MIT 2.853/2.854

Introduction to Manufacturing Systems

Inventory

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Storage

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  - ⋆ the separation in time from the acquisition or production of something and its consumption; and
  - ⋆ the occurrence of an event at one location (such as a machine failure or a power surge) which can prevent desired performance or do damage at another.
- Storage improves system performance by *decoupling* parts of the system from one another.

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- It reduces the propagation of disturbances, and thus reduces instability and the fragility of complex, expensive systems.

Manufacturing Inventory

Motives/benefits

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  - ⋆ reduced set-ups
- Helps manage uncertainty:
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  - ⋆ Unreliable deliveries of raw materials.

Manufacturing Inventory
Costs

- Financial: raw materials are paid for, but no revenue comes in until the item is sold.

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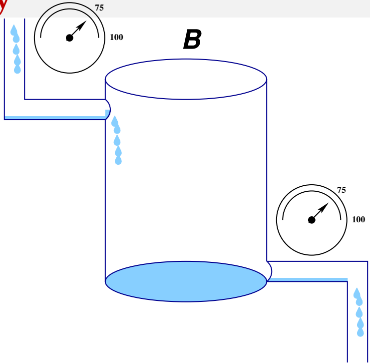
- Holding cost (warehouse space)
- "Shrinkage" = damage/theft/spoilage/loss

Variability and Storage

No variability



75 gal/sec in, 75 gal/sec out constantly

Variability and Storage

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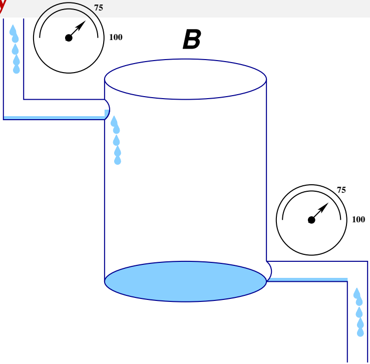

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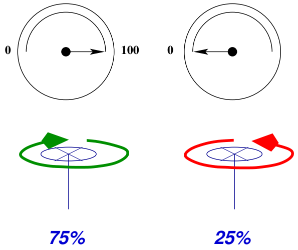
If we start with an empty tank, the tank is always empty (except for splashing at the bottom).

Variability and Storage
Variability from random valves

Consider a random valve:



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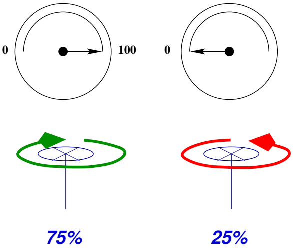

- The average period when the valve is open is 15 minutes.

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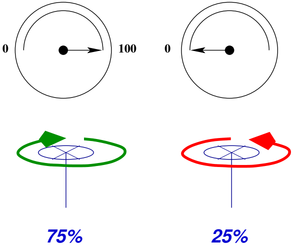

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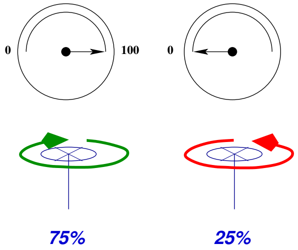

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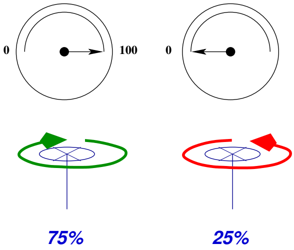

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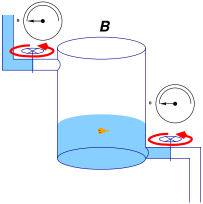
Four possibilities for two *unsynchronized* valves:

0 in, 0 out



0 in, 100 gal/sec out

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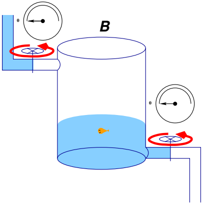

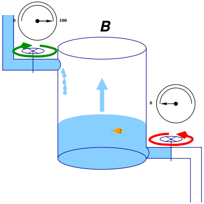

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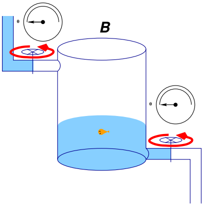

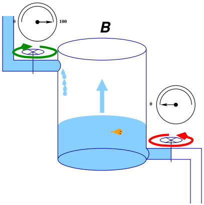

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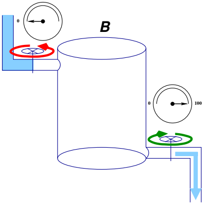

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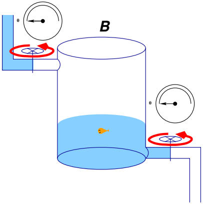

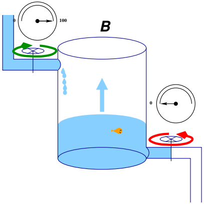

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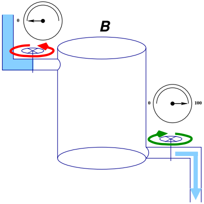

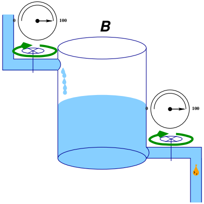

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2. *Variability causes inventory!!*

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  - ⋆ Rivers are dammed and reservoirs are created to control the flow of water — to reduce the variability of the water supply.
  - ⋆ For solar and wind power to be successful, energy storage is required for when the sun doesn't shine and the wind doesn't blow.

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- Other topics

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Note that $w \geq 0$ so $F(w) = 0$ and $f(w) = 0$ for $w \leq 0$.

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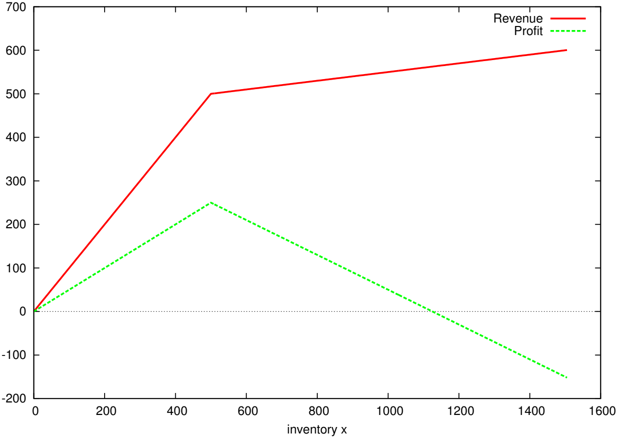

$$r = 1.; c = .5; s = .1; w = 500.$$

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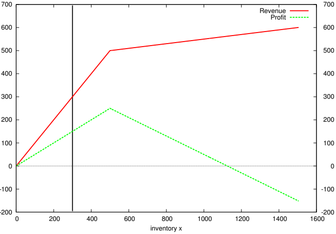
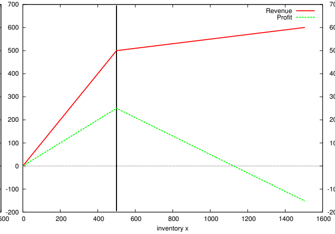


x too little



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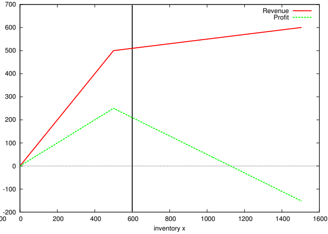

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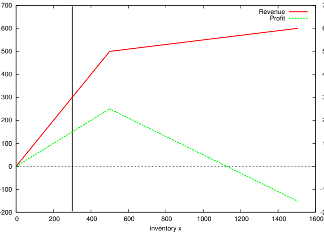

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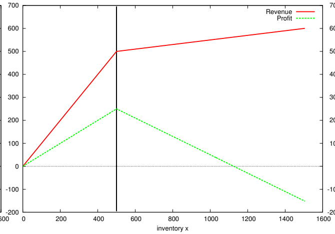

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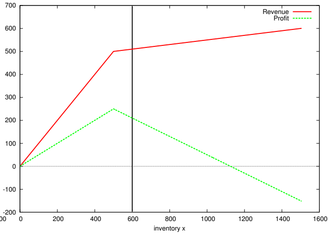

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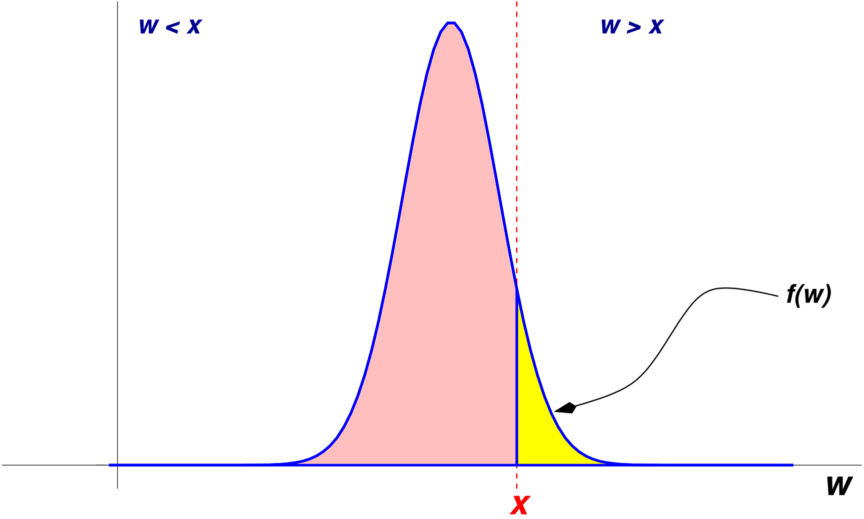
Note that if x is too large, the profit could be negative.

Newsvendor Problem
Normally distributed demand



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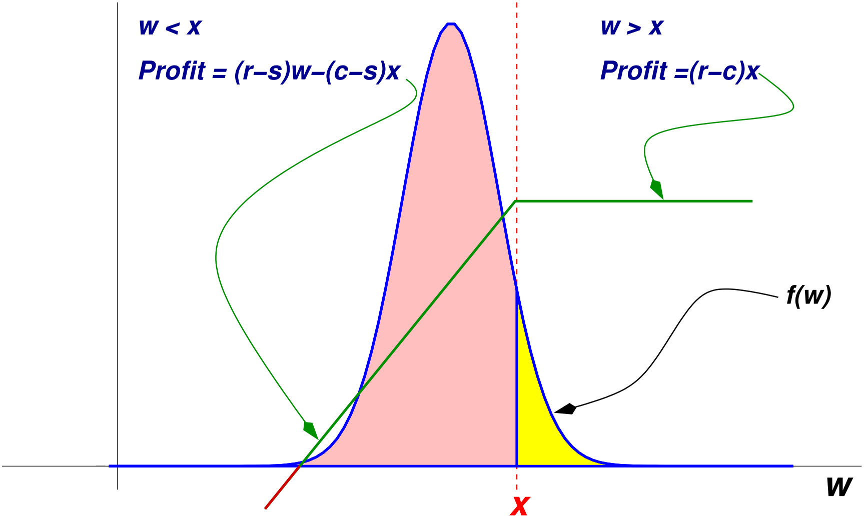

Newsvendor Problem

Expected profit

Expected Profit $= EP(x) = E_w P(x, w) =$

$$\int_{-\infty}^{\infty} P(x, w) f(w) dw =$$

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$$\int_{-\infty}^{x} [(r - s)w - (c - s)x] f(w) dw +$$

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$$\int_{x}^{\infty} (r-c) x f(w) dw$$

Newsvendor Problem
Expected profit

or $EP(x)$

$$= (r-s)\int_{-\infty}^{x} wf(w)dw - (c-s)x\int_{-\infty}^{x} f(w)dw$$

$$+(r-c)x\int_{x}^{\infty} f(w)dw$$

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$$= (r - s) \int_{-\infty}^{x} w f(w) dw$$

$$-(c - s) x F(x) +$$

Newsvendor Problem
Expected profit

or $EP(x)$

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$$+(r-c)x\int_{x}^{\infty} f(w)dw$$

$$= (r-s)\int_{-\infty}^{x} wf(w)dw$$

$$-(c-s)xF(x)+(r-c)x(1-F(x))$$

Newsvendor Problem
Expected profit

- When $x = 0$, EP is 0.

Newsvendor Problem

Expected profit

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- When $x \to \infty$,

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Expected profit

- When $x = 0$, EP is 0.

- When $x \to \infty$,
  - ⋆ the first term goes to a finite constant, $(r - s)Ew$;

Newsvendor Problem
Expected profit

- When $x = 0$, EP is 0.

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Therefore when $x \rightarrow \infty$, $EP \rightarrow -\infty$.

Newsvendor Problem

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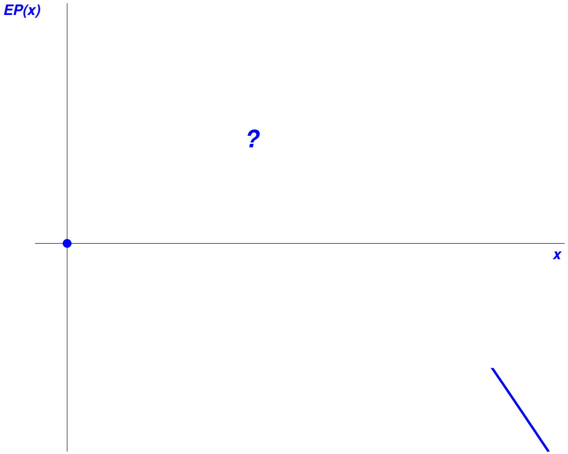

Newsvendor Problem
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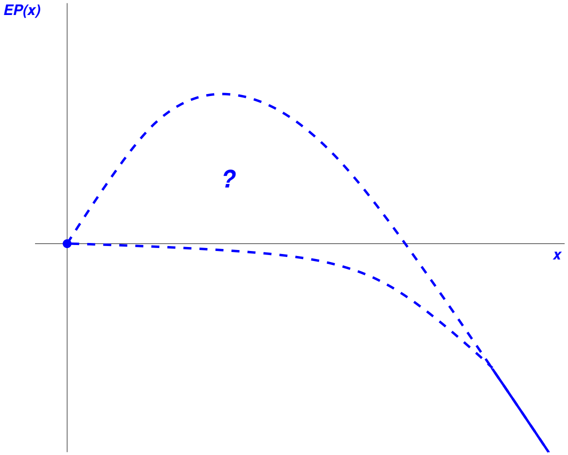

Newsvendor Problem

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Note also that $\dfrac{d^2EP}{dx^2} = -(r-s)f(x)$.

Newsvendor Problem
Expected profit

Note that

- $d^2EP/dx^2 = -(r-s)f(x) < 0$ for all $x > 0$. Therefore EP is concave and has a maximum.

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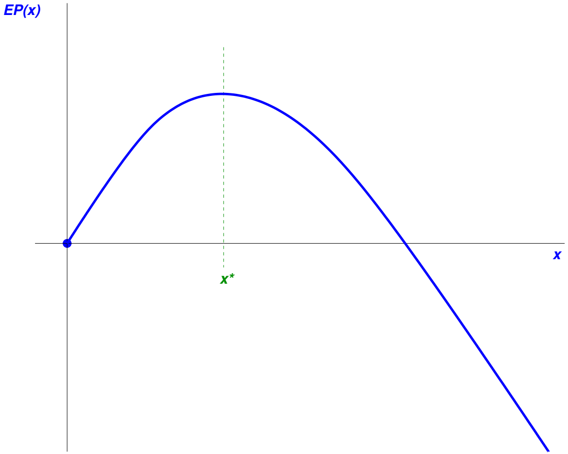
- x^* satisfies $\dfrac{dEP}{dx}(x^*) = 0$.

Newsvendor Problem
Expected profit



Newsvendor Problem
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Therefore $$\boxed{F(x^*) = \frac{r-c}{r-s}.}$$

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Recall the definition of $F()$: $F(x^*)$ is the probability that $W \leq x^*$.

Newsvendor Problem

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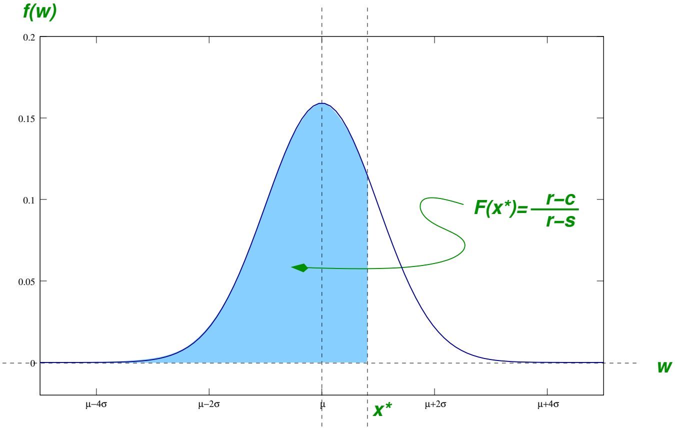
Recall the definition of $F()$: $F(x^*)$ is the probability that $W \leq x^*$. So the equation in the box means

Buy enough stock to satisfy demand $100K$% of the time, where

$$K = \frac{r-c}{r-s}.$$

Newsvendor Problem

Expected profit



Newsvendor Problem

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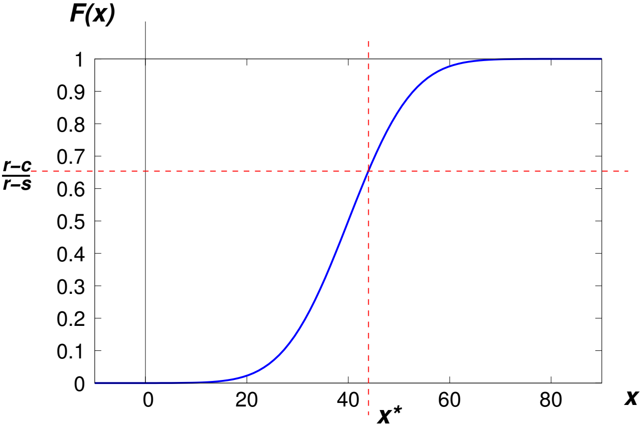

Newsvendor Problem

Expected profit

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$r - c > 0$ is the marginal profit when $x < w$.

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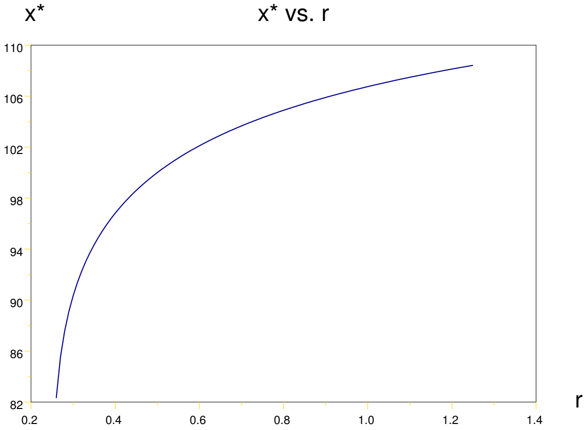
Choose x^* so that the fraction of time you do not buy too much is

$$\frac{\text{marginal profit}}{\text{marginal profit} + \text{marginal loss}}$$

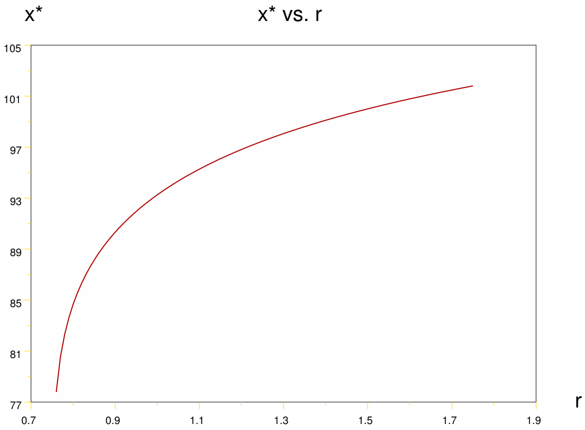
Newsvendor Problem

Example 1: r varying, $c = .25, s = 0, \mu_w = 100, \sigma_w = 10$



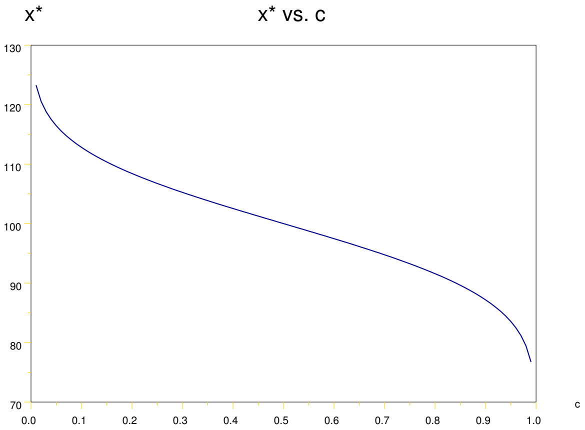
Newsvendor Problem

Example 2: r varying, $c = .75, s = 0, \mu_w = 100, \sigma_w = 10$



Newsvendor Problem

$r = 1$, c varying, $s = 0, \mu_w = 100, \sigma_w = 10$



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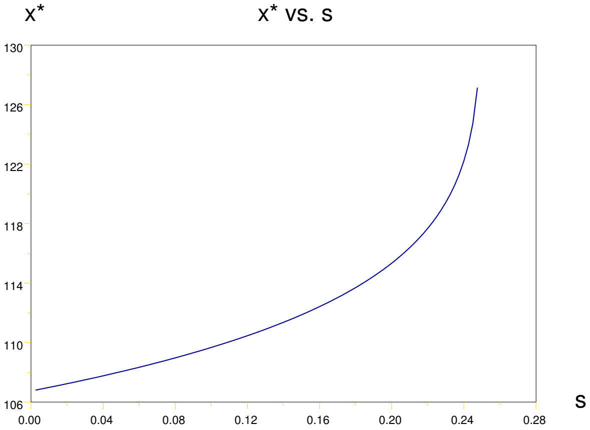

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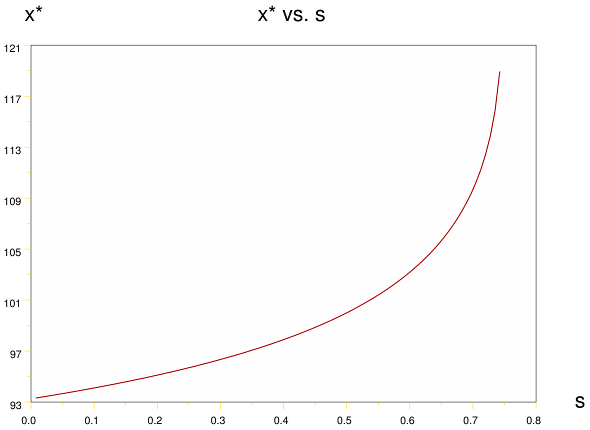

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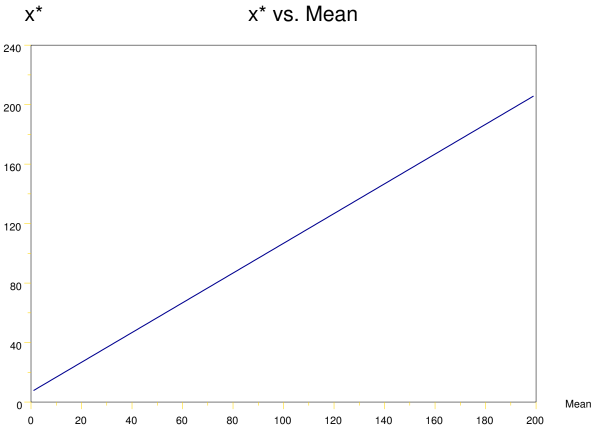

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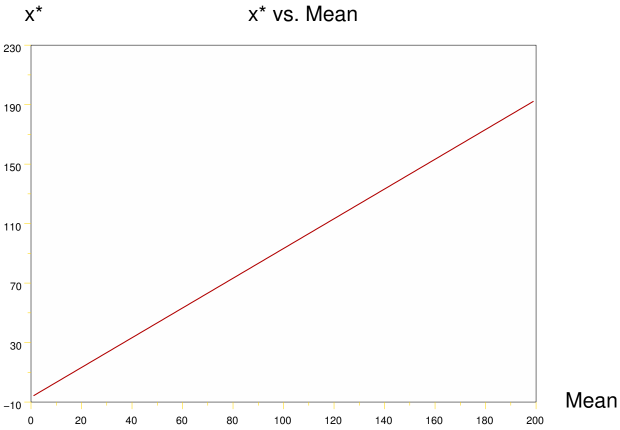

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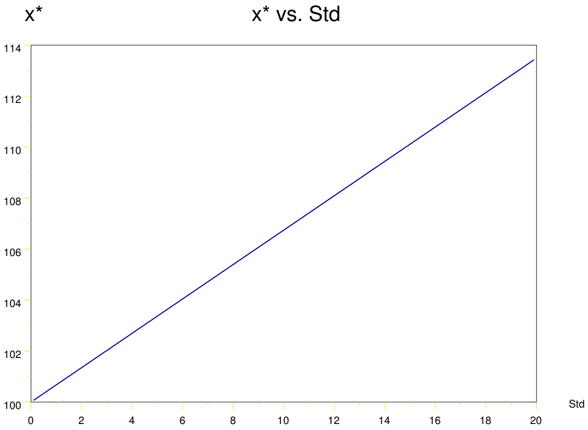

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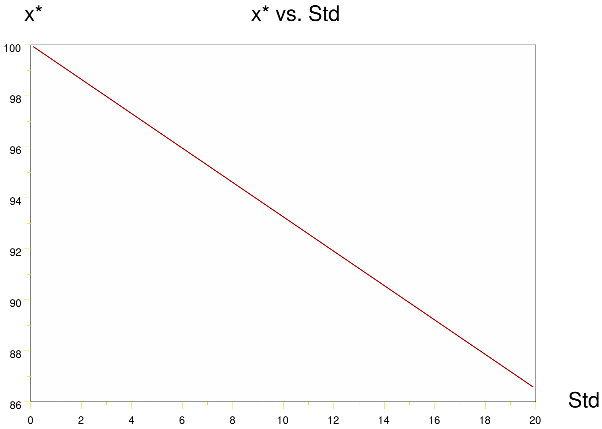

Newsvendor Problem

Why does x^* look linear in μ_w and σ_w?

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- x^* is the solution to $F(x^*) = \dfrac{r-c}{r-s}$.
- Assume demand w is $N(\mu_w, \sigma_w)$. Then, for any demand w,

$$F(w) = \Phi\left(\frac{w-\mu_w}{\sigma_w}\right)$$

where Φ is the standard normal cumulative distribution function.

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For what values of μ_w and σ_w is x^* closest to linear in μ_w and σ_w?

Answer: When $\mu_w >> \sigma_w$. If so, the normal distribution predicts a very small probability for w being negative. Therefore the normal is a good approximation for the truncated normal.

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x^* Increasing or Decreasing in σ_w

What determines whether x^* increases or decreases in σ_w?

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Newsvendor Problem

A General Strategy

Question:

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Answer: Yes.

"Build enough inventory to satisfy demand $100K\%$ of the time, for some K."

Economic Order Quantity
How much to order and when

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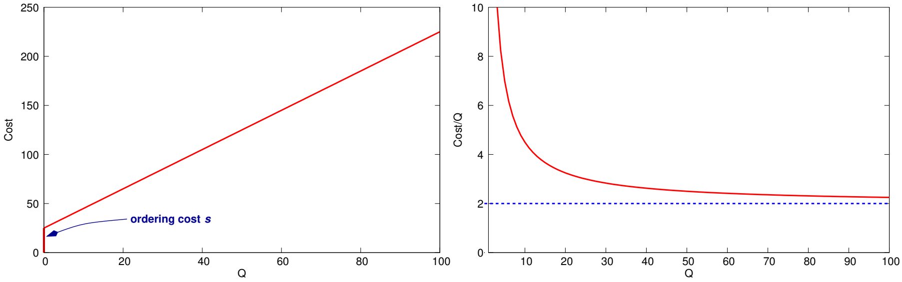
- It costs h to store one unit for one year. h is the *holding cost* (\$/unit/year).
- If the company orders Q units, it must pay $s + cQ$ for the order. s is the *ordering cost* (\$). c is the unit cost (\$/unit).

Economic Order Quantity

Assumptions



Total cost and cost per unit of an order

$$s = 25, c = 2$$

Economic Order Quantity
Problem

- Find a strategy for ordering materials that will minimize the total cost *per year.*

Economic Order Quantity
Problem

- Find a strategy for ordering materials that will minimize the total cost *per year.*

- There are two costs to consider: the *annual* ordering cost and the *annual* holding cost.

Economic Order Quantity
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Economic Order Quantity
Scenario

- At time 0, inventory level is 0.
- Q units are ordered and the inventory level jumps instantaneously to Q.
- Material is depleted at rate λ.
- Since the problem is totally deterministic, we can wait until the inventory goes to zero before we order next. There is no danger that the inventory will go to zero earlier than we expect it to.

Economic Order Quantity
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- Because of the very simple assumptions, we can assume that the optimal strategy does not change over time.

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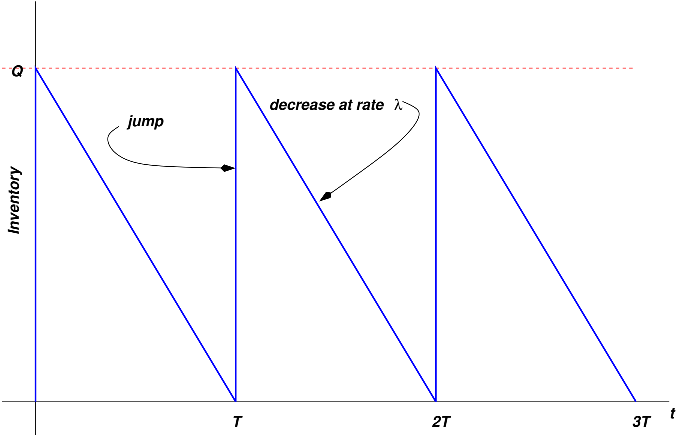
- Because of the very simple assumptions, we can assume that the optimal strategy does not change over time.
- Therefore the policy is to order Q each time the inventory goes to zero.
- We must determine the optimal Q.

Economic Order Quantity
Scenario



$$T = Q/\lambda \text{ (years)}$$

Economic Order Quantity
Formulation

- The number of orders in a year is $1/T = \lambda/Q$. Therefore, the ordering cost in a year is $s\lambda/Q$.

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- The average inventory is $Q/2$. Therefore the average inventory holding cost is $hQ/2$.
- Therefore, we must minimize the annual cost

$$C = \frac{hQ}{2} + \frac{s\lambda}{Q}$$

over Q.

Economic Order Quantity
Formulation

Then

$$\frac{dC}{dQ} = \frac{h}{2} - \frac{s\lambda}{Q^2} = 0$$

Economic Order Quantity
Formulation

Then

$$\frac{dC}{dQ} = \frac{h}{2} - \frac{s\lambda}{Q^2} = 0$$

or

$$Q^* = \sqrt{\frac{2s\lambda}{h}}$$

Economic Order Quantity
Examples

In the following graphs, the base case is

- $\lambda = 3000$
- $s = .001$
- $h = 6$

Economic Order Quantity

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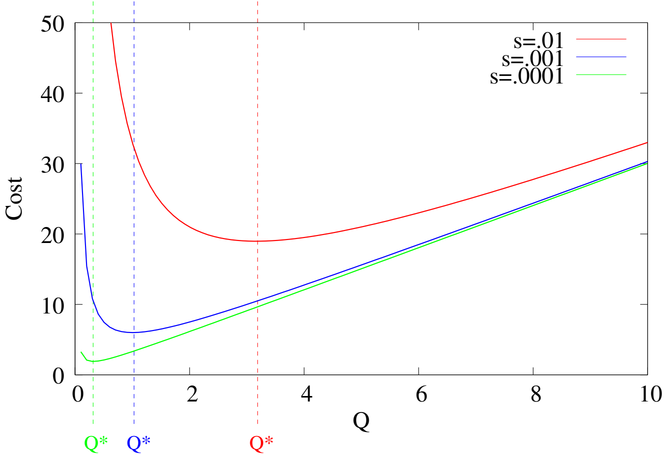
Note that

$$Q^* = 1$$

Economic Order Quantity
Examples



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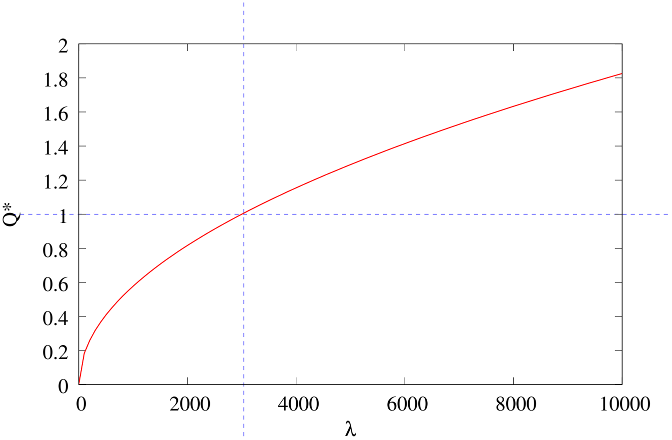

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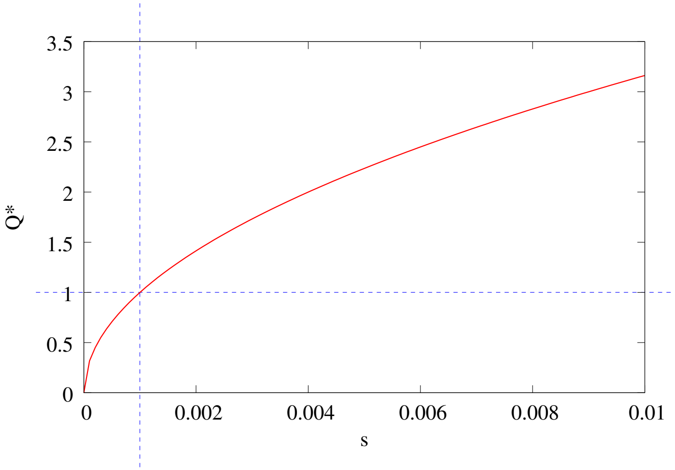

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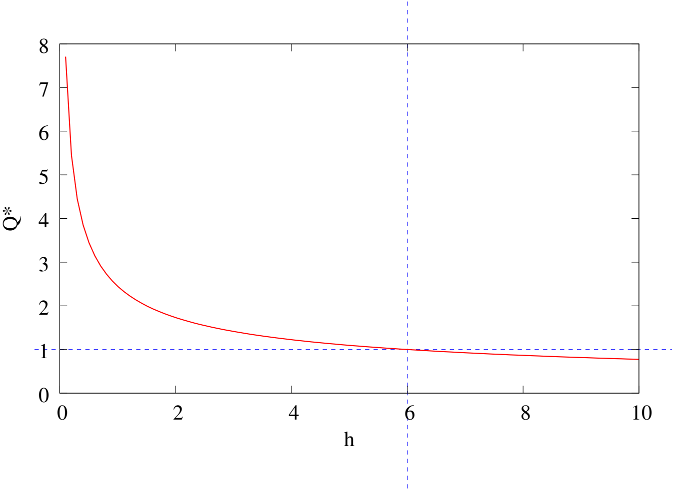

Supplier Lead Time + Random Demand

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Other possible issues: Lead time could be random, production quantity could be random, etc.

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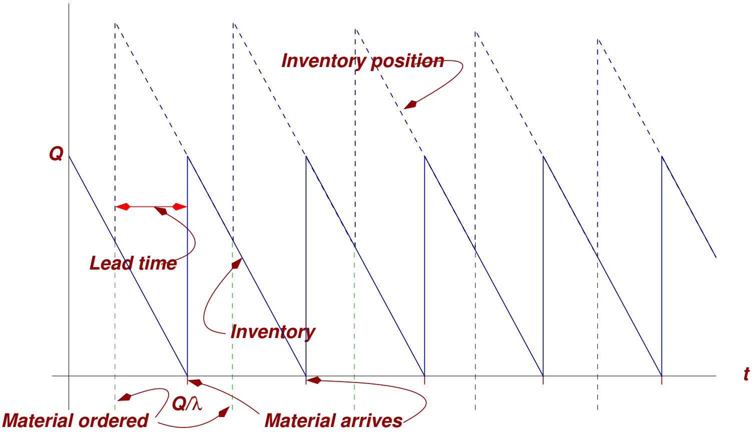
Inventory Position



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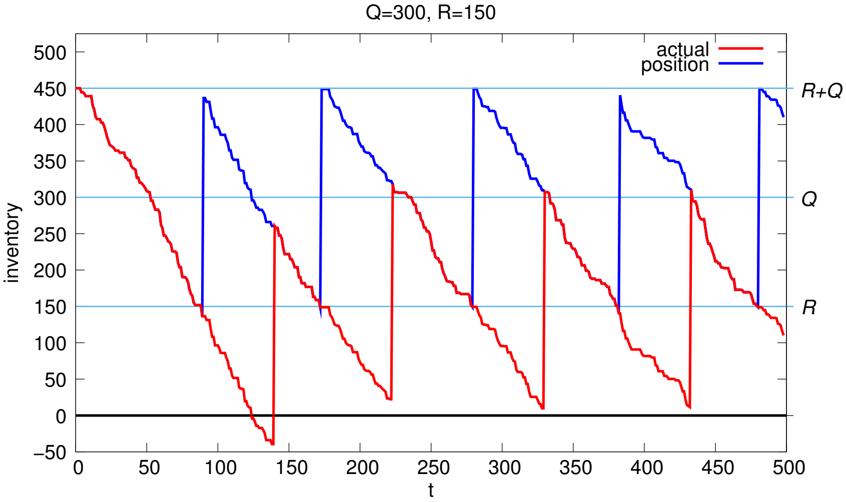
- *Problem:* find Q and R to minimize ordering + holding costs while not losing too many sales due to stockout.

Random Demand

Q, R Policy



Random Demand

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- The order is placed when the inventory level goes below R.
- The factory would like R to be greater than the demand D that arrives before the raw material arrives.
- But since the demand is random, the best we can do is to choose R such that

$$P(D > R) \leq \epsilon$$

where $P()$ is the probability and ϵ is a small number.

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Assume that D is $N(\mu, \sigma)$, a normal random variable with mean μ and standard deviation σ.

That is, the probability distribution of the demand is

$$P(D < d) = \Phi\left(\frac{d - \mu}{\sigma}\right)$$

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A graph of R^* as a function of ϵ with $\mu = 1000$ and $\sigma = 100$ is on the next slide.

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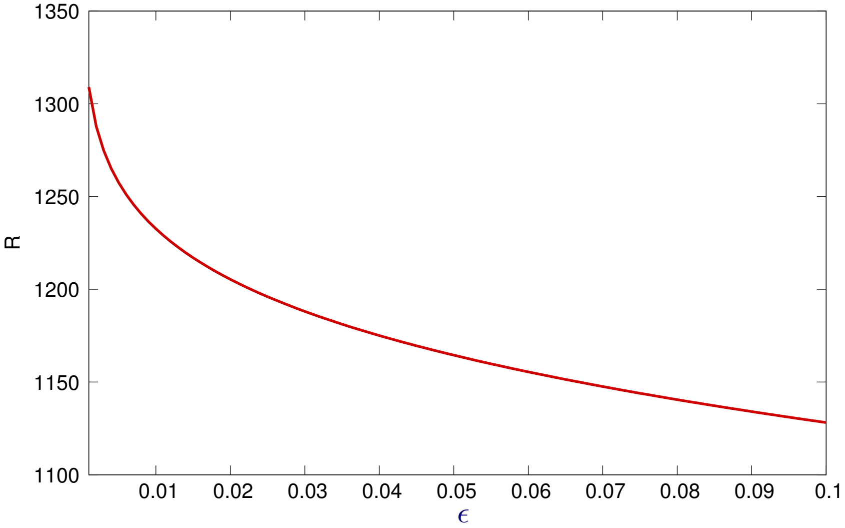

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- Assume that the holding cost for material in transit is the same as for inventory in the factory. The average inventory position is $(R + Q)/2$ and the average inventory holding cost is $h(R + Q)/2$.

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- Therefore, we choose Q to minimize the approximate expected annual cost

$$C = \frac{h}{2}\left(R + Q\right) + \frac{s\lambda}{Q}$$

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$$Q^* = \sqrt{\frac{2s\lambda}{h}}$$