirical evidence and strategic insights that enhance the understanding of fixed-income investments. The findings not only inform investment strategy formulation but also pave the way for future research exploring the evolving setting of the fixed-income market.

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1 Introduction

1.1 Motivation

As the advisory team at the wealth management firm XYZ, our primary motivation is to adeptly manage portfolios of large pension funds. This dissertation is poised to illustrate our judicious allocation of substantial investment capital - USD 500 million - within the fixed income bond market between the period 1st January 2015 to 31st December 2022. We direct our analysis towards two predominant portfolio frameworks: the fixed income Exchange-Traded Funds (ETFs) portfolio and the homogeneous bond laddering portfolio. Through this comparative analysis, we endeavor to uncover the intrinsic advantages inherent in each framework, thereby furnishing our investors with nuanced recommendations. Integral to our approach is the construction of three distinct risk profiles for both the ETF and homogeneous bond laddering portfolios - aggressive (high return and high-risk profile, HRHR), moderate (medium return and medium-risk profile, MRMR), and defensive (low return and low-risk profile, LRLR) growth strategies - to accommodate the diverse risk preferences and financial objectives of our investors. Ultimately, our objective is to facilitate a seamless connection between an investors risk tolerance and their investment strategy, ensuring a congruent alignment with their financial ambitions within the appropriate risk-return framework.

Challenges

- To identify what assets or which fixed-income ETFs to invest in;
- To identify criteria to rank them, into risk profiles;
- To solve the optimization problems, such as how much capital to allocate to each of the assets, and consider factors like target benchmark, management fees, etc.;
- To contrast these advanced statistical approaches in managing risk and maximizing returns, offering valuable insights for strategic investment planning in the fixed income domain.

Notation

$X_{P,t}$: Portfolio return at time t

$Var(X_{P,t})$: Variance of portfolio return at time t
μ_t: Expected return at time t
σ_t^2: Variance of return at time t
$\sigma_{ij,t}$: Covariance between two returns at time t
$\rho_{ij,t}$: Correlation coefficient between two returns at time t
π_t: Portfolio weight at time t
S_t: Sharpe ratio
rf_t: Risk-free rate
Σ_t: Covariance matrix of returns at time t
VaR: Value at risk
Ω: Omega ratio

T: Term to maturity/duration of bonds
Δt: Investment schedule
N: Number of rungs ($N = 1, 2, \ldots, i$)
F: Face value
ytm: Yield to maturity
τ: Reduced maturity from t to maturity time (date)
$y_t(\tau)$: Yield rate based on reduced maturity τ
r_t: Daily interest rate
p: Investment per rung K: Initial capital

1.2 Literature Review

In recent years, fixed income markets have become a cornerstone of strategic investing, maintaining stability amid the turmoil in global financial markets. The industry includes a range of bonds, mortgage-backed securities (MBS) and fixed-income ETFs that provide investors with effective protection from the volatility of stock investments. The appeal of fixed-income investments is their ability to provide predictable returns, diversification, and, in some cases, protection against inflation, making them a preferred choice for conservative investors and those approaching retirement [9].

However, fixed income is not always so stable and is more determined by the ebb and flow of macroeconomic policies and interest rate trends. The period 2015 to 2022 presents a unique set of challenges and opportunities for fixed income investors. After the 2008-2009 financial crisis, central banks around the world dabbled in low interest rates and quantitative easing policies, primarily aimed at stimulating economic growth. While these policies promote growth, they also require strategic adjustments to fixed income portfolios to adapt to the changing yield landscape and mitigate risks associated with interest rate fluctuations [3]. Above all, we introduce two important investment strategies - fixed income ETF and bond laddering - in the fixed income market, which are examined under the lens of theoretical frameworks and practical applications, offering a comprehensive understanding of their roles in modern portfolio management.

Initially, Markowitz Portfolio Theory (1952) laid the foundation for our optimization of fixed income portfolio allocation. Specifically, this suggests that investors can maximize returns at any given level of risk. It emphasizes the importance of balancing risk and return through diversified investments [5].

Fixed income ETFs offer professionally managed portfolios that provide diversification into the fixed income market. A fixed income ETF is a portfolio of bonds. These ETFs combine the advantages of bonds with the flexibility and liquidity of stocks, providing investors with a versatile vehicle to access the fixed income market [8] as well as providing practical solutions.

When discussing portfolios of fixed income ETF, we consider two portfolios. The first is the global minimum variance (GMV) portfolio. It further refines this concept by identifying portfolio allocations that minimize overall risk. GMV portfolios are particularly relevant to fixed income investments, where the primary goal is typically to reduce risk rather than generate maximum returns. These portfolios demonstrate the advantages of risk mitigation strategies in fixed income investing, highlighting the importance of strategic asset allocation [4].

The second one is Tangency portfolio and Sharpe ratio. It introduces an optimization framework for maximizing return per unit of risk. This approach is also critical for evaluating many kinds of fixed income portfolios, providing a way to assess the efficiency and performance of these investment strategies [1].

So far, whether it is Markowitz portfolio theory or GMV and Tangency portfolios, their use in practical applications has demonstrated their impact on fixed income portfolio performance, providing insights into the strategic allocation of assets to minimize risk and optimize returns.

Definitely, considering only one investment strategy is not what we want to show in this dissertation. A bond ladder, as a portfolio of fixed income securities, involves purchasing bonds with staggered maturities, creating a ladder effect that helps manage interest rate risk. This approach provides an ongoing stream of income as the bonds mature periodically, allowing the proceeds to be reinvested at prevailing interest rates. The structure, purpose, and benefits of bond ladders have been widely discussed, with Strickland (2001) highlighting their ability to reduce interest rate sensitivity and provide predictable income streams [7].

To better create our own bond ladder, the Nielsen-Siegel model plays a key role, providing a sophisticated method of yield curve analysis and bond valuation. The application of this model in interpreting the yield curve provides valuable insights into bond valuation, allowing investors to make informed decisions based on expected changes in interest rates and their impact on bond prices.

The two major investment strategies mentioned above will be reflected and expanded in our subsequent report. Yet, more importantly, bond laddering and fixed income ETF are both popular strategies for investing in the fixed-income market, and each has its own unique advantages and limitations. A bond ladder can manage interest rate risk and provide stable cash flow by purchasing bonds with different maturities, making it suitable for investors who require regular income. However, this strategy may have limited returns in a low interest rate environment and is relatively complex to manage, requiring investors to continuously adjust and monitor bond portfolios. In contrast, fixed income ETF offer better diversification and liquidity, allowing investors to easily access the broad bond market at a lower cost. They are highly transparent and easy to trade but may be subject to risks of market price fluctuations, tracking errors, and underperformance when interest rates rise.

One thing to note is that due to time limitations, we start using the bond laddering strategy with its simplest and most fundamental version - homogeneous bond laddering. To some extent, building a homogeneous bond ladder is based on many fixed and limited conditions and selected parameter values. For example, we only consider the bond with the same maturity over our entire investment horizon. This also means that we do not need to consider the weight issue. Compared with higher-order and more complete laddering strategies, homogeneous bond laddering lacks diversity, but the liquidity of cash flow is well reflected. In future work, we will further refine the ladder strategy, such as making different investment purchases at different ladders based on real-time market economic changes or purchasing multiple bonds with different durations in the same ladder.

1.3 Contribution

Our contributions to fixed income portfolio management are comprehensive and innovative, including the development of data collection and analysis, asset selection, methodologies for constructing both ETFs and homogeneous bond laddering portfolios, portfolio optimization, and performance and risk assessment. We meticulously gather and analyze data on fixed income ETF and U.S. Treasury bonds, using data bootstrapping and applying sophisticated models like Nelson-Siegel to ensure our strategies are grounded in precise, relevant data. Our distinctive approach to ETF screening and selection, leveraging a risk-weighted scoring system, aligns closely with various risk profiles, enhancing the tailored nature of our investment solutions. Through strategies like the Global Minimum Variance (GMV) and Tangency Portfolios, we demonstrate our capability to adapt to diverse investment goals, focusing on risk minimization and return maximization. Furthermore, our exploration of homogeneous bond ladders introduces a strategy for stable returns while managing risk effectively. Our comprehensive performance assessment and the comparison of ETF and bond ladder portfolios illustrate the advantages of our methodologies over traditional management approaches, significantly enriching fixed income investing with tailored, strategic insights.

2 Methodology

Portfolio optimization is a fundamental concept in finance and investment management, focusing on the process of selecting the best mix of assets to achieve specific objectives, such as maximizing returns or minimizing risk, within a given set of constraints.

Since we have selected three groups of ETFs based on return and risk, our next goal becomes determining the optimal weights for each asset that we invest at the beginning of 2015. For this quarter, we only construct Global Minimum Variance Portfolio and Max Sharpe Ratio - Tangency Portfolio to determine the weights.

2.1 Portfolio Introduction

We consider a time series of financial assets, where for each asset i at time t, we define the following:

- $X_{i,t}$: the random return on a unit invested in asset i at time t.
- $\mu_{i,t} = E[X_{i,t}]$: the expected return of asset i at time t.
- $\sigma^2_{i,t} = \text{Var}[X_{i,t}]$: the variance of return of asset i at time t.
- $\sigma_{ij,t} = E[(X_{i,t} - \mu_{i,t})(X_{j,t} - \mu_{j,t})]$: the covariance between the return $X_{i,t}$ and return $X_{j,t}$.
- $\pi_{i,t}$: the proportion of wealth invested in asset i at time t, with $\sum_{i=1}^{n} \pi_{i,t} = 1$, which indicates full investment.
- $\rho_{ij,t} = \frac{\sigma_{ij,t}}{\sigma_{i,t}\sigma_{j,t}}$: the correlation coefficient between $X_{i,t}$ and $X_{j,t}$, where $-1 \leq \rho_{ij,t} \leq 1$.

We assume the investor invests all the wealth for one period in some combination of available assets. In our calculation, we assume these to be known: $E[X_{i,t}]$, $\sigma^2_{i,t}$, and $\sigma_{ij,t}$ for all i, j. Throughout we will consider risk measured by the variance of the portfolio return.

2.1.1 Global Minimum Variance Portfolio

Our goal is to minimize risk (portfolio variance) subject to the constraint that the sum of weights at time t is equal to 1. Thus, the optimization problem is formulated as follows:

Minimize

$$Var(X_{P,t}) = \sum_{i=1}^{n} \pi_{i,t}^2 \sigma_{i,t}^2 + \sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} \pi_{i,t} \pi_{j,t} \sigma_{ij,t}$$

Subject to

$$\sum_{i=1}^{n} \pi_{i,t} = 1$$

Firstly, we calculate the covariance between each pair of assets:

$$\sigma_{ij,t} = \rho_{ij,t} \sigma_{i,t} \sigma_{j,t}$$

Let $\boldsymbol{\Sigma}_t$ denote the variance/covariance matrix of returns at time t, $\boldsymbol{\mu}_t$ denotes the expected return of each asset at time t, and $\boldsymbol{\pi}_t$ denotes the weights of each asset at time t.

$$\boldsymbol{\Sigma}_t = \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{1,2,t} & \dots & \sigma_{1,n,t} \\ \sigma_{2,1,t} & \sigma_{2,t}^2 & \dots & \sigma_{2,n,t} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n,1,t} & \sigma_{n,2,t} & \dots & \sigma_{n,t}^2 \end{bmatrix}, \boldsymbol{\mu}_t = \begin{bmatrix} E[X_{1,t}] \\ E[X_{2,t}] \\ \vdots \\ E[X_{n,t}] \end{bmatrix}, \mathbf{1}_n = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \boldsymbol{\pi}_t = \begin{bmatrix} \pi_{1,t} \\ \pi_{2,t} \\ \vdots \\ \pi_{n,t} \end{bmatrix}.$$

The Lagrangian in matrix form is given by:

$$L(\boldsymbol{\pi}_t, \lambda) = \frac{1}{2} \boldsymbol{\pi}_t^\top \boldsymbol{\Sigma}_t \boldsymbol{\pi}_t + \lambda(1 - \boldsymbol{\pi}_t^\top \mathbf{1}_n) \tag{1}$$

Solving the First Order Condition (FOC) for the optimal solution:

$$\frac{\partial L}{\partial \boldsymbol{\pi}_t} = 0 = \boldsymbol{\Sigma}_t \boldsymbol{\pi}_t - \lambda \mathbf{1}_n, \tag{2}$$

$$\frac{\partial L}{\partial \lambda} = 0 = 1 - \boldsymbol{\pi}_t^\top \mathbf{1}_n. \tag{3}$$

Thus, we find:

$$\lambda = \frac{1}{\mathbf{1}_n^\top \boldsymbol{\Sigma}_t^{-1} \mathbf{1}_n}, \tag{4}$$

and consequently we get a closed form of the weights expression given time t,

$$\boldsymbol{\pi}_{GMV,t} = \frac{\boldsymbol{\Sigma}_t^{-1} \mathbf{1}_n}{\mathbf{1}_n^\top \boldsymbol{\Sigma}_t^{-1} \mathbf{1}_n}. \tag{5}$$

In practice, when calculating the covariance matrix and its inverse, we may need to resort to robust methods of covariance estimation. In the later case study, we compare the difference in covariance by MCD method and sample method.

2.1.2 Max Sharpe Ratio - Tangency Portfolio

Tangency Portfolio is built based on the Sharpe Ratio, represented as S_t, evaluates how much extra return an investment gives over a risk-free return, considering its risk. It's calculated at time t by the formula:

$$S_t = \frac{R_{p,t} - rf_t}{\sigma_{p,t}}$$

where:

$$\begin{aligned} S_t &: \text{Sharpe Ratio at time } t \\ R_{p,t} &: \text{Return of the portfolio at time } t \\ rf_t &: \text{Risk-free return at time } t \\ \sigma_{p,t} &: \text{Risk of the portfolio at time t, measured by standard deviation} \end{aligned}$$

The term $R_{p,t} - rf_t$ is the extra return from the portfolio over the risk-free return at time t, and $\sigma_{p,t}$ shows the portfolio's risk at that time. A higher Sharpe Ratio means the portfolio's return is better compared to its risk.

For a Max Sharpe Ratio - Tangency Portfolio, unlike the GMV portfolio, we focus on getting the highest return for each unit of risk, based on the portfolio's Sharpe Ratio, with the given risk-free rate. In our study, we use the Federal Fund rate to be our risk-free rate. So, our goal at time t becomes:

Maximize

$$\frac{\boldsymbol{\pi}_t^\top \boldsymbol{\mu}_t - rf_t}{\sqrt{\boldsymbol{\pi}_t^\top \boldsymbol{\Sigma}_t \boldsymbol{\pi}_t}}$$

Subject to

$$\sum_{i=1}^{n} \pi_{i,t} = 1$$

where rf_t is the risk-free interest rate at time t.

We can then find the formula for the Max-Sharpe Tangency portfolio weights at time t:

$$\boldsymbol{\pi}_{MS,t} = \frac{\boldsymbol{\Sigma}_t^{-1}(\boldsymbol{\mu}_t - rf_t\mathbf{1})}{\mathbf{1}^\top \boldsymbol{\Sigma}_t^{-1}(\boldsymbol{\mu}_t - rf_t\mathbf{1})},$$

where $\boldsymbol{\mu}_t$ is the expected returns and rf_t is the risk-free rate at time t.

2.1.3 (Homogeneous) Bond Laddering Portfolio

In the world of fixed income investing, constructing a bond ladder portfolio is a prominent strategic approach aimed at reducing interest rate risk, ensuring liquidity, and ensuring a stable source of income. For the construction of a most basic bond ladder portfolio, we need to consider the following key parameters:

- T: Maturity term/duration of bonds
- Δt: Investment schedule
- N: Number of ladders/rungs
- Issuer
- F: Face value
- Coupon
- y_t: Yield rate
- r_t: Daily interest rate at time t
- K: Initial capital

The essence of a bond ladder is its ability to diversify bond holdings across different maturities. The structure of a bond ladder, including the chosen bond term or duration and the investment schedule (monthly, quarterly, or annually), is crucial for maintaining steady returns and ensuring the portfolio's liquidity. The complexity and diversification level of the portfolio are determined by the number of ladder rungs, affecting risk management and income stability. Additionally, the selection of bond issuers and the bonds' face value, coupon, and yield are essential for managing credit risk and optimizing the portfolio's return potential.

At the simplest level, the homogeneous bond laddering investment model employs the equal weight method, allocating a fixed proportion of capital to each bond, irrespective of its maturity. This approach simplifies portfolio management, allowing for easier rebalancing and strategic reinvestment, thus maintaining the ladder's benefits of liquidity and a consistent income stream. Whether adopting a general bond ladder or a homogeneous one, the strategy aims to secure liquid cash flow and manage risks through diversification and careful planning.

Now, we delve into details on how to construct a homogeneous bond ladder. The first step is to determine the parameters. Next, we will calculate the portfolio value evaluated daily:

$$X_{P,t}^{BL} = \sum_{i}^{N} p \cdot e^{-\tau_i y_t(\tau_i)} + (K - Np)(\text{Compounded Interest Rate})_t$$

where:

$$\text{Compounded Interest Rate} = (1 + r_1)(1 + r_2) \cdots (1 + r_{t-1})$$

$$i : \text{the number of rungs } 1, 2 \ldots, N$$

The first component of the formula is the bond value, which is calculated by summing the present values of all bonds in the rung i up to the rung N, each discounted by their respective yield to reduced maturity τ. The second component is the cash account value, which is the accumulated interest compounded over time. It's the product of the initial investment X, minus any principal that has been repaid (or in some cases, if the rung gets matured, we will add the face value gained), compounded at the interest rates over time.

Then, we can get daily portfolio return:

$$R_{P,t}^{BL} = \log\left(\frac{X_{P,t}^{BL}}{X_{P,t-1}^{BL}}\right)$$

To get the annual return or monthly return, what we do is to calculate the cumulative daily return for a time period (one year or one month), for example, for the annual simple return:

$$R_P^{BL} = e^{\sum_{t=1}^{365} R_{P,t}^{BL}} - 1$$

In terms of portfolio optimization, the application of Markowitz's portfolio theory in bond ladders effectively constructs ladders by optimizing the return-risk ratio, prioritizing non-selective investment-grade bonds, and focusing on returns and maturity dates, thereby improving the efficiency of the investment portfolio. However, due to time constraints, we have not yet been able to optimize the bond ladder portfolio. This will be incorporated into our future work.

2.2 Portfolio Assessment

Once the construction of portfolios is finished, in order to visualize the weights and construct a combination of two portfolios, we use Two Fund Theorem to build Opportunity Frontier. In addition, we take other risk measures to look back at a certain period of our portfolio performance. We take account of Value-at-Risk, Omega Ratio, and Sharpe Ratio of a rolling window for one year.

2.2.1 Two Fund Theorem

Consider we have two different groups of portfolios at a specific time t. The first one, A, may be a portfolio that minimizes risk, while the second one, B, focuses on a better return. We denote these return given time t of A and B to be

$$X_{P,t}^{A} = \sum_{i=1}^{n} \pi_{i,t}^{A} X_{i,t}$$

$$X_{P,t}^{B} = \sum_{i=1}^{n} \pi_{i,t}^{B} X_{i,t}$$

From here, we can create a new portfolio C as a linear combination of these two, meaning that we mix them in a way that can be adjusted dynamically. For some weight w, which can take any value from $-\infty$ to ∞, the portfolio C at time t is defined as:

$$X_{P,t}^{C} = \sum_{i=1}^{n} \left(w\pi_{i,t}^{A} + (1-w)\pi_{i,t}^{B}\right) X_{i,t}$$

The expected return of this combined portfolio is:

$$\mu_{P,t}^{C} = \sum_{i=1}^{n} \left(w\pi_{i,t}^{A} + (1-w)\pi_{i,t}^{B}\right) E[X_{i,t}] = w\mu_{P,t}^{A} + (1-w)\mu_{P,t}^{B}$$

By the Two Fund Theorem, Portfolio C at given time t has minimum variance among all portfolios with expected return $\mu_{P,t}^{C}$

Furthermore, when we choose different weights of A and B, we can achieve any possible returns given a minimum risk under the combination of A and B. This can be visualized under the concept of opportunity frontier.

2.2.2 Opportunity Frontier

The Opportunity Frontier at time t represents the set of portfolios that offer the expected return for a given level of risk. To compute the Efficient Frontier considering the combination of the Global Minimum Variance (GMV) portfolio and the Tangency portfolio at time t, the following steps are undertaken:

- Calculate the GMV portfolio, denoted as π_t^{GMV}, and determine its expected return $\mu_{P,t}^{GMV} = \pi_t^{GMV}\mu_t$ and its risk $\sigma_{P,t}^{GMV} = \sqrt{\left(\pi_t^{GMV}\right)^T \Sigma_t \left(\pi_t^{GMV}\right)}$.
- Calculate the Tangency portfolio, denoted as π_t^{MS}, and determine its expected return $\mu_{P,t}^{MS} = \pi_t^{MS}\mu_t$ and its risk $\sigma_{P,t}^{MS} = \sqrt{\left(\pi_t^{MS}\right)^T \Sigma_t \left(\pi_t^{MS}\right)}$.
- For a series of weights w ranging from 0 to 1, construct the mixed portfolios π_t using the formula $\pi_t = w\pi_t^{GMV} + (1-w)\pi_t^{MS}$, and calculate their expected returns and portfolio variances at time t.
- Plot $\mu_{P,t}$ against $\sigma_{P,t}$ for these portfolios to illustrate the Efficient Frontier at time t.

We can further prove that the relationship between the portfolio variance $\sigma_{p,t}^2$ and the mean $\mu_{p,t}$ at time t is expressed by

$$\sigma_{p,t}^2 = a_t\mu_{p,t}^2 + b_t\mu_{p,t} + c_t,$$

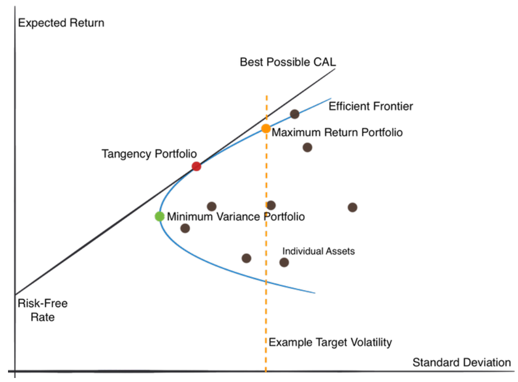


Figure 1: Efficient Frontier Example

indicating that the variance of the Efficient Frontier formed by these two efficient portfolios forms a parabolic curve in the mean-return space at any given time t. See Figure 1.

We can see that the entire blue line is called the opportunity set. This portfolio lies at the red dot where a line (Capital Market Line) from the risk-free rate is tangent to the Efficient Frontier. It represents the portfolio with the highest Sharpe ratio, offering the best return per unit of risk. The green dot represents the global minimum variance, which has the least risk in all the combinations on the blue line. If we only take the blue line that is above the minimum variance portfolio point, we will get the efficient frontier curve. This curve shows the set of optimal investment portfolios that offer the highest expected return for a given level of risk.

2.2.3 Risk Measures

- **Value at Risk (VaR)**: The VaR measures the potential loss in value of a risky asset or portfolio over a given time period, provided a specified confidence interval. The VaR at a confidence level α for a portfolio P_i at time t is defined as:

  $$\mathrm{VaR}_{\alpha}^{P_i}{}_t = F^{-1}_{R_t{}^{P_i}}(\alpha)$$

  where $F^{-1}_{R_t{}^{P_i}}(\alpha)$ is the quantile function, or the inverse of the cumulative distribution function (CDF) of returns, indicating the worst loss to expect with a confidence level α. See Figure 2, this histogram shows the distribution of profit and loss(PNL). The VaR is represented by the vertical dashed line, which partitions the area under the histogram. The point where the VaR line intersects the x-axis indicates the maximum expected loss over the given time period at this confidence level.

- **Omega Ratio**: The Omega Ratio is a risk-return performance measure of an investment asset or strategy. It is calculated as the ratio of the probability-weighted gains over a threshold T to the probability-weighted losses below that threshold. The formula is given by:

$$\Omega(T) = \frac{\int_T^\infty (1 - F(r))\, dr}{\int_{-\infty}^T F(r)\, dr}$$

where $F(r)$ is the CDF of returns and T is the threshold, often chosen as the Federal Fund Rate. The numerator integrates the complement of the CDF over the range of returns greater than T, representing potential gains, while the denominator integrates the CDF over the range of returns less than T, representing potential losses. See Figure 2. This chart shows two areas under a curve of returns distribution. The area to the right (red area) represents the returns distribution above the threshold, signifying the probability and magnitude of gains over T. The area to the left (green area) represents the returns distribution below T, denoting the probability and magnitude of losses. The Omega Ratio is the ratio of the area of the red region to the green one. An Omega Ratio greater than 1 indicates that the investment offers more probability-weighted gains than losses above the chosen threshold, signaling a better investment outcome. If it is less than 1, then the investment is underperforming.

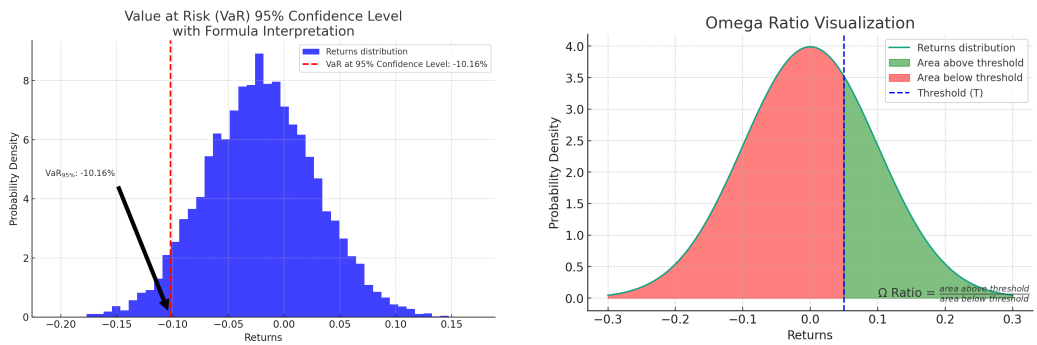


Figure 2: VaR and Omega

3 Data Analysis

This data and empirical analysis section focuses on the preparation and examination of data within the fixed-income market, incorporating a comprehensive analysis of U.S. fixed-income Exchange-Traded Funds (ETFs) and U.S. Treasury bonds. Fixed-income instruments play a crucial role in investment portfolios, offering investors the potential for stable returns and diversification benefits. By examining the close prices and returns of U.S. fixed-income ETFs and Treasury bonds, this analysis aims to provide valuable insights into the behavior and performance of these assets over time. The primary objective is to explore the general trends and characteristics of the datasets, laying the foundation for strategic analysis and subsequent optimization.

3.1 ETFs' Data Collection and Processing

To ensure the accuracy and reliability of the analysis, the datasets for this study were meticulously gathered from several reputable sources, including Trading View, ETF.com, masterdatareports.com, and ETFdb.com. The primary focus was on daily closing prices of the top 100 fixed-income ETFs based on Assets Under Management (AUM) from the beginning of 2014 to the end of 2023. This extensive time frame allows for a comprehensive examination of the performance and behavior of fixed-income ETFs across various market conditions.

In addition to closing prices, other relevant data points were collected to provide a more holistic view of the fixed-income ETF landscape. These data points include Assets Under Management (AUM), expense ratio, dividend yield, Environmental, Social, and Governance (ESG) score, net cash flow, and trading expense. By incorporating these additional metrics, the analysis can account for the impact of factors beyond price performance on the overall attractiveness and suitability of fixed-income ETFs for portfolio inclusion.

The ETF dataset comprises daily data points for 98 fixed-income ETFs over an 8-year period from 2015 to 2022, after excluding missing values and refining the original set of 404 metrics. The primary focus remains on closing prices, while the additional metrics provide valuable context for the analysis. The bond dataset, on the other hand, consists of daily yield data for U.S. Treasury bonds with varying maturities, ranging from one month to 30 years. The bond data places a significant emphasis on the period from 2013 to 2023, allowing for a comprehensive examination of the U.S. Treasury yield curve and its evolution over time.

To prepare the raw datasets for analysis, extensive cleaning and transformation processes were undertaken. Initial steps involved excluding ETFs that did not fit the study's criteria and integrating U.S. 10-Year Yields as a reference point for comparison. After the cleaning process, the dataset included 96 ETFs, a one-time ticker, and a U.S. 10-Year yield data column as an earlier comparison. Data handling procedures were employed to remove etfs that are 30 percent or more missing during a year.

3.1.1 Further Analysis

The year 2022 stands out in the financial markets due to significant changes in interest rates, prompting a closer examination of how fixed-income ETF portfolios have adapted to these conditions. By analyzing the return plots across three risk profilesHigh-Risk High-Return (HRHR), Medium-Risk Medium-Return (MRMR), and Low-Risk Low-Return (LRLR)we observe distinct behaviors reflective of the portfolios' risk management strategies and sensitivity to interest rate fluctuations.

The return plots for both HRHR and MRMR portfolios exhibit a consistent range of maximum and minimum returns, indicative of a stable investment strategy that has not significantly altered its structure in response to the changing interest rates. This stability suggests that the high and medium-risk portfolios were well-positioned to absorb the impact of interest rate changes without dramatic shifts in performance. Contrary to the HRHR

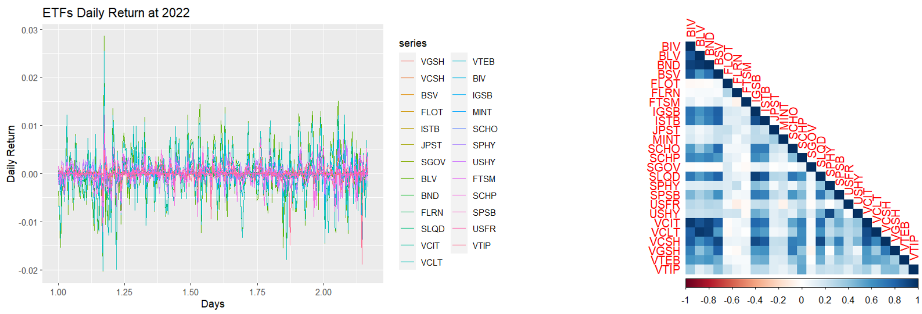


Figure 3: High-Risk High-Return Composition and Cross Corrlation

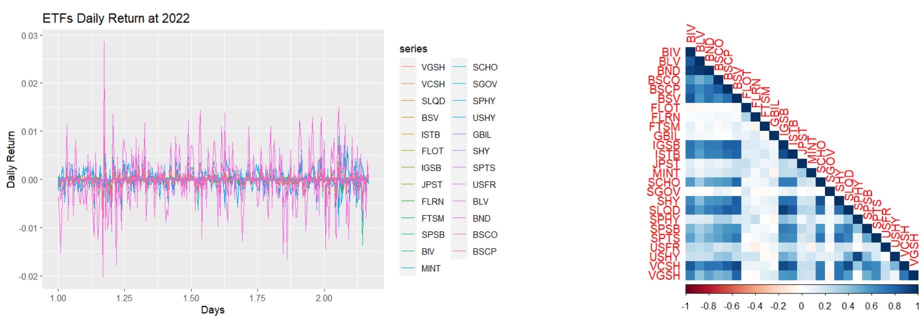


Figure 4: Mid-Risk Mid-Return Composition and Cross Corrlation

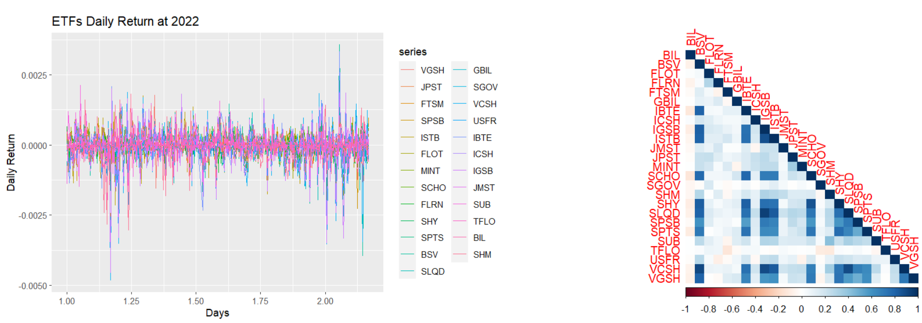


Figure 5: Low-Risk Low-Return Composition and Cross Corrlation

and MRMR portfolios, the LRLR portfolio displays a noticeable shrinkage in volatility, as seen in the tightening of the return plot's range. This reduction in volatility indicates a structural adjustment in the LRLR portfolio, likely through the reallocation of assets or changes in investment strategies to mitigate risk amidst rising interest rates. The LRLR portfolio's response highlights its sensitivity to market conditions and the proactive measures taken to preserve capital and maintain lower volatility.

Cross-section correlation plots for 2022 reveal nuanced interactions between different fixed-income investments within the portfolios, shedding light on how these relationships have evolved in the context of interest rate changes. As we transition from the HRHR to the MRMR portfolio, there is an observable decrease in relative correlation among investments. This shift suggests that the MRMR portfolio employs a more diversified strategy, possibly integrating a broader range of fixed-income assets with varying sensitivities to interest rates. The diversification aims to balance risk and return by spreading exposure across different segments of the fixed-income market. The most striking observation comes from the LRLR portfolio, where a clear structural decrease in cross-correlation is evident. This pronounced reduction in correlation points to a significant reallocation of assets within the LRLR portfolio, aimed at minimizing risk through investments in fixed-income securities less correlated to each other. By doing so, the LRLR portfolio demonstrates a strategic response to the rising interest rates, prioritizing capital preservation and stability in an uncertain market environment.

The analysis of return and cross-section correlation plots for the year 2022 provides valuable insights into the behavior of fixed-income ETF portfolios under varying risk profiles amidst significant interest rate changes. While HRHR and MRMR portfolios maintained their structure and exhibited resilience, the LRLR portfolio adapted by reducing volatility and enhancing diversification to mitigate risk. These observations underscore the importance of dynamic portfolio management and the strategic allocation of assets to navigate through periods of financial market volatility effectively.

3.2 Bonds' Data Collection and Processing

The US Treasury Bond market represents a cornerstone of global finance, offering insights into government fiscal policy, investor sentiment, and macroeconomic trends. This section delves into the empirical analysis of the US Treasury Bond dataset, highlighting its structure, the challenges posed by missing data, and the observable shifts in yield curves over recent years.

Our study focuses on a comprehensive dataset comprising 13 US Treasury bonds, spanning maturity terms from 1 month to 30 years, meticulously collected to offer a panoramic view of the fixed-income landscape. The dataset's structure embodies a wide spectrum of maturity terms, providing a rich basis for analyzing the yield curve's evolution. Key aspects of data processing include the volume of over 3000 daily data points per bond, encapsulating price movements and yield changes over the specified period; the variety of yields across a broad range of maturities, from short-term bills to long-term bonds, alongside other relevant metrics; the veracity of data sourced from Trading View, cross-validated with official Treasury data, despite some discrepancies, such as minimal negative yields not officially recorded in Treasury statistics; the variability of significant fluctuations in yield curves reflecting responses to monetary policy changes, economic outlook shifts, and global financial market dynamics; and the velocity of daily data updates capturing the fast-paced changes in bond yields, crucial for real-time analysis and forecasting.

The preprocessing of the bond dataset focused on addressing the challenges posed by missing data and anomalies, given the dataset's characteristics. A significant portion of the dataset was affected by missing values, particularly for bonds with shorter maturities. To tackle this issue, the data cleaning and preprocessing steps involved a two-stage approach.

In the first stage, bootstrapping techniques were employed to estimate the missing values. This involved resampling the available data to generate plausible values for the missing data points, creating a more complete dataset for further analysis.

Following the bootstrapping stage, the Nelson-Siegel model was applied to estimate a more realistic yield curve. The Nelson-Siegel model is a widely used parametric model that captures the key components of the yield curve, namely the level, slope, and curvature. By fitting the model to the available data, it becomes possible to estimate yields for any given maturity term at a specific date, ensuring continuity in the time series and enabling a more coherent analysis of yield curve shapes.

During the preprocessing, special attention was given to the few instances of slightly negative yields observed in the dataset. These negative yields diverge from the general expectation of non-negative US Treasury yields and were marked for future investigation to understand their implications and potential causes.

$$y_{t_i}(\tau) = L_{t_i} + S_{t_i}\frac{1-e^{-\lambda\tau}}{\lambda\tau} + C_{t_i}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \epsilon$$

Figure 6: Nelson-Siege Model

The application of bootstrapping and the Nelson-Siegel model in the data cleaning and preprocessing steps enables a more accurate examination of bond yields over time. By addressing missing data and estimating a realistic yield curve, this approach lays the foundation for constructing a bond ladder portfolio in the later stages of the analysis.

The descriptive analysis revealed notable shifts in the yield curve's shape over recent years, mirroring changes in economic conditions and monetary policy. These shifts were especially pronounced during periods of financial turmoil or significant policy announcements by the Federal Reserve. The analysis identified three main yield curve shapes: the normal yield curve, typically observed during periods of economic expansion, where longer-term bonds exhibit higher yields than shorter-term instruments; the inverted yield curve, emerging in times of economic uncertainty, when short-term yields surpass those of longer-term bonds, often considered a recession predictor; and the flat yield curve, occurring when there's little difference between short-term and long-term yields, indicating a transitional phase in the economy.

The preliminary insights from the descriptive analysis underscored the yield curve's sensitivity to macroeconomic forces and its role as a barometer for economic health. While US Treasury bonds have historically maintained positive yields, the dataset's minor negative values invite further scrutiny, possibly attributed to market anomalies or data inaccuracies. The observed variability in yield curves, ranging from normal to inverted shapes, reflects the complex interplay between investor expectations, fiscal policy, and global economic events.

3.2.1 Further Analysis

The visual analysis of the bond portfolio through various plots provides a comprehensive overview of the yield curve dynamics and correlation structures across different maturities of U.S. Treasury bonds. The 3D yield curve plots, with the x-axis representing bond maturity years, the y-axis indicating days since January 1, 2014, and the z-axis denoting the yield rate, offer a rich perspective on the evolution of bond yields over time. The cross-section correlation plot further illuminates the interconnectedness among bonds of various maturities.

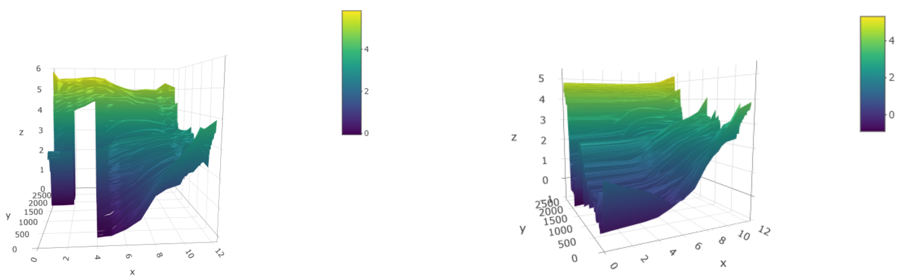


Figure 7: Orginal Yield Curve (Left) and Adjusted Yield Curve (Right)

The 3D yield curve plot for the original dataset reveals pronounced gaps in data, especially for bonds with shorter maturities. This missingness highlights the challenges in tracking the performance of these rapidly maturing securities, possibly due to their high turnover and the nuanced fluctuations in their yields. Despite these gaps, the plot encapsulates the shifts that U.S. Treasury bonds have experienced over the observed period, with the yield curve adapting to varying economic conditions, interest rate policies, and market sentiments.

Comparing the original yield curve to the fitted (bootstrapped and Nelson-Siegel) yield curves demonstrates the efficacy of our methods in bridging the data gaps. The fitted curve closely mirrors the original, maintaining the overall trend and smoothing out data discontinuities. However, irregular movements in bond yields appear at the extremities, particularly for bonds nearing or surpassing 30 years to maturity. These fluctuations suggest the need for more refined fitting techniques, possibly incorporating advanced spline methods, to accurately capture the yield dynamics at these longer durations without introducing artifacts.

The cross-section correlation plot underscores a strong interconnectedness across bonds of various maturities, reflecting the market's perception of risk and return across the term structure. As the maturity difference widens, the correlation among bonds diminishes, suggesting that bonds with significantly different maturities respond differently to market conditions. This

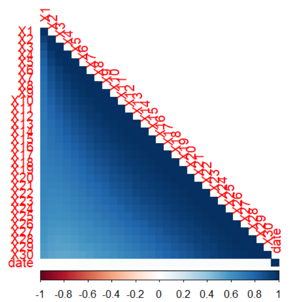


Figure 8: Simulated Bond Yield Cross Corrlation

trend highlights an opportunity for diversification and provides insights into strategic portfolio construction to minimize risk.

It is important to note that the fitting techniques employed, notably the Nelson-Siegel model, play a pivotal role in shaping the correlation structure observed in the plots. While these methods effectively smooth out yield curves and address missing data, they also influence the correlation dynamics among bonds of varying maturities. This influence warrants further exploration to understand the implications of fitting choices on the perceived relationships among different bond maturities.

The analysis of these plots sheds light on the intricate dynamics of the U.S. Treasury bond market, revealing patterns and relationships that are pivotal for investors, policymakers, and researchers. By addressing data gaps and refining our yield curve modeling approaches, future work can enhance our understanding of bond market behaviors and inform more effective bond portfolio management strategies. The inclusion of bond maturity years on the x-axis, days since January 1, 2014, on the y-axis, and yield rates on the z-axis provides a comprehensive framework for visualizing and interpreting the complex interplay of time, maturity, and yields in the U.S. Treasury bond market.

4 Fixed Income ETF Portfolio

4.1 Methodology of Asset Selection

Our methodology and project can be split up into 2 parts. The first part is to determine which ETFs we should add to our portfolios. The first thing we need to do is to understand and get enough background info on the different fixed-income markets and ETFs that are available. Once we understand this, we will create a scoring system that includes a risk-adjusted weighted score that will rank the ETFs for each risk profile. Once we have the top 25 ETFs based on our scoring system we will move to part 2 of our methodology and collect more data for these ETFs and create an optimized portfolio for each risk profile. We will then assess our portfolios and adjust if needed.

4.1.1 Screening Selection

The first part of our methodology is figuring out which ETFs we should select to give scores. There are thousands of different ETFs to choose from so to narrow it down we decided to take data from ETF.com which provides real-life ETF news and analysis. Using this website we are able to filter ETFs by many different factors. We decided to view only Fixed income North American ETFs. We decided to only view these ETFs as North American fixed-income ETFs focus primarily on US treasuries, corporate bonds, municipal bonds, and mortgage-backed securities. ETFs in different regions also have different market structures and regulatory frameworks. Once we filter by these factors we will sort by AUM and take the top 100 ETFs with the highest AUM. AUM stands for assets under management and refers to the total market value of all assets in an ETF. It is a key indicator of the size, popularity, and reliability of an ETF so that is why we are sorting our ETFs by this factor. With our screening process completed, we will take the top 100 ETFs and decide on a scoring criteria.

4.1.2 Scoring Criteria Selection

To rank these 100 ETFs we have created a scoring criteria with different metrics that we have calculated. We have a total of 12 metrics and all of these metrics can be placed into a category of what it is measuring. Our 4 categories are return performance, risk-adjusted return performance, volatility, and diversification.

Starting with our return performance category, this category evaluates how effectively a bond has performed in generating returns for its holder over a specific period. These metrics help investors assess the profitability and efficiency of their bond investments. We have decided to include four metrics to measure this category. The first one is the Expense Ratio which is the management fee an investor must pay to own an ETF and is a percent of your investment. Annual Dividend Yield is the amount of money the bond issuer pays shareholders for owning a share of the asset divided by its current price. Mean net cash flow for a bond refers to the difference between the cash inflows such as coupon payments and principal repayment and outflows such as the purchase of a bond associated with owning that particular bond over a specified period.

Our next category is risk-adjusted return performance. This category gauges the efficiency of a bond's return concerning the level of risk taken to achieve that return. These metrics help investors assess how well a bond has performed concerning the risks involved in holding that bond. The two metrics we have included for this category are the Sharpe Ratio and the Sortino Ratio. The Sharpe ratio divides a portfolios excess returns by a measure of its volatility to assess risk-adjusted performance. It quantifies the excess return generated per unit of risk (volatility). A higher Sharpe Ratio indicates better risk-adjusted performance: a higher return for the same level of risk or lower risk for the same level of return. The Sortino ratio is a variation of the Sharpe Ratio but differs in that it only considers the standard deviation of the downside risk, rather than that of the entire (upside plus downside) risk. Because the Sortino ratio focuses only on the negative deviation of a portfolios returns from the mean, it is thought to give a better view of a portfolios risk-adjusted performance since positive volatility is a benefit.

The next category to view is the Volatility category. This category assesses the level of fluctuation or variability in a bond's price or returns over a specific period. These metrics help investors understand the degree of risk or uncertainty associated with a bond's performance. The first metric of this category is the omega ratio which is a ratio that assesses the risk and return of an investment at a specific expected return level. It helps us determine the likelihood of winning versus losing, with a higher ratio indicating better chances of success. Next up is VaR (Value at risk). Value at Risk measures the potential loss an asset can have in a given period with a particular confidence level. VaR helps investors estimate the worst expected loss under normal market conditions. Expected Shortfall, also known as Conditional Value at Risk, is a risk measure that assesses the average magnitude of losses beyond the VaR. Expected Shortfall provides further insight into the potential severity of losses in the tail end of the distribution of possible outcomes. While VaR measures the maximum potential loss at a certain probability level, Expected Shortfall goes beyond that by estimating the average value of losses that occur beyond the VaR threshold. Max Drawdown is a measure used to evaluate the largest decline or loss experienced by an investment or a portfolio from its peak value to its lowest point over a specific period. It quantifies the worst possible loss an investor would have endured by holding an investment from its highest historical value to the lowest subsequent value before a new peak is achieved.

Our last category is Diversification and this category evaluates the degree of variety or spread within a bond portfolio, assessing the extent to which the portfolio is exposed to various bond issuers, sectors, or other factors. These metrics help in understanding the level of risk reduction achieved through diversifying across different types of bonds. The first metric in this category is the HHI (Herfindahl-Hirschman Index). HHI measures the size of ETFs relative to the size of the industry they are in and the amount of competitiveness. The HHI is calculated by squaring the market share of each firm competing in a market and then summing the resulting numbers. The ESG score of a bond measures the Environmental, Social, and Governance performance of the issuer or the specific bond itself. ESG stands for Environmental, Social, and Governance criteria, which are used to evaluate the sustainability and ethical impact of an investment. The Environmental area evaluates how the issuer manages its impact on the environment. It includes factors such as carbon emissions, resource usage, pollution, renewable energy initiatives, and adherence to environmental regulations. The Social aspect focuses on the issuer's social impact. It considers aspects such as labor practices, human rights, diversity, and community relations The last area is Governance and it examines the governance structure, transparency, accountability, and ethical standards of the issuer. We have decided to use MSCIs ESG score. MSCI is a leading provider of investment decision support tools and research for investors.

Category	Metrics
Returns	Expense Ratio
	Annual Dividend Yield
	Mean Net Cash Flow
Risk Adjusted Returns	Mean Sharpe Ratio
	Mean Sortino Ratio
Volatility	Omega
	Mean VaR
	Mean Expected Shortfall
	Mean Max Drawdown
Diversification	HHI
	ESG Score
	Number of Holdings

Table 1: Metrics used in the scoring criteria and their corresponding category

4.1.3 Scoring Rule

With our scoring criteria created our next step was to give each ETF a score for each of its metrics. We decided to use a 2-value scoring system and gave a value of 1 to an ETF if that ETF had a metric that was in the top 25 percent of all the 100 ETFs else we would give a value of 0 if it was in the bottom 75 percent. Depending on the metric and what the metric measures, a score of 1 would either go to the lowest 25 percent of values or the highest 25 percent of values of all ETFs we have collected. Our ETF readjustment occurs yearly so we will be able to add or remove ETFs to our portfolio at the beginning of every year. Because of this, we will have to recalculate the values of these metrics for each ETF every year. This will make sure that our ETF scoring rule is using the most up-to-date data. This means that every year an ETF could receive different scores for the same metric depending on whether or not it performed better or worse based on the metric we are measuring.

For the following metrics, we prefer values that are as low as possible: Expense Ratio, Mean Sortino Ratio, Omega, Mean Max Drawdown, and HHI. For the following metrics, we prefer values as high as possible: Annual Dividend Yield, Mean Net Cash Flow, Mean Sharpe Ratio, Mean VaR, Mean Expected Shortfall, ESG Score, and Number of Holdings.

Some examples of our scoring rule are given below. For our expense ratio metric, a low expense ratio is better than a higher one and would give you a higher return if everything else was fixed. Because of this, we will give ETFs a score of 1 if the expense ratio is in the lowest 25 percent of all 100 ETFs. For our ESG score, we would prefer higher ESG scores as they represent ETFs that are more diverse. We would give a score of 1 to an ETF if that ETF is in the top 25 percent of all the ESG scores.

Category	**Metric**	**Values With a Score of 1**
Returns	Expense Ratio	Bottom 25%
	Annual Dividend Yield	Top 25%
	Mean Net Cash Flow	Top 25%
Risk Adjusted Returns	Mean Sharpe Ratio	Top 25%
	Mean Sortino Ratio	Bottom 25%
Volatility	Omega	Bottom 25%
	Mean VaR	Top 25%
	Mean Expected Shortfall	Top 25%
	Mean Max Drawdown	Bottom 25%
Diversification	HHI	Bottom 25%
	ESG Score	Top 25%
	Number of Holdings	Top 25%

Table 2: Scores an ETF will get for each a metric

Metric	**2015**	**2016**	**2017**
Expense Ratio	1	1	1
Annual Dividend Yield	1	1	1
Mean Net Cash Flow	0	0	0
Mean Sharpe Ratio	0	0	1
Mean Sortino Ratio	1	0	0
Omega	1	0	0
Mean VaR	0	0	0
Mean Expected Shortfall	0	0	0
Mean Max Drawdown	0	0	0
HHI	1	1	1
ESG Score	0	0	0
Number of Holdings	1	1	1

Table 3: Score for ETF: BLV

For the ETF BLV, we can see the corresponding scores for each metric for the years 2015, 2016, and 2017. BLV gets a score of one every year for the expense ratio, dividend yield, HHI, and number of holdings metrics. Throughout these three years, these metrics have remained consistent and have always been in the top 25 for this ETF. we also see that the Mean Sharpe Ratio gets a score of 0 in 2015 and 2016 and a score of 1 in 2017. In 2015 it had a Mean Sharpe Ratio of -.0438 and in 2016 it was .0207. In 2017 it increased to .0574. Due to the low values in 2015 and 2016, it was not in the top 25% to earn a score of 1 however because of the huge increase in 2017 it was able to receive a score of 1 for this year.

4.1.4 Risk Weighting Rule

Once each metric has been given a score for its ETF, our next step is to figure out which metrics are more important for our different risk profiles. Each of our risk profiles targets a different type of growth so we will give different weights to these metrics based on their risk profile. We have a total of 12 metrics and each metric will be given a weight of either 1, 2/3, or 1/3 based on their importance for each risk profile. For our high-risk and returns profile, we give more weight to our return category metrics as we are targeting ETFs that give a high rate of return. We also gave less weight to the volatility category compared to the low-risk and returns profile. We gave our high risk and returns profile a smaller weight because we are fine with that risk profile having greater risk and volatility than our low risk and returns profile.

Category	Metric	High Risk & Returns	Med Risk & Returns	Low Risk & Returns
Returns	Expense Ratio	1	1/3	1/3
	Annual Dividend Yield	1	1/3	1/3
	Mean Net Cash Flow	1	1/3	1/3
Risk Adjusted Returns	Mean Sharpe Ratio	2/3	1	2/3
	Mean Sortino Ratio	2/3	1	2/3
Volatility	Omega	1/3	1/3	1
	Mean VaR	1/3	1/3	1
	Mean Expected Shortfall	1/3	1/3	1
	Mean Max Drawdown	1/3	1/3	1
Diversification	HHI	2/3	2/3	2/3
	ESG Score	2/3	2/3	2/3
	Number of Holdings	2/3	2/3	2/3

Table 4: Metrics with their corresponding weights for each risk profile

4.1.5 Scoring plus risk weighting rule

Now that we have our weights for each metric we will then multiply these weights by the original score that metric got and sum up this value for all of the metrics. Our S_i will represent the score for each metric and W_i will be the risk weight of that metric. For instance, S_1 will represent the score of the expense ratio category of an ETF and W_1 will be the risk weight for the expense ratio category. We will have different weights for each risk profile so we will do this for each risk profile and calculate the total score for each ETF.

Multiply the risk weight (1/3, 2/3, or 1) for each metric by the specific metric score (0 or 1) for each ETF and sum up the scores for each ETF ticker

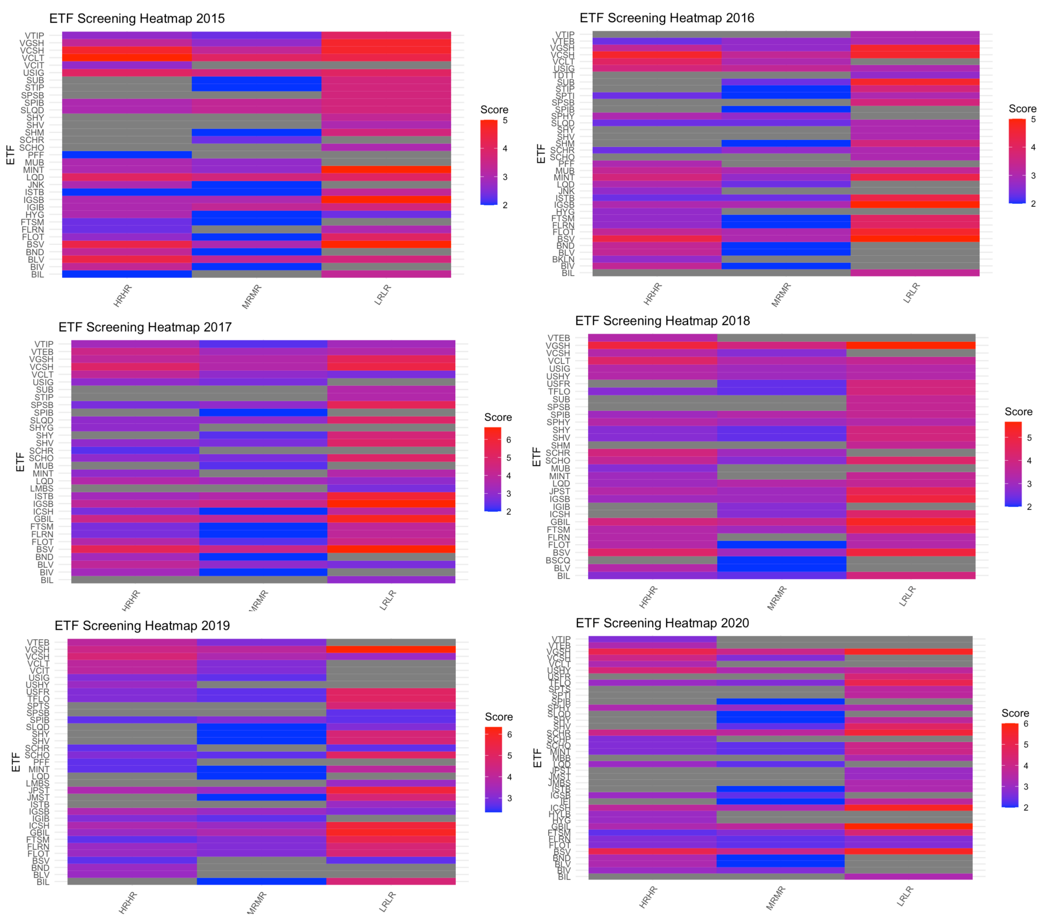
Total Score for one ETF:

$$\sum_{i=1}^{12} S_i w_i$$

Where

S_i = the score for metric i, W_i = the risk weight for metric i

Below are heatmaps of some ETFs and their corresponding scores for each risk profile. Because we are re-selecting ETFs to be added or removed from our portfolio every year, we will have different scores for each year. We will use these scores to decide on which ETFs to add to our portfolio.



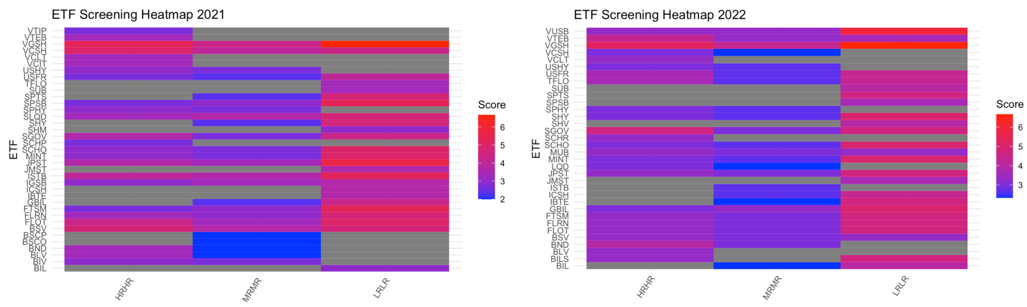


Figure 9: Heatmap of scores of ETFs

Comparing the scores from one risk profile to another we see that there are some differences in the scores of each ETF. Seeing how the scores differ from risk profile to risk profile shows that our scoring and risk-weighted rule was effective and is useful in determining which ETFs will perform better in the different risk profiles. We also see that throughout the years, the scores of each ETF remain relatively consistent. This means that there was not much sudden change on the metrics from year to year. When there were changes in the score this means that they either lost or gained a point on the metrics we used to calculate ther scores.

4.1.6 ETF selection for risk profiles

With our scores for each ETF for all of our risk profiles, we can now move on to our ETF selection step. For each of our risk profiles, we can have at most 25 and at least 3 ETFs. To select the 25 potential ETFs we can add to our profiles, we will determine the ETFs with the top 25 scores for each risk profile every year. With the ETFs selected, we will only focus on these ETFs for our portfolios and we can move on to part two of our methodology, portfolio optimization.

The heatmap below shows how our selection of ETFs for each risk profile has changed over the years. The light blue color indicates that that ETF had been selected for that year and if it is dark blue then that means that we did not select it for the corresponding year. ETFs that are light blue indicate that they received a high score and were in the top 25 scores for a year for the risk profile. If we see that an ETF appeared in our portfolio for one year then the following year it was not included then this means that it did not receive a high enough score that year to be included in our portfolio. This means that it could have received a score of 0 in one of the metrics above which caused it to have a lower score.

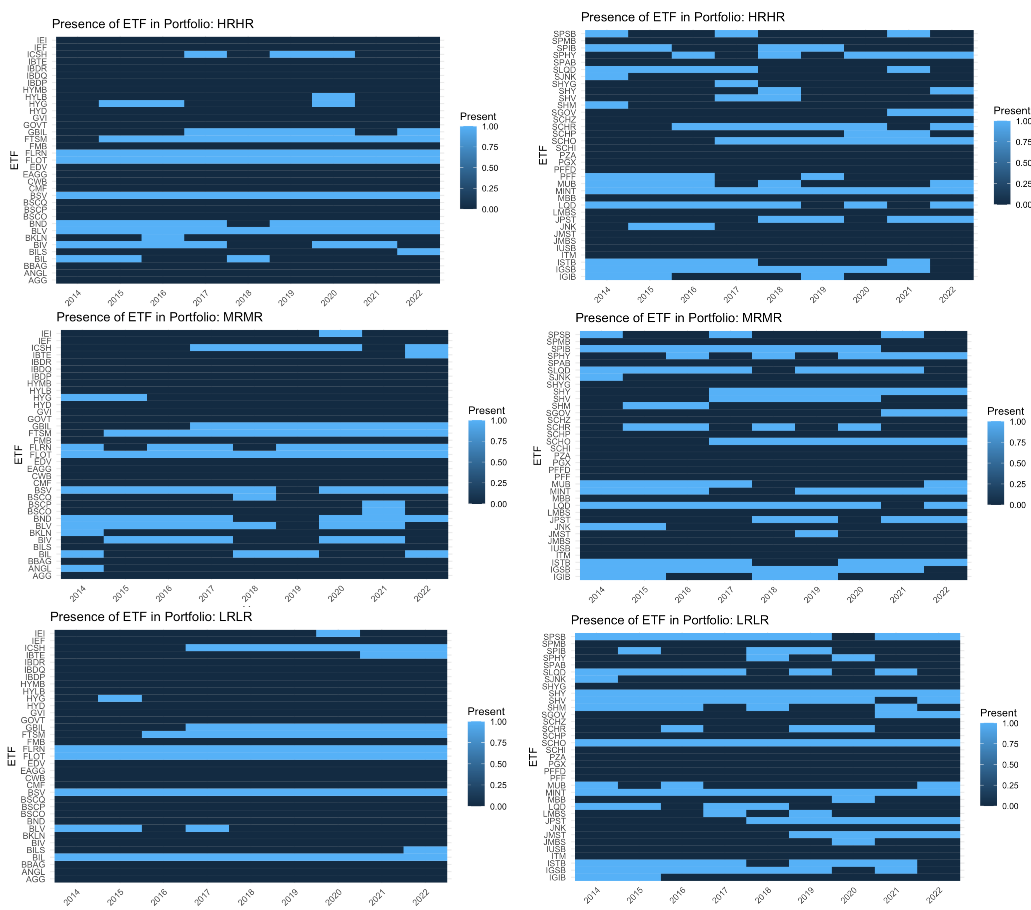


Figure 10: Heatmap of ETFs selected

With our scoring selection completed, we have our top 25 ETFs for each year.

4.2 Portfolio Analysis

4.2.1 Covariance comparison between Minimum Covariance Determinant and Sample methods

The Minimum Covariance Determinant (MCD) and Sample methods are two techniques used to estimate the covariance matrix of a dataset, which is essential in many statistical analyses, including financial risk modeling.

For MCD method, the idea behind minimizing the determinant is that the determinant of a covariance matrix measures how broad the distribution is. MCD therefore selects the subset of the data that is most tightly distributed. This is to exclude anomalies, which are likely to lie further away from the rest of the data. The MCD is particularly useful in situations where the data may contain anomalies or non-normal behavior that could skew the covariance matrix.

The Sample method, also known as empirical covariance, is the traditional way of estimating the covariance matrix. It takes the sample variance of each variable and the sample covariance between pairs of variables, assuming that every observation contributes equally to the estimate. While this method is straightforward, it can be sensitive to outliers.

In our case, we calculate the covariance of returns from 25 ETFs with a past one-year window in order to get the weights of two portfolios in closed form. Here, we are interested in whether using different covariance methods will produce a statistically different relationship between each ETF.

Therefore, we seek to compare the leading eigenvectors derived from the covariance matrices of 25 ETFs from High Risk High Return group over 252 trading days in 2015, which means that we have a one-year time series of covariance calculated using both the sample covariance and the Minimum Covariance Determinant (MCD) method. The analysis is structured as follows:

1. Perform Principal Component Analysis (PCA) on the covariance matrices obtained by both methods to extract the leading and the second eigenvectors. The leading eigenvector, associated with the largest eigenvalue, represents the direction of maximum variance in the data. The second eigenvector, corresponding to the second-largest eigenvalue, can then represent a secondary source of risk. So we then compare how each method captures the underlying risk factors.

2. For each ETF, compute the angles between the corresponding leading eigenvectors from the two methods. The angle θ_t between the first eigenvectors at time t from the normal covariance matrix, denoted $\mathbf{v}_{\text{normal},t}$, and the MCD method, denoted $\mathbf{v}_{\text{MCD},t}$, is calculated as follows. The cosine of the angle is given by the formula:

$$\cos(\theta_t) = \frac{\mathbf{v}_{\text{normal},t} \cdot \mathbf{v}_{\text{MCD},t}}{\|\mathbf{v}_{\text{normal},t}\| \|\mathbf{v}_{\text{MCD},t}\|}$$

   where $\mathbf{v}_{\text{normal},t} \cdot \mathbf{v}_{\text{MCD},t}$ is the dot product at time t, and $\|\mathbf{v}_{\text{normal},t}\|$ and $\|\mathbf{v}_{\text{MCD},t}\|$ are the magnitudes of the eigenvectors at time t. The angle θ_t is then determined by:

$$\theta_t = \arccos\left(\frac{\mathbf{v}_{\text{normal},t} \cdot \mathbf{v}_{\text{MCD},t}}{\|\mathbf{v}_{\text{normal},t}\| \|\mathbf{v}_{\text{MCD},t}\|}\right)$$

   The angles provide a measure of the similarity between the eigenvectors, with smaller angles indicating higher similarity on sensitivity to risks between our two covariance methods.

3. If an angle exceeds 90 degrees, it is adjusted to ensure the measure remains within the first quadrant, thereby standardizing the comparison.

4. Finally, we plot the angles for the first and second leading eigenvectors obtained from the two different covariance estimation methods on a circular histogram. See Figure 11, showing how frequently different angle values occur. Each bin represents an interval of 5 degrees, and the length of the bin from the center of the circle outward represents the count of angles within that interval.

We can see from the figures that the angles for leading eigenvectors calculated by MCD and Sample methods are close to 0 degree, and the angles for second eigenvectors are mostly in between 0 to 30 degrees, indicating a relatively high similarity.

This result is also intuitively appropriate since under fixed income market, lower volatility can result in fewer outliers and extreme values, making the robustness of the MCD method less impactful, as the sample covariance already provides a reasonably accurate representation of the data. Moreover, the returns of fixed income ETFs may be more homogeneous due to similar reactions to market conditions, such as interest rate or monetary policy changes in general. Homogeneity in data reduces the difference between the sample covariance and the MCD.

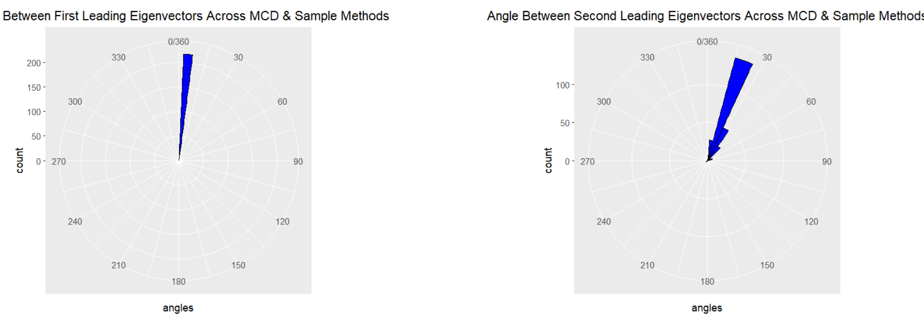


Figure 11: Histogram of Angles between Eigenvectors across MCD and Sample Methods

4.2.2 Portfolio Weights

First we can have a look at the monthly weights we calculated in one specific year of the Global Minimum Variance portfolio for High Risk High Return ETF group. As we have introduced in previous sections, the weights are rebalanced monthly so we can see in the Figure 12, we visualize the weights in long and short positions separately on the top two figures by month, and the bottom figure shows the proportion of long positions with respective to the overall investment.

We observe that a relatively high proportion of long positions in SGOV (an ETF that typically invests in short-term U.S. government securities) for the year 2022 in our GMV portfolio. Given that GMV aims to minimize risks and SGOV usually exhibits lower volatility compared to other assets. The GMV portfolio optimization process may heavily weight SGOV due to its stability. On the other hand, we can see that the GMV is holding a larger and larger short position in SLQD. We investigate that SLQD consists mainly of short-term, investment-grade corporate bonds.

We then turn to the weights calculated in 2022 of the Max Sharpe- Tangency Portfolio. We expect to see a different proportion of ETFs given the Tangency portfolio aiming to maximize returns. From Figure 13, we observe that the weights become more diversified among different ETFs and the relatively dominating weights in long positions is MINT, instead of SGOV from Month 1 to 8. Then interestingly, the dominating short weights become MINT and the dominating long weights become SGOV again. If we look into the 25 ETFs for High Risk High Return group, we can see that 19 out of 25 ETFs include a proportion of government-bond.

So it is not surprising to see that with the goal of maximizing the return, the Max-Sharpe Tangency portfolio has a relatively higher weights in MINT, which is PIMCO Enhanced Short Maturity Active ETF (includes government bonds, but mix). Compared with buying mostly government bonds, this action is more likely to gain a higher return with a higher risk.

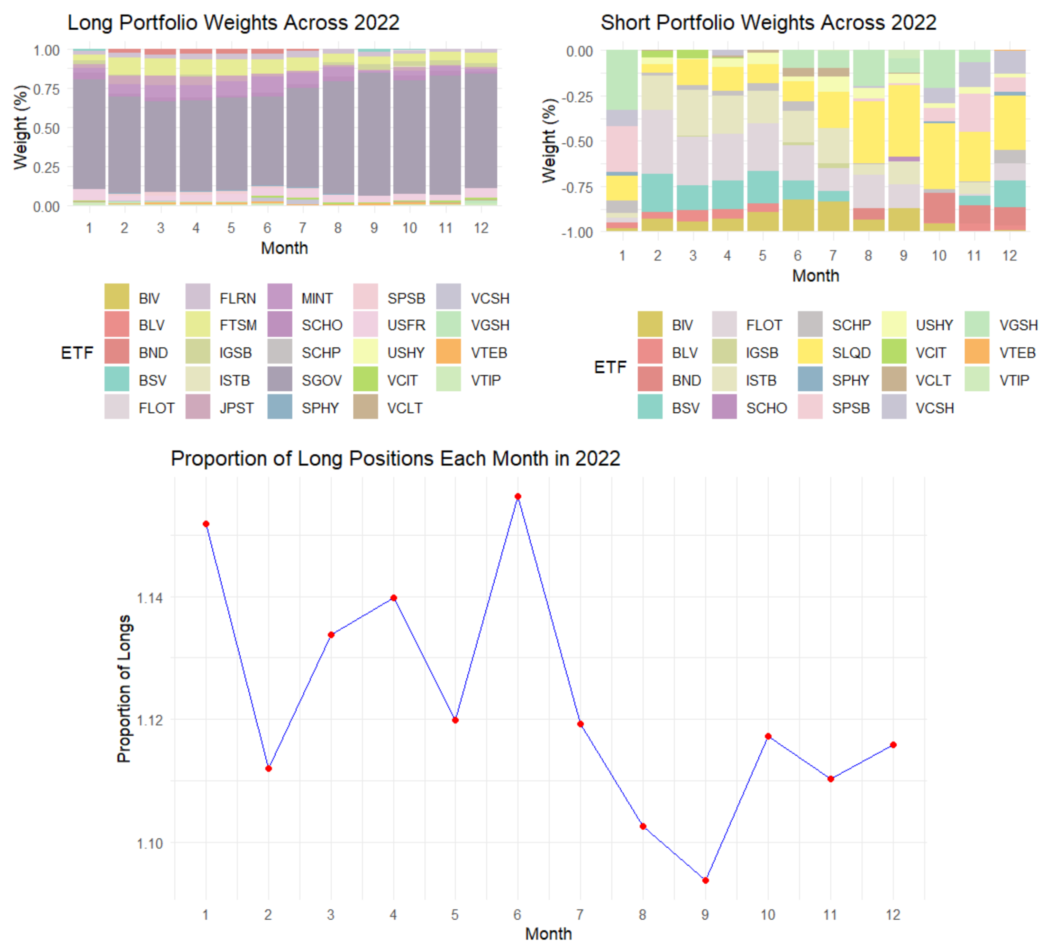


Figure 12: Top: GMV Portfolio Weights Long and Short positions for 2022. Bottom: Proportion of Long Positions Each Month in 2022.

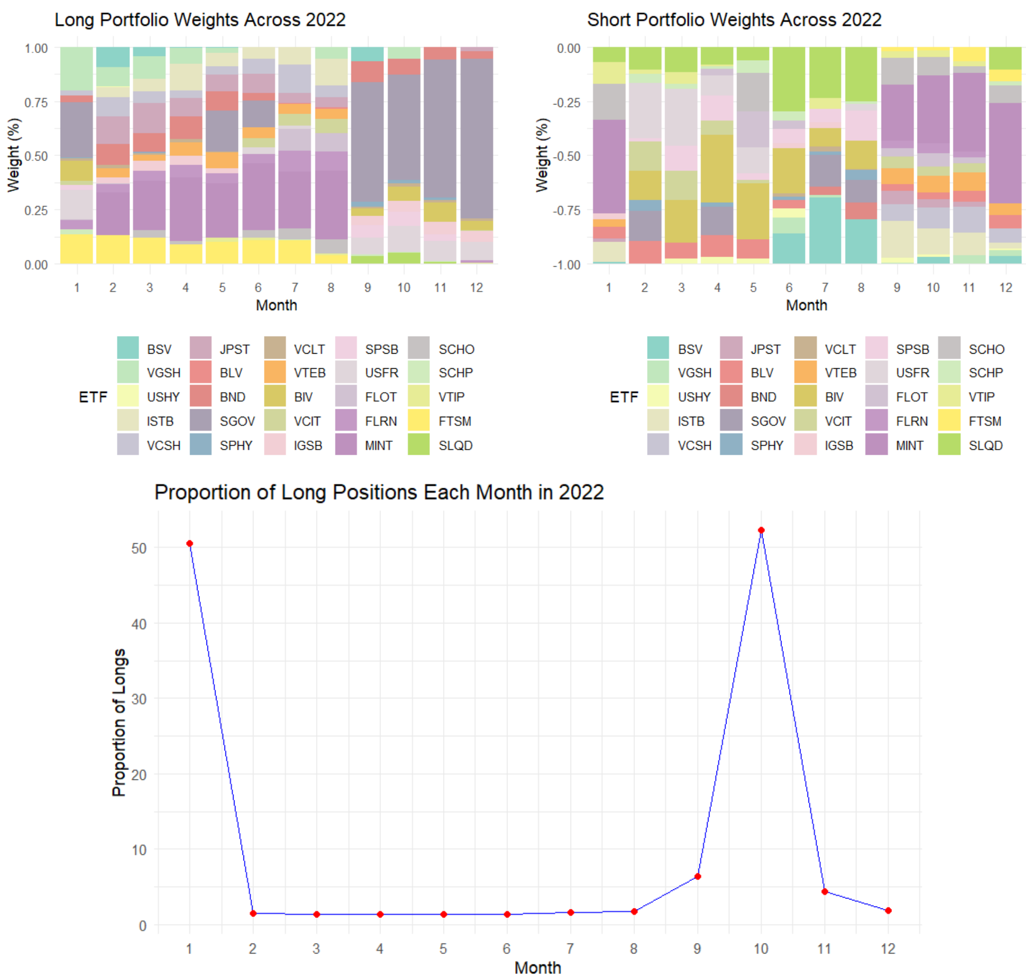


Figure 13: Top: Tangency Portfolio Long and Short positions for 2022. Bottom: Proportion of Long Positions in Tangency Portfolio Each Month in 2022.1 and 2022.12

4.2.3 Opportunity Frontier change

In Figure 14, we construct the Opportunity Frontier between 2022.1(Triangle points) and 2022.12(Square points) among the three ETF groups: High Risk High Return(Red color), Medium Risk Medium Return(Green color), and Low Risk Low Return(Blue color).

Firstly, we observe that the red lines in both months are always the outer curve, which implies that given the same expected risk, the investment in High Risk High Return ETF group can always gives us a better expected return, whereas the investment in Low Risk Low Return ETF group can often gives us a lower expected return. Moreover, we can see that from Month 1 to Month 12 in 2022, the opportunity frontier has shifted horizontally. This may because the correlations between the ETFs have shifted throughout the year. These shifts can change the shape and position of the frontier, indicating a higher risk level later in the year. It's also interestingly to see that the GMV portfolio is getting almost zero return in both months. The reason is that recall that the weight for GMV portfolio is greatly dominated by long positions in SGOV. If we plot the cumulative return for SGOV (Figure 15), we can see that the cumulative returns are close to 0 from the begining of 2021 to the mid of 2022 and then gets an upward trend to the end of 2022. So we can tell that the expected return for GMV is influenced by this trend.

This dominating proportion of nearly 80 percent leads us to the study of box constraints in the next section, where we perform constraints on the weights given different bounds to test on the diversification of the weights.

4.2.4 Concentration Comparison

First we aim to iteratively adjust the weights of ETFs within specified bounds. Given an initial set of weights, we take the following procedure:

1. If any weights fall below the lower bound or above the upper bound, they are set to the respective bound value.

2. We then calculate the excess or deficit amount caused by this bounding to maintain the sum of weights equal to one since we need full investment.

3. Next we excluding the outlier indices, as this reallocates the excess or deficit proportionally across the remaining weights. In other words, we

Figure 14: Opportunity Frontier Among Three ETF Groups in 2022

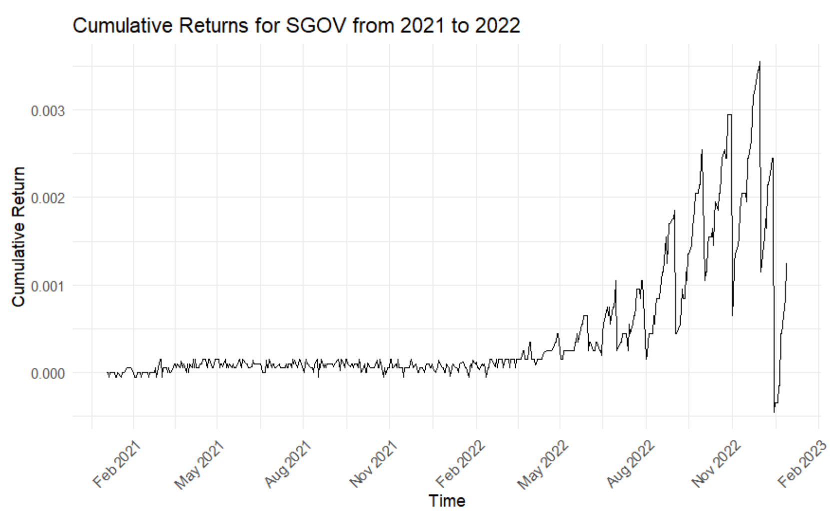


Figure 15: Cumulative Returns for SGOV from 2021 to 2022

redistribute the weights according to the previous weights in absolute value each ETF possesses.

4. This process repeats until all weights are within the bounds, ensuring that the total sum of weights for each column remains consistent.

We choose the upper bound and lower bound to be 20 percent and we get the following figure for the adjusted long and short positions GMV portfolio 2022 as before. See Figure 16. It's quite clear that the weights has a trend of equally distributed, since a few ETFs achieve the 20 percent bound as the adjustment goes iteratively. We now becomes more interested in the diversification measure as we adjust the bounds gradually larger both in GMV and Tangency portfolio.

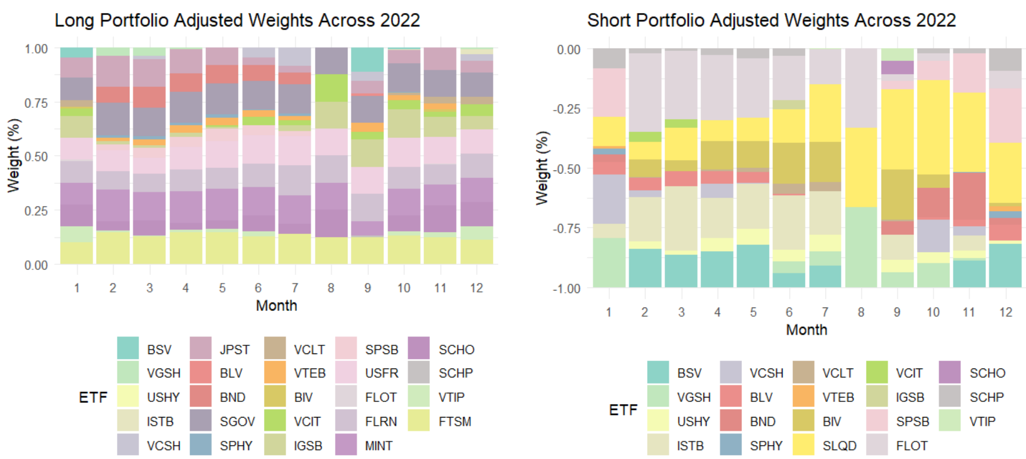


Figure 16: GMV Portfolio Long and Short positions for 2022 after Adjustment

The diversification measure D_t at time t is defined as the ratio of the portfolio's variance σ^2_{Port} to the weighted sum of the individual variances of the 25 ETFs in both portfolios. It is given by the formula:

$$D_t = \frac{\sigma^2_{Port}}{\sum_{i=1}^{25} w_i^2 \sigma_i^2}$$

This measure, D_t, reflects the level of diversification in the portfolio at time t. A higher value of D_t indicates a greater degree of diversification. We choose to plot D_t in 2022 for the original weights and the adjusted weights with bounds taking values of 20, 30, 40, and 50 percent.

From Figure 17, we can see that as the bounds grow more and more unrestricted, the diversification measure in GMV and Tangency portfolio becomes smaller in general. This is exactly what we expect at the beginning since introducing more equally distributed ETFs is less volatile than possessing one dominating ETF throughout a year.

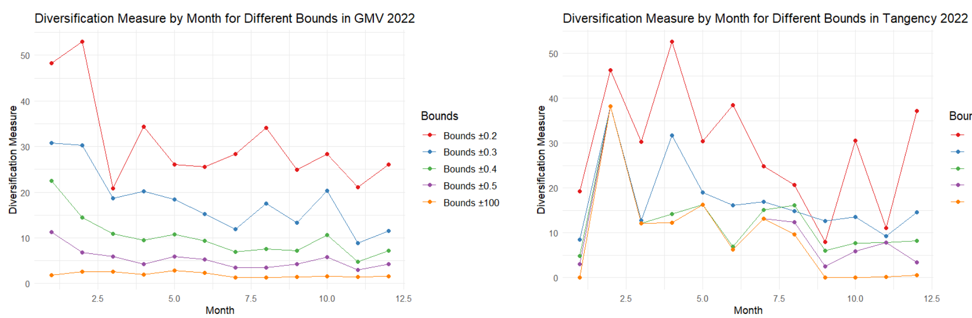


Figure 17: Diversification Measure for GMV and Tangency Portfolio 2022 with Different bounds

4.3 Portfolio Performance and Risk Analysis

For both GMV and Tangency portfolio, we plot the risk measures of VaR and Sharpe Ratio from 2015.1 to 2022.12 using a past one-year window among three ETF groups: High Risk High Return(Red color), Medium Risk Medium Return(Blue color), and Low Risk Low Return(Gree color).

See Figure 18. For GMV portfolio, we can observe that the High Risk High Return ETFs generally have more significant swings in both VaR and Sharpe Ratio compared to the Low Risk Low Return ETFs. This aligns with the expectation that higher-risk investments should exhibit greater volatility in both potential loss and risk-adjusted returns. Moreover, both risk measures show periods of increased decline. For example, the VaR for these three groups all expects a greater loss under the confidence interval of 95 percent starting with the 2020 year and they likely reflect the market's reaction to the COVID-19 pandemic. However, looking at the Sharpe Ratio, most of the times the GMV portfolio for High Risk High Return ETFs underperforms the other groups on a risk-adjusted basis, especially the case after the mid of 2021.

See Figure 19. For Tangency Portfolio in general, both the VaR and Sharpe Ratio are more volatile when compared to the GMV portfolio. This might suggest that the Tangency portfolio, while striving for the best risk-adjusted return, takes on more risk compared to the GMV portfolio. In addition, the Tangency portfolio's risk measured by VaR and performance seen in the Sharpe Ratio vary significantly across different ETF groups and over time. This is interpretable as the Tangency portfolio is optimized for the Sharpe Ratio and not for minimum variance, suggesting a trade-off between higher potential returns and increased potential risk.

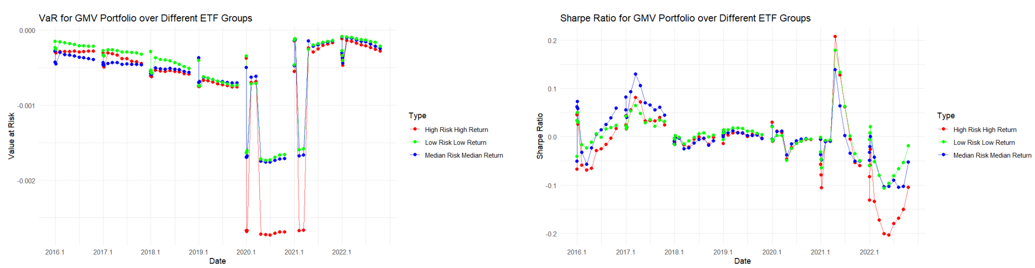


Figure 18: VaR and Sharpe Ratio for GMV Portfolio

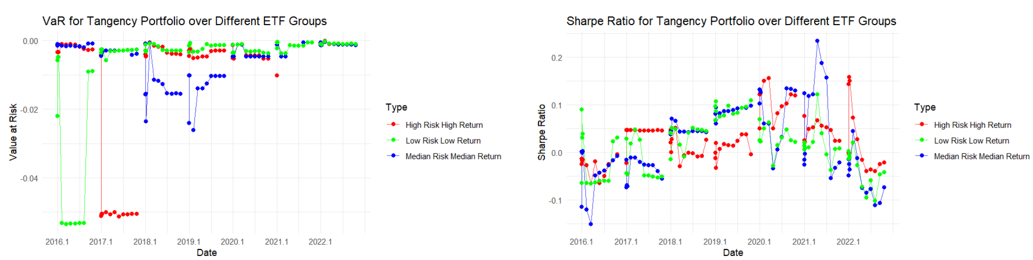


Figure 19: VaR and Sharpe Ratio for Tangency Portfolio

4.4 Dividends

A dividend is the distribution of a company or issuer's earnings to its shareholders and is determined by the issuer of the ETF. The issuer of the ETF will pay the dividend to the fund, and it will then be passed on through the portfolio of the owners' ETF. Each ETF has a specific distribution policy that outlines how and when dividends will be paid out to shareholders. Some ETFs will distribute dividends monthly, quarterly, or annually, while others may reinvest dividends and focus on capital appreciation.

The dividend yield is a financial ratio that measures the amount of cash dividends paid out to shareholders relative to the market value of a share. Its expressed as a percentage and is calculated by dividing the annual dividends per share by the price per share.

$$\text{Dividend yield} = \left(\frac{\text{Annual Dividends Per Share}}{\text{Price Per Share}}\right) \times 100\%$$

This ratio is commonly used by investors to evaluate the income generated from an investment in stocks or other dividend-paying assets, relative to its price. A higher dividend yield indicates that the investment generates a higher amount of income for each dollar invested, making it an attractive option for income-focused investors. However, its important to note that a very high dividend yield can sometimes indicate potential risks, such as a company in financial trouble that might not be able to sustain its dividend payments.

When shorting an ETF, you are borrowing shares of the ETF to sell them on the market with the hope of buying them back later at a lower price, thus profiting from the decrease in price. However, shorting an ETF comes with specific obligations regarding dividends. If the ETF pays a dividend while you are in a short position, you are responsible for paying the dividend amount to the entity from whom you borrowed the shares. This is because the actual owner of the ETF shares (the lender) is entitled to receive dividends.

4.4.1 Dividends in our Portfolio

In our investment strategy for managing our client portfolios, we have decided to adopt an approach to handling dividends from the ETFs in which we invest. Recognizing the varying dividend distribution schedules across

ETFs, monthly, quarterly, and annually, we have devised a method that simplifies the dividend reinvestment process, ensuring consistency and ease of management across our portfolios.

Our approach involves retrieving the annual dividend yield rate for each year for all ETFs. We utilized YCharts.com to obtain the historical dividend yields for the ETFs in our portfolios due to the reliability and availability of its data. YCharts is a comprehensive financial research platform widely recognized for its in-depth data, tools, and analysis capabilities across various investment instruments, including stocks, ETFs, mutual funds, and economic indicators. It offers users access to a vast array of financial information, including historical performance data, dividend yields, financial ratios, and much more, making it a valuable resource for investors and financial analysts.

Because dividend yield rates are not constant throughout the year, for simplicity we have decided to retrieve the annual dividend yield on the first trading day of that year. With our dividend yield rate retrieved for every year, our next step is to decide when to apply it to our ETFs. Because our selection of ETFs does not change within a year and will only change at the beginning of the year we have decided to reinvest our dividends at the beginning of the year as well. This means that we will apply our dividend yield to our ETFs current investment value at the end of December for every year. Once the annual dividends are calculated for each ETF, we will then sum up these amounts and these funds are then reinvested back into our portfolios, thereby increasing the capital available for reinvestment in the next year.

By doing so, we calculate the dividend payout as if all dividends were distributed at the end of the year, regardless of the actual dividend distribution schedule of each ETF. This method allows us to aggregate and reinvest dividends in a uniform manner, removing the complexities that arise from managing multiple distribution schedules. This method provides a streamlined, predictable framework for dividend reinvestment, contributing to the overall efficiency and effectiveness of our investment management approach.

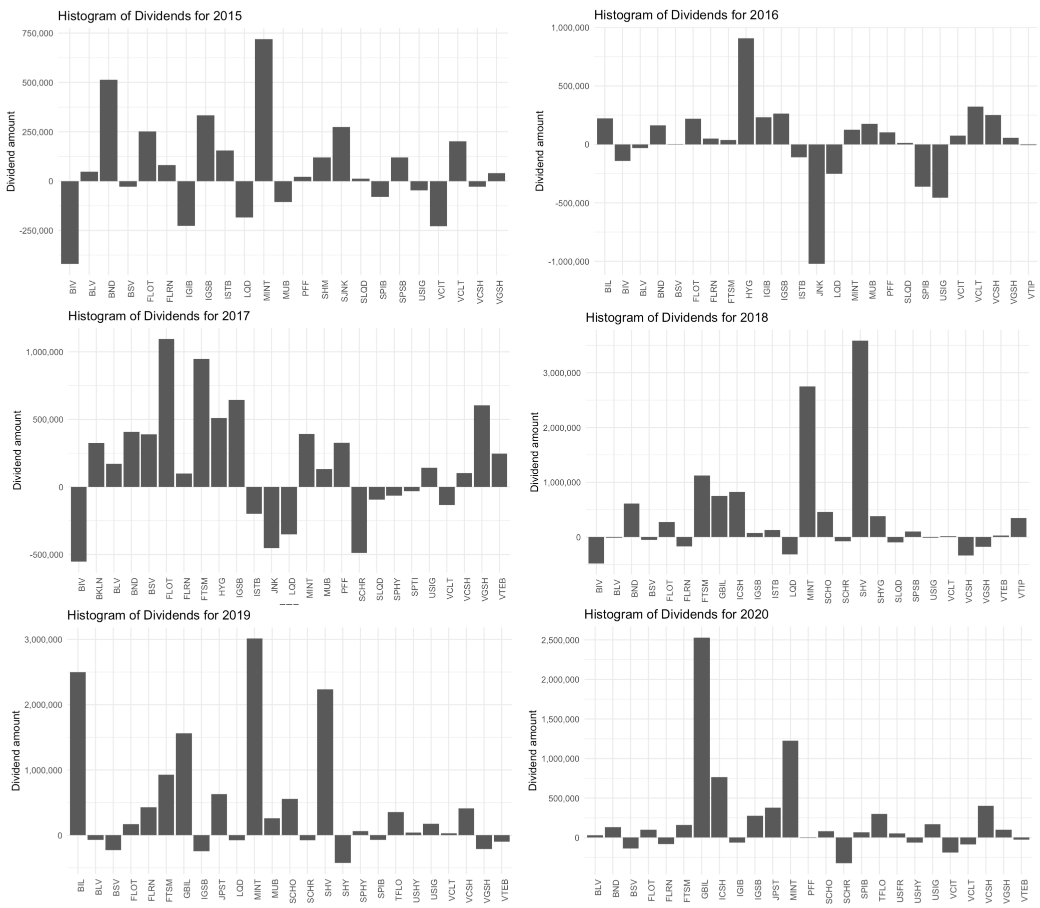
$$D_{iy} = a_{iy} \times b_{iy} \tag{6}$$

where a_{iy} is the value of ETF i at the end of year y, b_{iy} is the dividend yield rate for ETF i for year y, and D_{iy} is the dividend payout for ETF i for year y.

With our dividend payouts calculated for each ETF, we can take the sum of these values to calculate the total dividend payout for a specific year. We will then add this amount back into our capital to reinvest for the next year.

4.4.2 Dividend Analysis

The plots below show the dividend payments for each ETF for the high-risk and returns profile for every year.



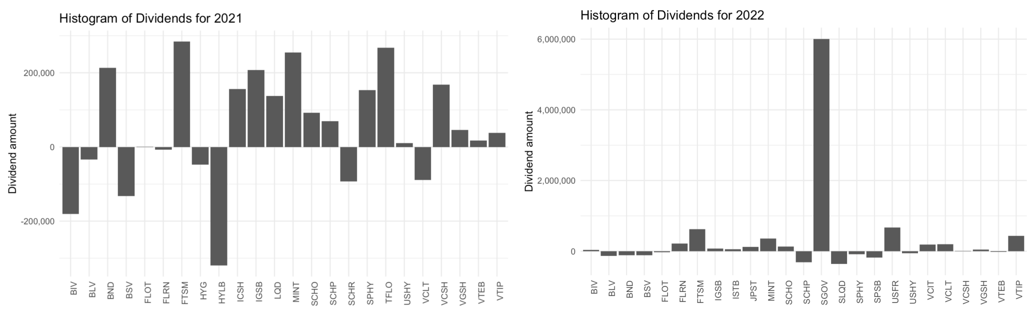


Figure 20: Dividend Payouts for High-Risk Profile

Looking at these plots one thing we can note is when we are shorting an ETF we will have a negative dividend payout of that ETF. This means that we will have to pay this dividend amount back to the owner of this ETF because we are not entitled to receive dividend payments when we are in a short position. Due to this, it might not always be wise to short a position even when the value of the ETF will decrease because you may miss out on dividend payments that could overcome this gain.

The dividend payments are a combination of two factors: the dividend yield and the weight allocation of the ETF. when multiplying these two factors, we will get the proportional dividend payments we will receive and we can see which ETFs will give us the most payments. In some years our dividend payments are very similar from one ETF to another while in other years we gain the most dividend payments from only a couple of ETFs. This is mostly due to our weight allocation of our ETFs. IF we take a look at 2022 we see that we receive a dividend payment of around 6 million dollars from the ETF SGOV which is significantly more than the other ETFs this year. The reason for this is mainly due to the weight of this ETF. In December 2022, the weight of this ETF was .8051 and the annual dividend yield rate was 1.45%. Although the annual dividend yield rate is relatively low, SGOV held the majority of our portfolio and due to this high amount, it also received high dividend payments.

A couple of takeaways to learn here is we should reduce the amount of shorting we do so we do not have to pay dividends. We should also focus on ETFs with high dividend yield rates. Our scoring criteria contained dividend yield rates as one of the metrics but we gave it a weight of 1/3 for the medium-risk and low-risk profiles. We can decide later on if changing this weight to 1 or even 2/3 will be more effective in increasing our dividend payments and thus increasing our total capital.

Once we calculate the dividend payments for all ETFs for a risk profile we can sum these all up and see how much we earn from dividends every year from all three of our risk profiles.

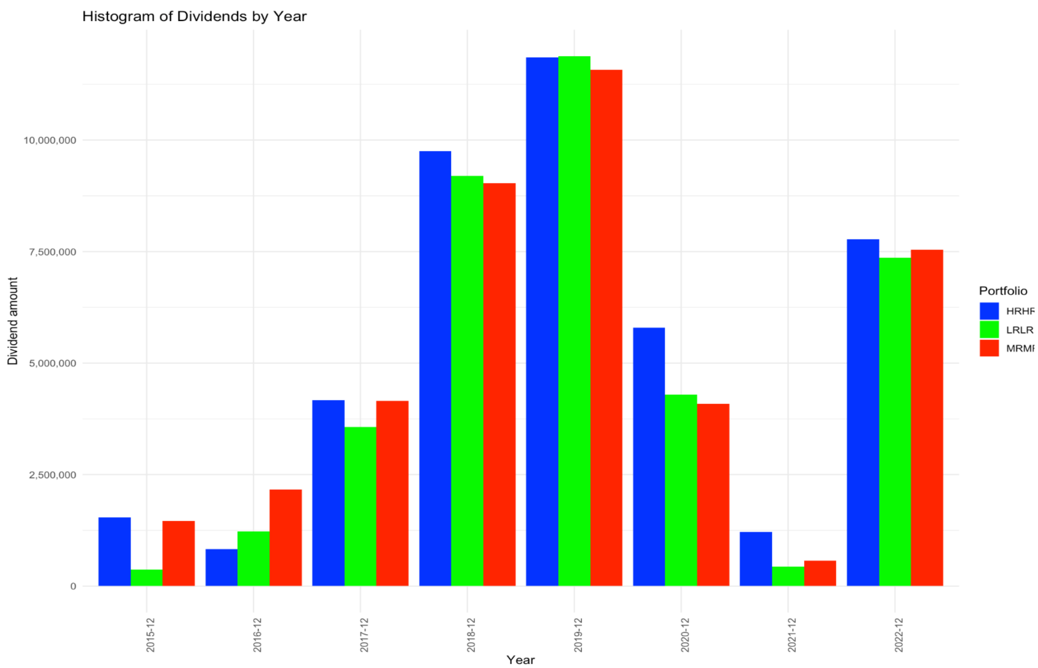


Figure 21: Dividend Payouts every year

Figure 22: Yield on 10-year US Treasury

In the plots above we can see our dividend payout increase every year up until 2019. then it decreases significantly in 2020 and 2021 then rebounds back up in 2022. The reason we see this drop in dividend payments is due to the interest rates over the years. Interest rates and dividend yield rates have a positive relationship so when interest rates go down so do dividend yield rates. During the COVID-19 pandemic, we saw very low interest rates which then decreased the dividend yields of ETFs. This explains why we see such low dividend payments in 2020 and 2021. In total, for the high-risk profile, we received \$43,114,647 in dividend payments. For the medium risk profile, we received \$40,268,479. And for the low-risk profile, we received \$37,864,529.

We also see that for some years there are big differences between the dividend payouts from each risk profile. This is due to the ETFs we selected and their annual dividend yield rates.

	2015	2016	2017	2018	2019	2020	2021	2022
HRHR	1.92%	2%	2.32%	3.01%	3.1%	2.29%	1.91%	3%
MRMR	1.75%	1.86%	2.1%	2.77%	2.9%	2.09%	1.67%	2.66%
LRLR	1.4%	1.5%	1.77%	2.44%	2.66%	1.87%	1.52%	2.57%

Table 5: Average annual dividend yield rate for ETFs selected

This table shows the average dividend yield rate for the ETFs in the corresponding risk profiles for every year. We see that the highest average dividend yield for all profiles was in 2019 and the lowest ones were in 2015 and 2021. Having the average annual dividend yield rate can explain why some profiles receive higher dividends than other profiles in the same year. Profiles with higher dividend yields will also receive higher dividend payments. However, this isn't always the case as some of these high dividend yields could be for ETFs where we have very low weight. The weights, selection of ETFs, and the corresponding dividend yield rates are the three factors that will decide on how much dividend payments we receive for each profile every year. Reallocating our weights towards ETFs with high dividend yields could help us out and increase our final capital.

4.5 Fees

The expense ratio for fixed-income ETFs is a measure of the fund's total annual operating expenses expressed as a percentage of the amount one invests. It includes various operational costs such as management fees, administrative fees, and other asset-based costs incurred by the fund. The expense ratio reduces the fund's overall return to investors and this is why it's an important figure for investors to consider; the higher the expense ratio, the more an investor's returns are potentially diminished.

For fixed-income ETFs, which typically aim to provide returns reflective of the bond market with relatively low risk, the expense ratio can be especially crucial. These funds often have narrower margins between their returns and risk-free rates compared to equity funds. Therefore, a high expense ratio can significantly reduce the income generated from the bonds held within the ETF. Additionally, in a low-interest-rate environment, the impact of the expense ratio becomes more pronounced, as the net yield after expenses may not significantly outpace inflation.

Even small differences can have a considerable impact on long-term returns, especially when compounded over many years. Hence, it's a critical factor in evaluating the efficiency and cost-effectiveness of an investment in a fixed-income ETF setting.

4.5.1 Fees in Our Portfolio

For the simplicity of our model, we have set the trading fee to .1% of the size of the trade when we change the position of an ETF. Because we are re-adjusting our weights monthly we will have monthly fees that we will have to pay off. Every time we change our allocation of our capital we will pay fees based on that change. This amount of fees will be reduced from our capital every month. It is important to be able to reduce our fees as low as possible while still earning the most capital. Our fees will be calculated by taking the absolute value of an ETF's position at the end of the month, minus the value it will be based on the new weight at the beginning of the next month. This difference will be the amount we have to trade to meet the new weight allocation.

4.5.2 Fees Analysis

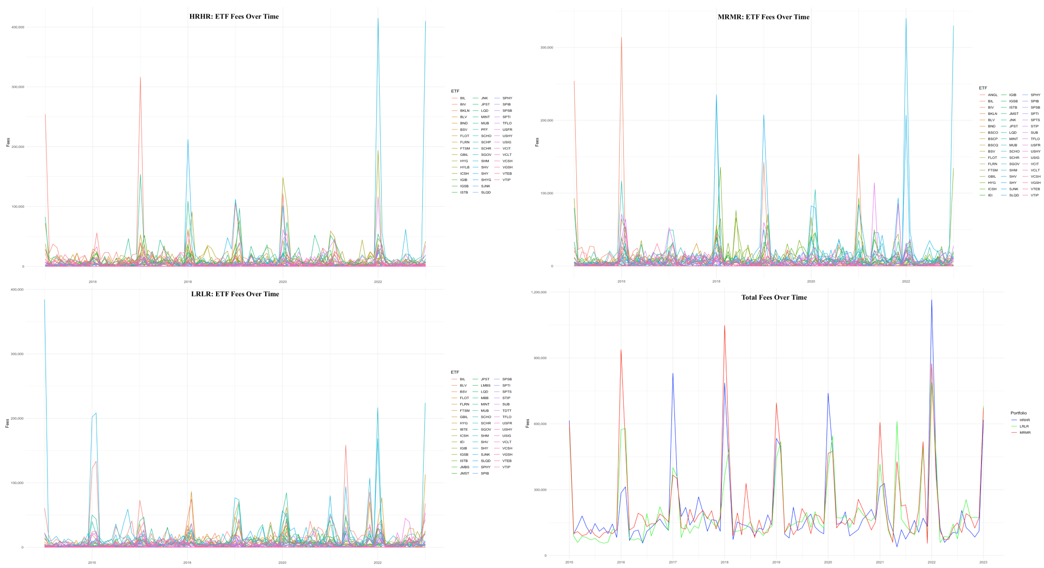


Figure 23: Fees of portfolios

The first three plots show how much we have to pay in fees for each ETF. For the most part, the fees are below 50,000. While in some months they are significantly greater. The amount of fees we pay is determined by the change in weight from one month to another. If an ETF has its weight changed drastically then we will have to pay fees on it. Therefore making drastic changes in the weights means that this choice should earn us more money than the amount of fees we pay doing this trade.

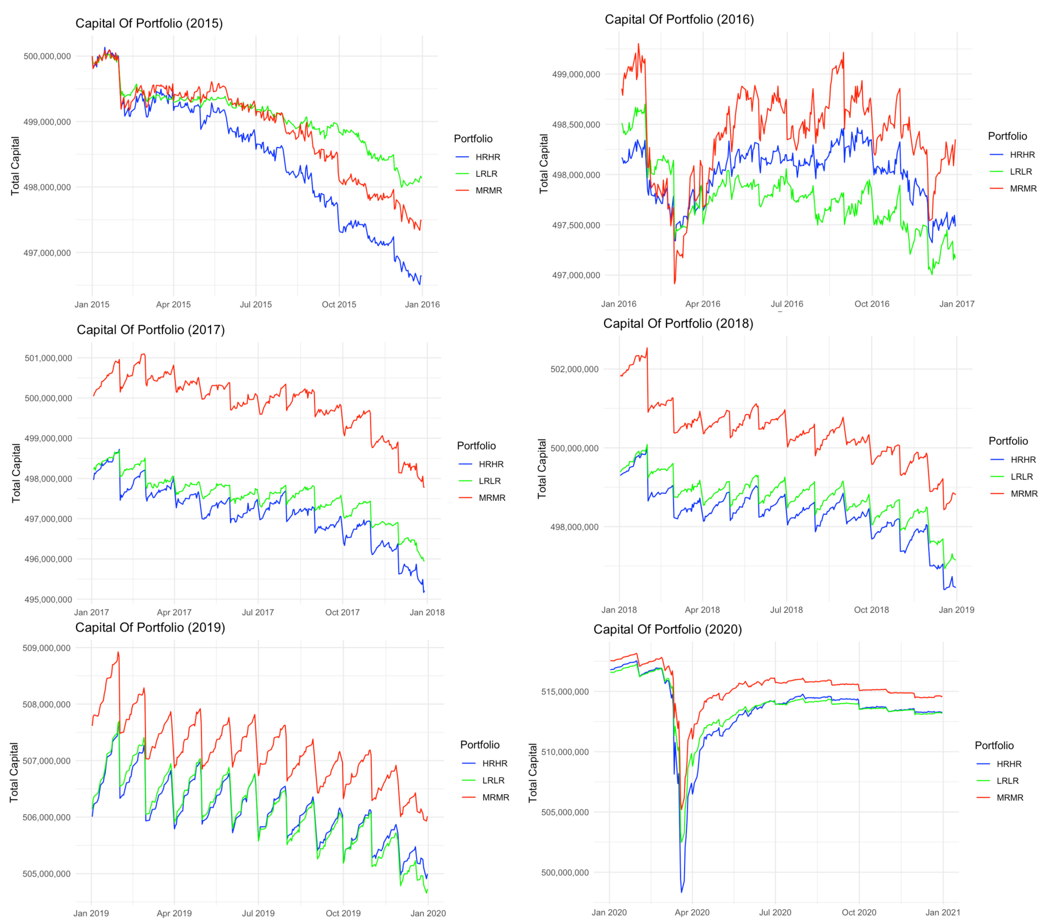
In the last plot, we see the total amount of fees we pay every month. We see a trend where the biggest amount of fees paid will be at the beginning of each year. This happens because our asset selection occurs at the beginning of the new year so this is when we can either remove or add new ETFs to our portfolio. We will select the ETFs yearly but readjust the weights of these ETFs monthly. Because we are readjusting monthly we will have some fees but they wont be as much as the beginning of the year.

Overall the process of adjusting and re-balancing our assets every month costs us a lot of money. For the high-risk profile, we pay a total of $18,739,531. For the medium-risk profile, we pay $21,616,358 and for the low-risk profile, we pay $18,925,478 One thing that we could do is to only rebalance yearly and keep the same weights throughout the year. This is an option that we will go over when comparing our portfolios to benchmarks.

4.6 Capital

With our dividends and fees calculated, we can finally calculate our final return and capital for each portfolio. Our starting capital was 500 million and our investment period went from January 1st, 2015 to December 31st, 2022.

4.6.1 Capital Over Time



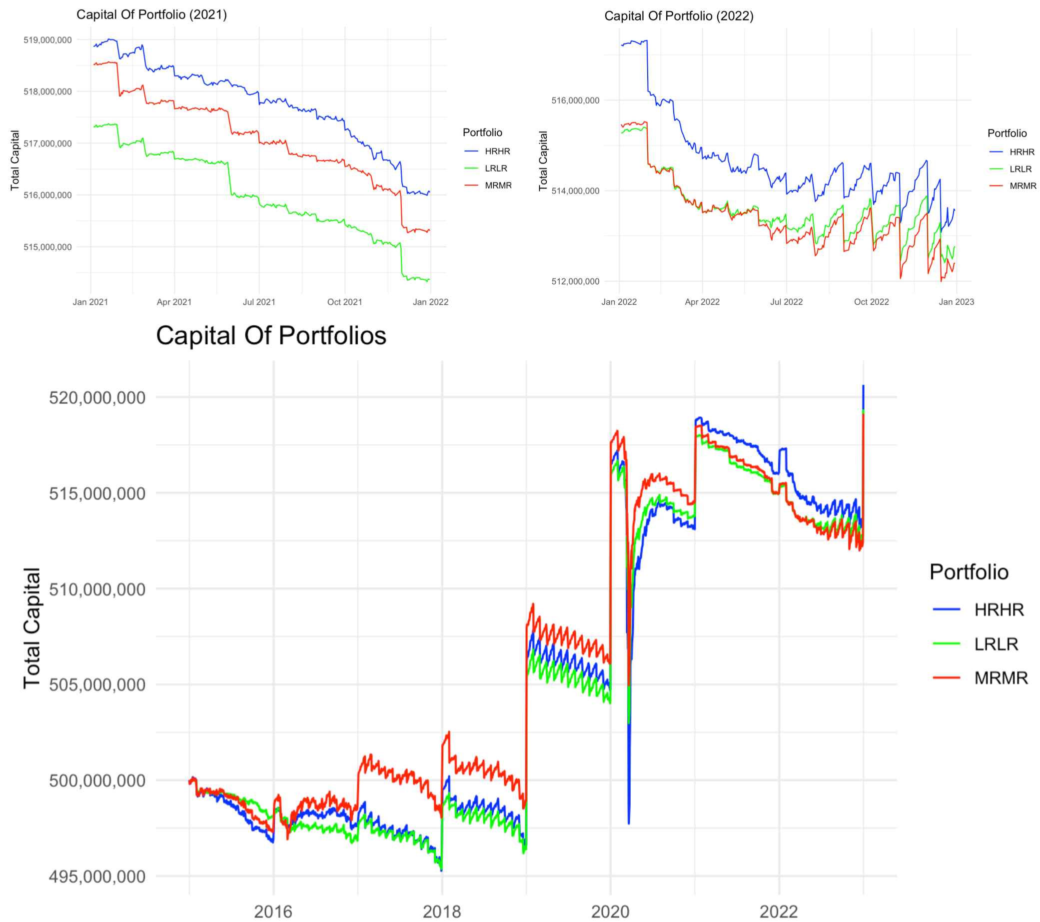


Figure 24: Capital of Portfolios

These plots show our total capital over time for each portfolio and as we can see, the small jumps between the years are mainly due to the monthly readjustment of weights where we have to pay a fee. Every time we readjust our position in an ETF we pay a fee so we see a sudden drop in our capital. In 2015 we see that our Low Risk profile first begins to outperform our other two profiles. However, at the beginning of 2016, we see that our medium-risk profile has outperformed the other two profiles. The reason we see this jump from the end of 2015 to the beginning of 2016 is due to the dividends we received at the end of 2015. These dividends are reinvested for the next trading year. Then in 2021, we see that our High Risk profile takes the lead and this is because we received much more dividends in this profile compared to the other ones and we also pay fewer fees at this time. Overall, the trends of our three portfolios are very similar and this may be due to the ETFs we had selected. Some ETFs may be in all three risk profiles.

At the beginning of 2020, we see that all portfolios dropped in value and this is due to the COVID-19 pandemic. During the COVID-19 pandemic, the values of many financial assets, including fixed-income ETFs, dropped sharply. This was a result of a combination of uncertainty and concerns about economic shutdowns and their impact on corporate earnings, and a broad shift toward risk aversion among investors. The global scale of the pandemic led to a liquidity crunch, prompting investors to sell off assets for cash, which put downward pressure on asset prices across the board. Even though fixed incomes are generally considered more stable than equity investments, the scale of the market disruption during the pandemic exceeded the traditional stability associated with fixed-income securities.

We also can note that every year we are losing money but then regaining at the end of every year due to dividends. There could be two reasons this could be happening. The first one is our selection of ETFs. The type of ETF we selected plays an important role in the returns we get. During this time we saw major changes in interest rates and different types of ETFs will be affected by interest rates differently. Some may be more sensitive than others. Another reason we have a loss of capital in a single year is due to our monthly fees. The fees we pay are a significant amount of money and it seems like the tradeoff of changing our weights to adjust for market changes is not worth it if we have to pay fees on these changes.

4.6.2 ETF Analysis

We can now see which ETFs play a major role in our capital and have the most weight in our profiles.

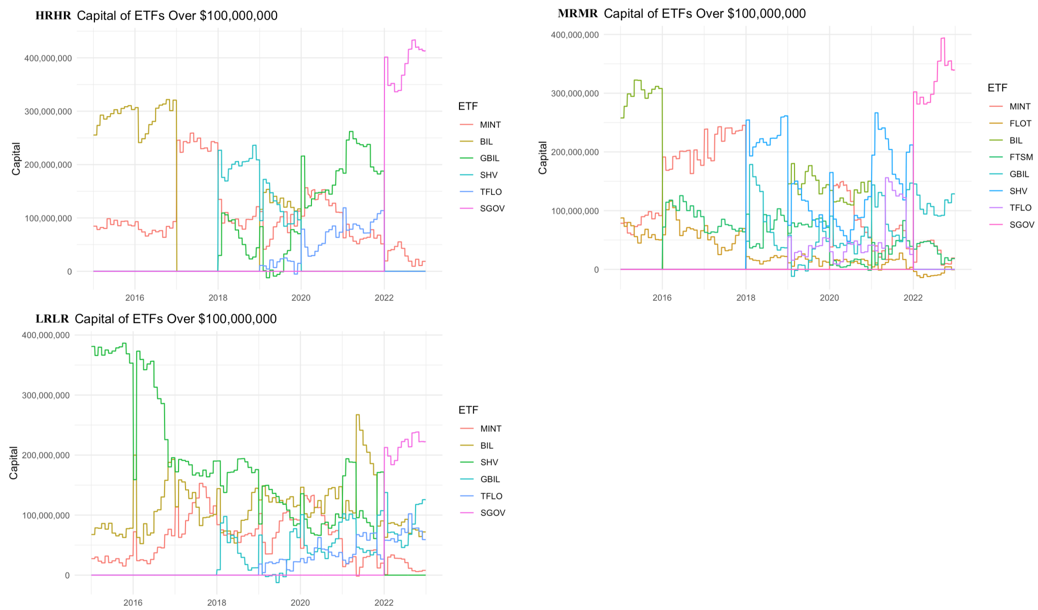


Figure 25: ETFs with Major Allocation in our Portfolios

These plots show all the ETFs that go over $100,000,000 in allocation and hold a majority of the weight of our portfolio.

Six ETFs play a significant role in our capital allocation and appear in all three of our profiles. The first one is MINT. This ETF focuses on the ultrashort segment of the maturity curve. It invests in corporate debt that is expected to mature within one year. BIL concentrates on ultra-short-term U.S. Treasury bills. Specifically, zero-coupon U.S. Treasury bills with maturities between one to three months. It carries minimal interest rate and credit risk and is designed for investors looking for stable and low-risk investment options rather than seeking high returns. GBIL focuses on U.S. Treasury securities with a short maturity of less than one year. This ETF is designed to provide exposure to the most liquid securities within this maturity range. As such, GBIL is intended for investors looking for a low-risk investment that has minimal interest rate risk and offers liquidity. SHV is the iShares Short Treasury Bond ETF and is focused on short-term U.S. Treasury bonds with maturities of one year or less. It's designed to offer investors exposure to the short end of the U.S. Treasury bond market. TFLO tracks U.S. Treasury floating rate bonds. This ETF provides exposure to Treasury bonds whose interest payments adjust with interest rates. This

ETF is useful because it has minimal interest-rate sensitivity, which should appeal to those concerned about potential rising rates. The last ETF is SGOV. SGOV focuses on ICE 0-3 Month US Treasury Securities Index whose components are rebalanced every month.

4.6.3 Final Return and Capital

	HRHR	**MRMR**	**LRLR**
Final Capital	\$520,634,038	\$519,125,350	\$519,359,312
Final Return	4.13%	3.83%	3.87%

Table 6: Final Capital and Returns

We end up with the highest final return for our High-Risk Profile. The medium and low-risk profiles are around 3.85% and are very close to each other. The main reason we see a big difference in our high-risk profile and our other two profiles is due to the dividend payments we received. For every single year, the high-risk profile has had a higher average annual dividend yield for the ETFs selected in this group. Dividends played a significant role in our capital and are the main reason we earn a positive gain.

4.6.4 Capital With no Monthly Re-adjustments

Now we can return back to the case if we decided not to readjust our weights monthly. The main reason we would decide to do this is to decrease the amount of fees we pay by a significant amount. If we didnt readjust monthly then we would keep the same weights as the beginning of each year and this would mean we wouldnt have to worry about any monthly fee payments. However, the downside of doing this is that we wont be able to adjust for any changes within the market throughout the year. Without regular rebalancing, the portfolio might miss opportunities to optimize returns or reduce risk. We will have to decide if this trade-off will be worth it.

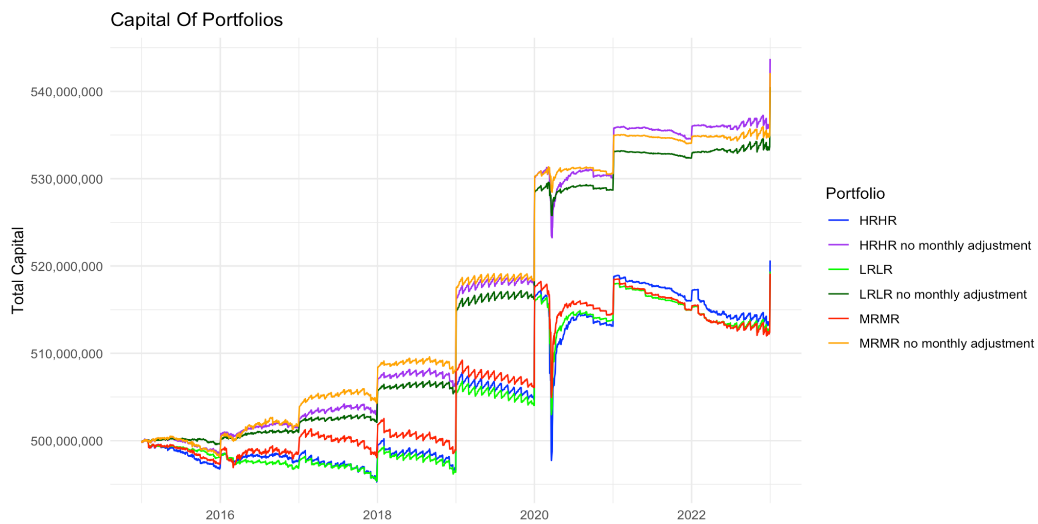


Figure 26: Original Portfolios and Portfolios with no monthly re-adjustments

This plot shows what our capital would look like if we decided not to adjust our weights monthly. We see that we outperform our original portfolios by a huge amount and the trade-off is worth it. For the most part, these portfolios do not lose value throughout the year like our original portfolio did. They remain relatively constant which means that the reason our capital was decreasing through a year was due to the high amount of fees we had to pay.

	HRHR	**MRMR**	**LRLR**
Final Capital	\$543,721,076	\$542,094,077	\$540,493,627
Final Return	8.74%	8.42%	8.1%

Table 7: Final Capital and Returns

In a fixed-income investment setting, where the securities involved typically offer stable returns with less volatility compared to equities, the necessity to frequently rebalance the portfolio, such as every month, is reduced. This is because the inherent stability in returns from these types of investments means that their relative value does not fluctuate dramatically over short periods. The potential benefits gained from frequent readjustments, in terms of optimizing returns or reducing risk, may not justify the costs or effort involved in doing so. If our portfolio included stocks then it would make more sense to have frequent re-adjustments because returns in the equity market are much more volatile than in the fixed-income market. Knowing this we can update our portfolio to only calculate the weights for the beginning of each year and keep these weights throughout the year.

4.7 Benchmarks

Benchmarks are essential tools in the investment world, serving as standard references against which the performance of ETFs and other investment choices can be evaluated. They are particularly important because they provide a concrete metric for comparing the effectiveness of an ETF's strategy, management, and overall return against a well-defined index or sector. This comparison allows investors to assess whether an ETF is meeting its stated objectives in terms of both risk and return. By aligning an ETFs performance with its benchmark, investors gain insights into the funds success in replicating the benchmark's performance or even outperforming it. This evaluation can inform decisions on portfolio adjustments, investment strategies, and risk management, enhancing the potential for achieving desired financial goals. Benchmarks not only help in measuring performance but also play a critical role in investment strategy development and portfolio management.

4.7.1 Choosing the Right Benchmark

We will be comparing our updated portfolio to three potential benchmarks. The three potential benchmarks we are interested in are SPY, DIA, and ULST.

The SPDR SP 500 ETF Trust (SPY) benchmarks against the SP 500 Index, a premier indicator of U.S. equities' performance, reflecting the operation of 500 leading companies in key sectors. SPY is the best-recognized and oldest US-listed ETF and typically tops rankings for largest AUM and greatest trading volume. It serves as a critical measure for the health and direction of the American stock market, offering insights into the large-cap sector's performance. By aligning SPY with this index, investors get a transparent view of its efficacy in tracking the broader market trends, aiding in strategic investment decisions within the landscape of U.S. equities.

DIA, also known as the SPDR Dow Jones Industrial Average ETF Trust, tracks a price-weighted index of the 30 large-cap US stocks. By investing in DIA, investors essentially get exposure to the stocks of these 30 companies in a single transaction, making it a convenient way to diversify their portfolio with some of the largest and most influential U.S. companies. The ETF is designed to provide a similar return to the DJIA by holding the same stocks in the same proportions as the index. DIA is extremely liquid, with huge assets and a long track record.

The SPDR SSgA Ultra Short Term Bond ETF (ULST) is designed to offer investors a way to gain exposure to ultra-short-term bonds, aiming to match the performance of the SSgA Ultra Short Term Bond Index. This ETF primarily invests in a diversified portfolio of high-quality, short-duration fixed-income securities, including corporate and government bonds. ULST offers a higher yield potential than what is typically available from cash investments or money market funds, with a relatively low level of risk due to the short maturity of its holdings. It offers the added benefits of liquidity and flexibility and given its focus on ultra-short-term bonds, ULST carries a lower risk profile, making it an attractive option for conservative investing.

Comparing our investment portfolio to the correct benchmark is crucial for several reasons. Firstly, benchmarks serve as a standard or reference point against which the performance of a portfolio can be measured. This comparison helps investors understand how well their investments are performing relative to a relevant market segment or strategy. When you choose the correct benchmark, you ensure that you're making a fair comparison. This is because the underlying assets, risk profiles, and expected returns differ based on the type of ETF we are interested in.

Equity benchmarks like SPY or DIA track the performance of large-cap U.S. stocks, reflecting the collective movements of the equity markets. They are influenced by factors such as corporate earnings, economic indicators, and market sentiment, which impact stock prices. In contrast, fixed-income investments, like those represented by ULST, are more sensitive to changes in interest rates, credit risk, and other factors specific to the bond market. Fixed-income investments typically offer lower risk and lower returns compared to equities and are often used for income generation or as a portfolio stabilizer.

Because our portfolio is concentrated on fixed-income ETFs, the most appropriate benchmark out of the three would be ULST or a similar fixed-income index that closely matches the portfolio's investment strategy, duration, and credit quality. This comparison would provide a more accurate assessment of the portfolio's performance, risk management, and strategic alignment with investment objectives.

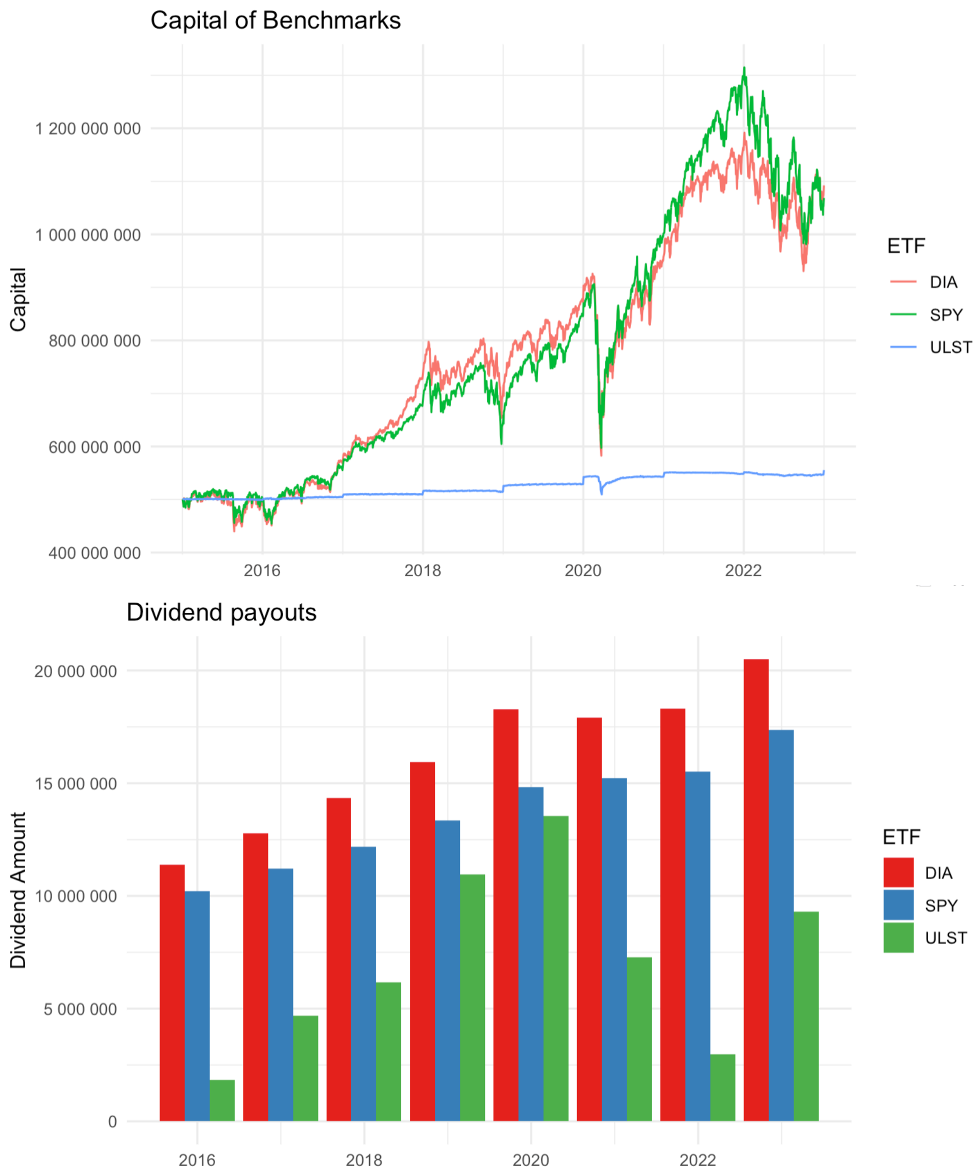


Figure 27: Capital and Dividend Payout of Benchmarks

	DIA	SPY	ULST
Final Capital	$1,092,272,583	$1,068,482,115	$555,273,068
Final Return	118.45%	113.7.%	11.05%

Table 8: Final Capital and Returns

The first plot shows what our capital and dividend payouts would turn out to be if we held a passive investment approach in the three benchmarks. Because DIA and SPY are in the equity market their returns are much more volatile and this is why we see them greatly outperform ULST. ULST remains more consistent due to the low volatility of its returns but this is why it will be a good comparison to our portfolios.

4.7.2 Comparing our Portfolios to ULST

To understand how our portfolios did we can compare them to the benchmark, ULST. Would we outperform ULST if we didnt update our portfolio to where we didn't adjust our weights every month?

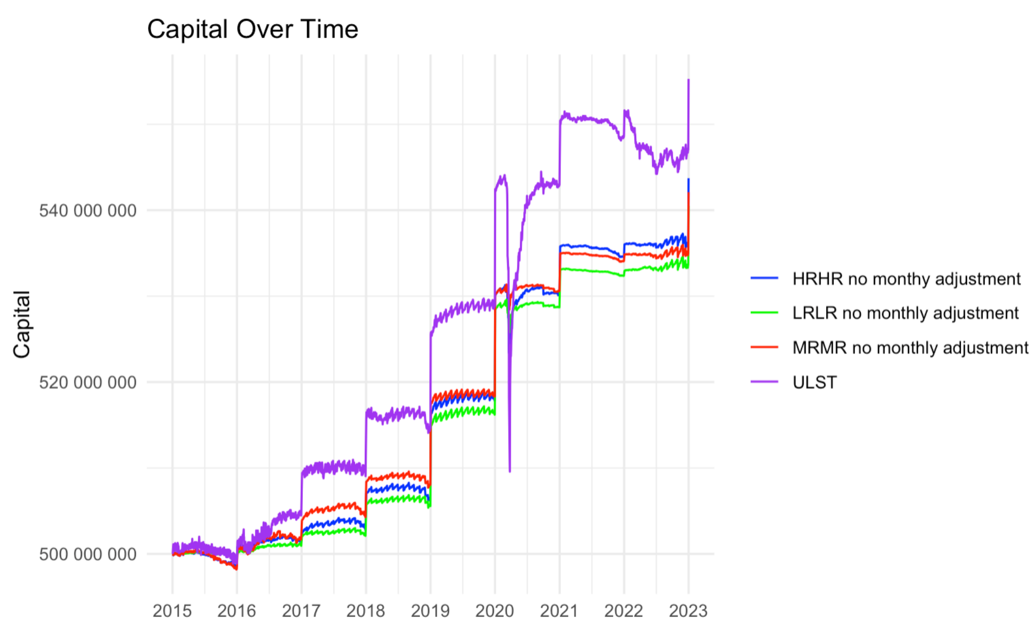


Figure 28: Capital of our Portfolio and ULST

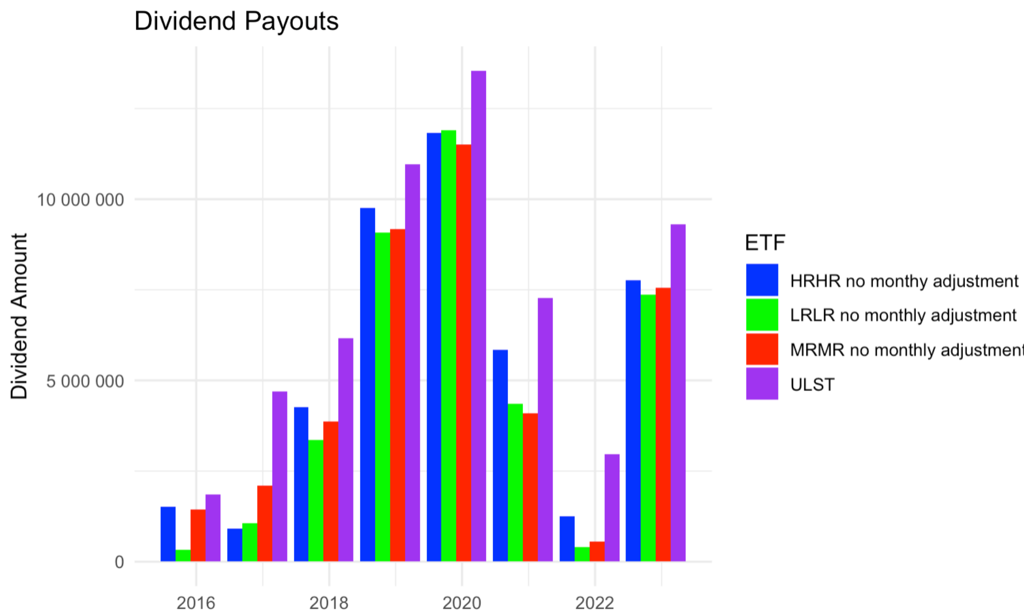


Figure 29: Capital of our Portfolio and ULST

Based on these plots we were not able to outperform ULST. One reason for this is because we are still paying yearly fees. Although we are not re-adjusting weights monthly, we are still re-adjusting weights yearly and we are either adding or removing new ETFs to our portfolio. ULST also receives more dividends because it is earning dividends on capital that would have been lost to yearly fees.

ULST also performs much better than our other portfolios due to the trend of interest rates. The relationship between interest rates and bond prices is inverse, which means that they move in opposite directions. When interest rates in the market go up, the prices of existing bonds tend to decrease. This is because new bonds are being issued with higher interest rates, making the older, lower-yielding bonds less attractive. Investors selling these older bonds will often have to do so at a discount to match the more attractive yields of new issuances. If interest rates fall, existing bonds with higher coupon rates than the new issues become more valuable. As a result, their prices increase because they offer a higher return than what's available on the market with new bonds. Up to 2019, we have seen an upward trend in interest rates which causes bond prices to decrease. This decrease will then cause our portfolio to lose value throughout the year.

However, Short-term bonds are generally less sensitive to interest rate changes compared to long-term bonds. This is because short-term bonds have a shorter duration until maturity. With short-term bonds, investors receive their principal back sooner than they would with long-term bonds. As a result, there is less time for interest rate changes to impact the bond's cash flows, reducing the overall volatility of the bonds price in response to rate fluctuations. Additionally, because investors get their investment back quicker, they can reinvest in new issues at the current rates sooner.

This explains why an ETF like ULST which focuses on short-term bonds performs relatively well. Understanding this we can update our portfolios to include more ETFs that include short-term bonds by updating our scoring criteria to give scores to ETFs that focus on short-term bonds.

5 Homogeneous Bond Laddering Portfolio

5.1 Portfolio Assumptions & Parameters

When we construct a homogeneous bond ladder portfolio, we assume that:

- Use the same term/duration of bonds along the whole investment horizon (2015-2022)
- $\$p$: we allocate the same investment amount per rung
- Issuer of bonds: US Treasury
- Zero-coupon bonds
- F: standard face value = \$1000 for each bond
- r: interest rate - daily federal fund rate
- K: initial capital = \$500mil
- No transaction costs or the cost is so small that we can ignore

The degree of freedom of this construction is three. In other words, there are three crucial and independent variables we need to consider.

- T (Duration/term) selection: US01MY, US02MY, US03MY, US04MY, US06MY, US01Y, US02Y, US03Y, US05Y, US07Y, US10Y, US20Y, US30Y
- Δt (Investment schedule) selection: Monthly, Quarterly, Semi-Annually, Annually
- N (Number of rungs) selection: 8, 16, 32, 96

$$N = \Delta t \times 8$$

One thing to note is that the maximum number of rungs is determined by the investment schedule selected and the total investment horizon. For example, if we choose 5-year US Treasury bonds and make an investment for them monthly in 8 years, we would have the number of rungs ranging from 2 to 96 (8×12). The number of rung 1 is not included because a one-rung ladder portfolio is like putting all your eggs in one basket. It is meaningless for bond laddering.

In addition, considering all possible choices of the number of rungs (that is, any integer value from 2 to the maximum number of ladders) can provide investors with a comprehensive range of options. This approach can theoretically find the optimal portfolio allocation to maximize returns or minimize risk. However, this approach does present several practical limitations in our project.

The first one is computationally intensive. As the number of steps increases, the number of possible combinations grows exponentially. This means a lot of calculations are needed to evaluate every possible combination. For large portfolios or high-frequency investment decisions, this computational need quickly becomes unfeasible.

Secondly, it is the time cost. Our time is restricted, so directly related to the huge amount of calculation is the time cost. Calculating and evaluating all possible combinations requires a significant amount of time, which can result in lost investment opportunities. Market conditions are rapidly changing, and waiting for calculations may mean missing the best opportunity to invest.

Choosing the maximum number of rungs as part of our homogeneous bond laddering strategy simplifies the calculation process and help reduce computational effort and time. Also, we also simplify decision-making process. A streamlined approach reduces the analytical burden, allowing investors to focus more on evaluating a few key parameters and trends rather than drowning in large amounts of data analysis. Furthermore, choosing the maximum means we fully invest throughout until the end of the investment horizon (the last day in 2022). This takes advantage of the full range of reinvestment opportunities, maintains a stable term structure, systematically reduces interest rate risk and provides a stable, predictable income stream.

Admittedly, choosing the maximum has its risks, primarily the possibility of missing out on superior portfolio allocations that may offer a better balance between risk and return.

5.2 Portfolio Assessment

As mentioned above, we have three parameters to determine and use in constructing the homogeneous bond laddering portfolio and each has a variety of choices. That results in many possible configurations (with different durations, investment schedules, and number of rungs selected) to establish homogeneous bond ladder portfolios. In order to obtain the optimal portfolio for each risk profile among a large number of configurations, we divide the assessment into the following steps

1. Determine the best configuration for each risk profile every year

2. Determine the best configuration for each risk profile for our homogeneous bond laddering portfolio over whole time period

5.2.1 Yearly Assessment

The analysis involves monthly log returns for various bond ladder configurations, with data collated for each year from 2015 to 2022.

The core assessment in this part is two key metrics: average excess return and value at risk (VaR), which serve as measures for performance and risk. By adjusting these indicators to an annual inflation rate of 2%, this method ensures an accurate assessment of the real growth of investments, providing a reliable basis for comparison and selection.

First, we transform cumulative log returns into monthly excess returns, adjusted for the assumed inflation rate. We then perform a comprehensive calculation of the average excess return and VaR for each bond ladder allocation. To ensure that the return on investment is favorable, we set a limit on the maximum drawdown. Allocations exceeding a maximum drawdown of 20% will be excluded from further screening.

To perform comprehensive calculations, we applied a scoring system. We normalize average excess returns and VaR scores because such normalization helps create a uniform platform for comparison, thereby enhancing the decision-making process. Furthermore, we assign specific weights to these standardized scores, as shown in Table 5.2.1.1. When allocating weights to investment allocations with different risk preferences, the high-risk high-return (HRHR) allocation gives a high weight of 3 to the average excess return, while a negative weight of -2.5 is set for VaR (value at risk), reflecting the high weight of A strong pursuit of returns, even if it requires taking higher risks. The medium risk medium return (MRMR) allocation takes a more balanced perspective and assigns weights of 1.8 and -1 to average excess return and VaR respectively, showing that while pursuing returns, it is also alert to risks. The low-risk, low-return (LRLR) allocation places an extreme emphasis on risk aversion. It only sets the weight of average excess return to

0.1, but gives VaR a weight of as high as 3, highlighting the extreme focus on potential losses and the relative neglect of income growth. , highlighting the importance of security and capital protection.

Table 5.2.1.1: Scoring weights for different risk profiles

Risk Profile	**Average Excess Return Weight**	**VaR Weight**
High Risk/High Return (HRHR)	3	-2.5
Medium Risk/Medium Return (MRMR)	1.8	-1
Low Risk/Low Return (LRLR)	0.1	3

The final step is to multiply the normalized average excess return and VaR by their corresponding weights to get our final composite score. We can observe the score distribution for each possible configuration for HRHR. See Figure 5.2.

We take the highest score to come up with the three best risk profiles for each year. For example, Table 5.2.1.2 indicates the best HRHR configurations each year. Most HRHR profiles in the table reflect the high returns and high risks brought by the long-term bonds in general.

Table 5.2.1.2: Best HRHR Configurations for each year

Year	**Number of Rungs**	**Maturity**	**Investment Schedule**
2015	16	US06MY	Semi-Annually
2016	8	US01Y	Annually
2017	8	US02Y	Annually
2018	8	US03Y	Annually
2019	96	US30Y	Monthly
2020	32	US30Y	Quarterly
2021	96	US03Y	Monthly
2022	96	US01MY	Monthly

5.2.2 Overall Performance & Risk Assessment

Obtaining the best configuration for each year for different risk profiles means that we carefully selected 8 configurations for each risk profile from a large number of possible configurations.

In this part, we will go one step further to get the three best configurations fitting each risk profile for the entire investment period. We use the configurations obtained in the previous part (5.2.1) to calculate the average annual return and variance. Generally, variance can reflect the risk

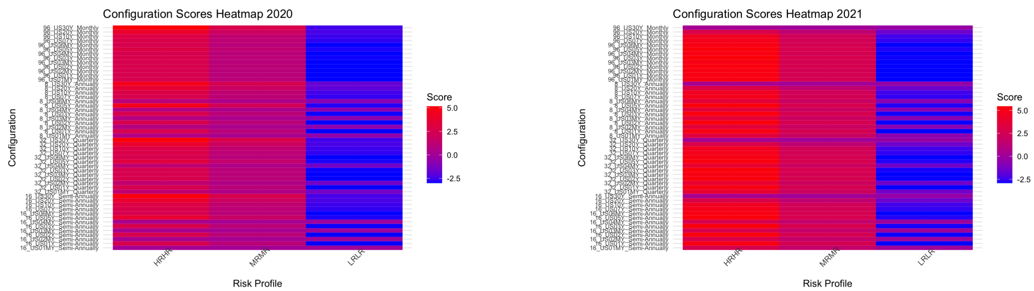


Figure 5.2: Heatmaps for configurations scores in 2020 and 2021

level of a portfolio. Finally, based on the characteristics of each risk profile (e.g. HRHR), Table 5.2 summarizes the three best homogeneous bond ladder configurations and their final return after assessment:

Table 5.2: Best Homogeneous Bond Ladder Portfolio Configurations for Different Risk Profiles

Risk Profile	**Number of Rungs**	**Maturity**	**Investment Schedule**	**Final Return**
HRHR	8	US02Y	Annually	20.54%
MRMR	8	US03Y	Annually	19.71%
LRLR	32	US06MY	Quarterly	10.90%

5.3 Portfolio Result Analysis

Figure 5.3.1 shows the daily value of a bond ladder portfolio from 2015 to 2022, categorized by different risk profiles: HRHR, MRMR, and LRLR. Each line represents the progression of a portfolio's value within a given risk category over time.

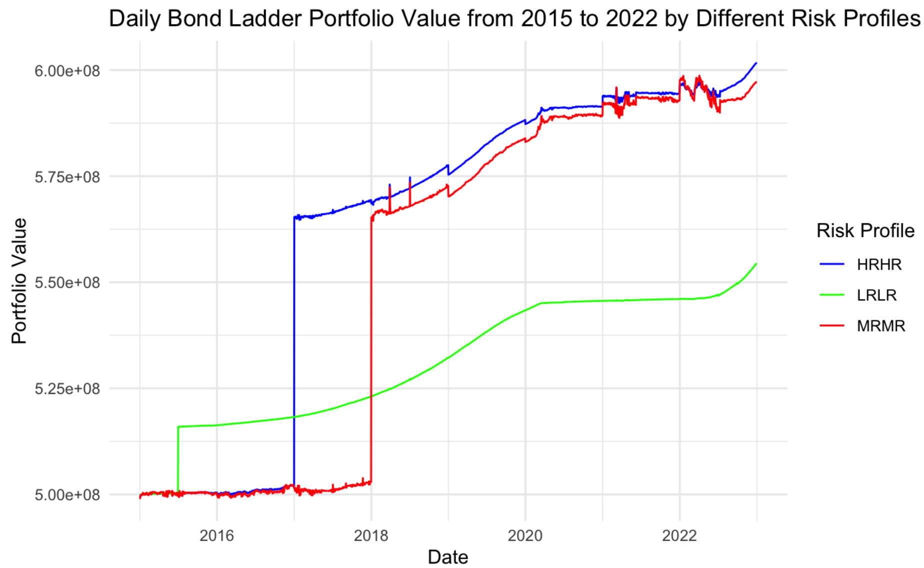


Figure 5.3.1

5.3.1 Portfolio Analysis

A striking feature of the plot is the sudden jump in the value for each risk condition. These jumps are consistent with the typical behavior of a bond ladder portfolio as bonds mature. As bonds mature, they repay their face value, often resulting in a significant increase in the total portfolio value if the bonds are held to maturity.

For the HRHR configuration (blue line), there is a sharp increase in value, which may indicate that the bonds in this configuration have higher face values, or that they were purchased at a significant discount to their face value and therefore rise sharply at maturity. The MRMR curve (red line) and LRLR curve (green line) also show increases, but these are less pronounced than the HRHR curve, possibly due to different bond par values, maturity dates, or frequency of maturities within the ladder.

It is also worth noting that the HRHR portfolio appears to have the highest volatility, with its value fluctuating more significantly than the other profiles. This is consistent with higher-risk strategies, where the potential for higher returns is accompanied by increased price volatility.

Overall, this chart effectively illustrates the impact of bond duration on the value of a homogeneous bond ladder portfolio and demonstrates the differences in volatility and returns among various investment strategies.

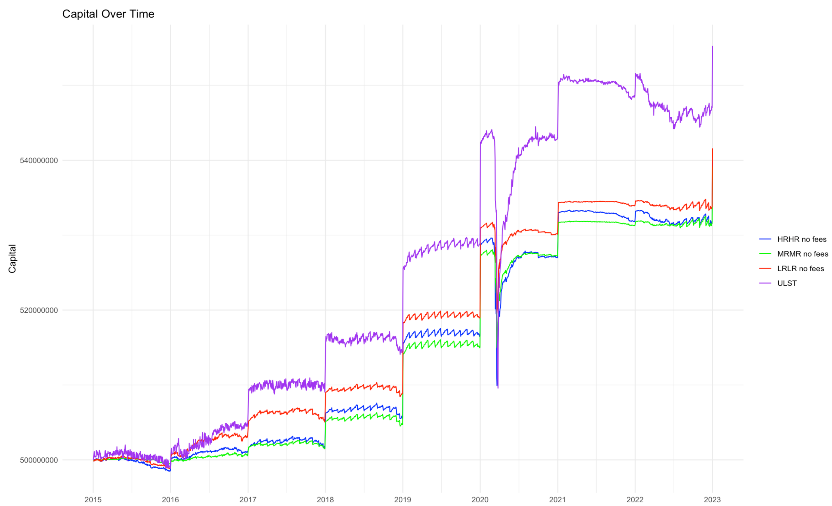
5.3.2 Compare to Benchmark ULST

As mentioned in the previous section, the reason to choose ULST as the benchmark that used to compare with our portfolios is that ULST remains more consistent due to its lower return volatility.

Thus, we focus on the progression of ULST (purple line) in Figure 5.3.2. We can discern homogeneous bond laddering portfolios' relative performance over the observed time period.

Examining capital growth over time for a homogeneous bond ladder portfolio against the ULST benchmark, the bond strategies (HRHR, MRMR and LRLR) consistently outperformed the benchmark. This is particularly noteworthy after 2018, when portfolio capital diverged significantly from ULST, signaling strong growth. This performance means that the homogeneous bond ladder, with its structured maturity and reinvestment strategies, can provide a more stable and valuable investment over the long term compared to the short-term focus of ULST. This trend is particularly evident when looking at the bond portfolio's relatively smooth growth line, which contrasts with the volatility indicated by ULST's sharp spike in capital.

The bond ladder portfolio is less sensitive to market swings than the ULST benchmark, as evidenced by ULST's dramatic capital swings around 2022. This highlights their reduced vulnerability to short-term market volatility. Despite the inherent risks of high-yield bonds in the HRHR and MRMR strategies, their performance shows a compelling balance between growth and stability. Homogenous bond ladders are less volatile than ULSTs, which may make them more attractive to investors seeking sustained growth without the sharp short-term risks that come with ultra-short-term securities.



Capital Over Time
Capital
540000000
520000000
500000000
2015
2016
2017
2018
2019
2020
2021
2022
2023
HRHR no fees
MRMR no fees
LRLR no fees
ULST

Figure 5.3.2

6 Comparative Analysis of Two Investment Strategies

Through the two figures and Table 6.1, we clearly describe the performance of two different investment strategies over an eight-year period. The first strategy (shown in Figure 6.2) uses a homogeneous bond ladder approach. When each bond matures, the principal is reinvested in a new bond at the end of the ladder. This approach tends to provide predictability and stability because it minimizes reinvestment risk associated with interest rate fluctuations. In the graph, this is reflected in smoother and more consistent growth trajectories, particularly for higher risk profiles (HRHR and MRMR), suggesting a steady accumulation of value over time.

The second strategy (shown in Figure 6.1) involves investing in fixed-income exchange-traded funds (ETFs). ETFs are known for their liquidity and flexibility because they trade on stock exchanges like stocks. This can lead to greater volatility, as the market price of an ETF can fluctuate throughout the trading day, influenced by market sentiment and other economic factors. In Figure 6.1, this is evident from the more pronounced peaks and troughs, indicating the variable performance typically associated with ETFs.

The biggest difference between the two investment strategies is that bond laddering is a self-managed portfolio, while ETF, a portfolio of bonds, is a portfolio managed by professionals. That's because as the bond matures, the bond ladder may require more hands-on management to maintain the ladder structure. ETFs are passively managed most of the time, requiring investors to make less active decisions. Specifically, the differences between the two are reflected in the following aspects.

The first is volatility. Homogeneous bond ladder portfolios are less volatile than ETF portfolios, with fewer large increases or decreases in value over time. This suggests that bond ladders may be more suitable for investors looking for stability.

In terms of performance, according to Table 6.1, we can more intuitively find that the bond ladder portfolio outperforms the ETF portfolio in terms of final capital and return, especially for medium-risk and high-risk conditions. This is likely due to the compounding effect of reinvesting the proceeds from maturing bonds at potentially higher interest rates over time.

In terms of liquidity, ETFs generally offer greater liquidity because they can be bought and sold throughout the trading day. This can be advantageous for investors who value or need to be able to quickly adjust their positions based on market movements.

Additionally, bond ladders can mitigate interest rate risk to a certain extent because the periodic maturities of bonds provide the opportunity to reinvest at prevailing interest rates. ETFs, on the other hand, may be more directly affected by changes in interest rates, reflecting direct changes in the market's valuation of the underlying bonds.

In all, while the bond ladder portfolio appears to perform better in terms of growth and returns over the observation period, its suitability will depend on the investor's personal risk tolerance, liquidity needs and investment horizon. Although returns may be lower and volatility higher, ETFs may be popular with those looking for flexibility and ease of trading. Each strategy has unique advantages and disadvantages that should be carefully weighed against an investor's financial goals and market prospects. In future work, we want to introduce a combination of two strategies to create an optimal portfolio.

Table 6.1: Final Capital & Return Comparison

Risk Profile	**Portfolio Type**	**Final Capital**	**Final Return**
HRHR	ETF	\$543,721,076	8.74%
MRMR	ETF	\$542,094,077	8.42%
LRLR	ETF	\$540,493,627	8.1%
HRHR	Homo BL	\$601,803,629	20.54%
MRMR	Homo BL	\$597,232,478	19.71%
LRLR	Homo BL	\$554,468,901	10.90%

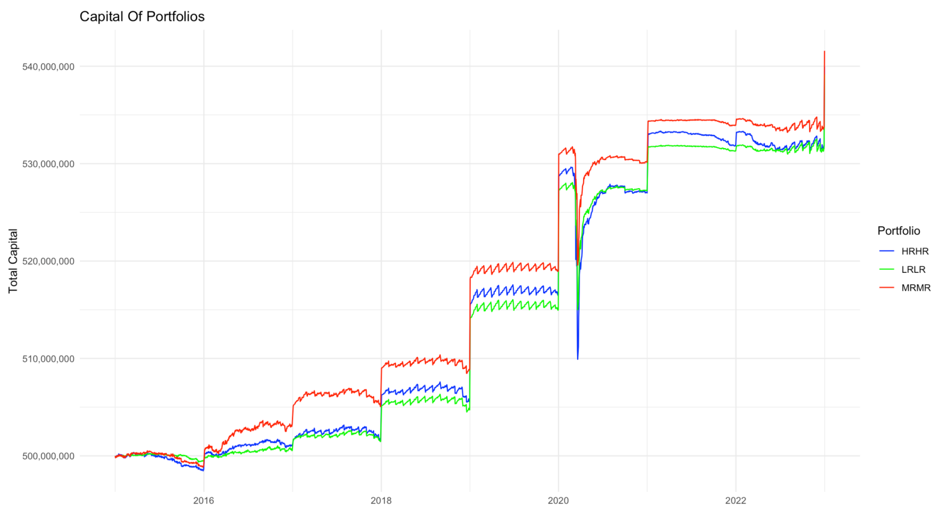


Figure 6.1

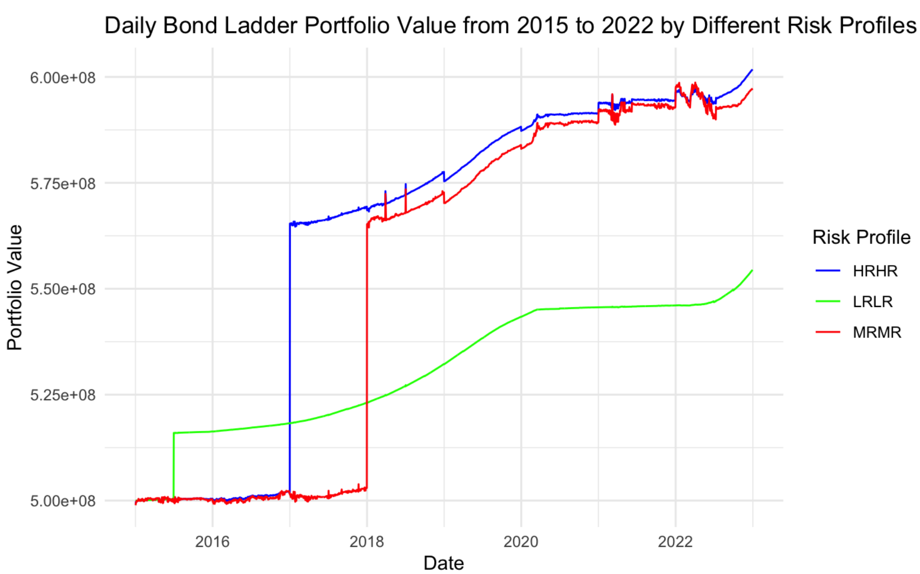


Figure 6.2

7 Conclusion & Future Work

In our dissertation, we have constructed and provided a detailed comparative analysis of two major investment strategies in the fixed income market over eight years: fixed income ETFs and homogeneous bond ladder portfolios. Our findings illuminate the unique characteristics and performance metrics associated with each strategy, providing important insights into their respective strengths and challenges in different markets and risk profiles.

7.1 Summary

The homogeneous bond ladder strategy, which features the systematic reinvestment of principal from maturing bonds into new bonds at the end of the ladder, has demonstrated commendable stability and predictability. This approach has proven particularly effective in managing reinvestment risk associated with interest rate fluctuations. Our analysis clearly shows that homogeneous bond ladder portfolios, particularly those in the medium and high risk categories, consistently outperform in terms of value accumulation over time, outperforming in terms of final capital and returns Fixed income ETFs. ETF portfolios, while having the advantages of liquidity and flexibility due to their stock-like tradability, also exhibit higher volatility, which demonstrates their sensitivity to immediate market sentiment and economic factors.

7.2 Future Work

Our current analysis lays the foundation for future research developments. We plan to delve deeper into the synergies of combining bond ladders and ETF-based strategies to form optimized portfolios. This effort will involve broadly exploring potential risk-adjusted returns and creating more robust portfolios that can more effectively withstand market uncertainty. Before that, we will also further promote the bond ladder strategy. The current notice bond ladder strategy is not portfolio optimization to some extent. What we do is simplify portfolio selection. In the future, we will try to incorporate more bond options, such as different durations. At the same time, we will also choose whether new investments are more suitable bonds based on real-time market conditions to make the investment portfolio more diversified (homogeneous bond ladders actually lack diversity).

7.3 Recommendations to Investors

Based on our results, we offer the following recommendations to investors:

1. For ETF portfolios, it is recommended to diversify beyond just the top 100 ETFs based on AUM. Expanding your investment universe to include more diverse fixed income, commodities and currencies (FICC) ETFs, particularly in the short-term space, can provide better risk-adjusted returns.

2. Investors can consider reducing the frequency of investment adjustments - from monthly adjustments to quarterly, semi-annual or even annual adjustments, which can reduce transaction costs and prevent overreaction to short-term market fluctuations.

3. Imposing variable constraints on global minimum variance (GMV) and tangent portfolios can provide more tailored risk exposures. These restrictions may include annual fee limits, trading volume caps (buy and sell limits), and specific holding periods that can align investment decisions more closely with investors' risk appetite and return expectations.

4. In the case of bond portfolios, time-series forecasts based on past trends and interest rate expectations should play a key role in the portfolio construction process. Adjustments based on buy and sell limits, coupled with the inclusion of non-domestic investments such as Japanese, German and Austrian bonds, may provide additional diversification benefits and exposure to different interest rate environments. The model fit dictated by the Akaike Information Criterion (AIC) should be optimized to ensure the most reliable yield curve estimates.

5. For bond ladder selection, more complex choices of investment weights and schedules are critical. This requires a dynamic investment approach that considers maturity, credit quality and market conditions, rather than a static ladder structure, to improve ladder performance in different market situations.

6. For more professional investors, it is recommended that one could benefit from a hybrid approach, leveraging the stability of bond ladders and the liquidity of ETFs to achieve a balanced and diversified portfolio.

7. Last, investors are encouraged to weigh these recommendations based on their own personal investment circumstances. A detailed understanding of one's financial goals, coupled with a customized approach that incorporates these recommendations, can lead to an optimized investment strategy that successfully balances return potential with risk tolerance.

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