[Week 2 Problem 1]

(1)

$$E[W] = \sum_i W_i P(W_i) = 1.5 \times 0.2 + 3 \times 0.25 + 6 \times 0.4 + 7 \times 0.15 = 4.5$$

$$E[W^2] = \sum_i W^2{}_i P(W_i) = 24.45$$

$$Var[W] = E[W^2] - E[W]^2 = 4.2$$

$$StDev[W] = \sqrt{Var(W)} = 2.0494$$

(2)

$$F(t) = P(W \le t) = \sum_{W_i < t} P(W_i)$$

Given the random variable W, we have

$$F(t) = \begin{cases} 0 & t < 1.5 \\ 0.2 & 1.5 \le t < 3 \\ 0.45 & 3 \le t < 6 \\ 0.85 & 6 \le t < 7 \\ 1 & t \ge 7 \end{cases}$$

The plot function of F(t) is shown in Figure 6:

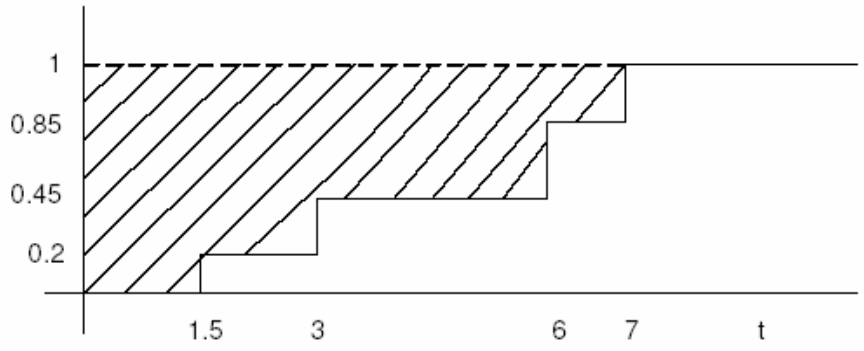

Figure 6 : F(t) vs t

The area of shaded region is

$$S = 1.5 \times 0.2 + 3 \times (0.45 - 0.2) + 6 \times (0.85 - 0.45) + 7 \times (1 - 0.85) = 4.5 = E(W)$$