

Escher's Tessellations with Sketchpad

Indiana MAA
GOSHEN COLLEGE
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Larry Hautzinger, May Lee, Nicole Nigh

Topics in Euclidean Geometry

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Dr. Vesna Kilibarda

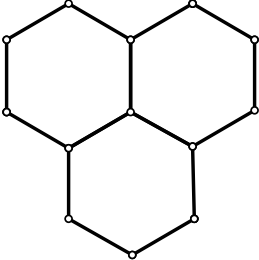
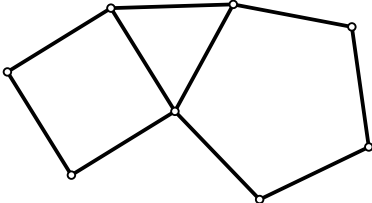
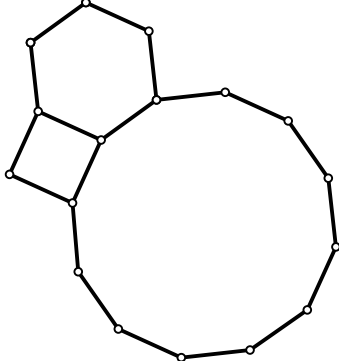
What is a Tessellation?

- Geometric patterns on a surface that can be filled with contiguous figures without any gaps or overlap
- Comes from the Greek word *tesseres*, which means four (in reference to the squares used in the first tessellations)

What Can Be Tessellated?

- Made by repeated use of one polygon (regular tessellation)
- Formed by using more than one polygon – must be regular polygons (semi-regular tessellation)
- Constructed from curved figures

Vertex Figures

| Polygons | Figures |
|--------------------------------------|--|
| $n_1 = 6$ $n_2 = 6$ $n_3 = 6$ |  |
| $n_1 = 3$ $n_2 = 4$ $n_3 = 5$ |  |
| $n_1 = 4$ $n_2 = 6$ $n_3 = 12$ |  |

Tessellation with 3 Polygons (PROOF)

$$180\left(1 - \frac{2}{n_1}\right) + 180\left(1 - \frac{2}{n_2}\right) + 180\left(1 - \frac{2}{n_3}\right) = 360$$

$$\left(1 - \frac{2}{n_1}\right) + \left(1 - \frac{2}{n_2}\right) + \left(1 - \frac{2}{n_3}\right) = 2$$

$$3 - \left(\frac{2}{n_1} + \frac{2}{n_2} + \frac{2}{n_3}\right) = 2 \qquad \left(\frac{2}{n_1} + \frac{2}{n_2} + \frac{2}{n_3}\right) = 1$$

$$\boxed{\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}\right) = \frac{1}{2}}$$

Tessellation with Polygons

| Number of Polygons Surrounding Vertex | Simplified Equation |
|---------------------------------------|--|
| 3 | $\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} \right) = \frac{1}{2}$ |
| 4 | $\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_4} \right) = 1$ |
| 5 | $\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_4} + \frac{1}{n_5} \right) = \frac{3}{2}$ |
| 6 | $\left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_4} + \frac{1}{n_5} + \frac{1}{n_6} \right) = 2$ |

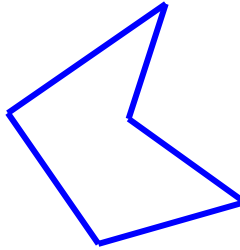
What is Symmetry?

A figure is symmetrical if you can rotate it, flip it or move it in such a way that it is identical to the original figure.

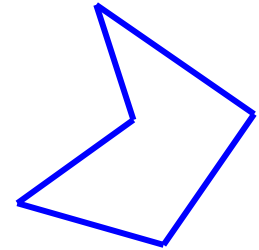
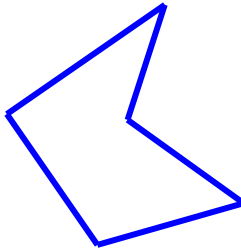
Types of Symmetry



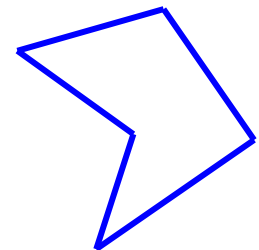
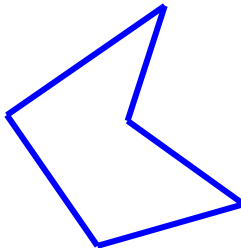
Translation



Reflection



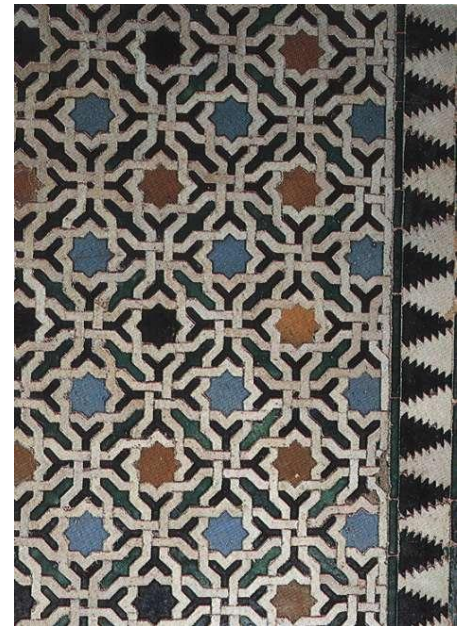
Rotation



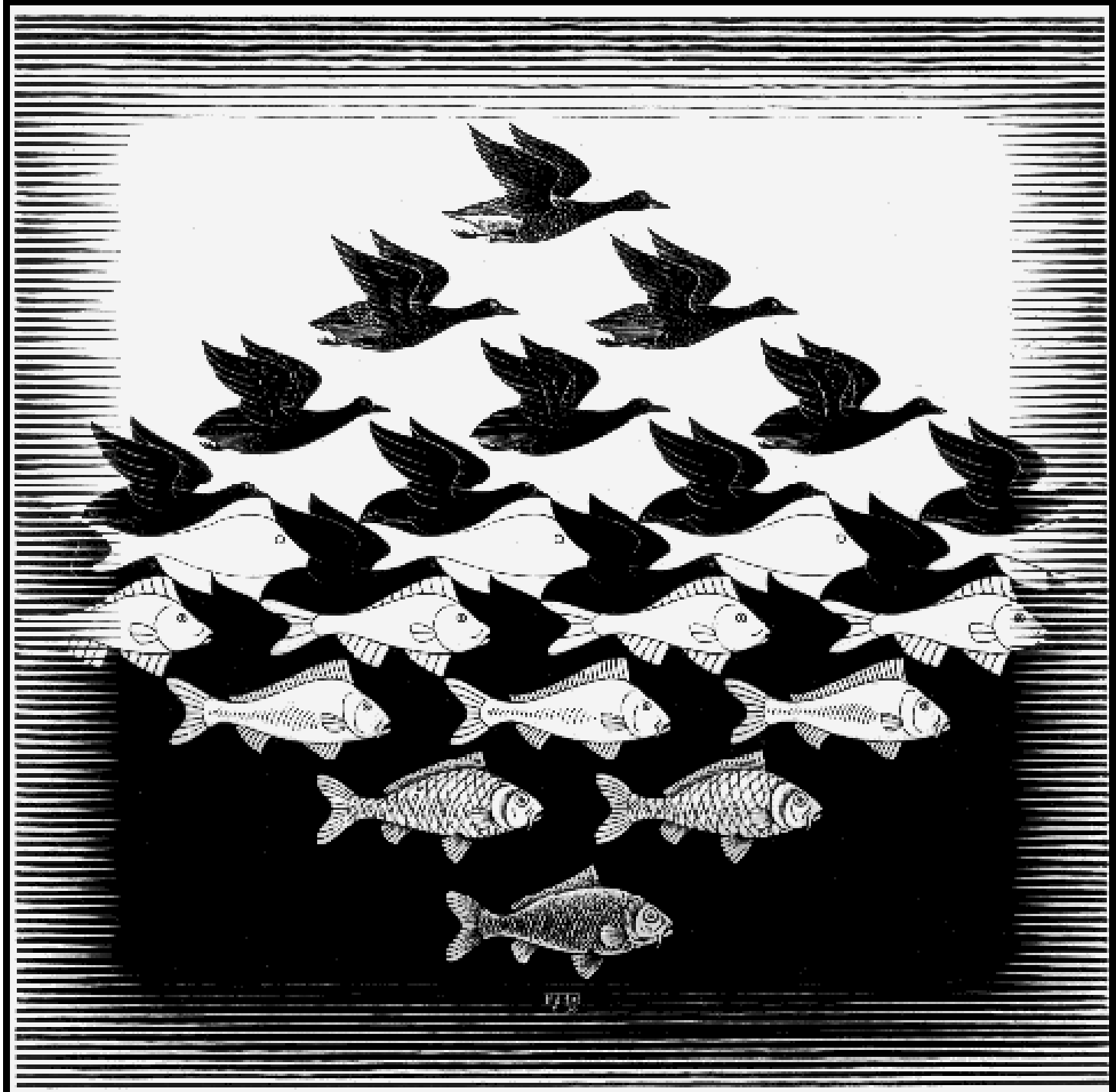
Escher's Tessellations

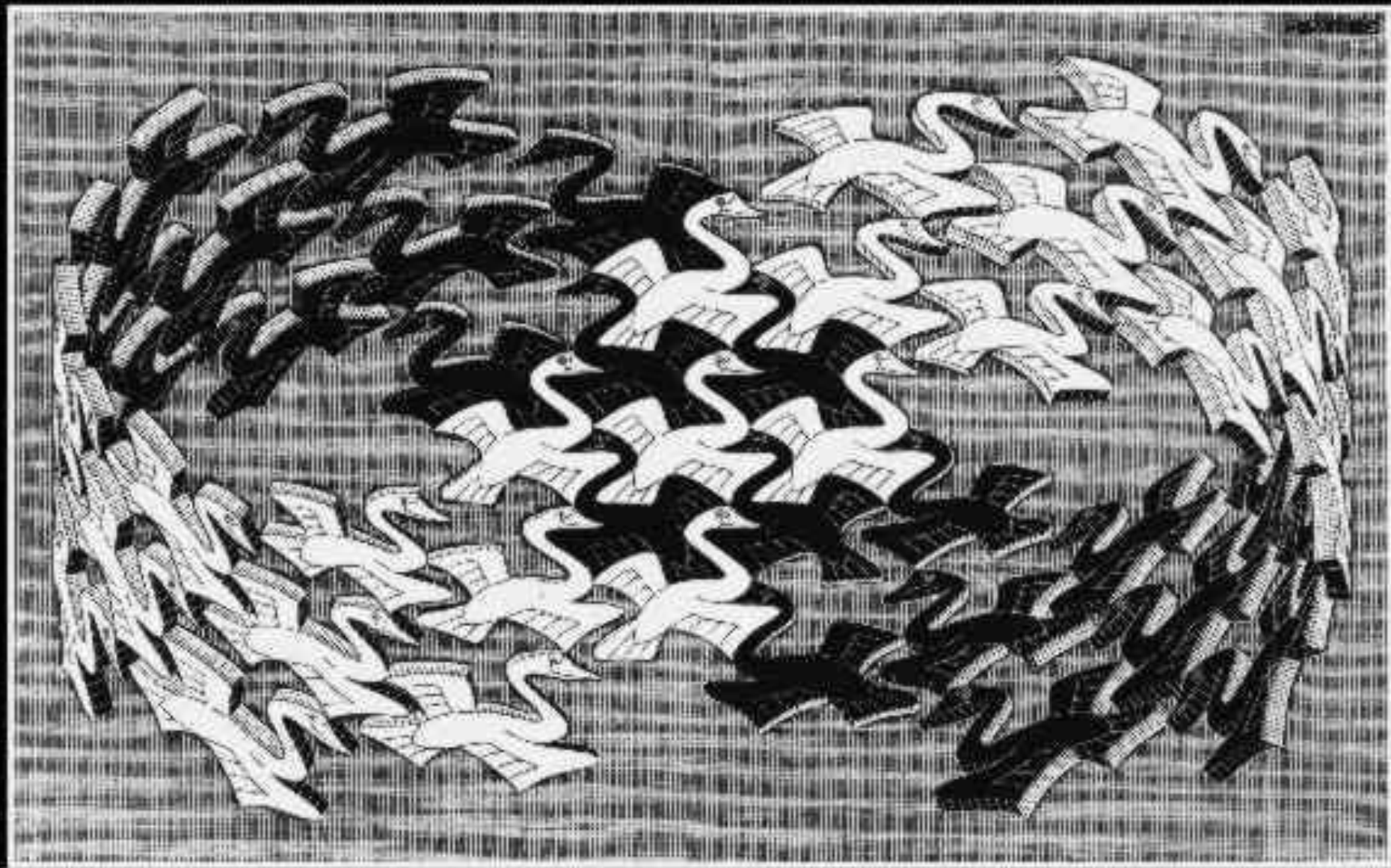
Very Brief History

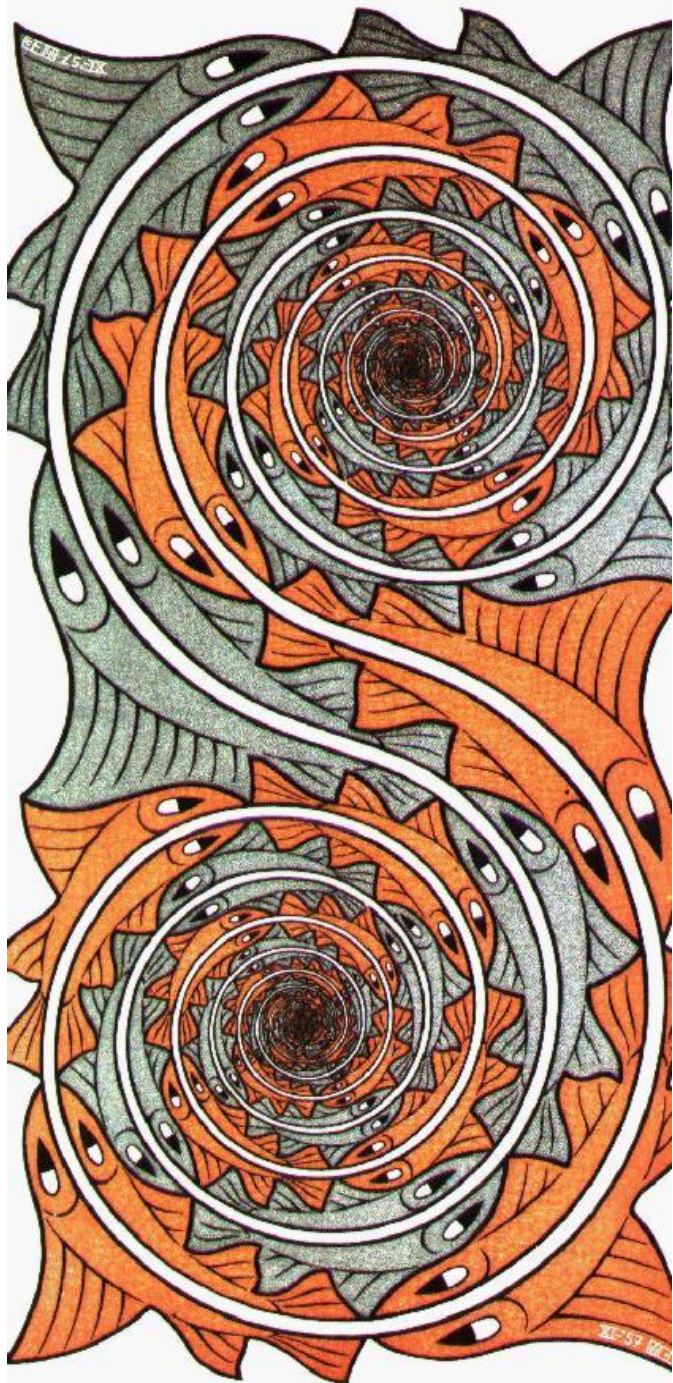
- ◇ Mesopotamian Valley Sumerians in 400 BCE decorated homes and temples with geometric patterns, using tiles made of burnt clay
- ◇ M.C. Escher was born in 1898 and dropped out of high school to study graphic arts
- ◇ Trip in 1922 to see Moorish mosaics in Alhambra, Spain started Escher on tessellations

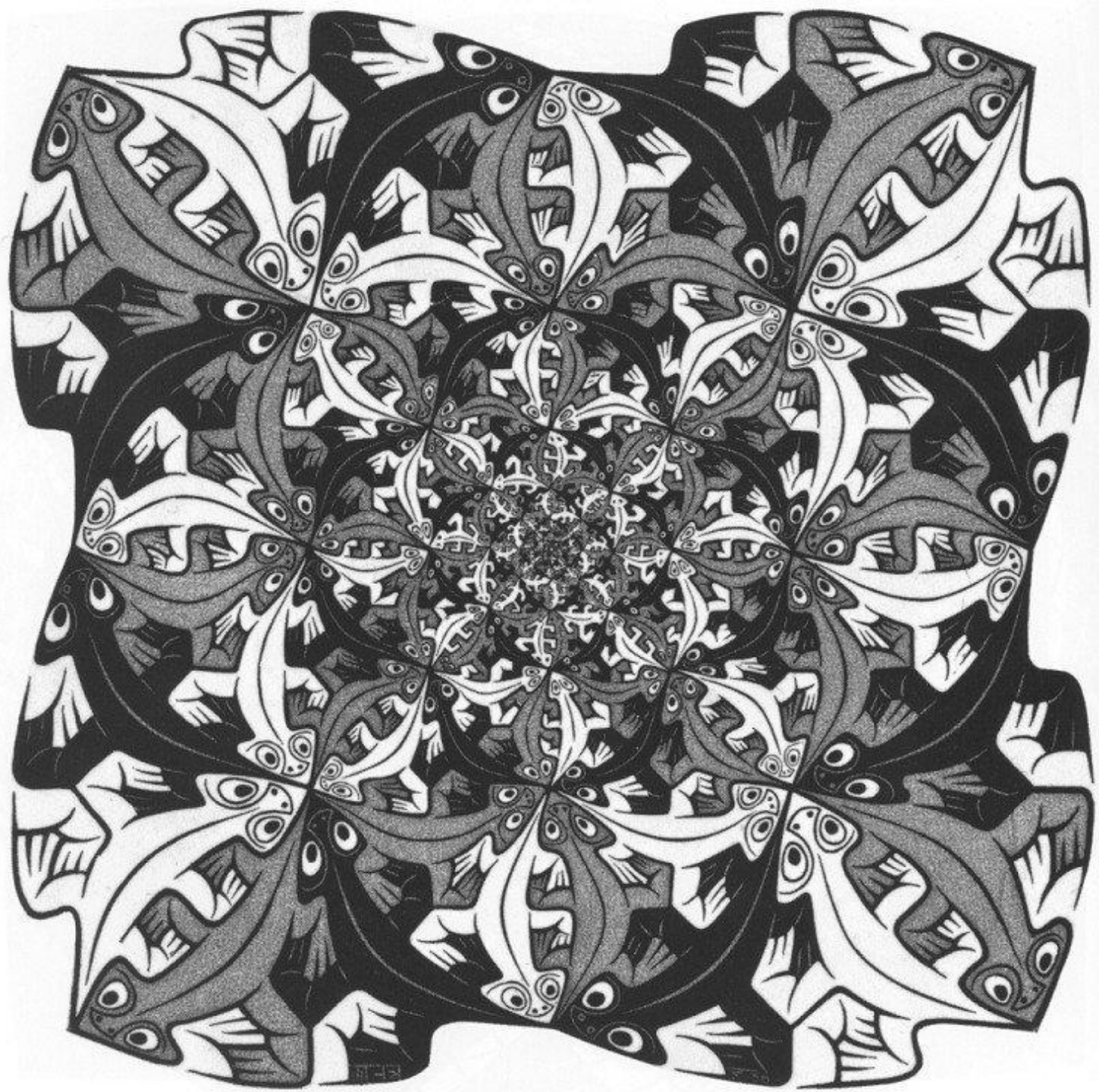


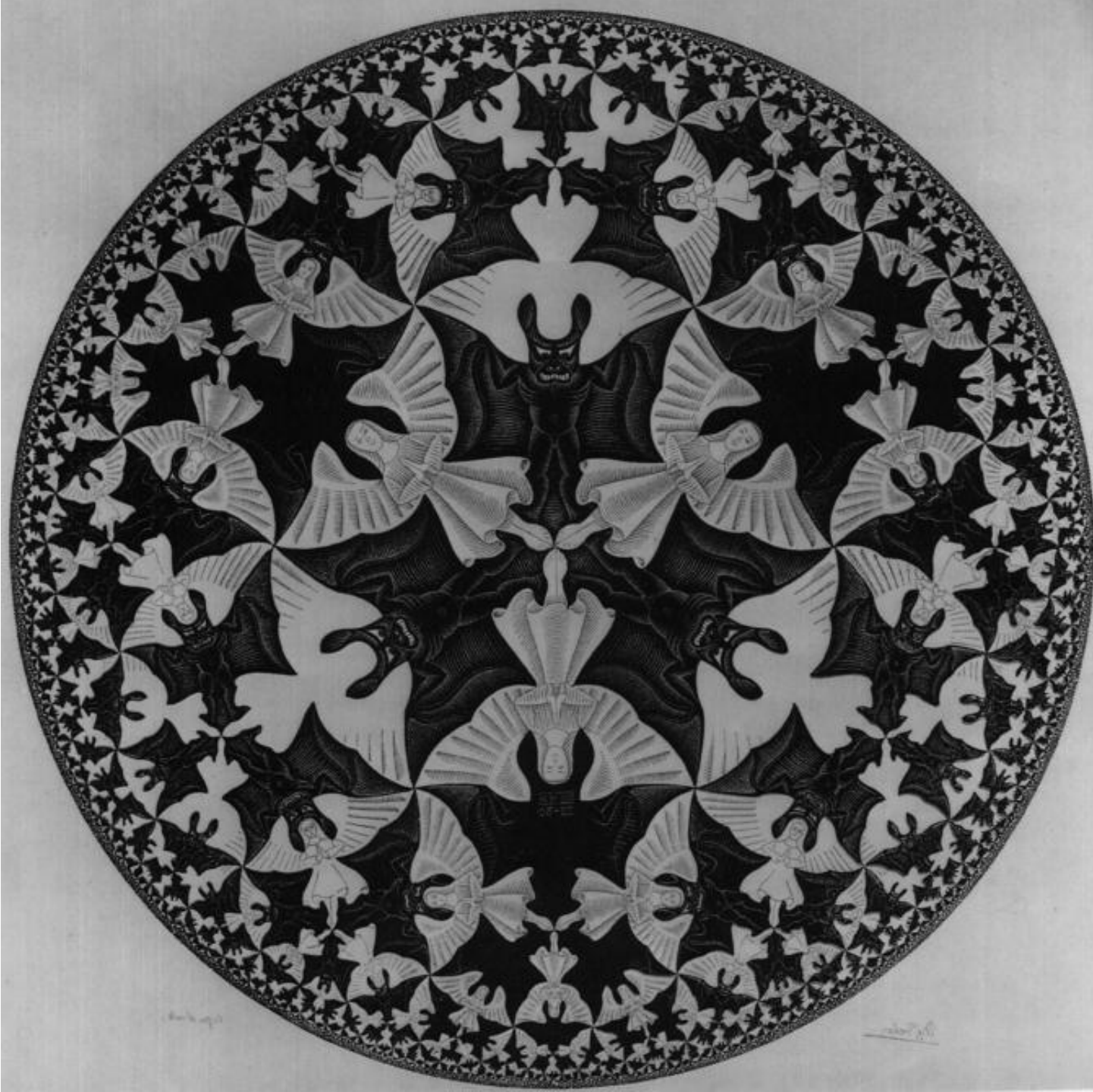








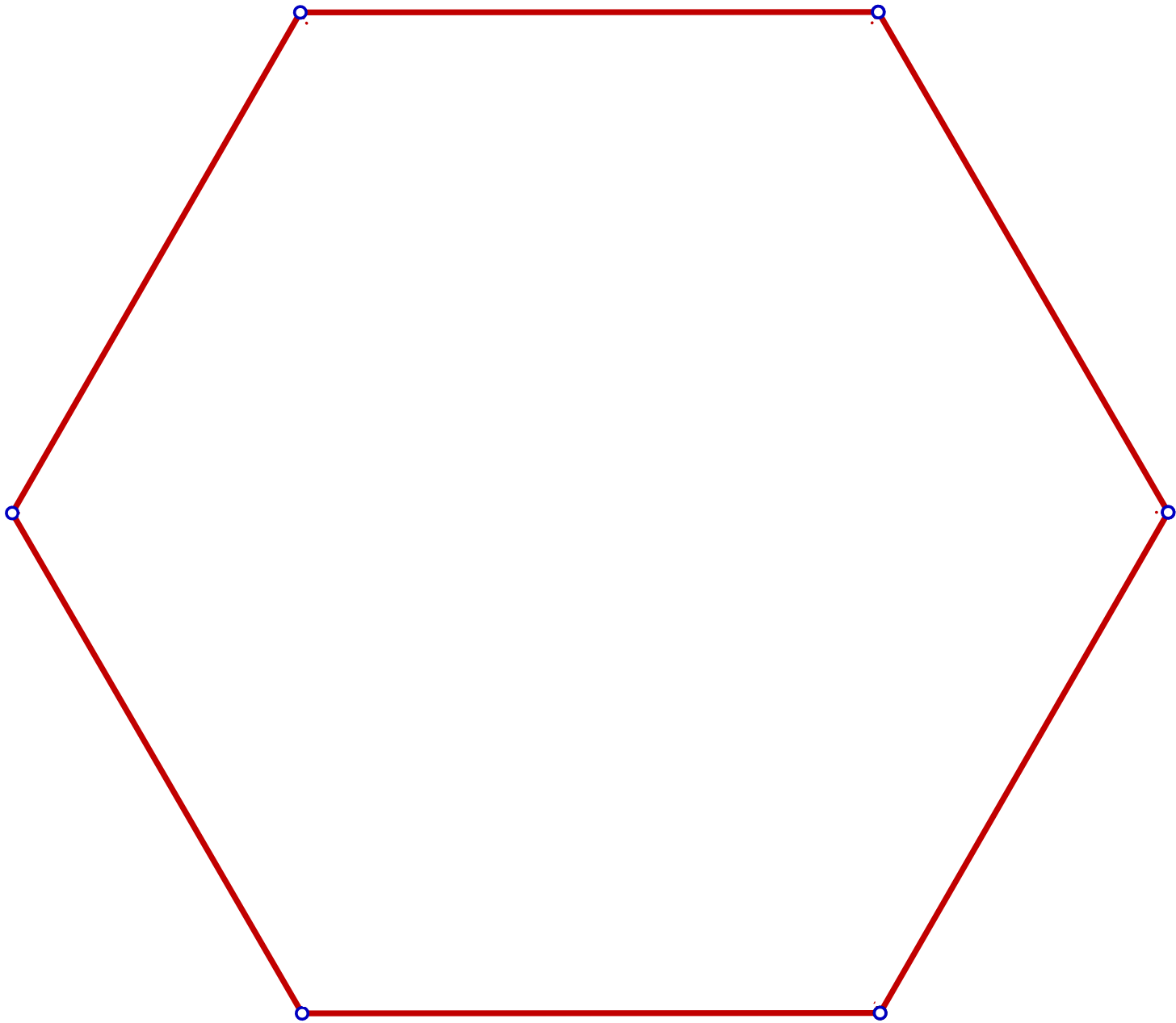


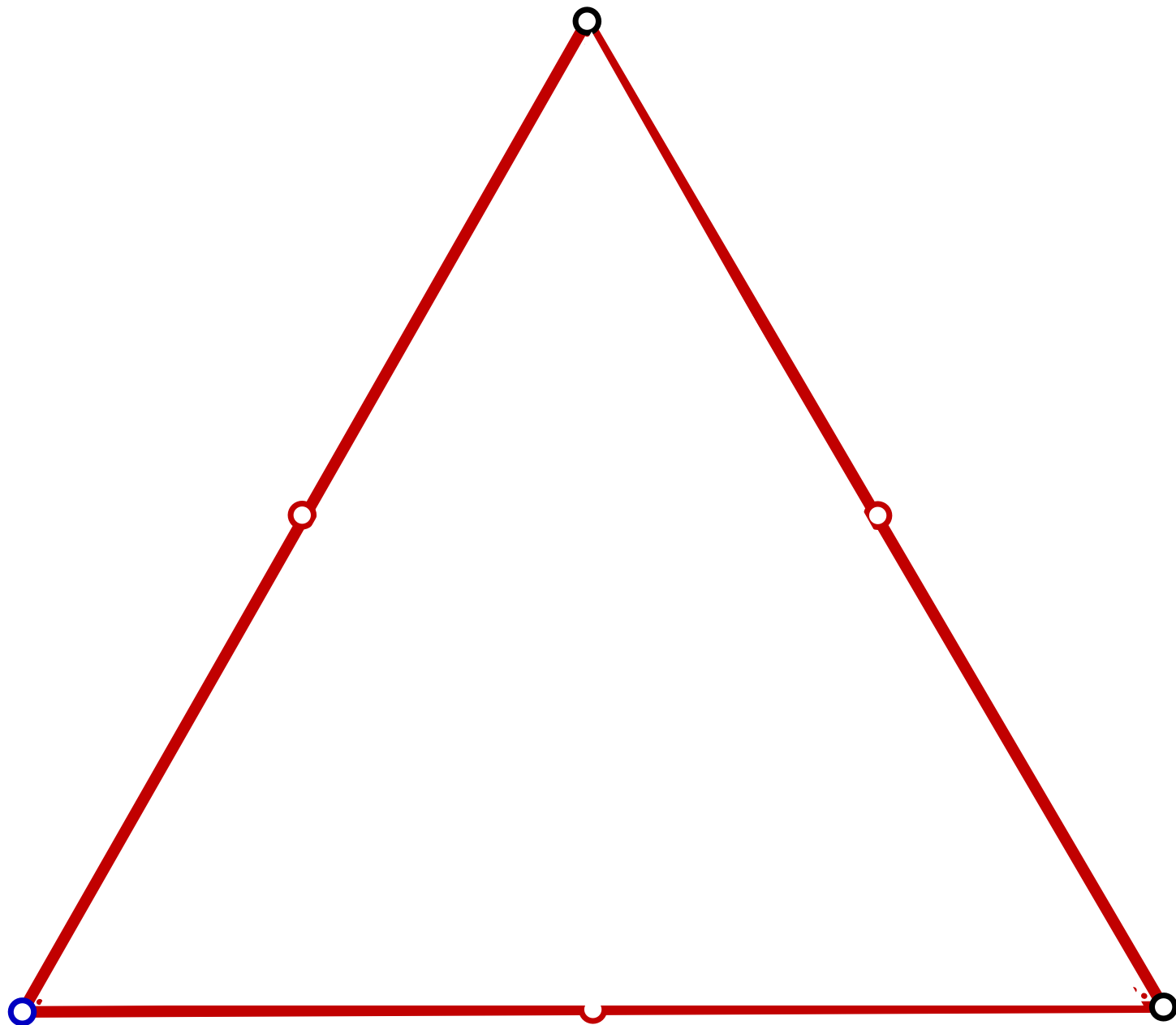


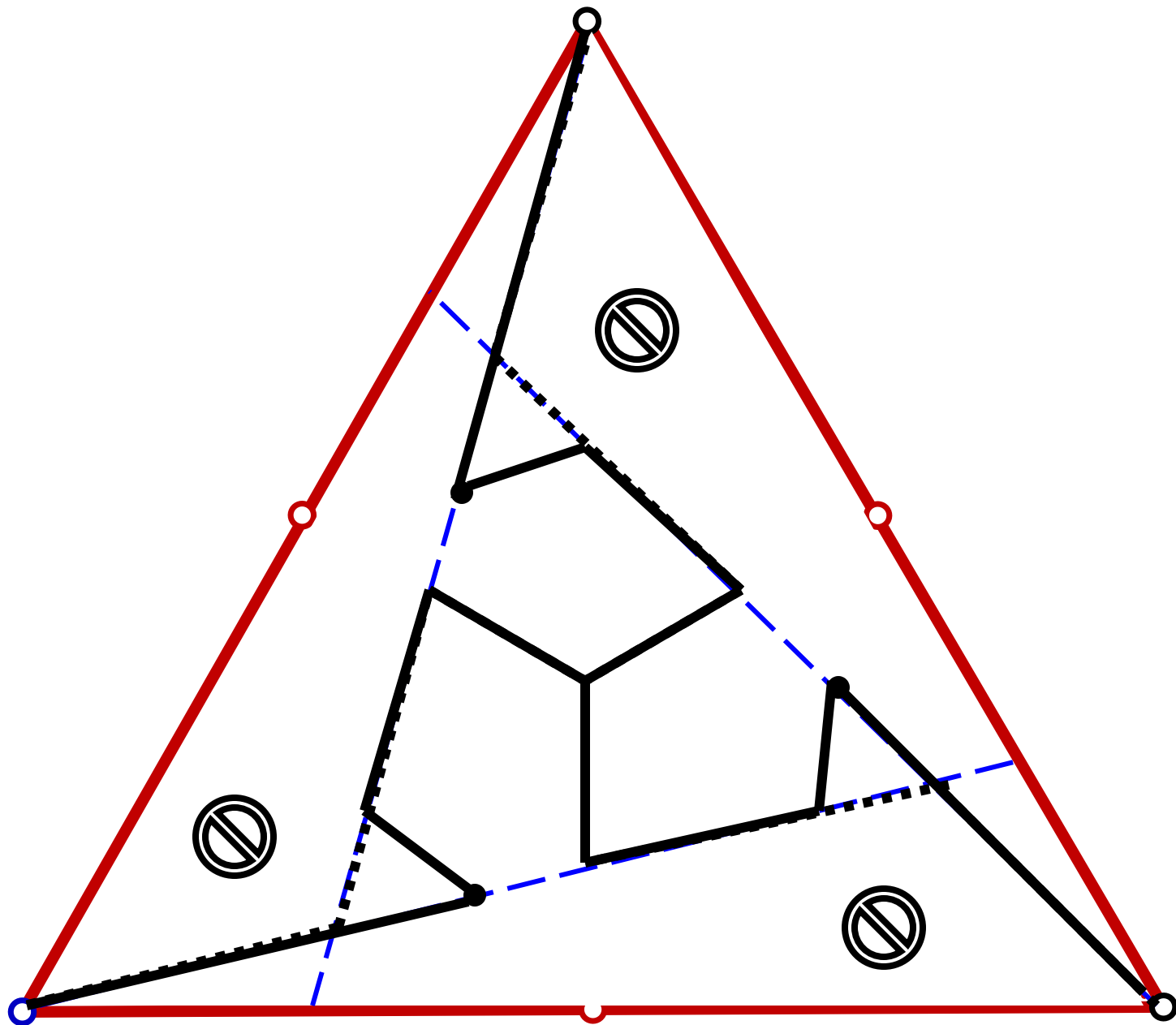


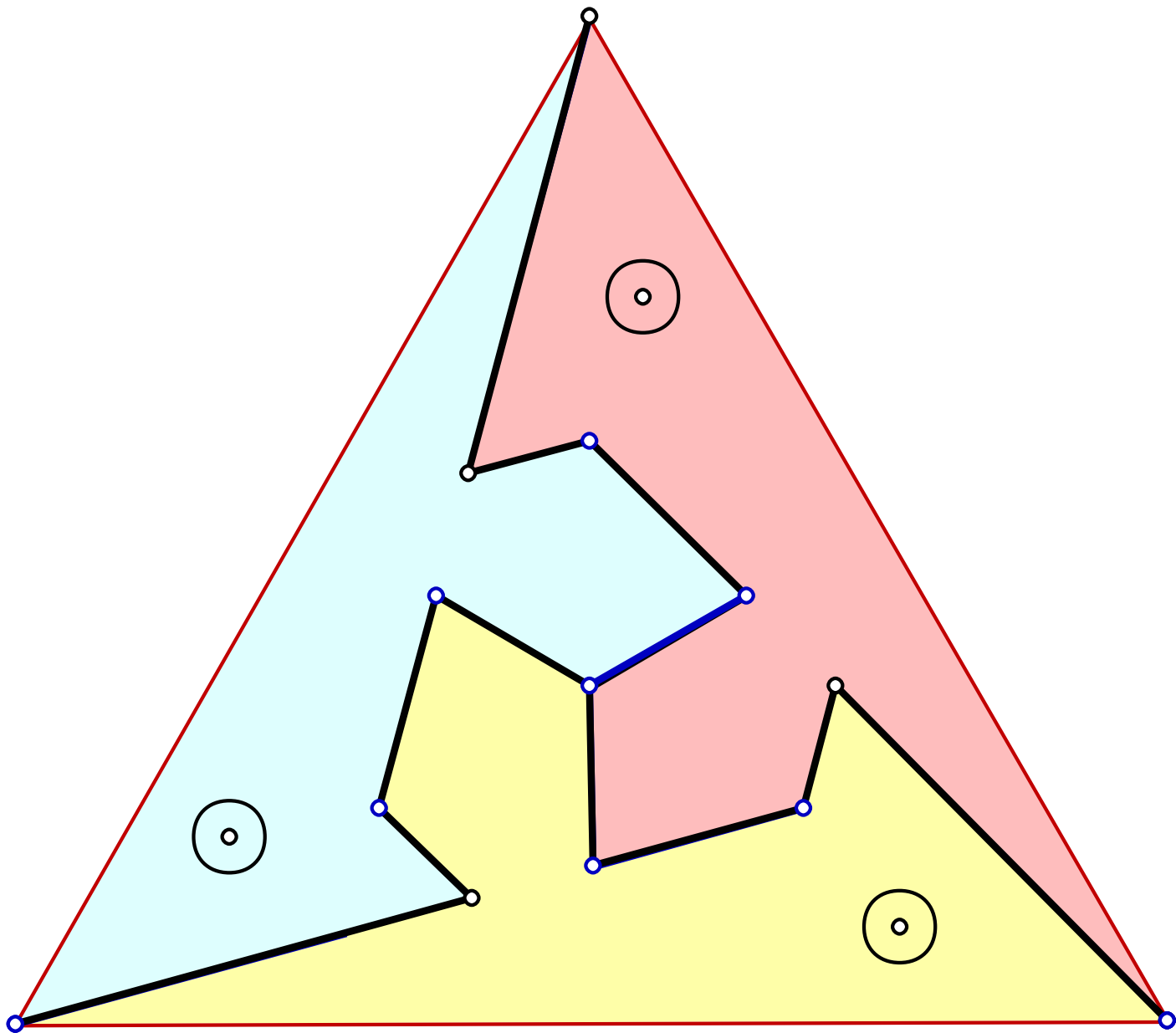
Tessellating With Sketchpad

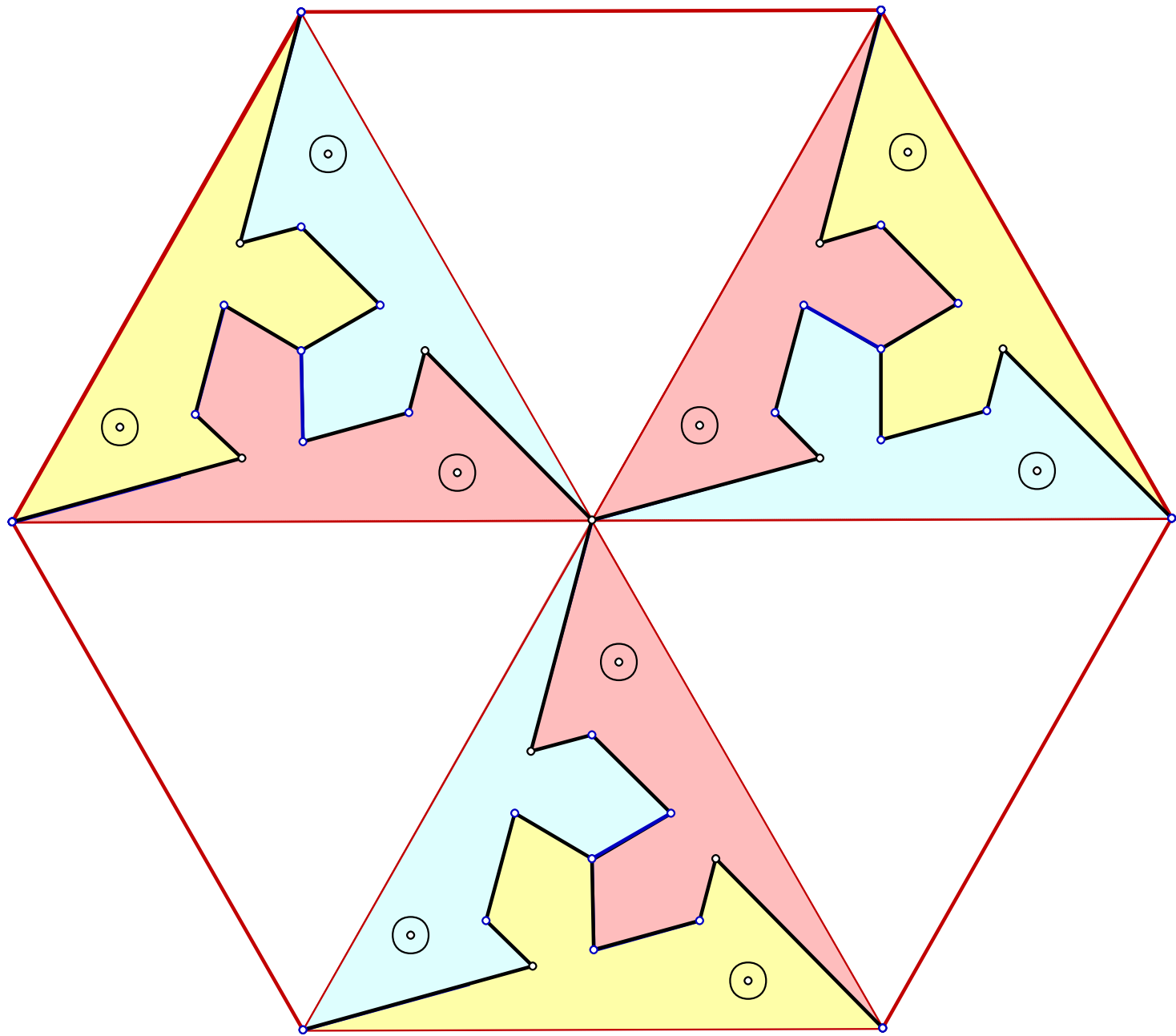


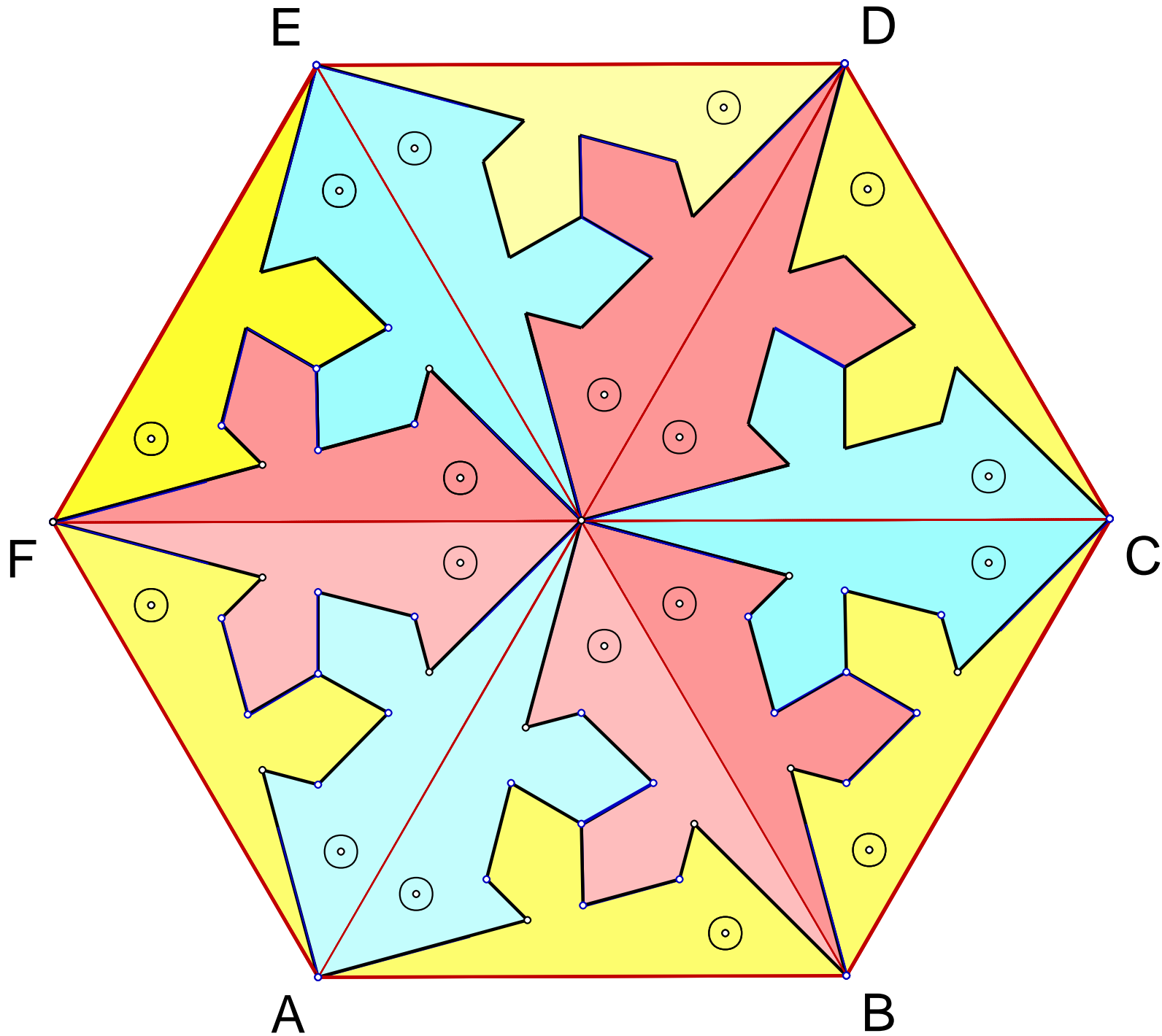


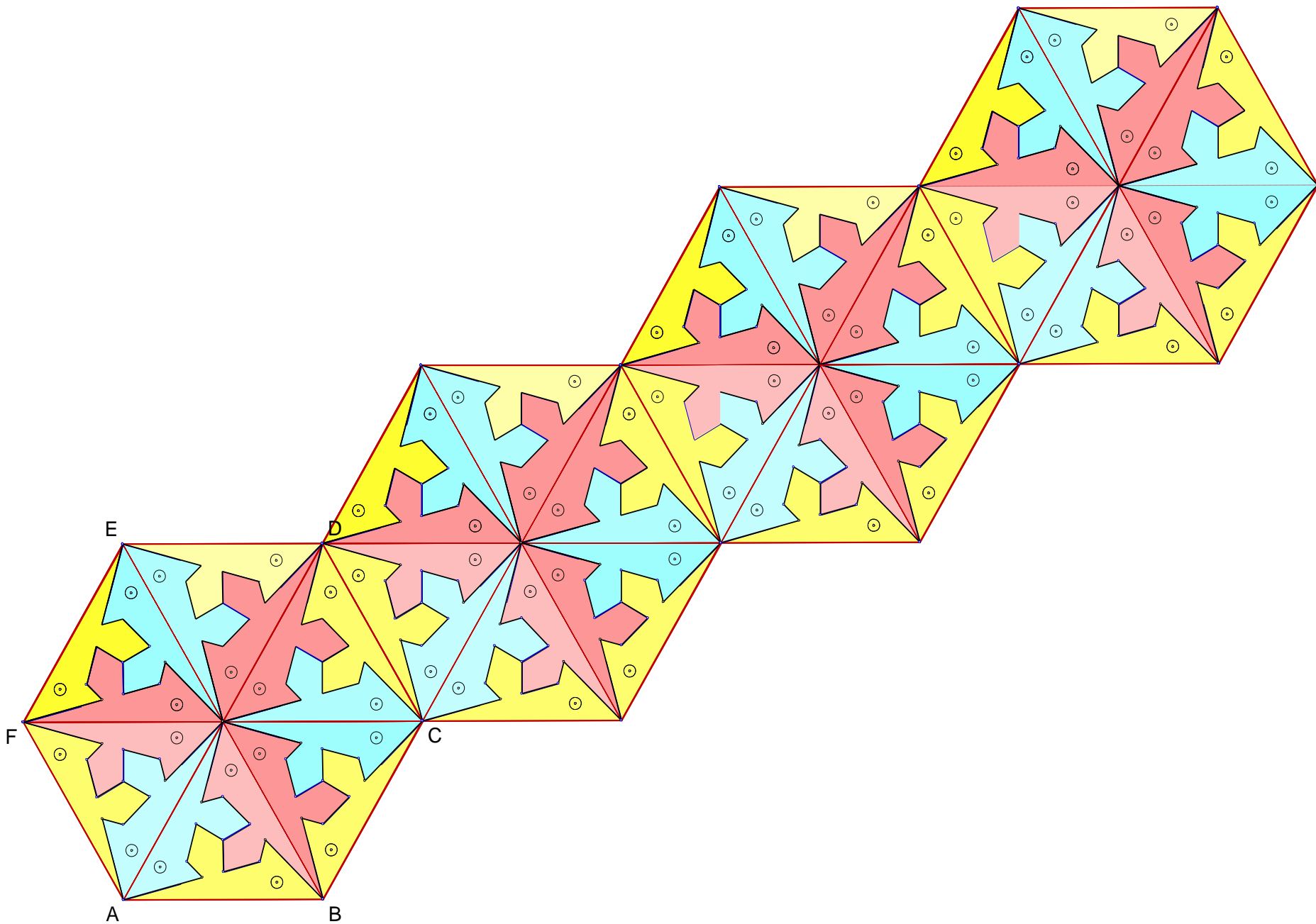


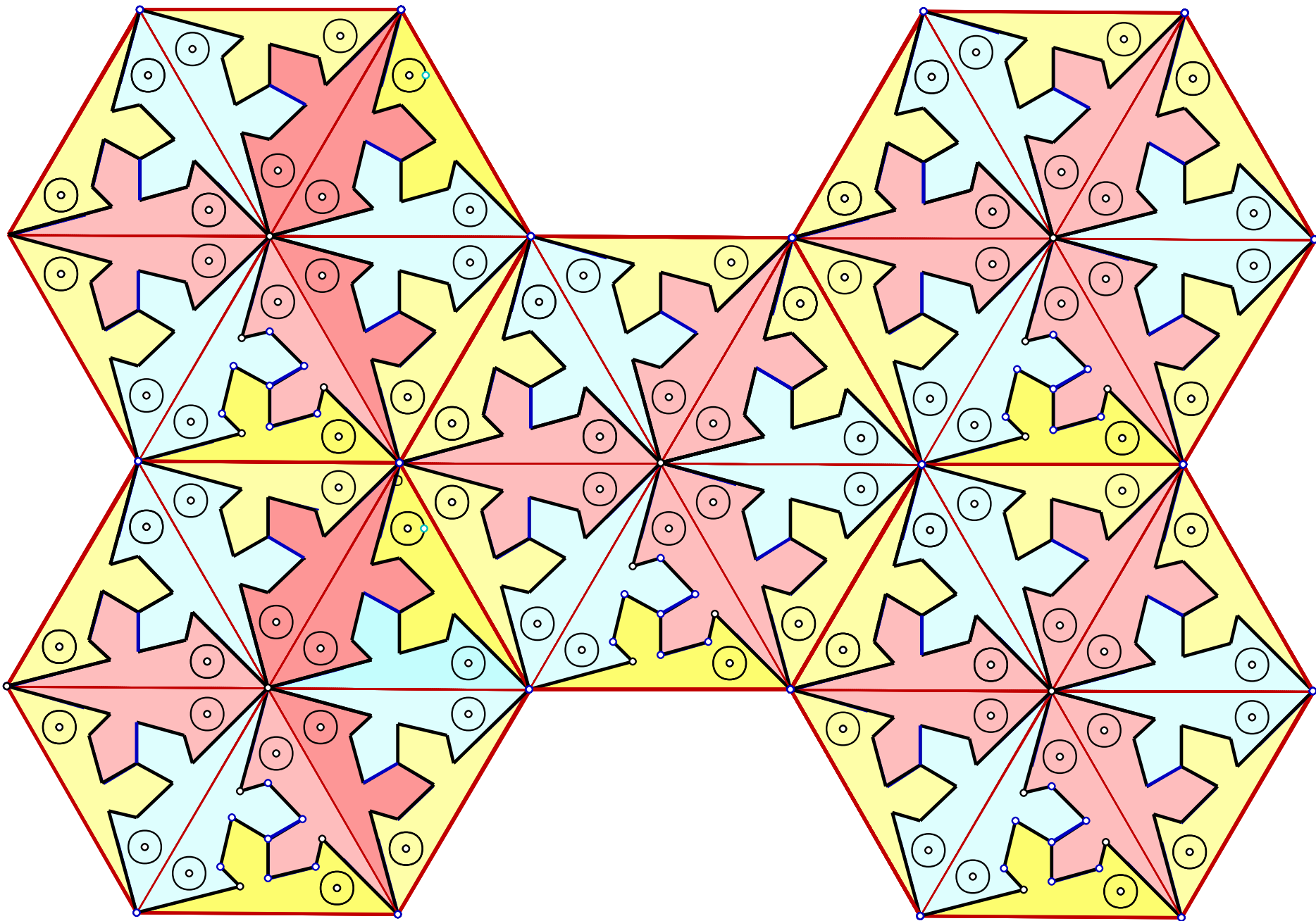


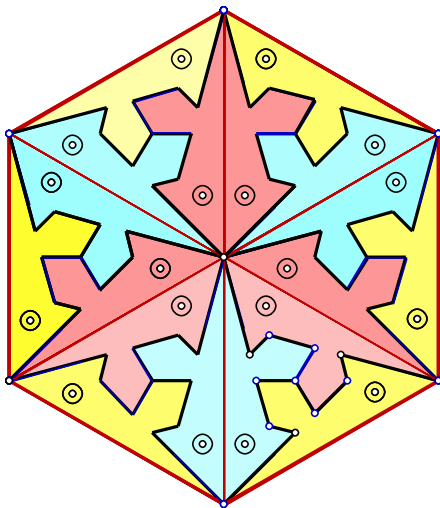












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