MIT 2.853/2.854

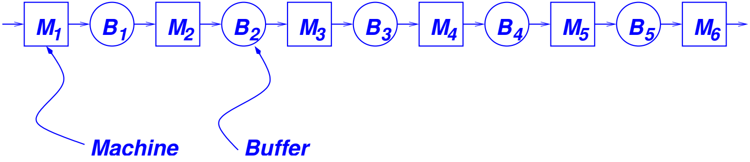
Introduction to Manufacturing Systems

Single-part-type, multiple stage systems

Stanley B. Gershwin
Laboratory for Manufacturing and Productivity
Massachusetts Institute of Technology
gershwin@mit.edu

Flow Lines

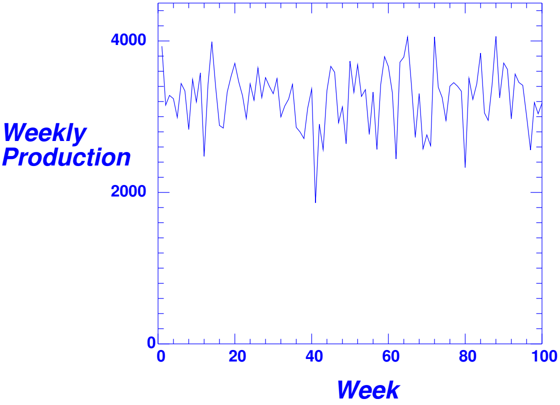
... also known as a Production or Transfer Lines.



- Machines are unreliable.
- Buffers are finite.
- In many cases, the operation times are constant and equal for all machines.

Flow Lines

Output Variability



Production output from a simulation of a transfer line.

Single Reliable Machine

- If the machine is perfectly reliable, and its average operation time is τ, then its maximum production rate is $\mu = 1/\tau$.

- *Note:*
  - ⋆ Sometimes *cycle time* is used instead of *operation time,* but *BEWARE:* cycle time has (at least) two meanings!
  - ⋆ The other meaning is the time a part spends in a system. If the system is a single, reliable machine, the two meanings are the same.

Single Unreliable Machine
Failures and Repairs

- A machine is either *up* (operational) or *down* (being repaired or maintained).
- MTTF = mean time to fail.
- MTTR = mean time to repair
- MTBF = MTTF + MTTR

Single Unreliable Machine

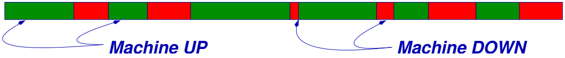
Production rate

- If the machine is unreliable, and
  - its average operation time is τ,
  - its mean time to fail is MTTF,
  - its mean time to repair is MTTR,

  then its maximum production rate is

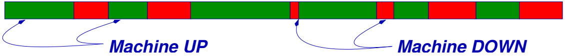
$$\frac{1}{\tau}\left(\frac{\text{MTTF}}{\text{MTTF}+\text{MTTR}}\right)$$

Single Unreliable Machine
Proof



- Average production rate, while machine is up, is $1/\tau$.
- Average *duration* of an up period is MTTF.
- Average *production* during an up period is MTTF$/\tau$.
- Average *duration* of up-down period: MTTF + MTTR.
- Average *production* during up-down period: MTTF$/\tau$.

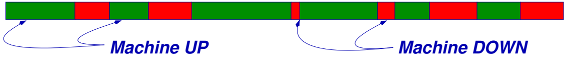
Single Unreliable Machine
Proof



- Therefore, the average production rate is

$$\frac{\mathsf{MTTF}/\tau}{\mathsf{MTTF}+\mathsf{MTTR}} = \frac{1}{\tau}\left(\frac{\mathsf{MTTF}}{\mathsf{MTTF}+\mathsf{MTTR}}\right)$$

Single Unreliable Machine
Efficiency



$$\frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} = e,$$ the *efficiency* of the machine.

- e is the fraction of time the machine is up.
- e can be thought of as the production rate in units of parts per operation time...
- ... or the actual production rate divided by what the production rate would be if the machine never failed.

Single Unreliable Machine
Geometric Up- and Down-Times

- *Assumptions:* Operation time is constant. Failure and repair times are *geometrically* distributed.

- Let p be the probability that a machine fails during any given operation. Then MTTF $= 1/p$.

Single Unreliable Machine

Geometric Up- and Down-Times

- Let r be the probability that M gets repaired during any operation time when it is down. Then MTTR $= 1/r$.

- Then the *average production rate* of M, in parts per operation time, is

$$e = \frac{r}{r+p}.$$

Single Unreliable Machine

Production Rates

- The *capacity* of a machine is its maximum possible production rate.

- The machine really has *three* capacities:

  - ⋆ 1 when it is up *(short-term capacity)* ,
  - ⋆ 0 when it is down *(short-term capacity)* ,
  - ⋆ $e = r/(r+p)$ on the average *(long-term capacity)* .

Multiple-Machine Lines
Failure Assumptions

- These assumptions are important for machines in systems, in which a machine can be made idle by another machine's failure.
- Operation-Dependent Failures (ODF)
  - A machine can only fail while it is working — not idle.
  - Idleness occurs due to starvation or blockage.
  - This is the usual assumption.
- Time-Dependent Failures (TDF)
  - A machine can fail even if it is idle.

Infinite-Buffer Lines

- Starvation: Machine M_i is starved at time t if Buffer B_{i-1} is empty at time t.

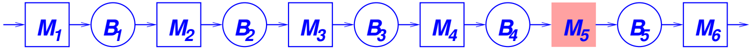
Assumptions:

- A machine is not idle if it is not starved.

- The first machine is never starved.

Infinite-Buffer Lines

Bottleneck



- The production rate of the line is the production rate of the *slowest* machine in the line — called the *bottleneck* .

- Slowest means *least average production rate.*

Infinite-Buffer Lines

Bottleneck

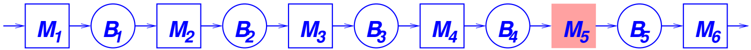
- The production rate is therefore

$$P = \min_i \frac{1}{\tau_i} \left(\frac{\text{MTTF}_i}{\text{MTTF}_i + \text{MTTR}_i} \right)$$

- and M_i is the bottleneck.

Infinite-Buffer Lines

Bottleneck



- Or, if all $\tau_i = 1$,

$$P = \min_i \left(\frac{\mathsf{MTTF}_i}{\mathsf{MTTF}_i + \mathsf{MTTR}_i} \right)$$

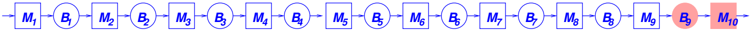
- and M_i is the bottleneck.

Infinite-Buffer Lines

Bottleneck

- The system is not in steady state.
- An increasing amount of inventory accumulates in the buffer upstream of the bottleneck.
- A finite amount of inventory appears downstream of the bottleneck.

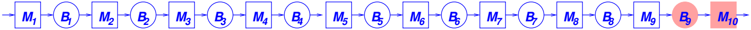
Infinite-Buffer Lines
Example 1

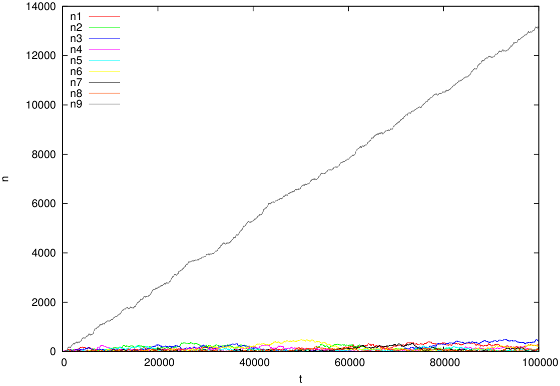


- Parameters:
  $r_i = .1, p_i = .01, i = 1, ..., 9; r_{10} = .1, p_{10} = .03.$
- Therefore, $e_i = .909, i = 1, ..., 9; e_{10} = .769.$

Infinite-Buffer Lines

Example 1





Infinite-Buffer Lines

Example 1

- *Question:* what is the the rate of growth of $n_9(t)$, the inventory in B_9?
- *Answer:*
  - ⋆ The rate that parts arrive at buffer 9 is .909.
  - ⋆ The rate that parts leave is .769.
  - ⋆ Therefore the rate of increase is $.909 - .769 =$ *0.14* parts per operation time.

Infinite-Buffer Lines

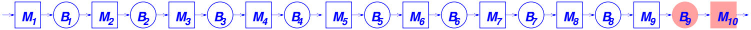
Example 1

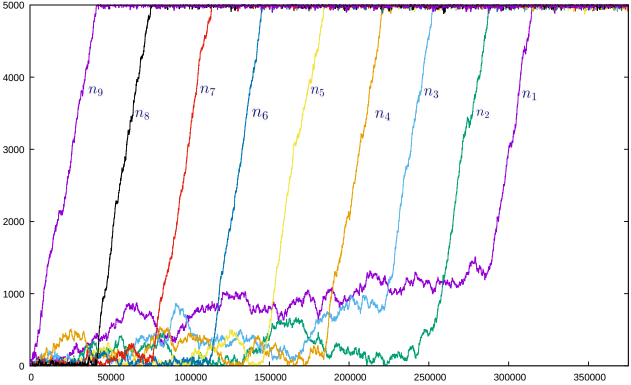
- *Question:* What happens when the buffers are large but finite?

- *Answer:* The last buffer gains material until it becomes full. Then the next to last buffer gains material until it becomes full. The process repeats until all buffers are full. See the graph on next slide.

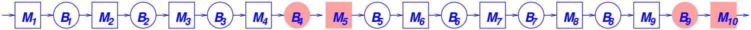
Infinite-Buffer Lines

Example 1, but with finite buffers





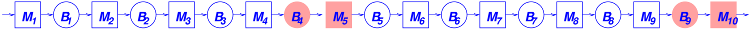
Infinite-Buffer Lines
Second Bottleneck



- The *second bottleneck* is the slowest machine upstream of the bottleneck.
- An increasing amount of inventory accumulates just upstream of it.
- A finite amount of inventory appears between the second bottleneck and the machine upstream of the first bottleneck.
- An increasing amount of inventory accumulates in the buffer just upstream of the first bottleneck.
- A finite amount of inventory appears downstream of the first bottleneck.

Infinite-Buffer Lines

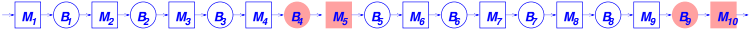
Example 2

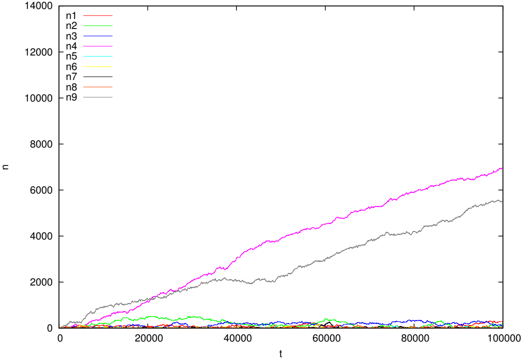


- Parameters: $r_i = .1, p_i = .01, i = 1, ..., 4, 6, ..., 9;$
  $r_5 = .1, p_5 = .02, r_{10} = .1, p_{10} = .03.$

- Therefore, $e_i = .909, i = 1, ..., 4, 6, ..., 9;$
  $e_5 = .833, e_{10} = .769.$

Infinite-Buffer Lines
Example 2





Infinite-Buffer Lines

Example 2

Rates of growth of $n_4(t)$ and $n_9(t)$

- The rate of growth of $n_4(t)$ is $.909 - .833 = .076$
- The rate of growth of $n_9(t)$ is $.833 - .769 = .064$
- Note that when t is large enough, $n_4(t) > n_9(t)$.
- It is tempting to believe that the easiest way to find the bottleneck of a line is to look for the greatest accumulation of inventory. *Is that correct?*

Infinite-Buffer Lines

Improvements

Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Simulation Note

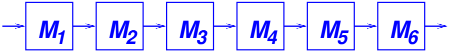
- The simulations shown here were *discrete-time* rather than *discrete-event.*
- Discrete-time simulations are easier to program, but less general, less accurate, and slower, than discrete-event simulations.
- Discrete-time simulations are easiest to write for systems where all event times are geometrically distributed.

Simulation Note

Discrete-time simulation

- Assume that some event occurs according to a geometric probability distribution and it has a mean time to occur of T time steps.
- Then the probability that it occurs in any given time step is $p = 1/T$.
- The discrete-time simulation logic is:
  - ⋆ At each time step, choose x, a U[0,1] random number.
  - ⋆ If $x \leq p$, the event has occurred. Change the state accordingly.
  - ⋆ If $x > p$, the event has not occurred. Change the state accordingly.

Zero-Buffer Lines



- If any one machine fails, or takes a very long time to do an operation, *all* the other machines must wait.
- Therefore the production rate is usually less — possibly much less – than the slowest machine.

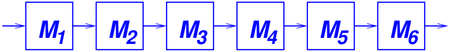
Zero-Buffer Lines

$$\rightarrow \boxed{M_1} \rightarrow \boxed{M_2} \rightarrow \boxed{M_3} \rightarrow \boxed{M_4} \rightarrow \boxed{M_5} \rightarrow \boxed{M_6} \rightarrow$$

- *Example:* Constant, unequal operation times, perfectly reliable machines.
  - ⋆ The operation time of the line is equal to the maximum operation time of the of all the machines, ...
  - ⋆ ... so the production rate of the line is the inverse of the maximum operation time.

- *Line balancing:* the assignment of tasks to machines in a way that minimizes the maximum operation time.

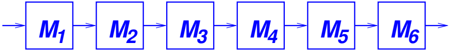
Zero-Buffer Lines
Constant, equal operation times, unreliable machines



- Assumption: Time-Dependent Failures (TDF).
- The operation time is the time unit.
- $e_i = \frac{\text{MTTF}_i}{\text{MTTR}_i + \text{MTTF}_i}$ is the probability that M_i is operational in any time unit.

Zero-Buffer Lines

Constant, equal operation times, unreliable machines



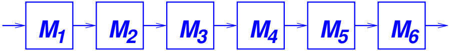
- A part is produced as often as *all* the machines are operational.
- Since machines fail independently of the states of the other machines, the probability that all machines are operational is $e_1 e_2 ... = \Pi_i e_i$.
- Therefore, the production rate of the line is

$$\boxed{E_{\text{TDF}} = \Pi_i e_i}$$

parts per operation time.

Zero-Buffer Lines

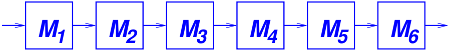
Constant, equal operation times, unreliable machines



- *Assumption:* Operation-Dependent Failures (ODF).
- The operation time is the time unit.
- *Assumption:* Failure and repair times are *geometrically* distributed.
- $p_i = 1/\text{MTTF}_i =$ probability of failure during an operation.
- $r_i = 1/\text{MTTR}_i =$ probability of repair during an operation time when the machine is down.

Zero-Buffer Lines

Production Rate



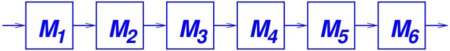
Buzacott's Zero-Buffer Line Formula:

Let k be the number of machines in the line. Then the production rate per operation time is

$$\boxed{E_{\mathsf{ODF}} = \frac{1}{1 + \displaystyle\sum_{i=1}^{k} \frac{p_i}{r_i}}}$$

Zero-Buffer Lines

Production Rate



- This reduces to the earlier formula (Slides **??** and **??**) when $k = 1$.

- According to the new formula, the *isolated production rate* of a single machine M_1 is

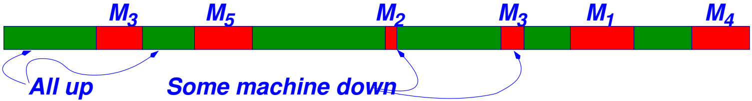
$$\frac{1}{1+\frac{p_1}{r_1}} = \frac{r_1}{r_1+p_1}.$$

which is the same as the earlier formula when the operation time τ is 1.

Zero-Buffer Lines
Proof of formula

- *Approximation:* At most, only <u>one</u> machine can be down at any time.

- Consider a long time interval of length T operation times during which Machine M_i fails m_i times $(i = 1, \ldots k)$.



- Without failures, the line would produce T parts.

Zero-Buffer Lines

Proof of formula

- The average repair time of M_i is $1/r_i$ each time it fails, so the total system down time is close to

$$D = \sum_{i=1}^{k} \frac{m_i}{r_i}$$

where D is the number of operation times in which a machine is down.

Zero-Buffer Lines

Proof of formula

- The total up time is approximately

$$U = T - \sum_{i=1}^{k} \frac{m_i}{r_i}$$

- where U is the number of operation times in which all machines are up.

Zero-Buffer Lines

Proof of formula

- Since the system produces one part per time unit while it is working, it produces U parts during the interval of length T.

- Note that, approximately,

$$m_i = p_i U$$

because M_i can only fail while it is operational.

Zero-Buffer Lines

Proof of formula

- Thus,

$$U = T - U \sum_{i=1}^{k} \frac{p_i}{r_i},$$

or,

$$\frac{U}{T} = E_{\text{ODF}} = \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}}$$

Zero-Buffer Lines

p_i and r_i and p_i/r_i

$$P = E_{\mathsf{ODF}} = \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}}$$

- Note that P is a function of the *ratio* p_i/r_i and not p_i or r_i separately.
- The same statement is true for the infinite-buffer line.
- However, the same statement is *not* true for a line with finite, non-zero buffers.

Zero-Buffer Lines

Improvements

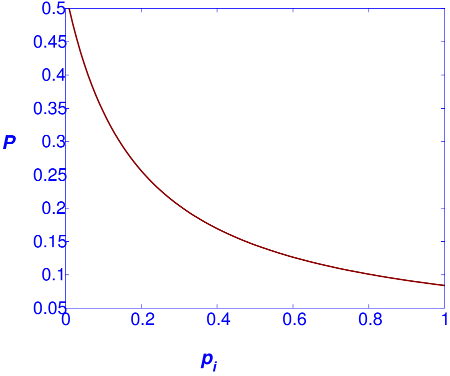
Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Zero-Buffer Lines

P as a function of p_i

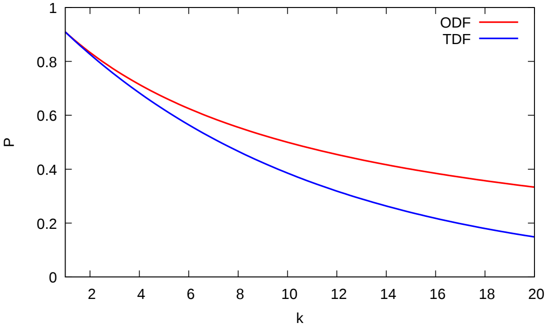
All machines are the same except M_i. As p_i increases, the production rate decreases.



Zero-Buffer Lines

P as a function of k

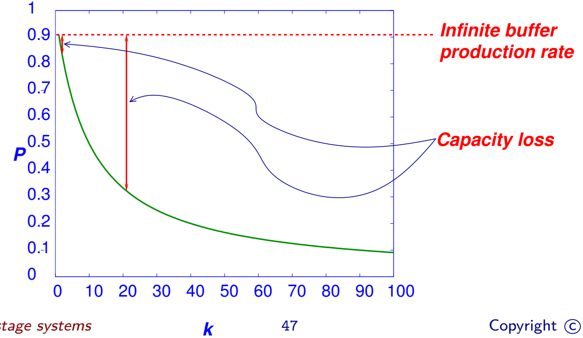
All machines are the same. This compares E_{TDF} and E_{ODF}.



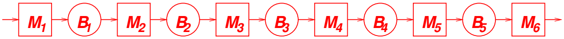
Zero-Buffer Lines
P as a function of k

This graph compares the production rates of an infinite-buffer line with that of a zero-buffer line.

All machines are the same. As k increases, the production rate of the zero-buffer line decreases. The production rate of the infinite-buffer line stays the same.



Finite-Buffer Lines



- Motivation for buffers: increase production rate.
- Cost
  - ⋆ in-process inventory/lead time
  - ⋆ floor space
  - ⋆ material handling mechanism

Finite-Buffer Lines

- Infinite buffers: delayed downstream propagation of disruptions *(starvation)* and *no* upstream propagation.

- Zero buffers: instantaneous propagation in both directions.

- Finite buffers: delayed propagation in both directions.

  ⋆ New phenomenon: *blockage.*

- Blockage: Machine M_i is blocked at time t if Buffer B_i is full at time t.

Finite-Buffer Lines

$$\rightarrow M_1 \rightarrow B_1 \rightarrow M_2 \rightarrow B_2 \rightarrow M_3 \rightarrow B_3 \rightarrow M_4 \rightarrow B_4 \rightarrow M_5 \rightarrow B_5 \rightarrow M_6 \rightarrow$$

- Difficulty:
  - No simple formula for calculating production rate, inventory levels, or other performance measures.
- Solutions:
  - Simulation
  - Analytical approximation
  - Exact numerical solution for short lines
  - *Exact analytical solution for two-machine lines only*

Two-Machine, Finite-Buffer Lines
Parameters

- p_1 is the probability of M_1 failing while doing an operation.
- r_1 is the probability of M_1 getting repaired during an operation time when it is down.
- p_2 is the probability of M_2 failing while doing an operation.
- r_2 is the probability of M_2 getting repaired during an operation time when it is down.
- N is the number of parts the buffer can hold.

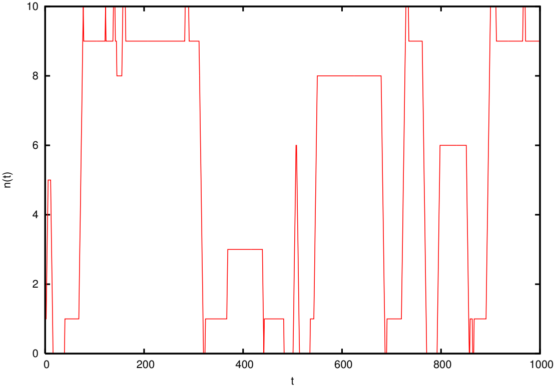
Two-Machine, Finite-Buffer Lines
Simulations

Graphs of $n(t)$, the number of parts in the buffer of a two-machine line, are shown in the next four slides for four examples. The parameters of each example are chosen so that the lines have different behaviors.

- Example 1: $r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 10$
  Both machines are the same. The buffer is small.
- Example 2: $r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 100$
  Same as Example 1 except the buffer is large.
- Example 3: $r_i = .1, i = 1, 2, p_1 = .02, p_2 = .01, N = 100$
  Same as Example 2 except the first machine fails more often.
- Example 4: $r_i = .1, i = 1, 2, p_1 = .01, p_2 = .02, N = 100$
  Same as Example 2 except the second machine fails more often.

Two-Machine, Finite-Buffer Lines

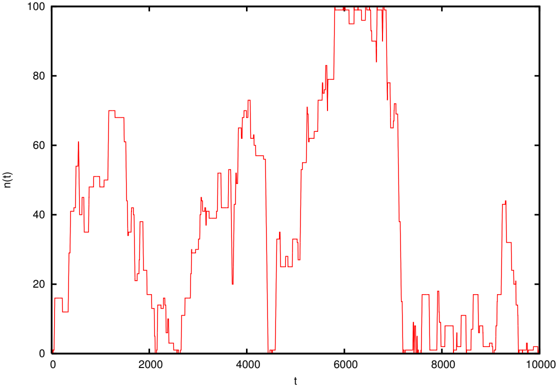
Simulations



Example 1: $r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 10$

Two-Machine, Finite-Buffer Lines

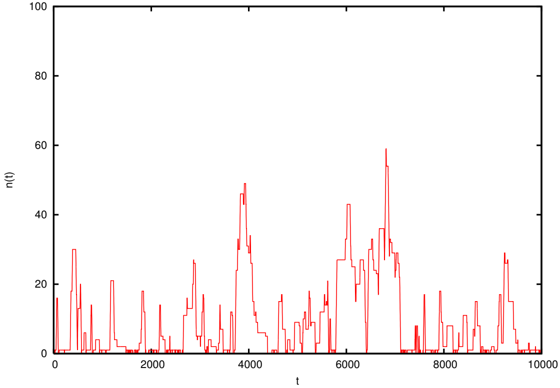
Simulations



Example 2: $r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 100$

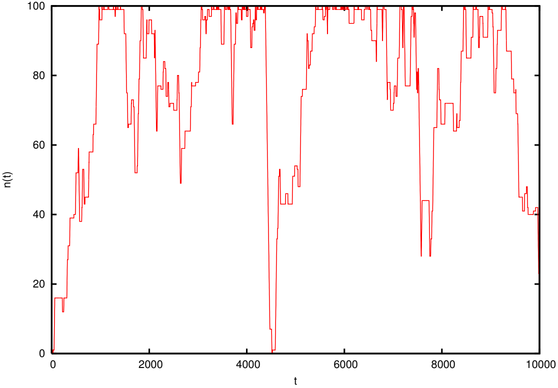
Two-Machine, Finite-Buffer Lines

Simulations



Example 3: $r_i = .1, i = 1, 2, p_1 = .02, p_2 = .01, N = 100$

Two-Machine, Finite-Buffer Lines
Simulations



Example 4: $r_i = .1, i = 1, 2, p_1 = .01, p_2 = .02, N = 100$

Two-Machine, Finite-Buffer Lines

Simulations

If we increase the buffer size of Example 4 from 100 to 200, what will happen to the production rate?

a. It will double it.

b. It will increase it substantially, but less than double.

c. It will increase it but only by a trivial amount.

d. It will have exactly zero effect.

e. None of the above.

Two-Machine, Finite-Buffer Lines
Simulations

If we increase the buffer size of Example 4 from 100 to 200, what will happen to the average inventory?

a. It will increase by something very close to 100.

b. It will double it.

c. It will increase it substantially, but less than double.

d. It will increase it but only by a trivial amount.

e. It will have exactly zero effect.

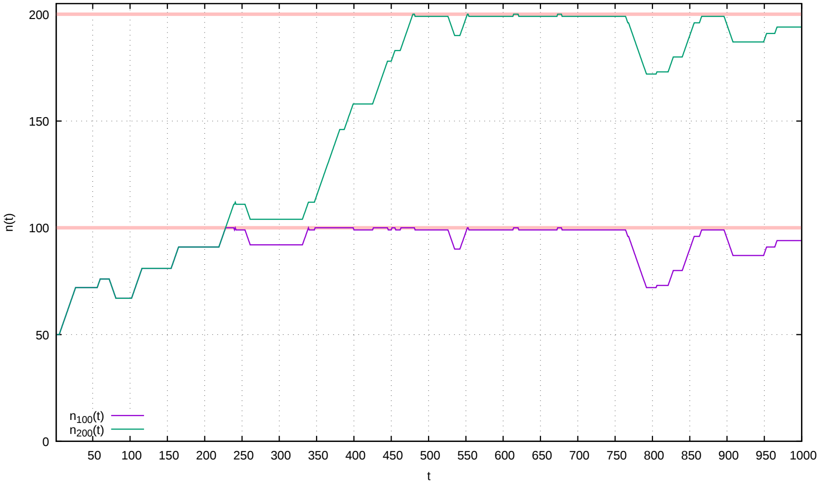
f. None of the above.

Two-Machine, Finite-Buffer Lines
Simulations

Insert animated simulation here!!

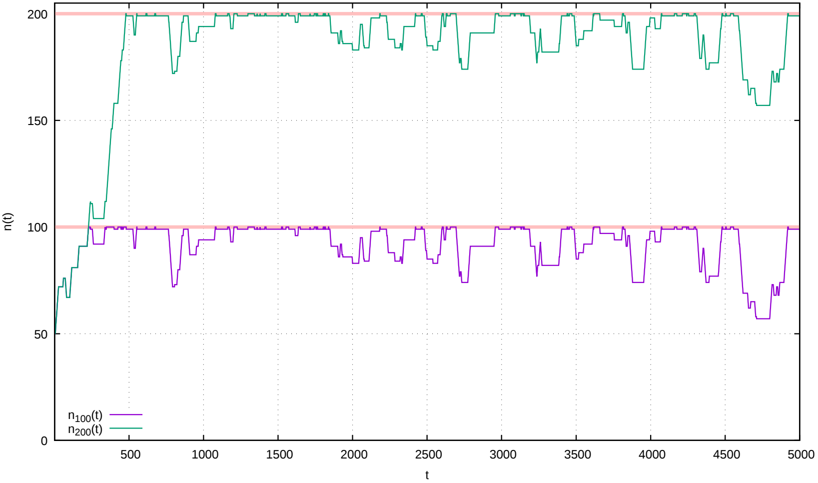
Two-Machine, Finite-Buffer Lines

Simulations

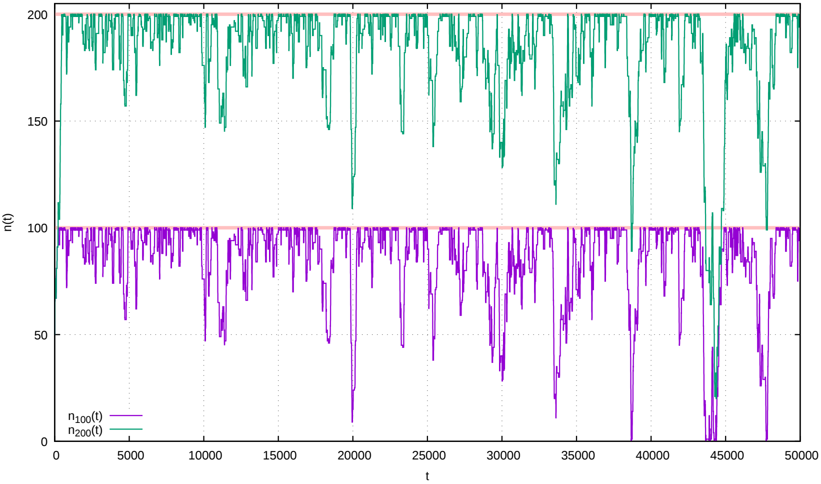


Two-Machine, Finite-Buffer Lines

Simulations



Two-Machine, Finite-Buffer Lines
Simulations



Two-Machine, Finite-Buffer Lines

Deterministic processing time model

- There are several models in the literature.
- We focus on the *Deterministic processing time,* or *Buzacott model:*
  - ⋆ deterministic processing time;
  - ⋆ geometric failure and repair times;
  - ⋆ operation-dependent failures;
  - ⋆ discrete state, discrete time.

Two-Machine, Finite-Buffer Lines
Markov Process Model

- *Discrete time-discrete state Markov process:*

$$\text{prob}\{X(t+1) = x(t+1) | X(t) = x(t), X(t-1) = x(t-1), X(t-2) = x(t-2), \ldots\} = \text{prob}\{X(t+1) = x(t+1) | X(t) = x(t)\}$$

- If we write $i = x(t+1), j = x(t)$, then

$$\text{prob}\{X(t+1) = i | X(t) = j\} = P_{ij}$$

is the transition matrix of the Markov process.

Two-Machine, Finite-Buffer Lines

State Space

- An analytical solution is available for a Markov process model of a two-machine line.

- Here, $X(t) = (n(t), \alpha_1(t), \alpha_2(t))$, where

  - $\star$ n is the number of parts in the buffer; $n = 0, 1, ..., N$.
  - $\star$ α_i is the repair state of M_i; $i = 1, 2$.
    - $\alpha_i = 1$ means the machine is *up* or *operational*;
    - $\alpha_i = 0$ means the machine is *down* or *under repair*.

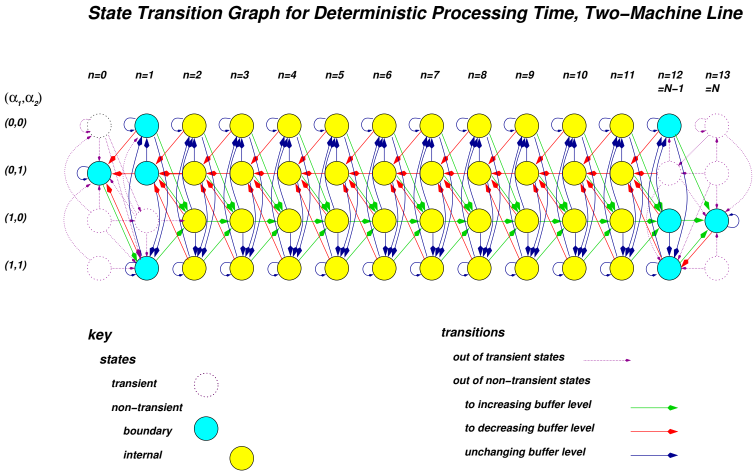
Two-Machine, Finite-Buffer Lines
State Space

Examples of transition probabilities P_{ij}:

- If $j = x(t) = (3, 0, 1)$ and $i = x(t+1) = (7, 1, 1)$ then
  $P_{ij} = \text{prob}\{X(t+1) = x(t+1)|X(t) = x(t)\} = 0$
  - ⋆ because the buffer level cannot change by more than 1 part in 1 time unit.
- If $j = x(t) = (3, 0, 1)$ and $i = x(t+1) = (2, 0, 1)$ then
  $P_{ij} = \text{prob}\{X(t+1) = x(t+1)|X(t) = x(t)\} = (1 - r_1)(1 - p_2)$
  - ⋆ because the probability that the first machine does not get repaired is $1 - r_1$, the probability that the second machine does not fail is $1 - p_2$, and the buffer level goes down by 1.
- If $j = x(t) = (3, 0, 1)$ and $i = x(t+1) = ($**3**$, 0, 1)$ then
  $P_{ij} = \text{prob}\{X(t+1) = x(t+1)|X(t) = x(t)\} = 0$
  - ⋆ because the buffer level *must* go down by 1.

Two-Machine, Finite-Buffer Lines

State Space



Two-Machine, Finite-Buffer Lines

Calculation of performance measures

- $\pi(n, \alpha_1, \alpha_2)$ is the steady-state probability that there are n parts in the buffer and the machine states are α_1 and α_2.

- To obtain $\pi(n, \alpha_1, \alpha_2)$ for all (n, α_1, α_2), let $i = (n, \alpha_1, \alpha_2)$ and $j = (n', \alpha'_1, \alpha'_2)$.

- Then

$$\pi_i = \sum_j P_{ij}\pi_j \quad \text{or} \quad \pi = P\pi$$

and

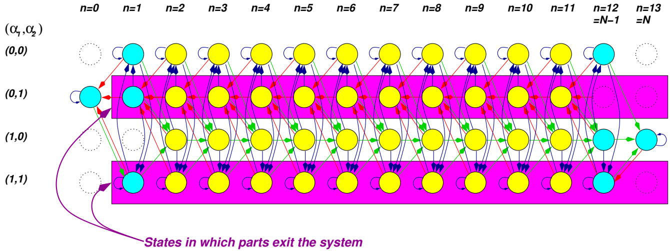
$$\sum_i \pi_i = 1 \quad \text{or} \quad \nu^T\pi = 1$$

- This is exactly what we discussed in the Markov Process videos, although the specific notation for this system is a little more complicated here.

Two-Machine, Finite-Buffer Lines

Calculation of performance measures

- To calculate the production rate in parts per operation time, add up the probabilities of the indicated states:



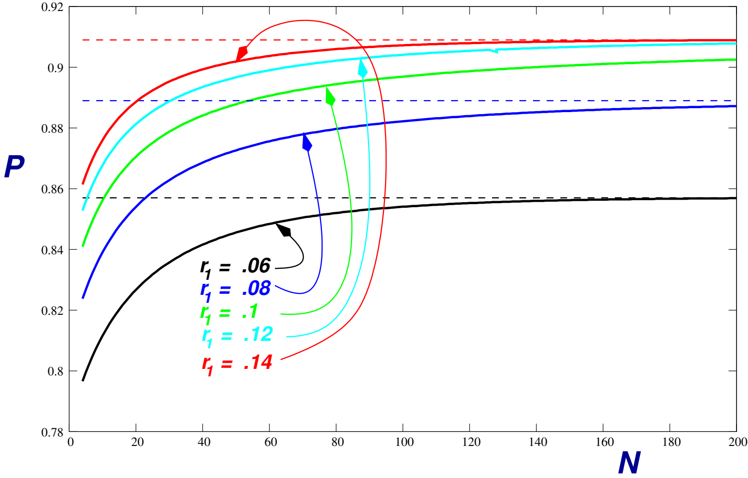
- To calculate the average inventory, evaluate

$$\bar{n} = \sum_{n=0}^{N} \sum_{\alpha_1=0}^{1} \sum_{\alpha_2=0}^{1} n\pi(n, \alpha_1, \alpha_2)$$

Two-Machine, Finite-Buffer Lines

Production rate vs. Buffer Size

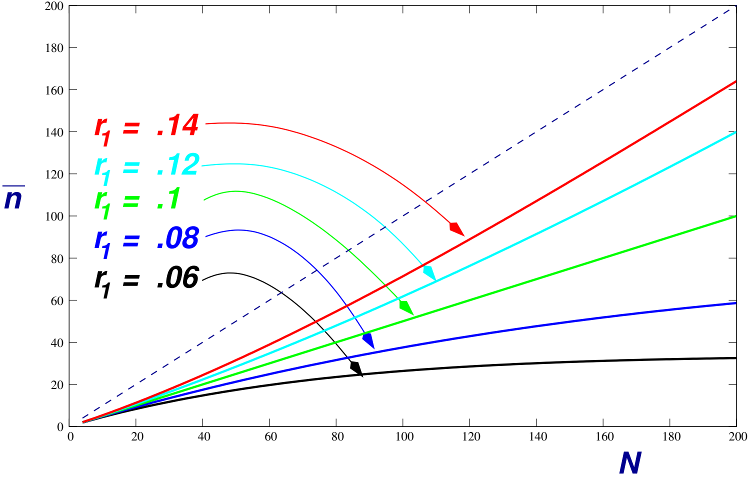
$p_1 = .01$
$r_2 = .1$
$p_2 = .01$



Two-Machine, Finite-Buffer Lines

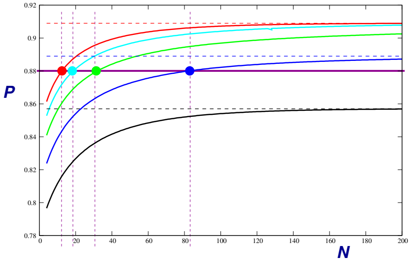
Average Inventory vs. Buffer Size

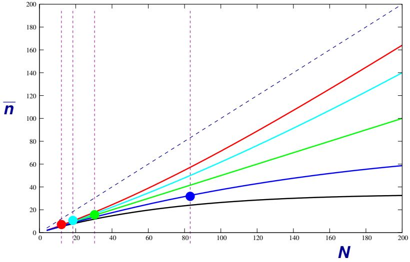
$p_1 = .01$
$r_2 = .1$
$p_2 = .01$



Two-Machine, Finite-Buffer Lines

Line Design





Problem: Select M_1 and N so that $P = .88$.

Solutions:

r_1	MTTR	N	$\bar{n}$
.14	7.14	13	7.08
.12	8.33	19	10.12
.10	10.00	32	16.00
.08	12.50	82	32.21

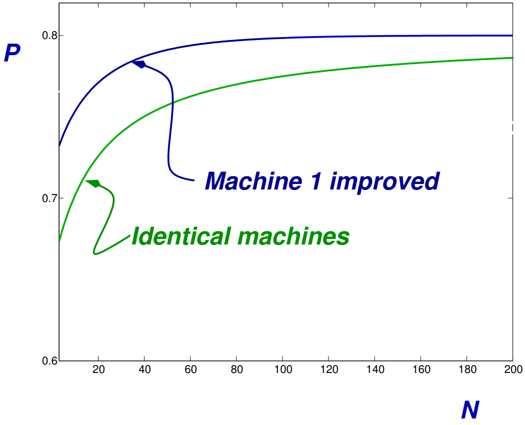
Two-Machine, Finite-Buffer Lines
Improvements

Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

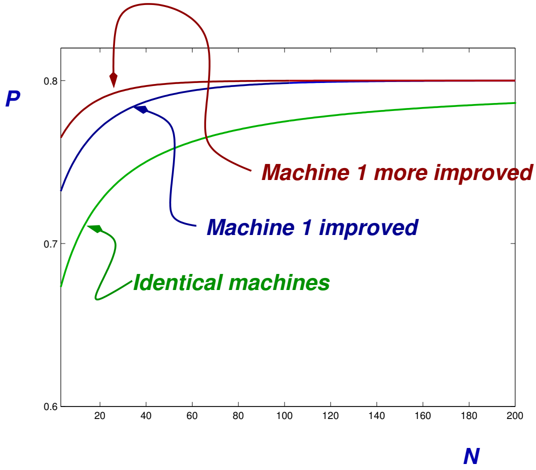
Two-Machine, Finite-Buffer Lines

Improvements to *non-bottleneck* machine.



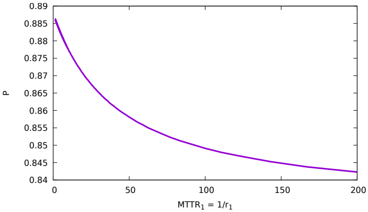
Two-Machine, Finite-Buffer Lines

Improvements to *non-bottleneck* machine.



Two-Machine, Finite-Buffer Lines
Failure frequency

Is it better to have short, frequent, disruptions or long, infrequent, disruptions?



- $r_2 = 0.1$, $p_2 = 0.009$, $N = 30$
- r_1 and p_1 vary together such that
  $$e_1 = \frac{r_1}{r_1+p_1} = \frac{\text{MTTF}}{\text{MTTF+MTTR}} = .9$$
- *Answer:* evidently, short, frequent failures.
- *Why?*

Introduction to Manufacturing Systems
Part 1

- I hope that you enjoyed this course and and that will find it useful.
- Part 2 continues the exploration of the issues that we have studied in Part 1: causes and consequences of variability in factories, and what to do about them.
- I hope you will come back for Part 2 and for the rest of the Principles of Manufacturing program.
- Please feel free to offer comments, criticisms, and suggestions for improvements.