Gravitational Solitons from Topology

Tasos Bouzikas

Supervised by: Bert Vercnocke (UvA)

Utrecht University

December 1, 2015

Universiteit Utrecht

Introduction - Black Holes

- Black Holes are solution to the Einstein Field Equations

Introduction - Black Holes

- Black Holes are solution to the Einstein Field Equations
- Black Holes in General Relativity ($d = 4$)
  - Schwarzschild Black Hole (M)
  - Reissner - Nordstrom Black Hole (Q)
  - Kerr Black Hole (J)

Introduction - Black Holes

- Black Holes are solution to the Einstein Field Equations
- Black Holes in General Relativity ($d = 4$)
  - Schwarzschild Black Hole (M)
  - Reissner - Nordstrom Black Hole (Q)
  - Kerr Black Hole (J)
- No-Hair Theorem: "All black hole solutions can be completely characterised by only three parameters: M, Q, J."

Introduction - Black Holes

- Black Holes are solution to the Einstein Field Equations

- Black Holes in General Relativity ($d = 4$)

  - Schwarzschild Black Hole (M)
  - Reissner - Nordstrom Black Hole (Q)
  - Kerr Black Hole (J)

- No-Hair Theorem: "All black hole solutions can be completely characterised by only three parameters: M, Q, J."

- Add extra dimensions $\rightarrow$ Superstring Theory ($d = 10$) (LATER!!!)

Introduction - Conserved Charges

- Way to define these 3 quantities: M, Q, J (conserved charges)

Introduction - Conserved Charges

- Way to define these 3 quantities: M, Q, J (conserved charges)

- Motivated by the easiest one: $Q_{\text{Electric Charge}}$

Introduction - Conserved Charges

- Way to define these 3 quantities: M, Q, J (conserved charges)
- Motivated by the easiest one: $Q_{\text{Electric Charge}}$
- Electrodynamics
  - Bianchi Identity: $dF = 0$

Introduction - Conserved Charges

- Way to define these 3 quantities: M, Q, J (conserved charges)
- Motivated by the easiest one: $Q_{\text{Electric Charge}}$
- Electrodynamics
  - Bianchi Identity: $dF = 0$
  - Equations of Motion: $d * F = *j \quad \rightarrow \quad d * j = 0$ (Conserved Current)

Introduction - Conserved Charges

- Way to define these 3 quantities: M, Q, J (conserved charges)
- Motivated by the easiest one: $Q_{\text{Electric Charge}}$
- Electrodynamics
  - Bianchi Identity: $dF = 0$
  - Equations of Motion: $d * F = *j \quad \rightarrow \quad d * j = 0$ (Conserved Current)
  - Conserved Charge:

$$Q = \int_{\Sigma} *j = \int_{\Sigma} d * F = \int_{\partial\Sigma} *F = Q_{\text{Electric Charge}}$$

Introduction - Conserved Charges

- General Relativity
  - We would like to have a similar expression for M and J

Introduction - Conserved Charges

- General Relativity
  - We would like to have a similar expression for M and J
  - (We are not going to deal with J in this project!)

Introduction - Conserved Charges

- General Relativity
  - We would like to have a similar expression for M and J
  - (We are not going to deal with J in this project!)
  - Define $j^{\mu} = R^{\mu\nu} K_{\nu} \rightarrow \quad \nabla_{\mu} j^{\mu} = 0$ (Conserved Current)

Introduction - Conserved Charges

- General Relativity
  - We would like to have a similar expression for M and J
  - (We are not going to deal with J in this project!)
  - Define $j^\mu = R^{\mu\nu} K_\nu \rightarrow \quad \nabla_\mu j^\mu = 0$ (Conserved Current)
  - Conserved Charge:

$$Q = \int_\Sigma *j = \int_\Sigma d * dK = \int_{\partial\Sigma} *dK$$

Introduction - Conserved Charges

- General Relativity
  - We would like to have a similar expression for M and J
  - (We are not going to deal with J in this project!)
  - Define $j^\mu = R^{\mu\nu} K_\nu \rightarrow \quad \nabla_\mu j^\mu = 0$ (Conserved Current)
  - Conserved Charge:

$$Q = \int_{\Sigma} *j = \int_{\Sigma} d * dK = \int_{\partial\Sigma} *dK$$

- Komar Integral

$$\boxed{Q_K = \kappa_d \int_{\partial\Sigma} *dK}$$

Introduction - Conserved Charges

- Simplest Example: The Schwarzschild Black Hole

Introduction - Conserved Charges

- Simplest Example: The Schwarzschild Black Hole
  - Schwarzschild Metric
  - Killing vector $K^\mu = (1, 0, 0, 0)$

Introduction - Conserved Charges

- Simplest Example: The Schwarzschild Black Hole
  - Schwarzschild Metric
  - Killing vector $K^\mu = (1, 0, 0, 0)$
  - By calculating $*dK$ and by choosing $\kappa_d = \frac{1}{8\pi G_4}$, we end up having:

Introduction - Conserved Charges

- Simplest Example: The Schwarzschild Black Hole
  - Schwarzschild Metric
  - Killing vector $K^\mu = (1, 0, 0, 0)$
  - By calculating $*dK$ and by choosing $\kappa_d = \frac{1}{8\pi G_4}$, we end up having:
- Komar Integral

$$\boxed{M = \frac{1}{8\pi G_4} \int_{\partial\Sigma} *dK}$$

Introduction - Conserved Charges

- Simplest Example: The Schwarzschild Black Hole
  - Schwarzschild Metric
  - Killing vector $K^\mu = (1, 0, 0, 0)$
  - By calculating $*dK$ and by choosing $\kappa_d = \frac{1}{8\pi G_4}$, we end up having:
- Komar Integral

$$\boxed{M = \frac{1}{8\pi G_4} \int_{\partial\Sigma} *dK}$$

- Remember: $j_\mu = R_{\mu\nu} K^\nu \xrightarrow[\text{Equation}]{\text{Einstein}} T_{\mu\nu} K^\nu \xrightarrow[\text{Component}]{\text{Temporal}} T_{00} \rightarrow M$

Overview

- Include Cosmological Constant Λ (AdS Spacetime)

Overview

- Include Cosmological Constant Λ (AdS Spacetime)
- Include Topology

Overview

- Include Cosmological Constant Λ (AdS Spacetime)
- Include Topology
- Include Matter Fields (Non - Empty Spacetime)

Overview

- Include Cosmological Constant Λ (AdS Spacetime)
- Include Topology
- Include Matter Fields (Non - Empty Spacetime)
- Interpret the result. Why 4 dimensions are not enough!

Overview

- Include Cosmological Constant Λ (AdS Spacetime)
- Include Topology
- Include Matter Fields (Non - Empty Spacetime)
- Interpret the result. Why 4 dimensions are not enough!
- Moving to String Theory

Overview

- $\boxed{\text{Include Cosmological Constant } \Lambda \text{ (AdS Spacetime)}}$
- Include Topology
- Include Matter Fields (Non - Empty Spacetime)
- Interpret the result. Why 4 dimensions are not enough!
- Moving to String Theory

Cosmological Constant Λ

- Hilbert - Einstein Action

$$S_{HE} = \frac{1}{2\kappa} \int d^d x \sqrt{-g} \Big(R - 2\Lambda \Big)$$

Cosmological Constant Λ

- Hilbert - Einstein Action

$$S_{HE} = \frac{1}{2\kappa} \int d^d x \sqrt{-g} \Big(R - 2\Lambda \Big)$$

- Einstein Equations: $R_{\mu\nu} = \frac{2\Lambda}{d-2} g_{\mu\nu}$

Cosmological Constant Λ

- Hilbert - Einstein Action

$$S_{HE} = \frac{1}{2\kappa}\int d^d x\sqrt{-g}\Big(R - 2\Lambda\Big)$$

- Einstein Equations: $R_{\mu\nu} = \frac{2\Lambda}{d-2} g_{\mu\nu}$

- Solution: AdS Spacetime

$$ds^2 = -\frac{r^2}{R^2}dt^2 + \frac{R^2}{r^2}dr^2 + r^2 d\Omega_2^2, \qquad \frac{1}{R^2} = \frac{2\Lambda}{(d-1)(d-2)}$$

Cosmological Constant Λ

- Hilbert - Einstein Action

$$S_{HE} = \frac{1}{2\kappa}\int d^d x\sqrt{-g}\Big(R - 2\Lambda\Big)$$

- Einstein Equations: $R_{\mu\nu} = \frac{2\Lambda}{d-2} g_{\mu\nu}$
- Solution: AdS Spacetime

$$ds^2 = -\frac{r^2}{R^2}dt^2 + \frac{R^2}{r^2}dr^2 + r^2 d\Omega_2^2, \qquad \frac{1}{R^2} = \frac{2\Lambda}{(d-1)(d-2)}$$

- The Komar integral diverges:

$$Q_K = \int_{\partial\Sigma} *dK = \int_{\partial\Sigma} d\Sigma_{\mu\nu}\nabla^{\mu}K^{\nu} = \int_{\partial\Sigma} d\Sigma_{tr}\frac{r}{R^2} \to \infty$$

Cosmological Constant Λ

- Way to deal with the divergence: Killing Potential $\omega^{\mu\nu}$

Cosmological Constant Λ

- Way to deal with the divergence: Killing Potential $\omega^{\mu\nu}$
  - Killing Equation: $\nabla_{(\mu}K_{\nu)} = 0 \rightarrow \nabla_{\mu}K^{\mu} = 0$

Cosmological Constant Λ

- Way to deal with the divergence: Killing Potential $\omega^{\mu\nu}$
  - Killing Equation: $\nabla_{(\mu}K_{\nu)} = 0 \rightarrow \nabla_{\mu}K^{\mu} = 0$
  - We define the **fully antisymmetric** Killing Potential $K^{\nu} = \nabla_{\mu}\omega^{\mu\nu}$

Cosmological Constant Λ

- Way to deal with the divergence: Killing Potential $\omega^{\mu\nu}$
  - Killing Equation: $\nabla_{(\mu}K_{\nu)} = 0 \rightarrow \nabla_{\mu}K^{\mu} = 0$
  - We define the **fully antisymmetric** Killing Potential $K^{\nu} = \nabla_{\mu}\omega^{\mu\nu}$
  - That way: $\nabla_{\mu}K^{\mu} = \nabla_{\mu}\nabla_{\nu}\omega^{\nu\mu} = 0$

Cosmological Constant Λ

- Way to deal with the divergence: Killing Potential $\omega^{\mu\nu}$
  - Killing Equation: $\nabla_{(\mu}K_{\nu)} = 0 \rightarrow \nabla_\mu K^\mu = 0$
  - We define the **fully antisymmetric** Killing Potential $K^\nu = \nabla_\mu \omega^{\mu\nu}$
  - That way: $\nabla_\mu K^\mu = \nabla_\mu \nabla_\nu \omega^{\nu\mu} = 0$
  - Gauge Freedom: $\tilde{\omega}_{\mu\nu} = \omega_{\mu\nu} + \nabla_\rho \lambda^{\rho\mu\nu}$

Cosmological Constant Λ

- Way to deal with the divergence: Killing Potential $\omega^{\mu\nu}$
  - Killing Equation: $\nabla_{(\mu} K_{\nu)} = 0 \rightarrow \nabla_\mu K^\mu = 0$
  - We define the **fully antisymmetric** Killing Potential $K^\nu = \nabla_\mu \omega^{\mu\nu}$
  - That way: $\nabla_\mu K^\mu = \nabla_\mu \nabla_\nu \omega^{\nu\mu} = 0$
  - Gauge Freedom: $\tilde{\omega}_{\mu\nu} = \omega_{\mu\nu} + \nabla_\rho \lambda^{\rho\mu\nu}$
  - Killing Potential works as a counterterm which we add to cancel the divergence.

$$Q_K = \int_{\partial\Sigma} d\Sigma_{\mu\nu} \left(\nabla^\mu K^\nu + \frac{2\Lambda}{(d-2)} \omega^{\mu\nu} \right)$$

Cosmological Constant Λ

- We solve $K^{\nu} = \nabla_{\mu}\omega^{\mu\nu}$ for AdS space

Cosmological Constant Λ

- We solve $K^\nu = \nabla_\mu \omega^{\mu\nu}$ for AdS space
- General Result

$$\omega^{rt}_{\mathrm{AdS}} = \frac{r}{d-1}$$

Cosmological Constant Λ

- We solve $K^\nu = \nabla_\mu \omega^{\mu\nu}$ for AdS space
- General Result

$$\omega^{rt}_{\text{AdS}} = \frac{r}{d-1}$$

- Substituting back:

$$Q_K = \int_{\partial\Sigma} d\Sigma_{tr} \left(\frac{r}{R^2} - \frac{2\Lambda}{d-2} \frac{r}{d-1} \right) = 0$$

Cosmological Constant Λ

- We solve $K^\nu = \nabla_\mu \omega^{\mu\nu}$ for AdS space
- General Result

$$\omega^{rt}_{\text{AdS}} = \frac{r}{d-1}$$

- Substituting back:

$$Q_K = \int_{\partial\Sigma} d\Sigma_{tr} \left(\frac{r}{R^2} - \frac{2\Lambda}{d-2} \frac{r}{d-1} \right) = 0$$

- We manage to cancel the divergence coming from Λ

Cosmological Constant Λ

- We solve $K^\nu = \nabla_\mu \omega^{\mu\nu}$ for AdS space
- General Result

$$\omega^{rt}_{\text{AdS}} = \frac{r}{d-1}$$

- Substituting back:

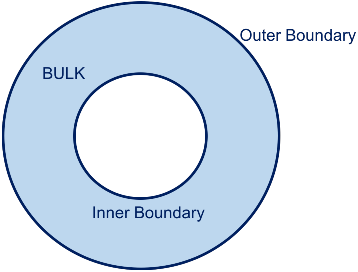
$$Q_K = \int_{\partial\Sigma} d\Sigma_{tr} \left(\frac{r}{R^2} - \frac{2\Lambda}{d-2} \frac{r}{d-1} \right) = 0$$

- We manage to cancel the divergence coming from Λ
- We can still interpret Komar Integral as mass from the left over $T_{\mu\nu}$

Overview

- Include Cosmological Constant Λ (AdS Spacetime) ✓
- $\boxed{\text{Include Topology}}$
- Include Matter Fields (Non - Empty Spacetime)
- Interpret the result. Why 4 dimensions are not enough!
- Moving to String Theory

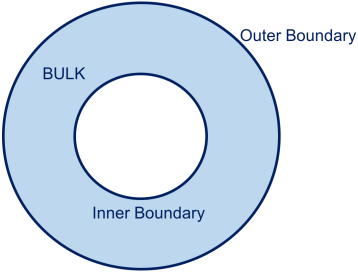
- Spaces with two boundaries



- Outer Boundary $\partial\Sigma^{\text{out}}$
- Bulk Σ
- Inner Boundary $\partial\Sigma^{\text{int}}$

Topology

- Spaces with two boundaries



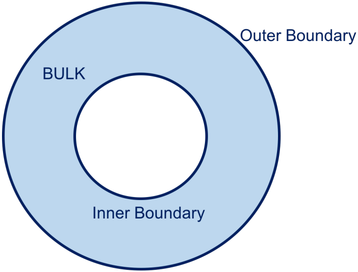
- Outer Boundary $\partial\Sigma^{\text{out}}$
- Bulk Σ
- Inner Boundary $\partial\Sigma^{\text{int}}$

- Stokes's Theorem:

$$\int_{\Sigma} dA_p = \int_{\partial\Sigma^{\text{out}}} A_p - \int_{\partial\Sigma^{\text{int}}} A_p$$

Topology

- Spaces with two boundaries



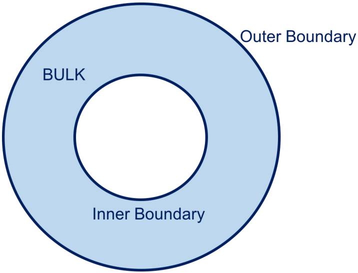
  - Outer Boundary $\partial\Sigma^{\text{out}}$
  - Bulk Σ
  - Inner Boundary $\partial\Sigma^{\text{int}}$

- Stokes's Theorem:

$$\int_{\Sigma} d * dK = \int_{\partial\Sigma^{\text{out}}} *dK - \int_{\partial\Sigma^{\text{int}}} *dK$$

Topology

- Spaces with two boundaries



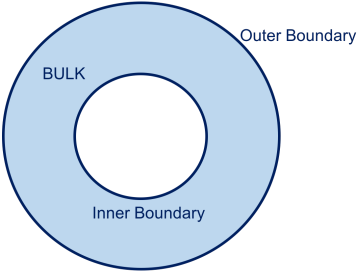
- Outer Boundary $\partial\Sigma^{\text{out}}$
- Bulk Σ
- Inner Boundary $\partial\Sigma^{\text{int}}$

- Stokes's Theorem:

$$\int_{\Sigma} d * dK = \underbrace{\int_{\partial\Sigma^{\text{out}}} *dK}_{Q_K} - \int_{\partial\Sigma^{\text{int}}} *dK$$

Topology

- Spaces with two boundaries

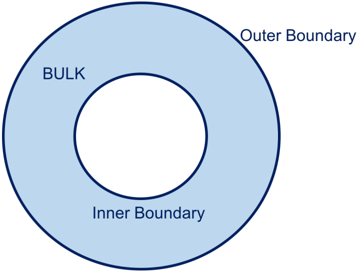


- Outer Boundary $\partial\Sigma^{\mathrm{out}}$
- Bulk Σ
- Inner Boundary $\partial\Sigma^{\mathrm{int}}$

$$Q_K = \int_{\Sigma} d * dK + \int_{\partial\Sigma^{\mathrm{int}}} * dK$$

Topology

- Spaces with two boundaries



- Outer Boundary $\partial\Sigma^{\text{out}}$
- Bulk Σ
- $\underbrace{\text{Inner Boundary } \partial\Sigma^{\text{int}}}_{\text{Event Horizon } \partial\Sigma^{\text{h}}}$

$$\boxed{Q_K = \int_{\Sigma} d * dK + \int_{\partial\Sigma^{\text{h}}} * dK}$$

Smarr Formula

Topology

- Smarr Formula

$$Q_K = \int_{\Sigma} d * dK + \int_{\partial\Sigma^{\mathrm{h}}} *dK$$

Topology

- Smarr Formula

$$Q_K = \cancel{\int_{\Sigma} d * dK} + \int_{\partial\Sigma^h} * dK$$

Topology

- Smarr Formula

$$Q_K = \cancel{\int_{\Sigma} d * dK} + \int_{\partial\Sigma^{\mathrm{h}}} *dK$$

- $Q_K \to$ Mass (or Energy dE)

Topology

- Smarr Formula

$$Q_K = \cancel{\int_{\Sigma} d * dK} + \int_{\partial\Sigma^h} *dK$$

- $Q_K \to$ Mass (or Energy dE)
- $\int_{\partial\Sigma^h} *dK \to$ Area dA (Most general case: dA, dQ, dJ)

Topology

- Smarr Formula

$$Q_K = \cancel{\int_\Sigma d * dK} + \int_{\partial\Sigma^{\mathrm{h}}} * dK$$

- $Q_K \to$ Mass (or Energy dE)
- $\int_{\partial\Sigma^{\mathrm{h}}} * dK \to$ Area dA (Most general case: dA, dQ, dJ)
- First Law of Black Hole Thermodynamics

$$dE \sim dA + dQ + dJ$$

Topology

- Smarr Formula

$$Q_K = \int_{\Sigma} d * dK + \cancel{\int_{\partial\Sigma^{\mathrm{h}}} * dK}$$

- $Q_K \to$ Mass (or Energy dE)

- $\int_{\partial\Sigma^{\mathrm{h}}} * dK \to$ Area dA (Most general case: dA, dQ, dJ)

- First Law of Black Hole Thermodynamics

$$dE \sim dA + dQ + dJ$$

Topology

- Smarr Formula

$$\underbrace{Q_K}_{\text{No Divergence}} = \int_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int_{\partial\Sigma^{\mathrm{h}}} \nabla^\mu K^\nu d\Sigma_{\mu\nu}$$

Topology

- Smarr Formula

$$\underbrace{Q_K}_{\text{No Divergence}} = \underbrace{\int_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int_{\partial\Sigma^{\mathrm{h}}} \nabla^\mu K^\nu d\Sigma_{\mu\nu}}_{\text{We expect no divergence}}$$

- Smarr Formula

$$\underbrace{Q_K}_{\text{No Divergence}} = \underbrace{\int_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int_{\partial\Sigma^{\mathrm{h}}} \nabla^\mu K^\nu d\Sigma_{\mu\nu}}_{\text{We expect no divergence}}$$

- Adding Killing Potential

$$Q_K = \int_{\Sigma} \left(R^{\mu\nu} K_\nu + \Lambda \nabla_\nu \omega^{\mu\nu} \right) d\Sigma_\mu + \int_{\partial\Sigma^{\mathrm{h}}} \left(\nabla^\mu K^\nu + \Lambda \omega^{\mu\nu} \right) d\Sigma_{\mu\nu}$$

Topology

- Smarr Formula

$$\underbrace{Q_K}_{\text{No Divergence}} = \underbrace{\int\limits_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int\limits_{\partial\Sigma^{\mathrm{h}}} \nabla^\mu K^\nu d\Sigma_{\mu\nu}}_{\text{We expect no divergence}}$$

- Adding Killing Potential

$$Q_K = \int\limits_{\Sigma} \left(R^{\mu\nu} K_\nu + \Lambda \underbrace{\nabla_\nu \omega^{\mu\nu}}_{-K^\mu} \right) d\Sigma_\mu + \int\limits_{\partial\Sigma^{\mathrm{h}}} \left(\nabla^\mu K^\nu + \Lambda \omega^{\mu\nu} \right) d\Sigma_{\mu\nu}$$

Topology

- Smarr Formula

$$\underbrace{Q_K}_{\text{No Divergence}} = \underbrace{\int_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int_{\partial\Sigma^{\mathrm{h}}} \nabla^\mu K^\nu d\Sigma_{\mu\nu}}_{\text{We expect no divergence}}$$

- Adding Killing Potential

$$Q_K = \int_{\Sigma} \Big(R^{\mu\nu} K_\nu + \Lambda \underbrace{\nabla_\nu \omega^{\mu\nu}}_{-K^\mu} \Big) d\Sigma_\mu + \int_{\partial\Sigma^{\mathrm{h}}} \Big(\nabla^\mu K^\nu + \Lambda \omega^{\mu\nu} \Big) d\Sigma_{\mu\nu}$$

- Remember: $R_{\mu\nu} = \Lambda g_{\mu\nu} \rightarrow R^{\mu\nu} K_\nu = \Lambda K^\mu$

Topology

- Smarr Formula

$$\underbrace{Q_K}_{\text{No Divergence}} = \underbrace{\int\limits_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int\limits_{\partial\Sigma^{\mathrm{h}}} \nabla^\mu K^\nu d\Sigma_{\mu\nu}}_{\text{We expect no divergence}}$$

- Adding Killing Potential

$$Q_K = \int\limits_{\Sigma} \left(R^{\mu\nu} K_\nu + \Lambda \underbrace{\nabla_\nu \omega^{\mu\nu}}_{-K^\mu} \right) d\Sigma_\mu + \int\limits_{\partial\Sigma^{\mathrm{h}}} \left(\nabla^\mu K^\nu + \Lambda \omega^{\mu\nu} \right) d\Sigma_{\mu\nu}$$

- Remember: $R_{\mu\nu} = \Lambda g_{\mu\nu} \rightarrow R^{\mu\nu} K_\nu = \Lambda K^\mu$
- Bulk $\rightarrow$ Free of divergences! ✓

Summing Up

- Putting everything together
  - Cosmological Constant $\rightarrow$ Killing Potential
  - Topology $\rightarrow$ Smarr Formula
  - Inner Boundary $=$ Event Horizon
  - No divergence at infinity and in the bulk!

$$\boxed{M = \int_{\Sigma} R^{\mu\nu} K_{\nu} d\Sigma_{\mu} + \int_{\partial\Sigma^{\mathrm{h}}} \left(\nabla^{\mu} K^{\nu} + \Lambda\omega^{\mu\nu}\right) d\Sigma_{\mu\nu}}$$

Overview

- Include Cosmological Constant Λ (AdS Spacetime) ✓
- Include Topology ✓
- $\boxed{\text{Include Matter Fields (Non - Empty Spacetime)}}$
- Interpret the result. Why 4 dimensions are not enough!
- Moving to String Theory

Matter Fields

- Non-Empty Spacetime $\rightarrow$ Contribution of the bulk to the mass!

Matter Fields

- Non-Empty Spacetime $\rightarrow$ Contribution of the bulk to the mass!
- We include Matter Fields
  - A set of scalar fields X^I.
  - A set of $U(1)$ gauge fields A^I.

Matter Fields

- Non-Empty Spacetime $\rightarrow$ Contribution of the bulk to the mass!
- We include Matter Fields
  - A set of scalar fields X^I.
  - A set of $U(1)$ gauge fields A^I.
- The Action

$$S = \int d^4x\sqrt{-g}\Big(R - \frac{1}{2}Q_{IJ}\partial_\mu X^I \partial^\mu X^J - \frac{1}{4}Q_{IJ}F^I_{\mu\nu}F^{J\mu\nu}\Big) - \int C_{IJ}F^I \wedge F^J$$

Matter Fields

- Non-Empty Spacetime → Contribution of the bulk to the mass!
- We include Matter Fields
  - A set of scalar fields X^I.
  - A set of $U(1)$ gauge fields A^I.
- The Action

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{2}Q_{IJ}\partial_\mu X^I\partial^\mu X^J - \frac{1}{4}Q_{IJ}F^I_{\mu\nu}F^{J\mu\nu}\right) - \underbrace{\int C_{IJ}F^I \wedge F^J}_{\text{Chern-Simons Term}}$$

- Chern-Simons Term: Purely Topological → Topological Current

Matter Fields

- Remember

$$M = \int_{\Sigma} R^{\mu\nu} K_{\nu} d\Sigma_{\mu} + \int_{\partial\Sigma^{\mathsf{h}}} \left(\nabla^{\mu} K^{\nu} + \Lambda\omega^{\mu\nu}\right) d\Sigma_{\mu\nu}$$

Matter Fields

- Remember

$$M = \int_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int_{\partial\Sigma^{\mathsf{h}}} \left(\nabla^\mu K^\nu + \Lambda\omega^{\mu\nu}\right) d\Sigma_{\mu\nu}$$

- We introduce dual field: $G_I = Q_{IJ} * F^J$

Matter Fields

- Remember

$$M = \int_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int_{\partial\Sigma^{\mathrm{h}}} \left(\nabla^\mu K^\nu + \Lambda\omega^{\mu\nu}\right) d\Sigma_{\mu\nu}$$

- We introduce dual field: $G_I = Q_{IJ} * F^J$

- Einstein Equations

$$R_{\mu\nu} = \frac{1}{2} Q_{IJ} \partial_\mu X^I \partial_\nu X^J + \frac{1}{4} Q_{IJ} F^I_{\mu\rho} F^{J\rho}_{\nu} + \frac{1}{4} Q^{IJ} G_{I\mu\rho} G^{\rho}_{J\nu}$$

Matter Fields

- Remember

$$M = \int_{\Sigma} R^{\mu\nu} K_\nu d\Sigma_\mu + \int_{\partial\Sigma^{\mathrm{h}}} \left(\nabla^\mu K^\nu + \Lambda\omega^{\mu\nu}\right) d\Sigma_{\mu\nu}$$

- We introduce dual field: $G_I = Q_{IJ} * F^J$
- Einstein Equations

$$R_{\mu\nu} = \frac{1}{2} Q_{IJ} \partial_\mu X^I \partial_\nu X^J + \frac{1}{4} Q_{IJ} F^I_{\mu\rho} F^{J\rho}_\nu + \frac{1}{4} Q^{IJ} G_{I\mu\rho} G^\rho_{J\nu}$$

- $R^{\mu\nu} K_\nu \rightarrow$ We need to calculate: $K^\mu \partial_\mu X$, $K^\mu F_{\mu\nu}$ and $K^\mu G_{\mu\nu}$

Matter Fields

- $\boxed{\textbf{Assumption: Matter Fields respect the isometries.}}$

Matter Fields

- $\boxed{\textbf{Assumption: Matter Fields respect the isometries.}}$
- If $\mathcal{L}_K g_{\mu\nu} = 0$, then:

$$\mathcal{L}_K X^I = 0 \qquad \mathcal{L}_K F^I = 0 \qquad \mathcal{L}_K G_I = 0$$

Matter Fields

- $\boxed{\textbf{Assumption: Matter Fields respect the isometries.}}$
- If $\mathcal{L}_K g_{\mu\nu} = 0$, then:

$$\mathcal{L}_K X^I = 0 \qquad \mathcal{L}_K F^I = 0 \qquad \mathcal{L}_K G_I = 0$$

- $\mathcal{L}_K X^I = 0 \rightarrow \boxed{K^\mu \partial_\mu X = 0}$ No contribution from scalar fields!

Matter Fields

- $\boxed{\textbf{Assumption: Matter Fields respect the isometries.}}$
- If $\mathcal{L}_K g_{\mu\nu} = 0$, then:

$$\mathcal{L}_K X^I = 0 \qquad \mathcal{L}_K F^I = 0 \qquad \mathcal{L}_K G_I = 0$$

- $\mathcal{L}_K X^I = 0 \to \boxed{K^\mu \partial_\mu X = 0}$ No contribution from scalar fields!
- $\mathcal{L}_K F^I = 0 \to d(i_K F^I) = 0 \implies \boxed{i_K F^I = \Lambda^I + d\lambda^I}$

Matter Fields

- $\boxed{\textbf{Assumption: Matter Fields respect the isometries.}}$
- If $\mathcal{L}_K g_{\mu\nu} = 0$, then:

$$\mathcal{L}_K X^I = 0 \qquad \mathcal{L}_K F^I = 0 \qquad \mathcal{L}_K G_I = 0$$

- $\mathcal{L}_K X^I = 0 \rightarrow \boxed{K^\mu \partial_\mu X = 0}$ No contribution from scalar fields!
- $\mathcal{L}_K F^I = 0 \rightarrow d(i_K F^I) = 0 \implies \boxed{i_K F^I = \Lambda^I + d\lambda^I}$
- $\mathcal{L}_K G_I = 0 \rightarrow \boxed{i_K G_I = -2C_{IJ}\Lambda^J - 2C_{IJ} d\lambda^J + H_I + dh_I}$

Matter Fields

- **Assumption: Matter Fields respect the isometries.**
- If $\mathcal{L}_K g_{\mu\nu} = 0$, then:

$$\mathcal{L}_K X^I = 0 \qquad \mathcal{L}_K F^I = 0 \qquad \mathcal{L}_K G_I = 0$$

- $\mathcal{L}_K X^I = 0 \rightarrow \boxed{K^\mu \partial_\mu X = 0}$ No contribution from scalar fields!
- $\mathcal{L}_K F^I = 0 \rightarrow d(i_K F^I) = 0 \implies \boxed{i_K F^I = \Lambda^I + d\lambda^I}$
- $\mathcal{L}_K G_I = 0 \rightarrow \boxed{i_K G_I = -2C_{IJ}\Lambda^J - 2C_{IJ} d\lambda^J + H_I + dh_I}$
- Important: Λ^I and H^I are closed but not exact!

Matter Fields

- Substituting to Smarr Formula

$$M = \int_{\Sigma} \Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H_I \wedge F^I + \int_{\partial\Sigma^{\mathrm{h}}} \text{"something"}$$

Matter Fields

- Substituting to Smarr Formula

$$M = \int_{\Sigma} \Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H_I \wedge F^I + \int_{\partial\Sigma^{\mathrm{h}}} \text{“something”}$$

- “something”: ∇K, $\Lambda\omega$, λ^I and h_I

Matter Fields

- Substituting to Smarr Formula

$$M = \int_{\Sigma} \Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H_I \wedge F^I + \int_{\partial\Sigma^{\mathrm{h}}} \text{“something”}$$

- “something”: ∇K, $\Lambda\omega$, λ^I and h_I

- Finally, we can remove the event horizon!

$$\boxed{M = \frac{1}{32\pi G_4} \int_{\Sigma} \Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H_I \wedge F^I}$$

Overview

- Include Cosmological Constant Λ (AdS Spacetime) ✓
- Include Topology ✓
- Include Matter Fields (Non - Empty Spacetime) ✓
- $\boxed{\text{Interpret the result. Why 4 dimensions are not enough!}}$
- Moving to String Theory

Interpreting the result

$$M = \frac{1}{32\pi G_4} \int\limits_{\Sigma} \left(\Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H^I \wedge F_I \right)$$

Interpreting the result

$$M = \frac{1}{32\pi G_4} \int\limits_{\Sigma} \left(\Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H^I \wedge F_I \right)$$

- $H^p(M) = \frac{\text{Group of Closed p-Forms}}{\text{Group of Exact p-Forms}}$

Interpreting the result

$$M = \frac{1}{32\pi G_4} \int_{\Sigma} \left(\Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H^I \wedge F_I \right)$$

- $H^p(M) = \frac{\text{Group of Closed p-Forms}}{\text{Group of Exact p-Forms}}$
- Cohomology $H^r(M)$ says which forms are allowed by the topology!

Interpreting the result

$$M = \frac{1}{32\pi G_4} \int_\Sigma \left(\Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H^I \wedge F_I \right)$$

- $H^p(M) = \frac{\text{Group of Closed p-Forms}}{\text{Group of Exact p-Forms}}$
- Cohomology $H^r(M)$ says which forms are allowed by the topology!
- e.g: Trivial Cohomology $H^1(M) = 0$

Interpreting the result

$$M = \frac{1}{32\pi G_4} \int_{\Sigma} \Big(\Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H^I \wedge F_I \Big)$$

- $H^p(M) = \frac{\text{Group of Closed p-Forms}}{\text{Group of Exact p-Forms}}$
- Cohomology $H^r(M)$ says which forms are allowed by the topology!
- e.g: Trivial Cohomology $H^1(M) = 0$
- Simply Connected Topological Space $\rightarrow H^1(M) = 0 \rightarrow M = 0$

Interpreting the result

$$M = \frac{1}{32\pi G_4} \int_{\Sigma} \left(\Lambda^I \wedge (G_I + 2C_{IJ} \wedge F^J) + H^I \wedge F_I \right)$$

- $H^p(M) = \frac{\text{Group of Closed p-Forms}}{\text{Group of Exact p-Forms}}$
- Cohomology $H^r(M)$ says which forms are allowed by the topology!
- e.g: Trivial Cohomology $H^1(M) = 0$
- Simply Connected Topological Space $\rightarrow H^1(M) = 0 \rightarrow M = 0$
- Adding extra dimensions...

Previous Papers

- 4-D: Just seen!

Previous Papers

- 4-D: Just seen!
- 5-D: (2014) Global Structure of Five-dimensional BPS Fuzzballs - Gibbons & Warner (arXiv:1305.0957v1)

Previous Papers

- 4-D: Just seen!
- 5-D: (2014) Global Structure of Five-dimensional BPS Fuzzballs - Gibbons & Warner (arXiv:1305.0957v1)
- 6-D: (2015) Structure of Six-Dimensional Microstate Geometries - Lange, Mayerson & Vercnocke (arXiv:1504.07987v1)

Previous Papers

- 4-D: Just seen!
- 5-D: (2014) Global Structure of Five-dimensional BPS Fuzzballs - Gibbons & Warner (arXiv:1305.0957v1)
- 6-D: (2015) Structure of Six-Dimensional Microstate Geometries - Lange, Mayerson & Vercnocke (arXiv:1504.07987v1)
- 11-D: (2014) Smarr's Formula in Eleven-Dimensional Supergravity - Haas (arXiv:1405.3708v1)

Previous Papers

- 4-D: Just seen!
- 5-D: (2014) Global Structure of Five-dimensional BPS Fuzzballs - Gibbons & Warner (arXiv:1305.0957v1)
- 6-D: (2015) Structure of Six-Dimensional Microstate Geometries - Lange, Mayerson & Vercnocke (arXiv:1504.07987v1)
- 10-D: Current Project - Superstring Theory
- 11-D: (2014) Smarr's Formula in Eleven-Dimensional Supergravity - Haas (arXiv:1405.3708v1)

Overview

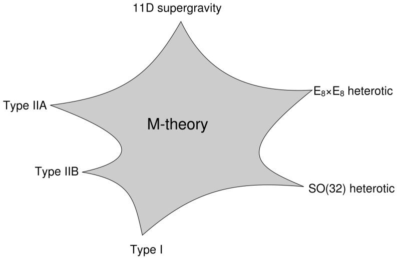
- Include Cosmological Constant Λ (AdS Spacetime) ✓
- Include Topology ✓
- Include Matter Fields (Non - Empty Spacetime) ✓
- Interpret the result. Why 4 dimensions are not enough! ✓
- $\boxed{\text{Moving to String Theory}}$

Superstring Theory

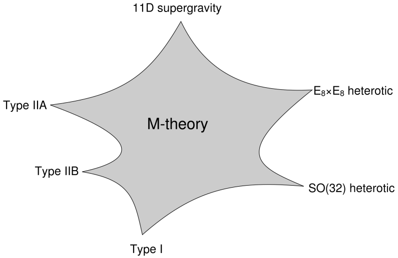
- Superstring Theory requires 10 dimensions.

Superstring Theory

- Superstring Theory requires 10 dimensions.
- Different types of Superstring Theories.



- Superstring Theory requires 10 dimensions.
- Different types of Superstring Theories.



- **Current Project:** IIA Superstring Theory $\xrightarrow[\text{Limit}]{\text{Low Energy}}$ IIA 10-D SUGRA

Type II Superstring Theory

- Fields in Type II Superstring Theory

- Fields in Type II Superstring Theory
  - **NS-NS Sector**

    Low Energy Limit $\rightarrow$ Massless Fields

$$\alpha_\mu^{-1}\alpha_\nu^{-1}\left|0\right\rangle \begin{cases} \text{Symmetric part without Trace} \rightarrow \text{Graviton } g_{\mu\nu} \\ \text{Antisymmetric part} \rightarrow \text{Antisymmetric 2-form } B_{\mu\nu} \\ \text{Trace} \rightarrow \text{Dilaton } \phi \end{cases}$$

Type II Superstring Theory

- Fields in Type II Superstring Theory
  - **NS-NS Sector**

    Low Energy Limit $\rightarrow$ Massless Fields

$$\alpha_\mu^{-1}\alpha_\nu^{-1}|0\rangle \begin{cases} \text{Symmetric part without Trace} \rightarrow \text{Graviton } g_{\mu\nu} \\ \text{Antisymmetric part} \rightarrow \text{Antisymmetric 2-form } B_{\mu\nu} \\ \text{Trace} \rightarrow \text{Dilaton } \phi \end{cases}$$

  - **R-R Sector**

    Generalisation of gauge fields: C_p with Field Strengths $F_p = dC_{p-1}$ $(p = 0, 1, \ldots 10)$

Type II Superstring Theory

- Fields in Type II Superstring Theory
  - **NS-NS Sector**

    Low Energy Limit $\rightarrow$ Massless Fields

$$\alpha_{\mu}^{-1}\alpha_{\nu}^{-1}\left|0\right\rangle \begin{cases} \text{Symmetric part without Trace} \rightarrow \text{Graviton } g_{\mu\nu} \\ \text{Antisymmetric part} \rightarrow \text{Antisymmetric 2-form } B_{\mu\nu} \\ \text{Trace} \rightarrow \text{Dilaton } \phi \end{cases}$$

  - **R-R Sector**

    Generalisation of gauge fields: C_p with Field Strengths $F_p = dC_{p-1}$ $(p = 0, 1, \ldots 10)$

- **Type IIA**: NS-NS: $g_{\mu\nu}$, B_2, ϕ R-R: C_1, C_3, (C_5), (C_7)

Type IIA Superstring Theory

- Constructing an action: $S_{IIA} = S_{\mathrm{NS}} + S_{\mathrm{RR}} + S_{\mathrm{CS}}$

Type IIA Superstring Theory

- Constructing an action: $S_{IIA} = S_{\mathrm{NS}} + S_{\mathrm{RR}} + S_{\mathrm{CS}}$
  - $S_{NS} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \Big(R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} e^{-\phi} |H_3|^2 \Big)$
  - $S_{RR} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \Big(-\frac{1}{2} e^{\frac{3\phi}{2}} |\tilde{F}_2|^2 - \frac{1}{2} e^{\frac{\phi}{2}} |\tilde{F}_4|^2 \Big)$
  - $S_{CS} = -\frac{1}{4\kappa^2} \int B_2 \wedge \tilde{F}_4 \wedge \tilde{F}_4$

Type IIA Superstring Theory

- Constructing an action: $S_{IIA} = S_{\mathrm{NS}} + S_{\mathrm{RR}} + S_{\mathrm{CS}}$
  - $S_{NS} = \frac{1}{2\kappa^2} \int d^{10}x\sqrt{-g}\Big(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}e^{-\phi}|H_3|^2\Big)$
  - $S_{RR} = \frac{1}{2\kappa^2} \int d^{10}x\sqrt{-g}\Big(-\frac{1}{2}e^{\frac{3\phi}{2}}|\tilde{F}_2|^2 - \frac{1}{2}e^{\frac{\phi}{2}}|\tilde{F}_4|^2\Big)$
  - $S_{CS} = -\frac{1}{4\kappa^2} \int B_2 \wedge \tilde{F}_4 \wedge \tilde{F}_4$
- Fast Forward: Repeat Process of 4 Dimensions ✓
  - Einstein Equation - Bianchi Identities - Equations of motion
  - Define the dual fields and rewrite everything in terms of them
  - $\mathcal{L}_K(\text{all fields}) = 0 \to$ Solve the equations
  - Substitute everything into Smarr Formula

Type IIA Superstring Theory

$$M = \int_{\Sigma} \Lambda_2 \wedge f(\text{fields}) + \Lambda_6 \wedge H_3^* + \sum_{n}^{1,3,5,7} \Omega_n \wedge \left(\tilde{F}_{9-n}^* + C_{6-n}^* \wedge H_3^* \right) \wedge e^{B_2}$$

Type IIA Superstring Theory

$$M = \int_{\Sigma} \Lambda_2 \wedge f(\text{fields}) + \Lambda_6 \wedge H_3^* + \sum_{n}^{1,3,5,7} \Omega_n \wedge \left(\tilde{F}_{9-n}^* + C_{6-n}^* \wedge H_3^* \right) \wedge e^{B_2}$$

- Λ_2 comes from H_3. Assumption: $H^2(M) \neq 0$
- Λ_6 comes from H_7. Assumption: $H^6(M) \neq 0$
- All Ω_n come from RR sector. Assumption: $H^{2n+1}(M) \neq 0$

Type IIA Superstring Theory

$$M = \int_{\Sigma} \Lambda_2 \wedge f(\text{fields}) + \Lambda_6 \wedge H_3^* + \sum_{n}^{1,3,5,7} \Omega_n \wedge \left(\tilde{F}_{9-n}^* + C_{6-n}^* \wedge H_3^* \right) \wedge e^{B_2}$$

- Λ_2 comes from H_3. Assumption: $H^2(M) \neq 0$
- Λ_6 comes from H_7. Assumption: $H^6(M) \neq 0$
- All Ω_n come from RR sector. Assumption: $H^{2n+1}(M) \neq 0$
- More Harmonic Forms give us more freedom on choosing the topology!

Type IIA Superstring Theory

$$M = \int_{\Sigma} \Lambda_2 \wedge f(\text{fields}) + \Lambda_6 \wedge H_3^* + \sum_{n}^{1,3,5,7} \Omega_n \wedge \left(\tilde{F}_{9-n}^* + C_{6-n}^* \wedge H_3^* \right) \wedge e^{B_2}$$

- Λ_2 comes from H_3. Assumption: $H^2(M) \neq 0$
- Λ_6 comes from H_7. Assumption: $H^6(M) \neq 0$
- All Ω_n come from RR sector. Assumption: $H^{2n+1}(M) \neq 0$
- More Harmonic Forms give us more freedom on choosing the topology!
- More dimensions can easily produce a non-zero mass!

Summary

- Komar Integral: A way to measure conserved quantities!
  - Modify in case of AdS spacetime. (Among solutions: Killing Potential)
  - Expand in order to include topology (Smarr Formula)

Summary

- Komar Integral: A way to measure conserved quantities!
  - Modify in case of AdS spacetime. (Among solutions: Killing Potential)
  - Expand in order to include topology (Smarr Formula)
- Add Matter Fields $\rightarrow$ Contribution of the bulk to the conserved charge!

Summary

- Komar Integral: A way to measure conserved quantities!
  - Modify in case of AdS spacetime. (Among solutions: Killing Potential)
  - Expand in order to include topology (Smarr Formula)
- Add Matter Fields $\rightarrow$ Contribution of the bulk to the conserved charge!
- Cohomology shows that 4-d case is not that interesting!

Summary

- Komar Integral: A way to measure conserved quantities!
  - Modify in case of AdS spacetime. (Among solutions: Killing Potential)
  - Expand in order to include topology (Smarr Formula)
- Add Matter Fields $\rightarrow$ Contribution of the bulk to the conserved charge!
- Cohomology shows that 4-d case is not that interesting!
- Type IIA Supersting Theory in $d = 10$

Summary

- Komar Integral: A way to measure conserved quantities!
  - Modify in case of AdS spacetime. (Among solutions: Killing Potential)
  - Expand in order to include topology (Smarr Formula)
- Add Matter Fields $\rightarrow$ Contribution of the bulk to the conserved charge!
- Cohomology shows that 4-d case is not that interesting!
- Type IIA Superstring Theory in $d = 10$
- Extra dimensions give us more freedom on choosing topological spaces!

Outlook

- Behaviour of the solutions to the problem of AdS Komar Integral in the bulk (e.g: G,H Formalism)

Outlook

- Behaviour of the solutions to the problem of AdS Komar Integral in the bulk (e.g: G,H Formalism)
- Repeat procedure for Type IIB Superstring Theory (Even Harmonic Forms)

Outlook

- Behaviour of the solutions to the problem of AdS Komar Integral in the bulk (e.g: G,H Formalism)
- Repeat procedure for Type IIB Superstring Theory (Even Harmonic Forms)
- Compactification of 10-d result $\rightarrow$ Obtain the 4-d, 5-d and 6-d results (Previous papers)

Outlook

- Behaviour of the solutions to the problem of AdS Komar Integral in the bulk (e.g: G,H Formalism)
- Repeat procedure for Type IIB Superstring Theory (Even Harmonic Forms)
- Compactification of 10-d result $\rightarrow$ Obtain the 4-d, 5-d and 6-d results (Previous papers)
- Apply specific geometries and calculate explicitly the mass!
  - e.g: Lin, Lunin, Maldacena Geometries ($AdS_5 \times S_5$)
  - e.g: Any geometry of the form ($AdS_p \times S_q$)
  - Expectation: $M \sim \sum_n Q_n$, where Q_n are the brane's charges

The End

Thanks for your attention!