[Week 5 Problem 1]

(1) Assume one year has 365 effective days (If you use different assumption, the result of calculation will vary). Use Newsboy model:

Cost per day: c = 4000/365 = 10.96 ($/day)
Revenue per day: r = 30 ($/day)
Salvage value per day: s = 0 ($/day)

Demand d is Poisson distributed with mean of 60.

$$P(d \le Q) \ge \frac{r-c}{r-s} = 0.6347$$

When Q=62, $P(d \le Q) = 0.63381$
When Q=63, $P(d \le Q) = 0.68043$

Therefore, we get the integer solution of Q as 63, which maximize the profit.

(2) Expected contribution at that location per year is (from textbook page 405)

$$\text{Revenue} = E\{(r-c)\min(Q,d) - (c-s)\max(0,Q-d)\}$$
$$= 19.04 \times \left(\sum_{d=0}^{63} d \times P(d) + \sum_{d=64}^{\infty} 63 \times P(d) \right) - 10.96 \times \left(\sum_{d=0}^{63} (Q-d) \times P(d) \right)$$
$$= 19.04 \times (38.028 + 20.133) - 10.96 \times 4.839$$
$$= 1054.35(\$/day)$$

Alternative equation leads to the same result.

$$\text{Revenue} = contirbution - \text{cost}$$
$$= 30 \times \left(\sum_{d=0}^{63} d \times P(d) + \sum_{d=64}^{\infty} 63 \times P(d) \right) - 4000 \times 63/365$$
$$= 30 \times (38.028 + 20.133) - 4000 \times 63/365$$
$$= 1054.419(\$/day)$$

[Problem 2]

(1) Use Newsboy's model to get the order quantity.

$$P(D \le q) \ge \frac{r-c}{r-s} = \frac{(4.5+8)-8}{(4.5+8)} = 0.36$$

D is normal distributed with mean of 200 and standard deviation of 50.

$$P(\frac{d-\mu}{\sigma} \le \frac{Q-\mu}{\sigma}) = P(Z \le -0.35) = 0.3632 > 0.36$$

$$\Rightarrow \frac{Q-\mu}{\sigma} = \frac{Q-200}{50} = -0.35$$

$$\Rightarrow q = 183$$

Therefore, the profit is maximized when we order 183 CDs per week.

(2) Suppose there is a 10% shrinkage rate, we here use 0.9Q. Then we have

$$P(\frac{d-\mu}{\sigma} \le \frac{0.9Q-\mu}{\sigma}) = P(Z \le -0.35) = 0.3632 > 0.36$$

$$\Rightarrow \frac{0.9Q-\mu}{\sigma} = \frac{0.9Q-200}{50} = -0.35$$

$$\Rightarrow q = 203$$

Therefore, the profit is maximized when we order 203 CDs per week.
To deal with shrinkage here, other reasonable approximations and methods can be accepted.