Data Engineering

LTAT.02.007

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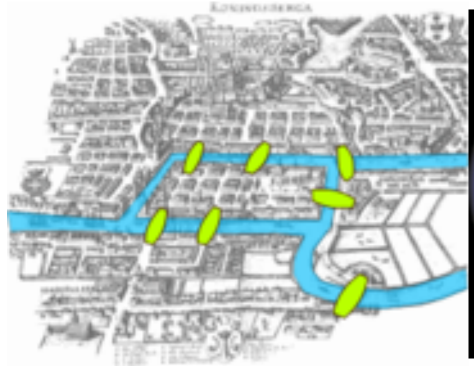
https://courses.cs.ut.ee/2020/dataeng

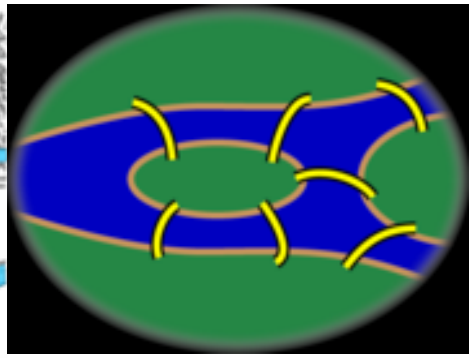
Forum

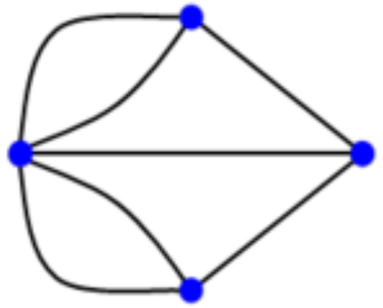
Moodle

History

Leonhard Euler's paper on "Seven Bridges of Königsberg", published in 1736.







Famous problems

- The traveling salesman problem: A traveling salesman is to visit a number of cities.
   - how to plan the trip so every city is visited once and just once and the whole trip is as short as possible ?

- Four color problem[100] : using only four colors, color any map of countries in such a way as to prevent two bordering countries from having the same color.
   - SOLVED ONLY 120 YEARS LATER!

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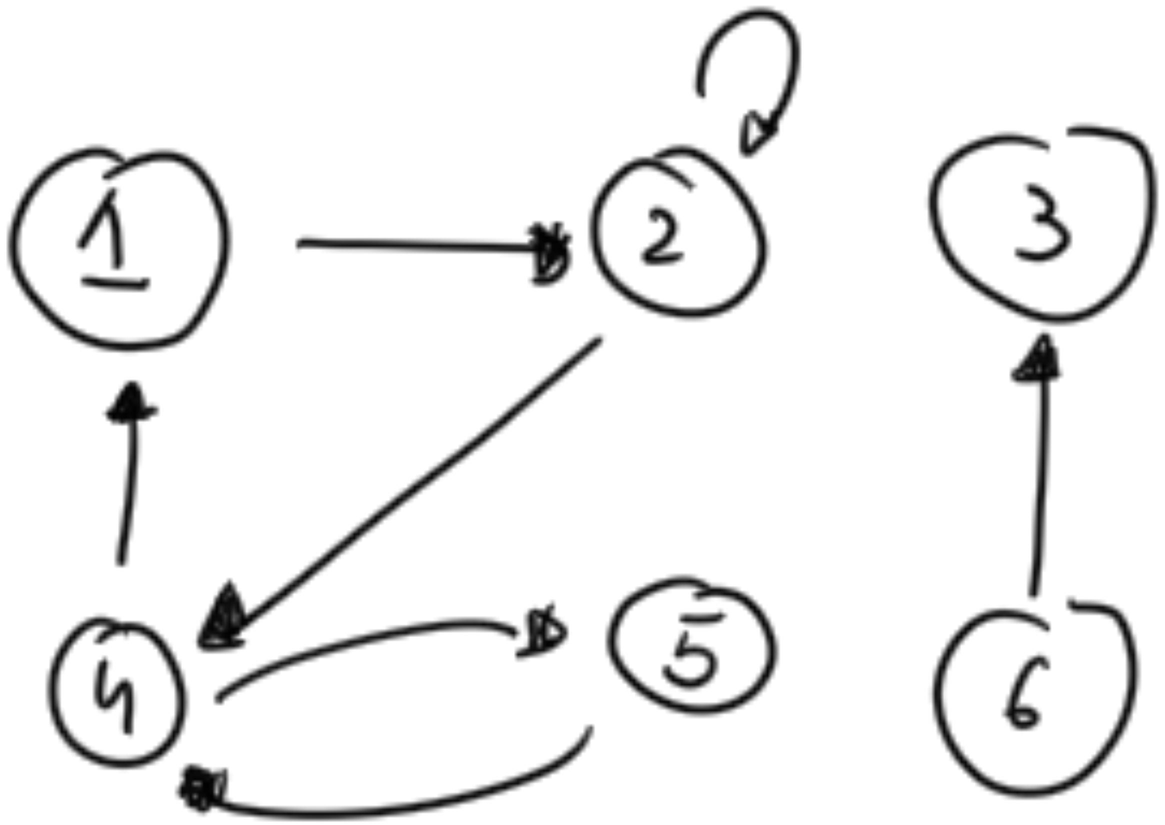
[100] Francis Guthrie,1852

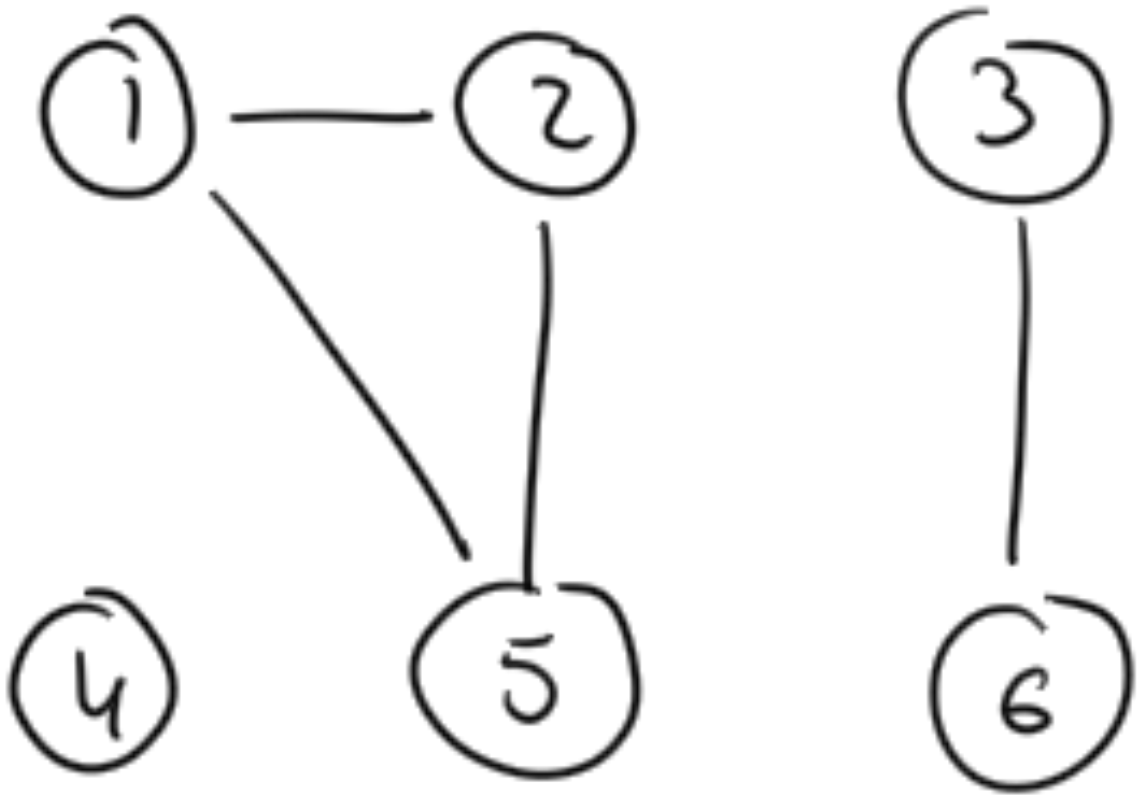
Other Examples of Graph Problems

- Cost of wiring electronic components
- Shortest route between two cities.
- Shortest distance between all pairs of cities in a road atlas.
- Matching / Resource Allocation
- Task scheduling
- Visibility / Coverage

What is a Graph?

Informally a *graph* is a set of nodes joined by a set of lines or arrows.





Graph

G is an ordered triple $G := (V, E, f)$

- V is a set of nodes, points, or vertices.
- E is a set, whose elements are known as edges or lines.
- f is a function
- maps each element of E
- to an unordered pair of vertices in V.

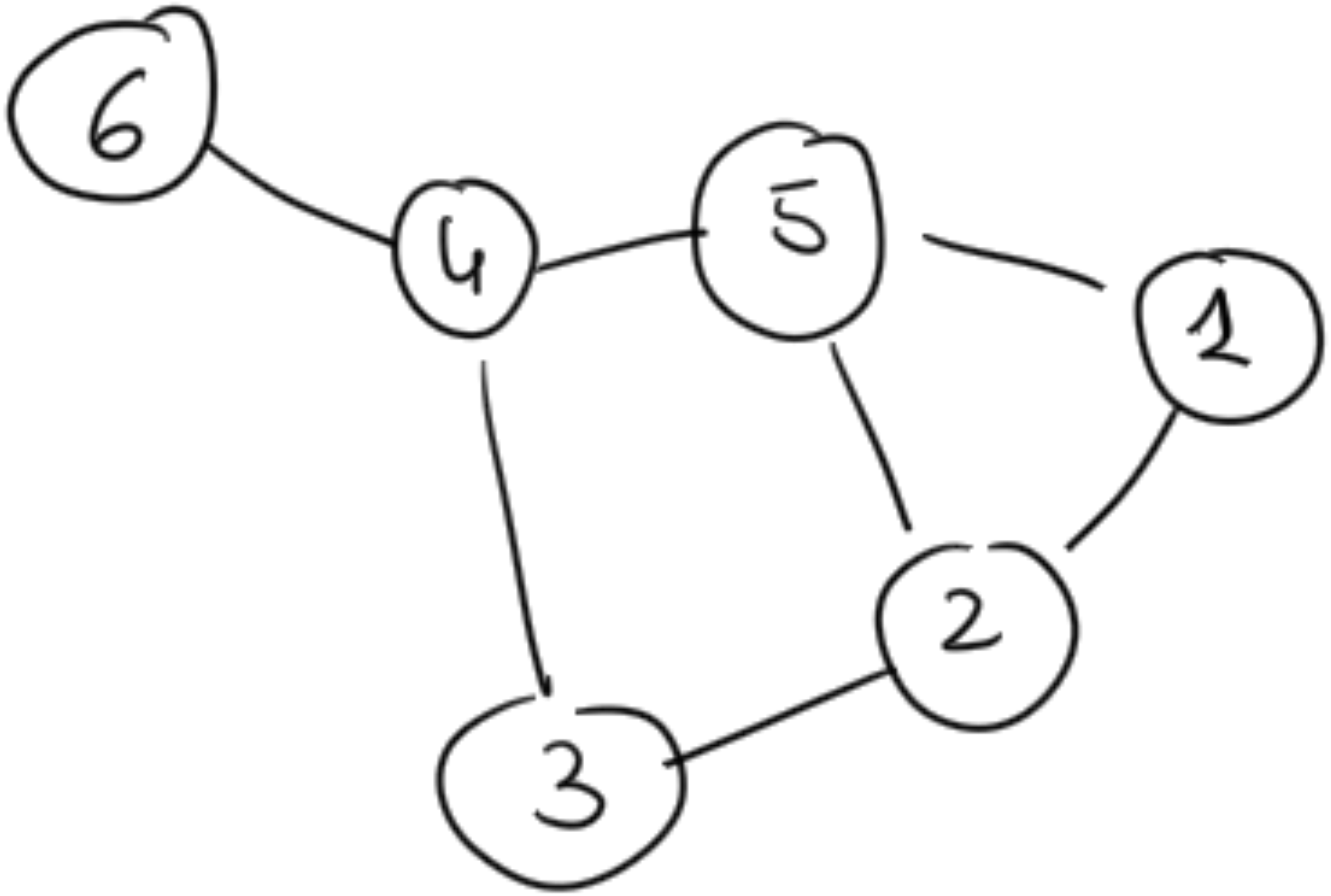
Vertexes and Edges

- A Vertex is a Basic Element
    - Drawn as a *node* or a *dot* .
    - The *Vertex set of a graph* *G is usually denoted by *V*
- An edge is *set* of two elements
    - Drawn as a line connecting two vertices, called end vertices, or endpoints.
    - The edge set of G is usually denoted by E(G), or E.

Example

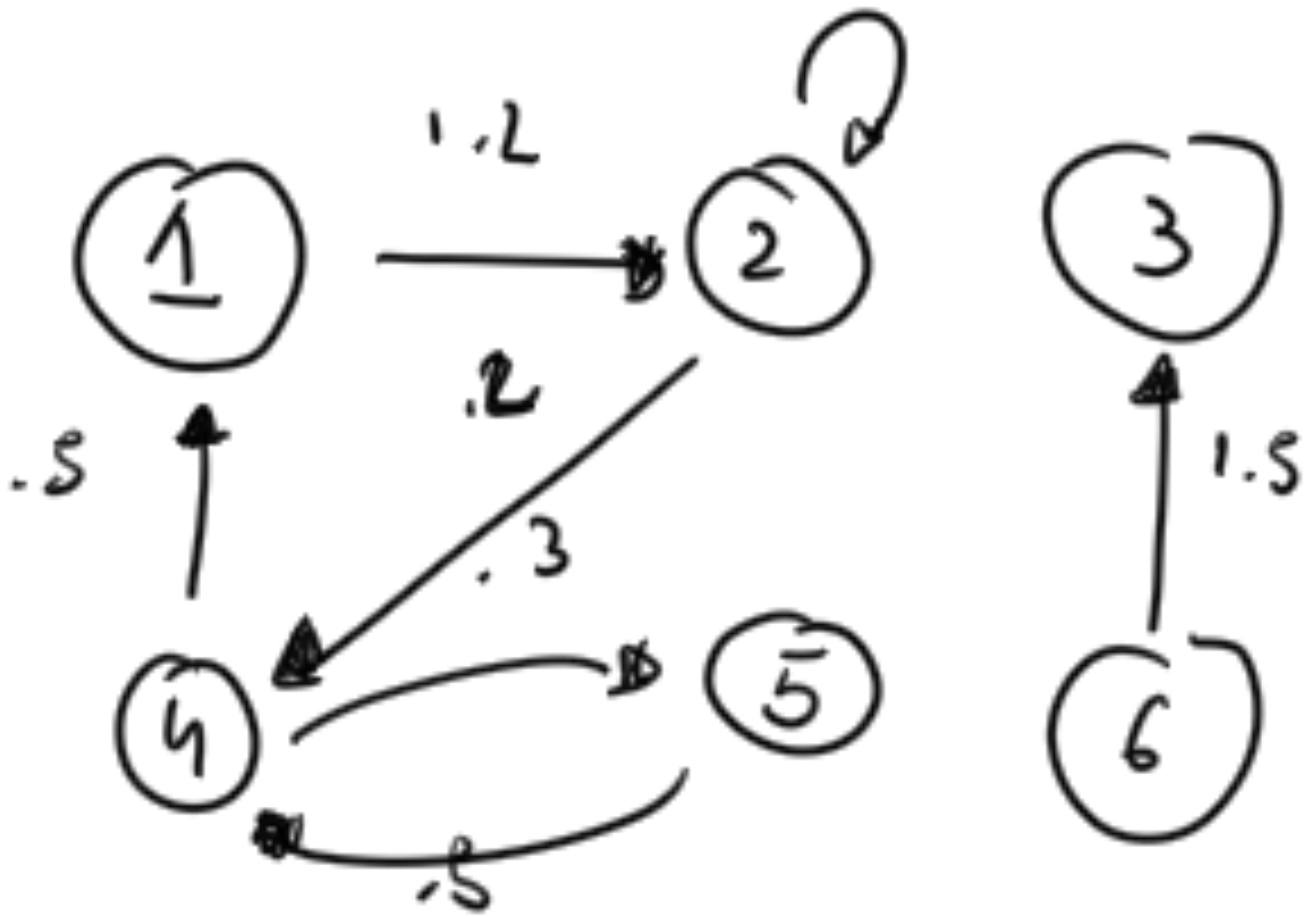
V:={1,2,3,4,5,6}

E:={{1,2},{1,5},{2,3},{2,5},{3,4},{4,5},{4,6}}



Directed Graph (digraph)

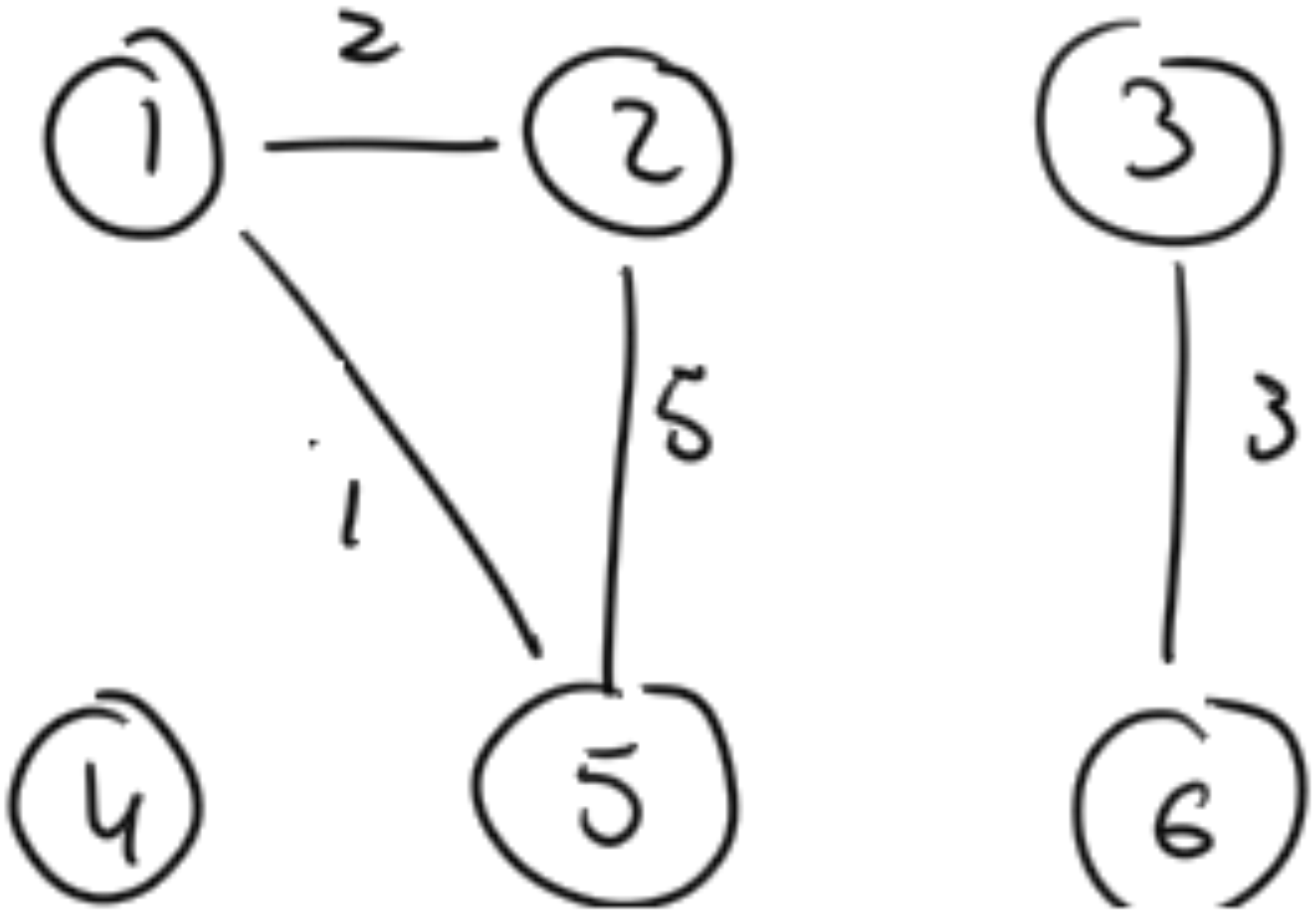
Edges have directions, i.e. an edge is an *ordered* pair of nodes



Weighted graphs

are graphs for which each edge has an associated *weight*, usually given by a _weight function

$$f_w : E \rightarrow R\ .$$

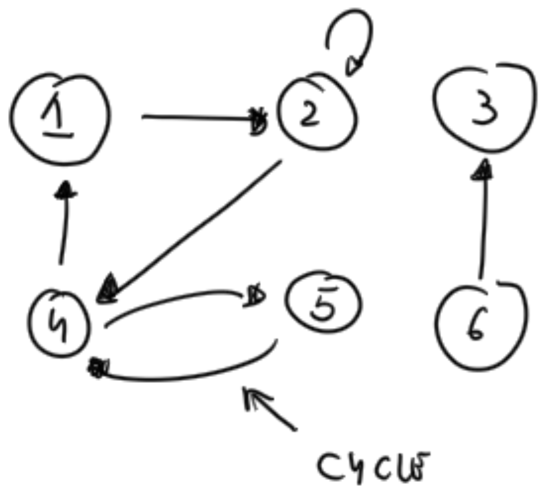


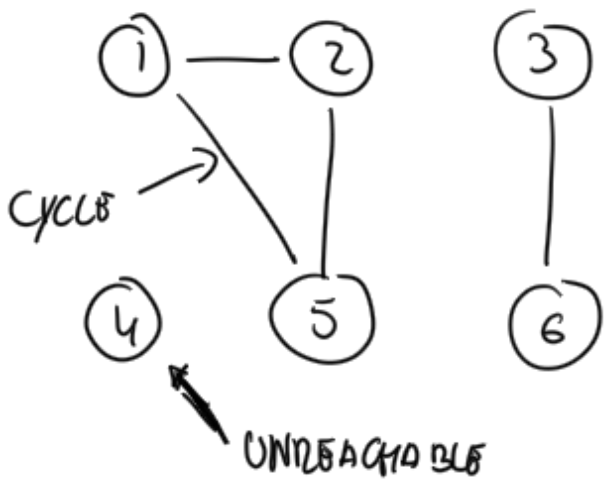
Path

A *path* is a sequence of vertices such that there is an edge from each vertex to its successor.

- A path is *simple* if each vertex is distinct.
- If there is path *p* from *u* to *v* then we say *v* is **reachable** from *u* via *p* .

Example: Simple path from 1 to 5= [1, 2, 4, 5]





Cycle

- A path from a vertex to itself is called a *cycle* .
- A graph is called *cyclic* if it contains a cycle;
  - otherwise it is called *acyclic*

Connectivity

- A graph is *connected* if
  - you can get from any node to any other by following a sequence of edges OR
  - any two nodes are connected by a path.
- A directed graph is *strongly connected* if there is a directed path from any node to any other node.

Sparsity/Density

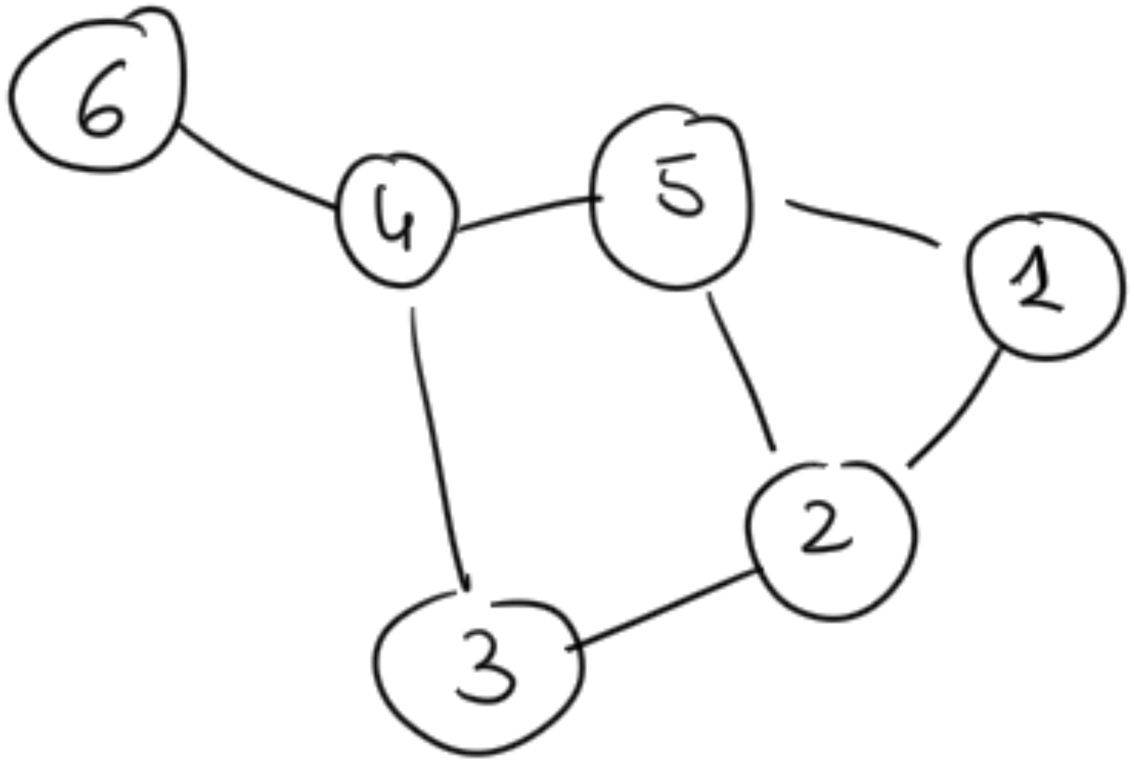
A graph is *sparse* if $|E| \approx |V|$

A graph is *dense* if $|E| \approx |V^2|$

Degree

Number of edges incident on a node

E.g., the degree of **5** is 3.



Degree (Directed Graphs)

In degree: Number of edges entering

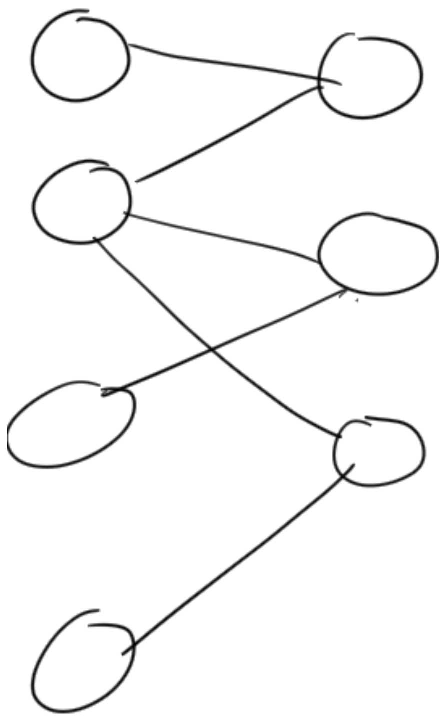
Out degree: Number of edges leaving

Degree =indegree+outdegree

Graph Types

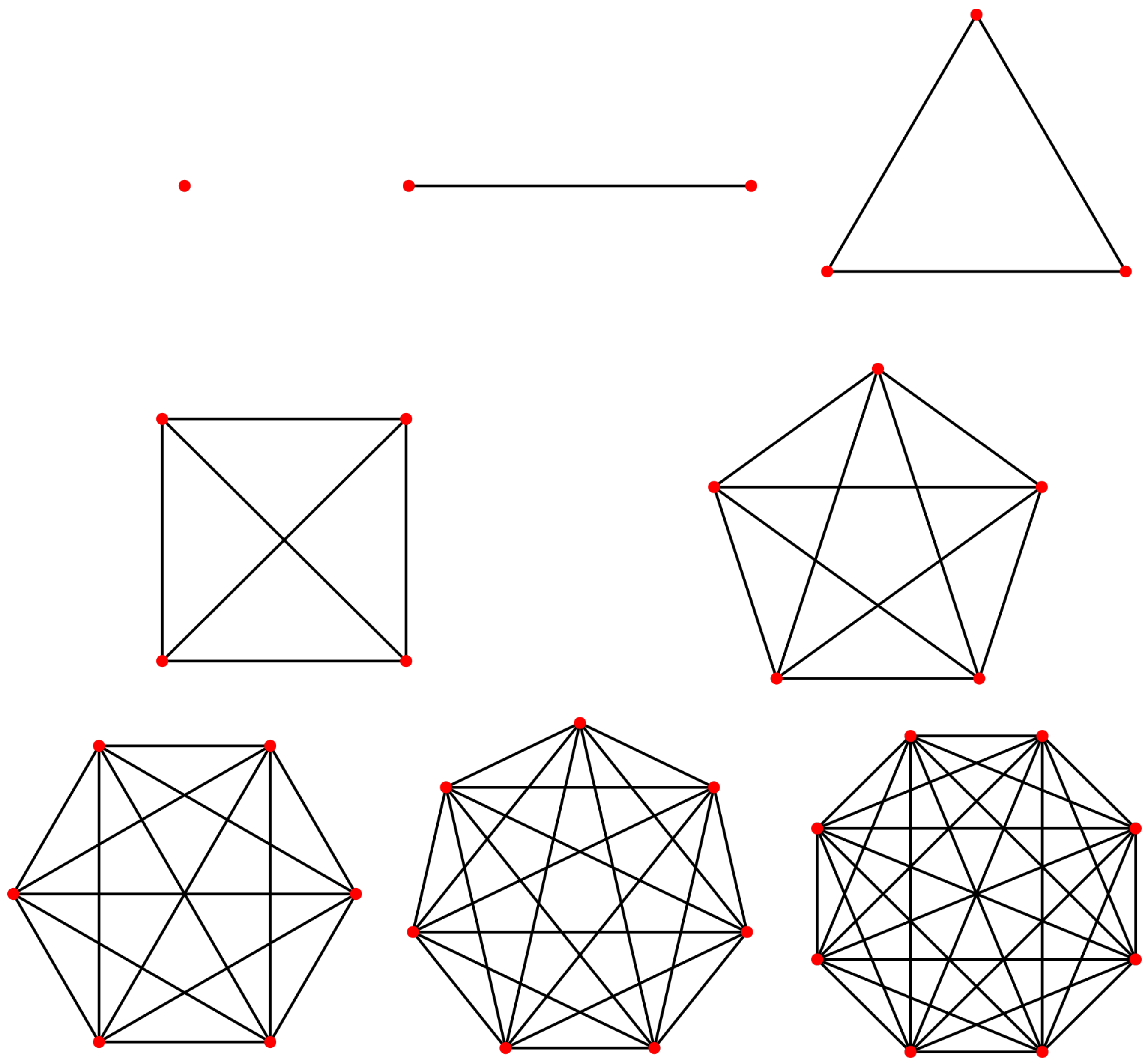
Bipartite graph

- V can be partitioned into 2 sets V_1 and V_2 such that $(u, v) \in E$ implies
    - either $u \in V_1$ and $v \in V_2$
    - or $^*v \in V_1$ and $u \in V_2$



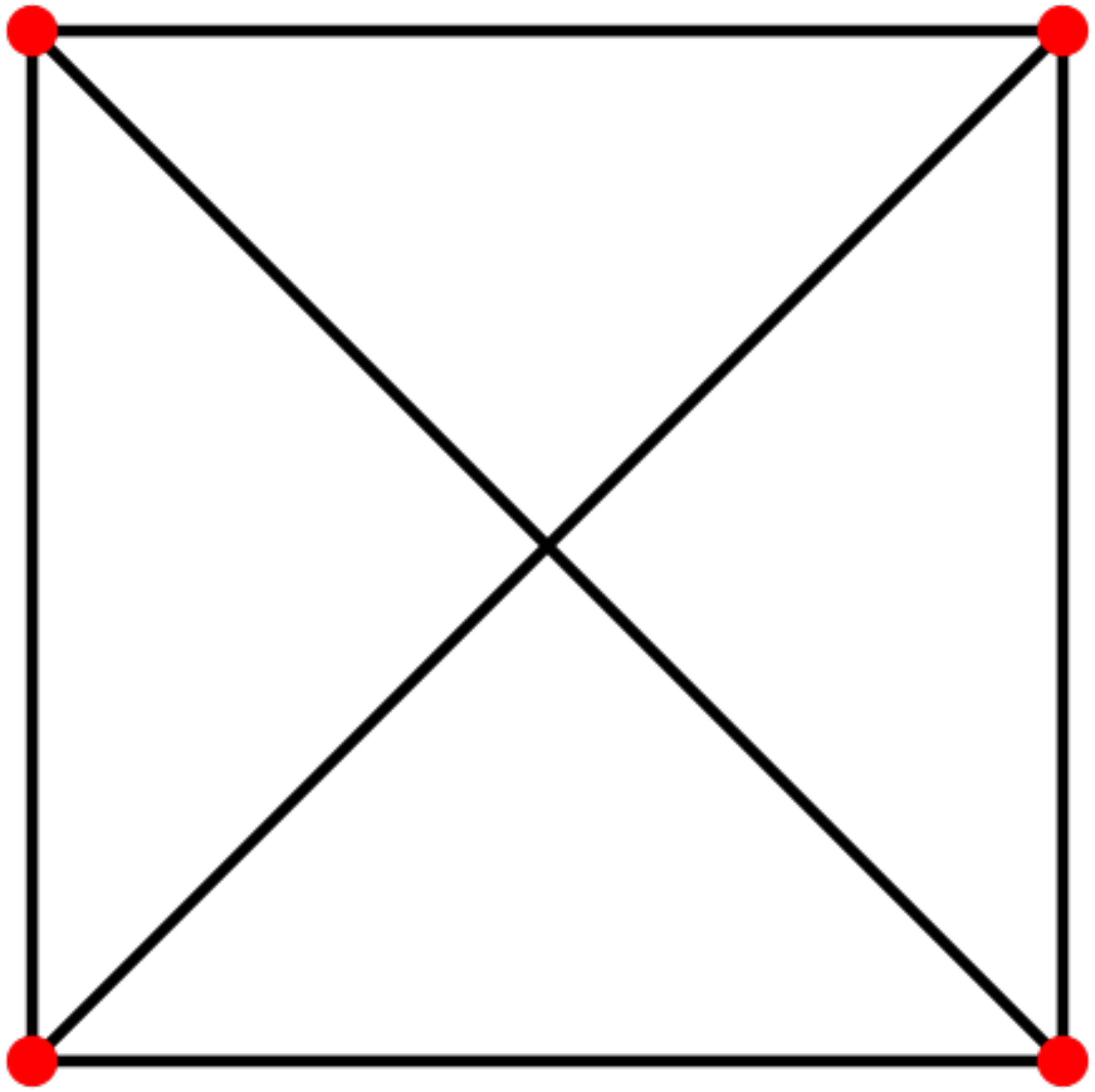
Complete Graph

- Denoted K_n
- Every pair of vertices are adjacent
- Has n(n-1) edges



Planar Graph

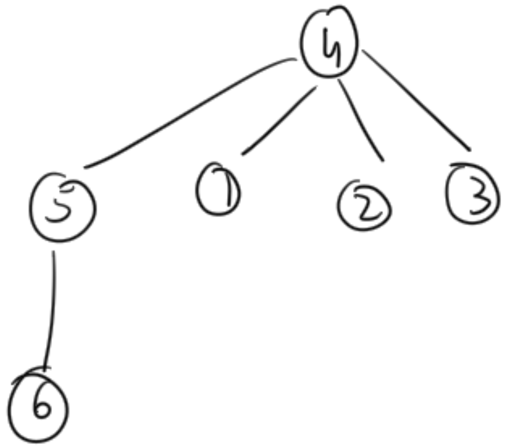
- Can be drawn on a plane such that no two edges intersect
- K_4 is the largest complete graph that is planar



Tree

Connected Acyclic Graph

Two nodes have *exactly* one path between them



Hypergraph

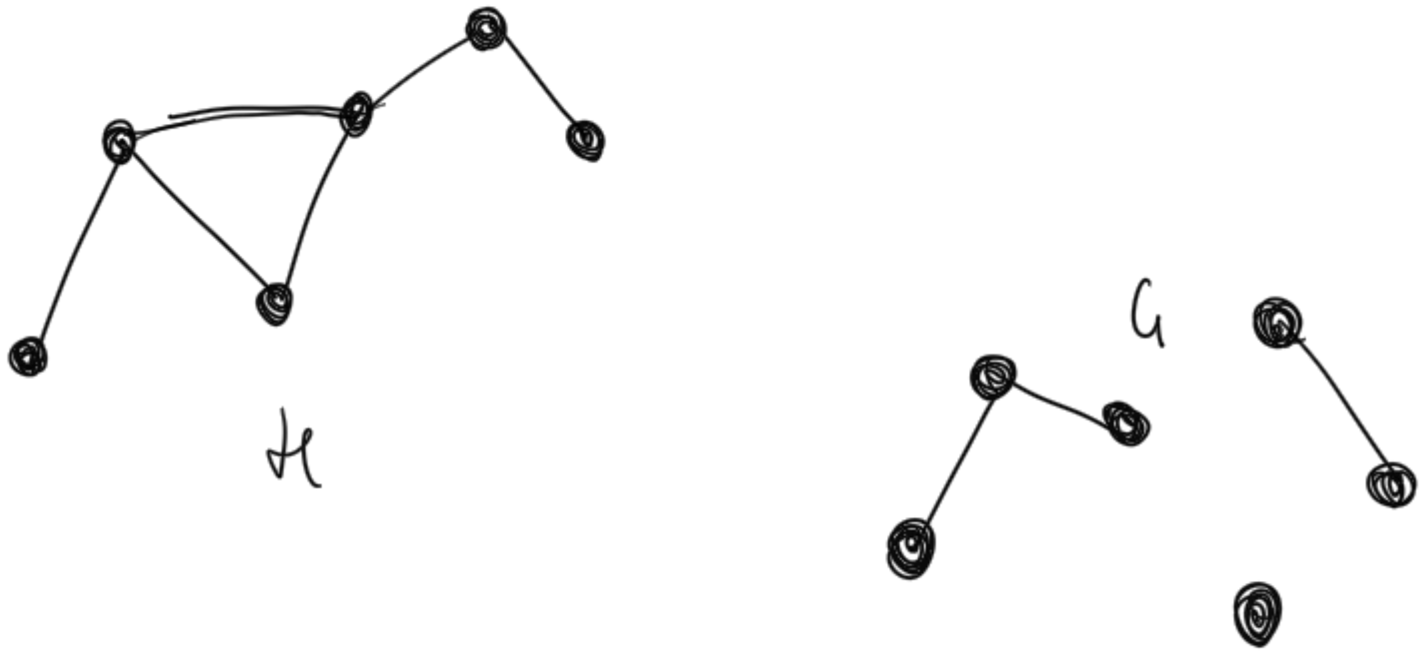
- Generalization of a graph,
  - edges can connect any number of vertices.
- Formally, an hypergraph is a pair (X,E) where
  - X is a set of elements, called nodes or vertices, and
  - E is a set of subsets of X, called hyperedges.
- Hyperedges are arbitrary sets of nodes,
  - contain an arbitrary number of nodes.

Subgraph

- Vertex and edge sets are subsets of those of G
  - a *supergraph* of a graph G is a graph that contains G as a subgraph.

Spanning subgraph

- Subgraph G has the same vertex set as H.
  - Possibly not all the edges
  - "G spans H".



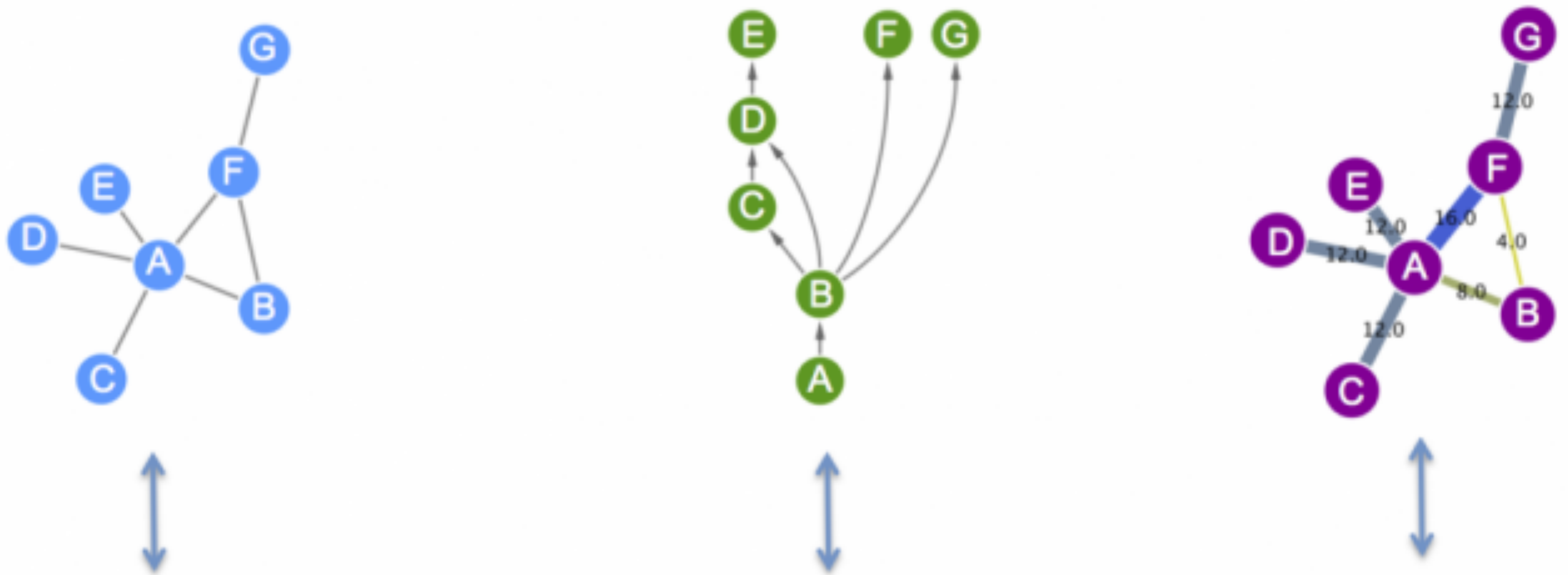
Graph ADT

- In computer science, a graph is an abstract data type (ADT)
- that consists of
  - a set of nodes and
  - a set of edges
  - establish relationships (connections) between the nodes.
- The graph ADT follows directly from the graph concept from mathematics.

Representation (Matrix)

- Incidence Matrix
  - E x V
  - [edge, vertex] contains the edge's data
- Adjacency Matrix
  - V x V
  - Boolean values (adjacent or not)
  - Or Edge Weights

Undirected	Directed	Weighted



Undirected

	A	B	C	D	E	F	G	Degree
A	0	1	1	1	1	1	0	5
B	1	0	0	0	0	1	0	2
C	1	0	0	0	0	0	0	1
D	1	0	0	0	0	0	0	1
E	1	0	0	0	0	0	0	1
F	1	1	0	0	0	0	1	3
G	0	0	0	0	0	1	0	1

Directed

	A	B	C	D	E	F	G	Out-degree
A	0	1	0	0	0	0	0	1
B	0	0	1	1	0	1	1	4
C	0	0	0	1	0	0	0	1
D	0	0	0	0	1	0	0	1
E	0	0	0	0	0	0	0	0
F	0	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0	0

Weighted

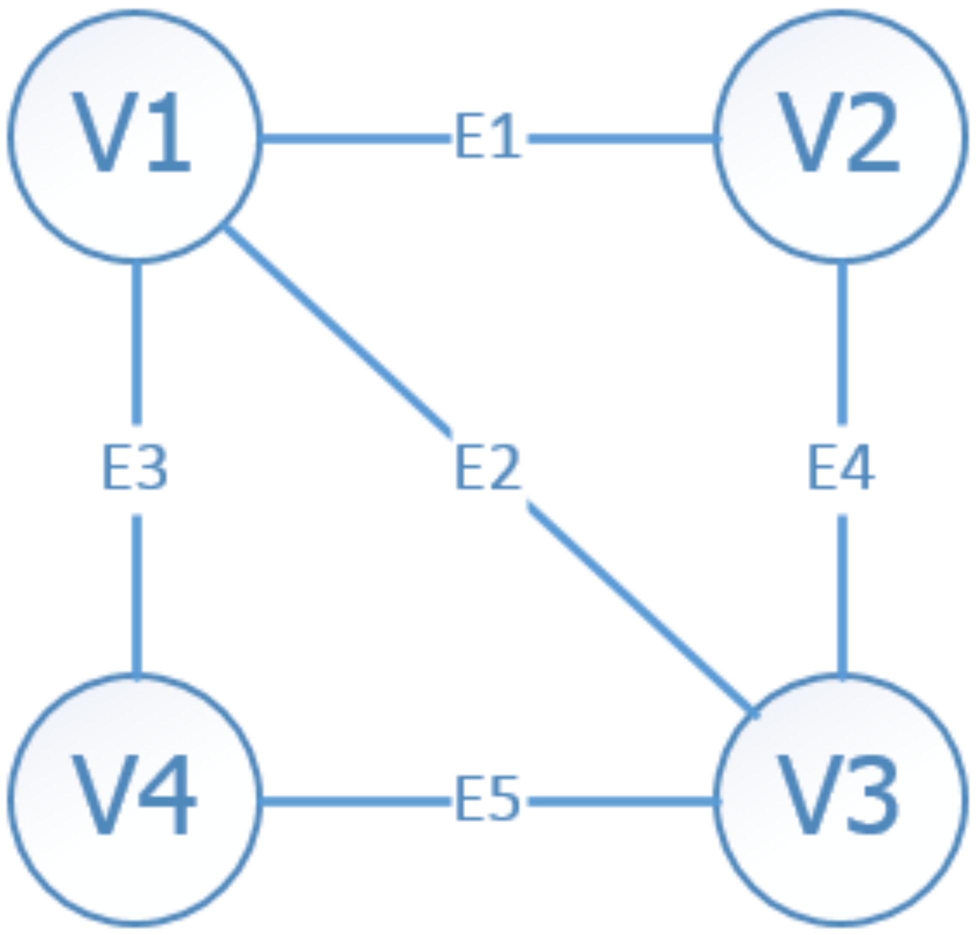
	A	B	C	D	E	F	G	Degree
A	0	8	12	12	12	16	12	72
B	8	0	0	0	0	4	0	12
C	12	0	0	0	0	0	0	12
D	12	0	0	0	0	0	0	12
E	12	0	0	0	0	0	0	12
F	16	4	0	0	0	0	12	32
G	12	0	0	0	0	12	0	24

Adjacency matrices

Representation (List)

- Edge List
  - pairs (ordered if directed) of vertices
  - Optionally weight and other data
- Adjacency List

Undirected Graph



Edge List

[[0,1],[0,2],[0,3],[1,2],[3,2]]

Adjacency Matrix

X	0	1	2	3
0	0	1	1	1
1	1	0	1	0
2	1	1	0	1
3	1	0	1	0

Adjacency List

[[1,2,3],
[0,2],
[0,1,3],
[0,2]]

Graph Algorithms

- Shortest Path
  - Single Source
  - All pairs (Ex. Floyd Warshall)
- Network Flow
- Matching
  - Bipartite
  - Weighted
- Topological Ordering
- Strongly Connected
- Biconnected Component / Articulation Point
- Bridge
- Graph Coloring
- Euler Tour
- Hamiltonian Tour
- Clique
- **Isomorphism**
- Edge Cover
- Vertex Cover
- Visibility