Consider the exponential growth model:

$$r_t = D_1 r_{xx} - \alpha r + 2\beta g$$
$$g_t = D_2 g_{xx} + \alpha r - \beta g$$

with observations made of only $n = r + g$.

So, we have $\qquad g, g_t, g_{xx}$

$$n_t = D_1 n_{xx} + (D_2 - D_1) g_{xx} + \beta g \qquad (1)$$
$$g_t = D_2 g_{xx} + \alpha n - (\alpha + \beta) g \qquad (2)$$

① Solve for g_t, g_{xx} in terms of g.

(1) gives:

$$g_{xx} = \frac{n_t}{D_2 - D_1} - \frac{D_1}{D_2 - D_1} n_{xx} - \frac{\beta}{D_2 - D_1} g \qquad (3)$$

(2) gives (after substituting (3) → (2)):

$$g_t = \frac{D_2}{D_2 - D_1} n_t - \frac{D_1 D_2}{D_2 - D_1} n_{xx} + \alpha n - \left(\frac{\beta D_2}{D_2 - D_1} + \alpha + \beta\right) g \qquad (4)$$

$$g = [\ ] n_t + [\ ] n + [\ ] n_{xx} + [\ ] t.$$

② Obtain another equation to allow us to eliminate g.

- Note that we can differentiate (1) or (2) w.r.t. t or xx so as not to introduce derivatives that we can't get from (3) or (4).

have

$$n_{tt} = D_1 n_{xxt} + (D_2 - D_1) g_{xxt} + \beta g_t \qquad (5)$$

from (3)

$$g_{xxt} = \frac{n_{tt}}{D_2 - D_1} - \frac{D_1}{D_2 - D_1} n_{xxt} - \frac{\beta}{D_2 - D_1} g_t \qquad (6)$$

(4) → (6)

$$(D_2 - D_1) g_{xxt} = n_{tt} - D_1 n_{xxt} - \frac{D_2 \beta}{D_2 - D_1} n_t + \frac{D_1 D_2 \beta}{D_2 - D_1} n_{xx} - \alpha \beta n + \beta \left(\frac{\beta D_2}{D_2 - D_1} + \alpha + \beta \right) g \qquad (7)$$

Thus, from (7) we get:

$$n_{tt} = D_1 n_{xxt} + n_{tt} - D_1 n_{xxt} - \frac{D_2 \beta}{D_2 - D_1} n_t + \frac{D_1 D_2 \beta}{D_2 - D_1} n_{xx} - \alpha \beta n + \beta \left(\frac{\beta D_2}{D_2 - D_1} + \alpha + \beta \right) g + \frac{\beta D_2}{D_2 - D_1} n_t$$