Source Coding with the PCM Method

PCM is a waveform coding method that converts an analog signal into digital data. Typically, the PCM method consists of three main parts: a sampler, a quantizer, and an encoder. The output of the encoder is a sequence of code words (symbols) of fixed length N bits. The purpose of this exercise is to familiarize you with the operation of the quantizer. Specifically, you are asked to implement a uniform and a non-uniform quantizer of N bits, i.e. 2N levels. The quantizers must be implemented as MATLAB functions.

Uniform Quantizer

[xq, centers] = my_quantizer (x, N, min_value, max_value)

x: the input signal in the form of a vector

- N: the number of bits to be used

- max_value: the maximum acceptable value of the input signal

- min_value: the minimum acceptable value of the input signal

- xq: the vector of the output signal encoded with the quantization levels represented by the integers {1, 2, ..., 2N}, where the largest positive quantization level corresponds to the integer '1' (as shown in Figure 1) and the smallest quantization level corresponds to '2N'. These integers can be represented binary with N bits.

- centres: the centres of the quantization regions where the 1st element of the centres vector must correspond to the smallest quantization level - '2N' and the last element to the largest quantization level - '1'.

In particular, the quantizer follows the following steps:

1. Limits the dynamic range of the input signal to the values [min_value max_value], setting samples outside the dynamic range to the corresponding extreme acceptable value.

2. It then calculates the quantization step D and the centers of each region in the centers vector.

3. It finds the region to which each sample of the input signal belongs, and outputs as output the vector xq = [1, 2, ..., 2N].

4. Vector xq can be used as a pointer to the vector centers and get the quantized signal.

An example of the quantization regions for N = 2 bits is shown in the next figure.

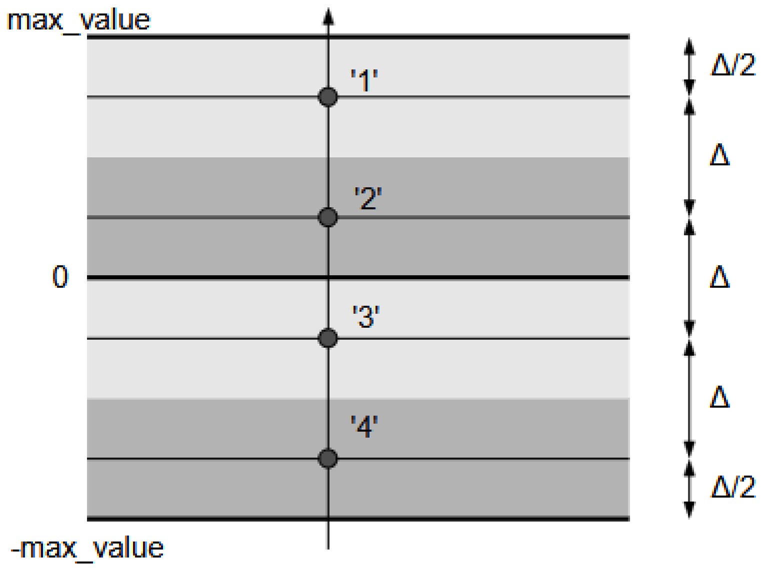

Figure 1: An example of a uniform quantizer layer for N = 2bits.

Non-Uniform Quantizer

For non-uniform quantization of the input vector, the Lloyd-Max algorithm will be used, which allows the design of an optimal quantizer for any number of levels.

[xq, centers, D] = Lloyd_Max(x, N, min_value, max_value)

The inputs are the same as in the case of the uniform quantizer. The outputs have a similar correspondence to the uniform one with an additional parameter D. Specifically, the outputs are as follows:

xq: the encoded vector after Kmax iterations of the algorithm,

- centers: the centers of the quantization regions after Kmax iterations of the algorithm

- D: vector containing the values [D1 D2 ... DKmax] where Di corresponds to the average distortion in iteration i of the algorithm.

Below is a brief description of the algorithm

First you choose a random set of quantization levels:

$$\left\{\tilde{x}_1^{(0)}, \tilde{x}_2^{(0)}, \ldots, \tilde{x}_M^{(0)}\right\}$$

In the exercise, choose these levels to correspond to the centres of the uniform quantizer.

In each iteration i of the Lloyd-Max Algorithm:

1. Calculate the boundaries of the quantum bands, which must be in the middle of the quantum levels, i.e:

$$T_k = (\tilde{x}_k^{(i)} + \tilde{x}_{k+1}^{(i)})/2, \quad 1 \le k \le M-1$$

In every iteration except the last, you must include min_value and max_value as centers in the vector of centers. That is, in every iteration the range of the signal must be [min_value, max_value].

2. Calculate the quantized signal based on these regions and measure the average distortion Di based on the given signal

3. The new quantum levels are the centroids of the bands:

$$\tilde{x}_k^{(i+1)} = E\left[x \middle| T_{k-1} < x < T_k\right]$$

4. Repeat the last 3 steps until:

$$\left|D_i - D_{i-1}\right| < \varepsilon$$

The value of e determines the number of Kmax iterations.

The centres after each iteration will be the same in number. What changes is their location, as the quantization regions will be defined as

[min_value, {centers(1) +centers(2)}/2], [{centers(1)+centers(2)}/2, {centers(2)+ centers(3)}/2] , ... , [{centers(end-1)+centers(end)}/2, max_value]

In case you detect a NaN when running a quantizer, the problem may be in the input and some normalization of the input data is needed.

Audio source

The source you will use is a voice signal that extends up to about 4 KHz. A digital audio signal is given in the form of a waveform (.wav) file which we will consider a satisfactory representation of the corresponding analog signal. The file 'speech.wav', contains samples of a speech signal with sampling rate fs = 8 KHz quantized with N = 16 bits (PCM coding).

To retrieve information about the above audio source, in MATLAB, use the command:

• audioinfo('speech.wav')

To load the signal into MATLAB, use the command:

• [y,fs] =audioread('speech.wav');

and to listen to it, the command

• sound(y,fs);

In case the source value range is larger than the range [min_value, max_value], then you need to normalize the source to this range.

Part A1 questions

1. Implement the PCM scheme using

    a. the uniform quantizer and

    b. the non-uniform one using the Lloyd-Max algorithm.

2. Encode the audio source for min_value = -1, max_value = 1 and N = 2, 4 and 8 bits. Evaluate the PCM scheme based on the following:

    i. Plot for the quantization scheme (b) how the SQNR varies with respect to the number of iterations of the Kmax algorithm (use any value of ε = [10-16, 10-6]). Plot on the same graph the SQNR variation curves for each different value of N so that the comparison can be made using different colors, line type and associated captions provided by the plot() function in Matlab.

    ii. For quantization scheme (b) compare the value of SQNR after Kmax iterations with that obtained using scheme (a) (uniform quantization). Calculate and comment on the performance of the two quantizers.

    iii. Evaluate the result for each quantizer using sound() (acoustic result) and the output waveforms. The comparison is to be made with respect to the original signal (without encoding) and for each value of N. For each N, the waveforms of the original and the encoded signal are to be plotted on the same graph with different colors and line type so that the comparison can be made.

    iv. For each type of quantizer, comment on the efficiency of PCM coding based on the Mean Squared Error (MSE) versus the values of N. Plot on the same graph the MSE curves for the two quantizers with different colors and line types so that the comparison can be made.

Footnote: The final SQNR calculated should be given in dB, i.e. in the form 10log10(.).