Chapter 3

Lexical Analysis

In this chapter we show how to construct a lexical analyzer To implement a lexical analyzer by hand it helps to start with a diagram or other description for the lexemes of each token We can then write code to identify each occurrence of each lexeme on the input and to return information about the token identi ed

We can also produce a lexical analyzer automatically by specifying the lex eme patterns to a *lexical analyzer generator* and compiling those patterns into code that functions as a lexical analyzer This approach makes it easier to mod ify a lexical analyzer since we have only to rewrite the a ected patterns not the entire program It also speeds up the process of implementing the lexical analyzer since the programmer speci es the software at the very high level of patterns and relies on the generator to produce the detailed code We shall introduce in Section 3 5 a lexical analyzer generator called *Lex* or *Flex* in a more recent embodiment

We begin the study of lexical analyzer generators by introducing regular expressions a convenient notation for specifying lexeme patterns We show how this notation can be transformed rst into nondeterministic automata and then into deterministic automata The latter two notations can be used as input to a driver that is code which simulates these automata and uses them as a guide to determining the next token This driver and the speci cation of the automaton form the nucleus of the lexical analyzer

3 1 The Role of the Lexical Analyzer

As the rst phase of a compiler the main task of the lexical analyzer is to read the input characters of the source program group them into lexemes and produce as output a sequence of tokens for each lexeme in the source program The stream of tokens is sent to the parser for syntax analysis It is common for the lexical analyzer to interact with the symbol table as well When the lexical analyzer discovers a lexeme constituting an identi er it needs to enter that lexeme into the symbol table In some cases information regarding the

From Chapter 3 of *Compilers: Principles, Techniques, and Tools*, Second Edition. Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman.

kind of identi er may be read from the symbol table by the lexical analyzer to assist it in determining the proper token it must pass to the parser

These interactions are suggested in Fig 3 1 Commonly the interaction is implemented by having the parser call the lexical analyzer The call suggested by the *getNextToken* command causes the lexical analyzer to read characters from its input until it can identify the next lexeme and produce for it the next token which it returns to the parser

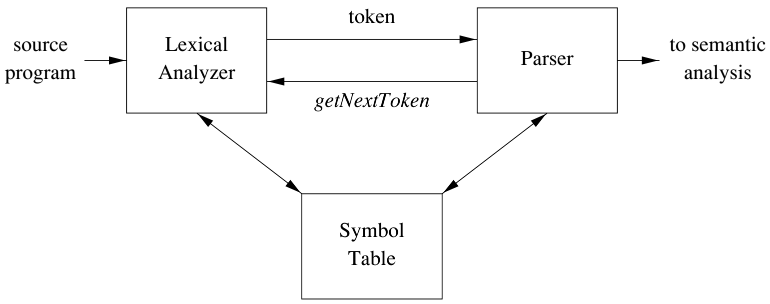


Figure 3 1 Interactions between the lexical analyzer and the parser

Since the lexical analyzer is the part of the compiler that reads the source text it may perform certain other tasks besides identi cation of lexemes One such task is stripping out comments and *whitespace* blank newline tab and perhaps other characters that are used to separate tokens in the input Another task is correlating error messages generated by the compiler with the source program For instance the lexical analyzer may keep track of the number of newline characters seen so it can associate a line number with each error message In some compilers the lexical analyzer makes a copy of the source program with the error messages inserted at the appropriate positions If the source program uses a macro preprocessor the expansion of macros may also be performed by the lexical analyzer

Sometimes lexical analyzers are divided into a cascade of two processes

a *Scanning* consists of the simple processes that do not require tokenization of the input such as deletion of comments and compaction of consecutive whitespace characters into one

b *Lexical analysis* proper is the more complex portion which produces to kens from the output of the scanner

3 1 1 Lexical Analysis Versus Parsing

There are a number of reasons why the analysis portion of a compiler is normally separated into lexical analysis and parsing syntax analysis phases

1 Simplicity of design is the most important consideration The separation of lexical and syntactic analysis often allows us to simplify at least one of these tasks For example a parser that had to deal with comments and whitespace as syntactic units would be considerably more complex than one that can assume comments and whitespace have already been removed by the lexical analyzer If we are designing a new language separating lexical and syntactic concerns can lead to a cleaner overall language design

2 Compiler e ciency is improved A separate lexical analyzer allows us to apply specialized techniques that serve only the lexical task not the job of parsing In addition specialized bu ering techniques for reading input characters can speed up the compiler signi cantly

3 Compiler portability is enhanced Input device speci c peculiarities can be restricted to the lexical analyzer

3 1 2 Tokens Patterns and Lexemes

When discussing lexical analysis we use three related but distinct terms

A *token* is a pair consisting of a token name and an optional attribute value The token name is an abstract symbol representing a kind of lexical unit e g a particular keyword or a sequence of input characters denoting an identi er The token names are the input symbols that the parser processes In what follows we shall generally write the name of a token in boldface We will often refer to a token by its token name

A *pattern* is a description of the form that the lexemes of a token may take In the case of a keyword as a token the pattern is just the sequence of characters that form the keyword For identi ers and some other tokens the pattern is a more complex structure that is *matched* by many strings

A *lexeme* is a sequence of characters in the source program that matches the pattern for a token and is identi ed by the lexical analyzer as an instance of that token

Example 3 1 Figure 3 2 gives some typical tokens their informally described patterns and some sample lexemes To see how these concepts are used in practice in the C statement

```
printf  Total    d n   score
```

both `printf` and `score` are lexemes matching the pattern for token **id** and `Total     d n` is a lexeme matching **literal** □

In many programming languages the following classes cover most or all of the tokens

TOKEN	INFORMAL DESCRIPTION	SAMPLE LEXEMES
if	characters `i` `f`	`if`
else	characters `e` `l` `s` `e`	`else`
comparison	or or or or or	
id	letter followed by letters and digits	`pi` `score` `D2`
number	any numeric constant	`3 14159 0 6 02e23`
literal	anything but surrounded by s	`core dumped`

Figure 3 2 Examples of tokens

1 One token for each keyword The pattern for a keyword is the same as the keyword itself

2 Tokens for the operators either individually or in classes such as the token `comparison` mentioned in Fig 3 2

3 One token representing all identi ers

4 One or more tokens representing constants such as numbers and literal strings

5 Tokens for each punctuation symbol such as left and right parentheses comma and semicolon

3 1 3 Attributes for Tokens

When more than one lexeme can match a pattern the lexical analyzer must provide the subsequent compiler phases additional information about the par ticular lexeme that matched For example the pattern for token **number** matches both 0 and 1 but it is extremely important for the code generator to know which lexeme was found in the source program Thus in many cases the lexical analyzer returns to the parser not only a token name but an attribute value that describes the lexeme represented by the token the token name in uences parsing decisions while the attribute value in uences translation of tokens after the parse

We shall assume that tokens have at most one associated attribute although this attribute may have a structure that combines several pieces of information The most important example is the token **id** where we need to associate with the token a great deal of information Normally information about an identi er e g its lexeme its type and the location at which it is rst found in case an error message about that identi er must be issued is kept in the symbol table Thus the appropriate attribute value for an identi er is a pointer to the symbol table entry for that identi er

> **Tricky Problems When Recognizing Tokens**
>
> Usually given the pattern describing the lexemes of a token it is relatively simple to recognize matching lexemes when they occur on the input However in some languages it is not immediately apparent when we have seen an instance of a lexeme corresponding to a token The following example is taken from Fortran in the xed format still allowed in Fortran 90 In the statement
>
> ```
> DO 5 I   1 25
> ```
>
> it is not apparent that the rst lexeme is `DO5I` an instance of the identi er token until we see the dot following the 1 Note that blanks in xed format Fortran are ignored an archaic convention Had we seen a comma instead of the dot we would have had a do statement
>
> ```
> DO 5 I   1 25
> ```
>
> in which the rst lexeme is the keyword `DO`

Example 3 2 The token names and associated attribute values for the Fortran statement

```
E   M   C    2
```

are written below as a sequence of pairs

id pointer to symbol table entry for `E`
assign_op
id pointer to symbol table entry for `M`
mult_op
id pointer to symbol table entry for `C`
exp_op
number integer value 2

Note that in certain pairs especially operators punctuation and keywords there is no need for an attribute value In this example the token **number** has been given an integer valued attribute In practice a typical compiler would instead store a character string representing the constant and use as an attribute value for **number** a pointer to that string □

3 1 4 Lexical Errors

It is hard for a lexical analyzer to tell without the aid of other components that there is a source code error For instance if the string `fi` is encountered for the rst time in a C program in the context

```
fi   a    f x
```

a lexical analyzer cannot tell whether `fi` is a misspelling of the keyword `if` or an undeclared function identi er Since `fi` is a valid lexeme for the token **id** the lexical analyzer must return the token **id** to the parser and let some other phase of the compiler probably the parser in this case handle an error due to transposition of the letters

However suppose a situation arises in which the lexical analyzer is unable to proceed because none of the patterns for tokens matches any pre x of the remaining input The simplest recovery strategy is panic mode recovery We delete successive characters from the remaining input until the lexical analyzer can nd a well formed token at the beginning of what input is left This recovery technique may confuse the parser but in an interactive computing environment it may be quite adequate

Other possible error recovery actions are

1 Delete one character from the remaining input

2 Insert a missing character into the remaining input

3 Replace a character by another character

4 Transpose two adjacent characters

Transformations like these may be tried in an attempt to repair the input The simplest such strategy is to see whether a pre x of the remaining input can be transformed into a valid lexeme by a single transformation This strategy makes sense since in practice most lexical errors involve a single character A more general correction strategy is to nd the smallest number of transforma tions needed to convert the source program into one that consists only of valid lexemes but this approach is considered too expensive in practice to be worth the e ort

3 1 5 Exercises for Section 3 1

Exercise 3 1 1 Divide the following C program

```
float limitedSquare x  float x
       returns x squared  but never more than 100
    return  x   10 0  x  10 0  100 x x
```

into appropriate lexemes using the discussion of Section 3 1 2 as a guide Which lexemes should get associated lexical values What should those values be

Exercise 3 1 2 Tagged languages like HTML or XML are di erent from con ventional programming languages in that the punctuation tags are either very numerous as in HTML or a user de nable set as in XML Further tags can often have parameters Suggest how to divide the following HTML document

```
Here is a photo of  B my house  B
 P  IMG SRC    house gif   BR
See  A HREF    morePix html  More Pictures  A  if you
liked that one  P
```

into appropriate lexemes Which lexemes should get associated lexical values and what should those values be

3 2 Input Bu ering

Before discussing the problem of recognizing lexemes in the input let us examine some ways that the simple but important task of reading the source program can be speeded This task is made di cult by the fact that we often have to look one or more characters beyond the next lexeme before we can be sure we have the right lexeme The box on Tricky Problems When Recognizing Tokens in Section 3 1 gave an extreme example but there are many situations where we need to look at least one additional character ahead For instance we cannot be sure we ve seen the end of an identi er until we see a character that is not a letter or digit and therefore is not part of the lexeme for **id** In C single character operators like or could also be the beginning of a two character operator like or Thus we shall introduce a two bu er scheme that handles large lookaheads safely We then consider an improvement involving sentinels that saves time checking for the ends of bu ers

3 2 1 Bu er Pairs

Because of the amount of time taken to process characters and the large number of characters that must be processed during the compilation of a large source program specialized bu ering techniques have been developed to reduce the amount of overhead required to process a single input character An impor tant scheme involves two bu ers that are alternately reloaded as suggested in Fig 3 3

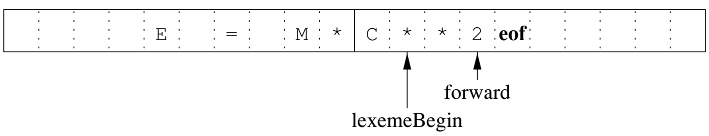


Figure 3 3 Using a pair of input bu ers

Each bu er is of the same size N and N is usually the size of a disk block e g 4096 bytes Using one system read command we can read N characters into a bu er rather than using one system call per character If fewer than N characters remain in the input le then a special character represented by **eof** marks the end of the source le and is di erent from any possible character of the source program

Two pointers to the input are maintained

1 Pointer `lexemeBegin` marks the beginning of the current lexeme whose extent we are attempting to determine

2 Pointer `forward` scans ahead until a pattern match is found the exact strategy whereby this determination is made will be covered in the balance of this chapter

Once the next lexeme is determined `forward` is set to the character at its right end Then after the lexeme is recorded as an attribute value of a token returned to the parser `lexemeBegin` is set to the character immediately after the lexeme just found In Fig 3 3 we see `forward` has passed the end of the next lexeme the Fortran exponentiation operator and must be retracted one position to its left

Advancing `forward` requires that we rst test whether we have reached the end of one of the bu ers and if so we must reload the other bu er from the input and move `forward` to the beginning of the newly loaded bu er As long as we never need to look so far ahead of the actual lexeme that the sum of the lexeme s length plus the distance we look ahead is greater than N we shall never overwrite the lexeme in its bu er before determining it

3 2 2 Sentinels

If we use the scheme of Section 3 2 1 as described we must check each time we advance `forward` that we have not moved o one of the bu ers if we do then we must also reload the other bu er Thus for each character read we make two tests one for the end of the bu er and one to determine what character is read the latter may be a multiway branch We can combine the bu er end test with the test for the current character if we extend each bu er to hold a *sentinel* character at the end The sentinel is a special character that cannot be part of the source program and a natural choice is the character **eof**

Figure 3 4 shows the same arrangement as Fig 3 3 but with the sentinels added Note that **eof** retains its use as a marker for the end of the entire input Any **eof** that appears other than at the end of a bu er means that the input is at an end Figure 3 5 summarizes the algorithm for advancing `forward` Notice how the rst test which can be part of a multiway branch based on the character pointed to by `forward` is the only test we make except in the case where we actually are at the end of a bu er or the end of the input

3 3 Speci cation of Tokens

Regular expressions are an important notation for specifying lexeme patterns While they cannot express all possible patterns they are very e ective in spec

Can We Run Out of Bu er Space

In most modern languages lexemes are short and one or two characters of lookahead is su cient Thus a bu er size N in the thousands is ample and the double bu er scheme of Section 3 2 1 works without problem However there are some risks For example if character strings can be very long extending over many lines then we could face the possibility that a lexeme is longer than N To avoid problems with long character strings we can treat them as a concatenation of components one from each line over which the string is written For instance in Java it is conventional to represent long strings by writing a piece on each line and concatenating pieces with a operator at the end of each piece

A more di cult problem occurs when arbitrarily long lookahead may be needed For example some languages like PL I do not treat key words as *reserved* that is you can use identi ers with the same name as a keyword like `DECLARE` If the lexical analyzer is presented with text of a PL I program that begins `DECLARE ARG1 ARG2` it cannot be sure whether `DECLARE` is a keyword and `ARG1` and so on are variables being de clared or whether `DECLARE` is a procedure name with its arguments For this reason modern languages tend to reserve their keywords However if not one can treat a keyword like `DECLARE` as an ambiguous identi er and let the parser resolve the issue perhaps in conjunction with symbol table lookup

ifying those types of patterns that we actually need for tokens In this section we shall study the formal notation for regular expressions and in Section 3 5 we shall see how these expressions are used in a lexical analyzer generator Then Section 3 7 shows how to build the lexical analyzer by converting regular expressions to automata that perform the recognition of the speci ed tokens

3 3 1 Strings and Languages

An *alphabet* is any nite set of symbols Typical examples of symbols are let ters digits and punctuation The set $\{0\ 1\}$ is the *binary alphabet* ASCII is an important example of an alphabet it is used in many software systems Uni

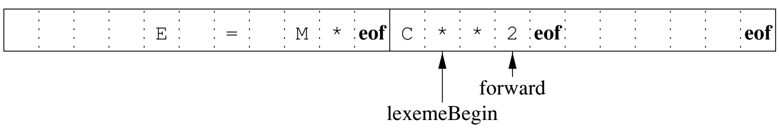


Figure 3 4 Sentinels at the end of each bu er

```
switch  forward     {
    case eof
        if  forward is at end of  rst bu er  {
            reload second bu er
            forward   beginning of second bu er
        }
        else if  forward is at end of second bu er  {
            reload  rst bu er
            forward   beginning of  rst bu er
        }
        else    eof within a bu er marks the end of input
            terminate lexical analysis
        break
    Cases for the other characters
}
```

Figure 3 5 Lookahead code with sentinels

> **Implementing Multiway Branches**
>
> We might imagine that the switch in Fig 3 5 requires many steps to ex ecute and that placing the case **eof** rst is not a wise choice Actually it doesn t matter in what order we list the cases for each character In practice a multiway branch depending on the input character is made in one step by jumping to an address found in an array of addresses indexed by characters

code which includes approximately 100 000 characters from alphabets around the world is another important example of an alphabet

A *string* over an alphabet is a nite sequence of symbols drawn from that alphabet In language theory the terms sentence and word are often used as synonyms for string The length of a string s usually written $|s|$ is the number of occurrences of symbols in s For example `banana` is a string of length six The *empty string* denoted is the string of length zero

A *language* is any countable set of strings over some xed alphabet This de nition is very broad Abstract languages like the *empty set* or { } the set containing only the empty string are languages under this de nition So too are the set of all syntactically well formed C programs and the set of all grammatically correct English sentences although the latter two languages are di cult to specify exactly Note that the de nition of language does not require that any meaning be ascribed to the strings in the language Methods for de ning the meaning of strings are discussed in Chapter 5

Terms for Parts of Strings

The following string related terms are commonly used

1 A *pre x* of string s is any string obtained by removing zero or more symbols from the end of s For example `ban` `banana` and are pre xes of `banana`

2 A *su x* of string s is any string obtained by removing zero or more symbols from the beginning of s For example `nana` `banana` and are su xes of `banana`

3 A *substring* of s is obtained by deleting any pre x and any su x from s For instance `banana` `nan` and are substrings of `banana`

4 The *proper* pre xes su xes and substrings of a string s are those pre xes su xes and substrings respectively of s that are not or not equal to s itself

5 A *subsequence* of s is any string formed by deleting zero or more not necessarily consecutive positions of s For example `baan` is a subsequence of `banana`

If x and y are strings then the *concatenation* of x and y denoted xy is the string formed by appending y to x For example if x `dog` and y `house` then xy `doghouse` The empty string is the identity under concatenation that is for any string s s s s

If we think of concatenation as a product we can de ne the exponentiation of strings as follows De ne s^0 to be and for all i 0 de ne s^i to be $s^{i \ 1}s$ Since s s it follows that s^1 s Then s^2 ss s^3 sss and so on

3 3 2 Operations on Languages

In lexical analysis the most important operations on languages are union con catenation and closure which are de ned formally in Fig 3 6 Union is the familiar operation on sets The concatenation of languages is all strings formed by taking a string from the rst language and a string from the second lan guage in all possible ways and concatenating them The *Kleene closure* of a language L denoted L is the set of strings you get by concatenating L zero or more times Note that L^0 the concatenation of L zero times is de ned to be { } and inductively L^i is $L^{i \ 1}L$ Finally the positive closure denoted L is the same as the Kleene closure but without the term L^0 That is will not be in L unless it is in L itself

OPERATION	DEFINITION AND NOTATION
Union of L and M	$L \quad M \quad \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$
Concatenation of L and M	$LM \quad \{st \mid s \text{ is in } L \text{ and } t \text{ is in } M\}$
Kleene closure of L	$L \qquad {}^{\infty}_{i\ 0}\, L^i$
Positive closure of L	$L \qquad {}^{\infty}_{i\ 1}\, L^i$

Figure 3 6 De nitions of operations on languages

Example 3 3 Let L be the set of letters $\{$`A B` `Z a b` `z`$\}$ and let D be the set of digits $\{$`0 1` `9`$\}$ We may think of L and D in two essentially equivalent ways One way is that L and D are respectively the alphabets of uppercase and lowercase letters and of digits The second way is that L and D are languages all of whose strings happen to be of length one Here are some other languages that can be constructed from languages L and D using the operators of Fig 3 6

1 $L \quad D$ is the set of letters and digits strictly speaking the language with 62 strings of length one each of which strings is either one letter or one digit

2 LD is the set of 520 strings of length two each consisting of one letter followed by one digit

3 L^4 is the set of all 4 letter strings

4 L is the set of all strings of letters including the empty string

5 $L\ L \quad D$ is the set of all strings of letters and digits beginning with a letter

6 D is the set of all strings of one or more digits

□

3 3 3 Regular Expressions

Suppose we wanted to describe the set of valid C identi ers It is almost ex actly the language described in item 5 above the only di erence is that the underscore is included among the letters

In Example 3 3 we were able to describe identi ers by giving names to sets of letters and digits and using the language operators union concatenation and closure This process is so useful that a notation called *regular expressions* has come into common use for describing all the languages that can be built from these operators applied to the symbols of some alphabet In this notation if *letter_* is established to stand for any letter or the underscore and *digit* is established to stand for any digit then we could describe the language of C identi ers by

$$\mathit{letter\_} \quad \mathit{letter\_} \mid \mathit{digit}$$

The vertical bar above means union the parentheses are used to group subex pressions the star means zero or more occurrences of and the juxtaposition of *letter_* with the remainder of the expression signi es concatenation

The regular expressions are built recursively out of smaller regular expres sions using the rules described below Each regular expression r denotes a language $L\ r$ which is also de ned recursively from the languages denoted by r s subexpressions Here are the rules that de ne the regular expressions over some alphabet and the languages that those expressions denote

BASIS There are two rules that form the basis

1 is a regular expression and L is $\{ \}$ that is the language whose sole member is the empty string

2 If a is a symbol in then $\mathbf{a}$ is a regular expression and $L\ \mathbf{a}$ $\{a\}$ that is the language with one string of length one with a in its one position Note that by convention we use italics for symbols and boldface for their corresponding regular expression [1]

INDUCTION There are four parts to the induction whereby larger regular expressions are built from smaller ones Suppose r and s are regular expressions denoting languages $L\ r$ and $L\ s$ respectively

1 $r \mid s$ is a regular expression denoting the language $L\ r \quad L\ s$

2 $r\ \ s$ is a regular expression denoting the language $L\ r\ L\ s$

3 r is a regular expression denoting $L\ r$

4 r is a regular expression denoting $L\ r$ This last rule says that we can add additional pairs of parentheses around expressions without changing the language they denote

As de ned regular expressions often contain unnecessary pairs of paren theses We may drop certain pairs of parentheses if we adopt the conventions that

a The unary operator has highest precedence and is left associative

b Concatenation has second highest precedence and is left associative

[1]However when talking about speci c characters from the ASCII character set we shall generally use teletype font for both the character and its regular expression

c | has lowest precedence and is left associative

Under these conventions for example we may replace the regular expression **a** | **b** **c** by **a**|**b** **c** Both expressions denote the set of strings that are either a single a or are zero or more b s followed by one c

Example 3 4 Let $\{a\ b\}$

1 The regular expression **a**|**b** denotes the language $\{a\ b\}$

2 **a**|**b** **a**|**b** denotes $\{aa\ ab\ ba\ bb\}$ the language of all strings of length two over the alphabet Another regular expression for the same language is **aa**|**ab**|**ba**|**bb**

3 **a** denotes the language consisting of all strings of zero or more a s that is $\{\ a\ aa\ aaa\ \}$

4 **a**|**b** denotes the set of all strings consisting of zero or more instances of a or b that is all strings of a s and b s $\{\ a\ b\ aa\ ab\ ba\ bb\ aaa\ \}$ Another regular expression for the same language is **a b**

5 **a**|**a b** denotes the language $\{a\ b\ ab\ aab\ aaab\ \}$ that is the string a and all strings consisting of zero or more a s and ending in b

□

A language that can be de ned by a regular expression is called a *regular set* If two regular expressions r and s denote the same regular set we say they are *equivalent* and write r s For instance **a**|**b** **b**|**a** There are a number of algebraic laws for regular expressions each law asserts that expressions of two di erent forms are equivalent Figure 3 7 shows some of the algebraic laws that hold for arbitrary regular expressions r s and t

LAW	DESCRIPTION
$r\|s \quad s\|r$	| is commutative
$r\|\ s\|t \quad r\|s\ \|t$	| is associative
$r\ st \quad rs\ t$	Concatenation is associative
$r\ s\|t \quad rs\|rt \quad s\|t\ r \quad sr\|tr$	Concatenation distributes over |
$r \quad r \quad r$	is the identity for concatenation
$r \quad r\|$	is guaranteed in a closure
$r \quad r$	is idempotent

Figure 3 7 Algebraic laws for regular expressions

3 3 4 Regular De nitions

For notational convenience we may wish to give names to certain regular ex pressions and use those names in subsequent expressions as if the names were themselves symbols If is an alphabet of basic symbols then a *regular de nition* is a sequence of de nitions of the form

$$\begin{array}{ll} d_1 & r_1 \\ d_2 & r_2 \\ & \\ d_n & r_n \end{array}$$

where

1 Each d_i is a new symbol not in and not the same as any other of the d s and

2 Each r_i is a regular expression over the alphabet $\{d_1 \; d_2 \quad d_{i \; 1}\}$

By restricting r_i to and the previously de ned d s we avoid recursive de ni tions and we can construct a regular expression over alone for each r_i We do so by rst replacing uses of d_1 in r_2 which cannot use any of the d s except for d_1 then replacing uses of d_1 and d_2 in r_3 by r_1 and the substituted r_2 and so on Finally in r_n we replace each d_i for i 1 2 n 1 by the substituted version of r_i each of which has only symbols of

Example 3 5 C identi ers are strings of letters digits and underscores Here is a regular de nition for the language of C identi ers We shall conventionally use italics for the symbols de ned in regular de nitions

$$\begin{array}{rl} \mathit{letter\_} & \mathtt{A} \mid \mathtt{B} \mid \quad \mid \mathtt{Z} \mid \mathtt{a} \mid \mathtt{b} \mid \quad \mid \mathtt{z} \mid \mathtt{\_} \\ \mathit{digit} & \mathtt{0} \mid \mathtt{1} \mid \quad \mid \mathtt{9} \\ \mathit{id} & \mathit{letter\_} \quad \mathit{letter\_} \mid \mathit{digit} \end{array}$$

□

Example 3 6 Unsigned numbers integer or oating point are strings such as `5280 0 01234 6 336E4` or `1 89E 4` The regular de nition

$$\begin{array}{rl} \mathit{digit} & \mathtt{0} \mid \mathtt{1} \mid \quad \mid \mathtt{9} \\ \mathit{digits} & \mathit{digit} \; \mathit{digit} \\ \mathit{optionalFraction} & \quad \mathit{digits} \mid \\ \mathit{optionalExponent} & \quad \mathtt{E} \quad \mid \quad \mid \quad \mathit{digits} \quad \mid \\ \mathit{number} & \mathit{digits} \; \mathit{optionalFraction} \; \mathit{optionalExponent} \end{array}$$

is a precise speci cation for this set of strings That is an *optionalFraction* is either a decimal point dot followed by one or more digits or it is missing the empty string An *optionalExponent* if not missing is the letter E followed by an optional or sign followed by one or more digits Note that at least one digit must follow the dot so *number* does not match `1` but does match `1 0`

□

3 3 5 Extensions of Regular Expressions

Since Kleene introduced regular expressions with the basic operators for union concatenation and Kleene closure in the 1950s many extensions have been added to regular expressions to enhance their ability to specify string patterns Here we mention a few notational extensions that were rst incorporated into Unix utilities such as `Lex` that are particularly useful in the speci cation lexical analyzers The references to this chapter contain a discussion of some regular expression variants in use today

1 *One or more instances* The unary post x operator represents the positive closure of a regular expression and its language That is if r is a regular expression then r denotes the language L r The operator has the same precedence and associativity as the operator Two useful algebraic laws r r | and r rr r r relate the Kleene closure and positive closure

2 *Zero or one instance* The unary post x operator means zero or one occurrence That is r is equivalent to $r|$ or put another way L r L r $\{ \}$ The operator has the same precedence and associativity as and

3 *Character classes* A regular expression $a_1|a_2|$ $|a_n$ where the a_i s are each symbols of the alphabet can be replaced by the shorthand a_1a_2 a_n More importantly when a_1 a_2 a_n form a logical se quence e g consecutive uppercase letters lowercase letters or digits we can replace them by a_1 a_n that is just the rst and last separated by a hyphen Thus **abc** is shorthand for **a**|**b**|**c** and **a z** is shorthand for **a**|**b**| |**z**

Example 3 7 Using these shorthands we can rewrite the regular de nition of Example 3 5 as

$$\begin{array}{rl} letter\_ & \texttt{A Za z\_} \\ digit & \texttt{0 9} \\ id & letter\_ \quad letter\_ \mid digit \end{array}$$

The regular de nition of Example 3 6 can also be simpli ed

$$\begin{array}{rl} digit & \texttt{0 9} \\ digits & digit \\ number & digits \quad digits \quad \texttt{E} \quad digits \end{array}$$

□

3 3 6 Exercises for Section 3 3

Exercise 3 3 1 Consult the language reference manuals to determine *i* the sets of characters that form the input alphabet excluding those that may only appear in character strings or comments *ii* the lexical form of numerical constants and *iii* the lexical form of identi ers for each of the following languages a C b C c C d Fortran e Java f Lisp g SQL

Exercise 3 3 2 Describe the languages denoted by the following regular ex pressions

a **a a|b a**

b **|a b**

c **a|b a a|b a|b**

d **a ba ba ba**

e **aa|bb ab|ba aa|bb ab|ba aa|bb**

Exercise 3 3 3 In a string of length n how many of the following are there

a Pre xes

b Su xes

c Proper pre xes

d Substrings

e Subsequences

Exercise 3 3 4 Most languages are *case sensitive* so keywords can be written only one way and the regular expressions describing their lexemes are very simple However some languages like SQL are *case insensitive* so a keyword can be written either in lowercase or in uppercase or in any mixture of cases Thus the SQL keyword `SELECT` can also be written `select` `Select` or `sElEcT` for instance Show how to write a regular expression for a keyword in a case insensitive language Illustrate the idea by writing the expression for select in SQL

Exercise 3 3 5 Write regular de nitions for the following languages

a All strings of lowercase letters that contain the ve vowels in order

b All strings of lowercase letters in which the letters are in ascending lexi cographic order

c Comments consisting of a string surrounded by and without an intervening unless it is inside double quotes d All strings of digits with no repeated digits *Hint* Try this problem rst with a few digits such as $\{0\ 1\ 2\}$

e All strings of digits with at most one repeated digit

f All strings of a s and b s with an even number of a s and an odd number of b s

g The set of Chess moves in the informal notation such as `p k4` or `kbp qn`

h All strings of a s and b s that do not contain the substring *abb*

i All strings of a s and b s that do not contain the subsequence *abb*

Exercise 3 3 6 Write character classes for the following sets of characters

a The rst ten letters up to j in either upper or lower case

b The lowercase consonants

c The digits in a hexadecimal number choose either upper or lower case for the digits above 9

d The characters that can appear at the end of a legitimate English sentence e g exclamation point

The following exercises up to and including Exercise 3 3 10 discuss the extended regular expression notation from `Lex` the lexical analyzer generator that we shall discuss extensively in Section 3 5 The extended notation is listed in Fig 3 8

Exercise 3 3 7 Note that these regular expressions give all of the following symbols *operator characters* a special meaning

Their special meaning must be turned o if they are needed to represent them selves in a character string We can do so by quoting the character within a string of length one or more e g the regular expression matches the string We can also get the literal meaning of an operator character by preceding it by a backslash Thus the regular expression also matches the string Write a regular expression that matches the string

Exercise 3 3 8 In `Lex` a *complemented character class* represents any char acter except the ones listed in the character class We denote a complemented class by using as the rst character this symbol caret is not itself part of the class being complemented unless it is listed within the class itself Thus `A Za z` matches any character that is not an uppercase or lowercase letter and represents any character but the caret or newline since newline cannot be in any character class Show that for every regular expression with complemented character classes there is an equivalent regular expression with out complemented character classes

EXPRESSION	MATCHES	EXAMPLE
c	the one non operator character c	`a`
$\backslash c$	character c literally	
s	string s literally	
	any character but newline	`a b`
	beginning of a line	`abc`
	end of a line	`abc`
s	any one of the characters in string s	`abc`
s	any one character not in string s	`abc`
r	zero or more strings matching r	`a`
r	one or more strings matching r	`a`
r	zero or one r	`a`
$r\{m\ n\}$	between m and n occurrences of r	`a{1 5}`
r_1r_2	an r_1 followed by an r_2	`ab`
$r_1 \mid r_2$	an r_1 or an r_2	`a b`
r	same as r	`a b`
$r_1\ r_2$	r_1 when followed by r_2	`abc 123`

Figure 3 8 `Lex` regular expressions

Exercise 3 3 9 The regular expression $r\{m\ n\}$ matches from m to n occur rences of the pattern r For example `a{1 5}` matches a string of one to ve a s Show that for every regular expression containing repetition operators of this form there is an equivalent regular expression without repetition operators

Exercise 3 3 10 The operator matches the left end of a line and matches the right end of a line The operator is also used to introduce complemented character classes but the context always makes it clear which meaning is in tended For example `aeiou` matches any complete line that does not contain a lowercase vowel

a How do you tell which meaning of is intended

b Can you always replace a regular expression using the and operators by an equivalent expression that does not use either of these operators

Exercise 3 3 11 The UNIX shell command `sh` uses the operators in Fig 3 9 in lename expressions to describe sets of le names For example the lename expression `o` matches all le names ending in `o` `sort1` matches all le names of the form `sort1` c where c is any character Show how `sh` lename

EXPRESSION	MATCHES	EXAMPLE
$'s'$	string s literally	
$\backslash c$	character c literally	
	any string	`o`
	any character	`sort1`
s	any character in s	`sort1 cso`

Figure 3 9 Filename expressions used by the shell command `sh`

expressions can be replaced by equivalent regular expressions using only the basic union concatenation and closure operators

Exercise 3 3 12 SQL allows a rudimentary form of patterns in which two characters have special meaning underscore _ stands for any one character and percent sign stands for any string of 0 or more characters In addition the programmer may de ne any character say e to be the escape character so an e preceding _ or another e gives the character that follows its literal meaning Show how to express any SQL pattern as a regular expression given that we know which character is the escape character

3 4 Recognition of Tokens

In the previous section we learned how to express patterns using regular expres sions Now we must study how to take the patterns for all the needed tokens and build a piece of code that examines the input string and nds a pre x that is a lexeme matching one of the patterns Our discussion will make use of the following running example

stmt		**if** *expr* **then** *stmt*
	|	**if** *expr* **then** *stmt* **else** *stmt*
	|	
expr		*term* **relop** *term*
	|	*term*
term		**id**
	|	**number**

Figure 3 10 A grammar for branching statements

Example 3 8 The grammar fragment of Fig 3 10 describes a simple form of branching statements and conditional expressions This syntax is similar to that of the language Pascal in that **then** appears explicitly after conditions

For **relop** we use the comparison operators of languages like Pascal or SQL where is equals and is not equals because it presents an interesting structure of lexemes

The terminals of the grammar which are **if then else relop id** and **number** are the names of tokens as far as the lexical analyzer is concerned The patterns for these tokens are described using regular de nitions as in Fig 3 11 The patterns for *id* and *number* are similar to what we saw in Example 3 7

digit	`0 9`			
digits	*digit*			
number	*digits*	*digits*	`E`	*digits*
letter	`A Za z`			
id	*letter*	*letter* | *digit*		
if	`if`			
then	`then`			
else	`else`			
relop	| | | | |			

Figure 3 11 Patterns for tokens of Example 3 8

For this language the lexical analyzer will recognize the keywords `if` `then` and `else` as well as lexemes that match the patterns for *relop* *id* and *number* To simplify matters we make the common assumption that keywords are also *reserved words* that is they are not identi ers even though their lexemes match the pattern for identi ers

In addition we assign the lexical analyzer the job of stripping out white space by recognizing the token *ws* de ned by

$$ws \quad \textbf{blank} \mid \textbf{tab} \mid \textbf{newline}$$

Here **blank tab** and **newline** are abstract symbols that we use to express the ASCII characters of the same names Token *ws* is di erent from the other tokens in that when we recognize it we do not return it to the parser but rather restart the lexical analysis from the character that follows the whitespace It is the following token that gets returned to the parser

Our goal for the lexical analyzer is summarized in Fig 3 12 That table shows for each lexeme or family of lexemes which token name is returned to the parser and what attribute value as discussed in Section 3 1 3 is returned Note that for the six relational operators symbolic constants `LT` `LE` and so on are used as the attribute value in order to indicate which instance of the token **relop** we have found The particular operator found will in uence the code that is output from the compiler □

LEXEMES	TOKEN NAME	ATTRIBUTE VALUE
Any *ws*		
`if`	**if**	
`then`	**then**	
`else`	**else**	
Any *id*	**id**	Pointer to table entry
Any *number*	**number**	Pointer to table entry
	relop	`LT`
	relop	`LE`
	relop	`EQ`
	relop	`NE`
	relop	`GT`
	relop	`GE`

Figure 3 12 Tokens their patterns and attribute values

3 4 1 Transition Diagrams

As an intermediate step in the construction of a lexical analyzer we rst convert patterns into stylized owcharts called transition diagrams In this section we perform the conversion from regular expression patterns to transition dia grams by hand but in Section 3 6 we shall see that there is a mechanical way to construct these diagrams from collections of regular expressions

Transition diagrams have a collection of nodes or circles called *states* Each state represents a condition that could occur during the process of scanning the input looking for a lexeme that matches one of several patterns We may think of a state as summarizing all we need to know about what characters we have seen between the *lexemeBegin* pointer and the *forward* pointer as in the situation of Fig 3 3

Edges are directed from one state of the transition diagram to another Each edge is *labeled* by a symbol or set of symbols If we are in some state s and the next input symbol is a we look for an edge out of state s labeled by a and perhaps by other symbols as well If we nd such an edge we advance the *forward* pointer and enter the state of the transition diagram to which that edge leads We shall assume that all our transition diagrams are *deterministic* meaning that there is never more than one edge out of a given state with a given symbol among its labels Starting in Section 3 5 we shall relax the condition of determinism making life much easier for the designer of a lexical analyzer although trickier for the implementer Some important conventions about transition diagrams are

1 Certain states are said to be *accepting* or *nal* These states indicate that a lexeme has been found although the actual lexeme may not consist of all positions between the *lexemeBegin* and *forward* pointers We always

indicate an accepting state by a double circle and if there is an action to be taken typically returning a token and an attribute value to the parser we shall attach that action to the accepting state

2 In addition if it is necessary to retract the *forward* pointer one position i e the lexeme does not include the symbol that got us to the accepting state then we shall additionally place a near that accepting state In our example it is never necessary to retract *forward* by more than one position but if it were we could attach any number of s to the accepting state

3 One state is designated the *start state* or *initial state* it is indicated by an edge labeled start entering from nowhere The transition diagram always begins in the start state before any input symbols have been read

Example 3 9 Figure 3 13 is a transition diagram that recognizes the lexemes matching the token **relop** We begin in state 0 the start state If we see as the rst input symbol then among the lexemes that match the pattern for **relop** we can only be looking at or We therefore go to state 1 and look at the next character If it is then we recognize lexeme enter state 2 and return the token **relop** with attribute `LE` the symbolic constant representing this particular comparison operator If in state 1 the next character is then instead we have lexeme and enter state 3 to return an indication that the not equals operator has been found On any other character the lexeme is and we enter state 4 to return that information Note however that state 4 has a to indicate that we must retract the input one position

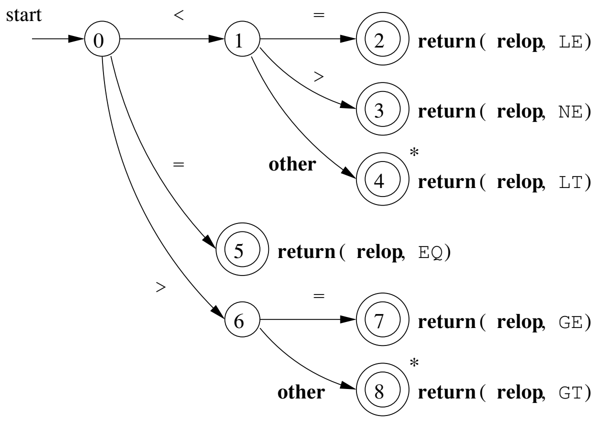


Figure 3 13 Transition diagram for **relop**

On the other hand if in state 0 the rst character we see is then this one character must be the lexeme We immediately return that fact from state 5

The remaining possibility is that the rst character is Then we must enter state 6 and decide on the basis of the next character whether the lexeme is if we next see the sign or just on any other character Note that if in state 0 we see any character besides or we can not possibly be seeing a `relop` lexeme so this transition diagram will not be used □

3 4 2 Recognition of Reserved Words and Identi ers

Recognizing keywords and identi ers presents a problem Usually keywords like `if` or `then` are reserved as they are in our running example so they are not identi ers even though they *look* like identi ers Thus although we typically use a transition diagram like that of Fig 3 14 to search for identi er lexemes this diagram will also recognize the keywords `if then` and `else` of our running example

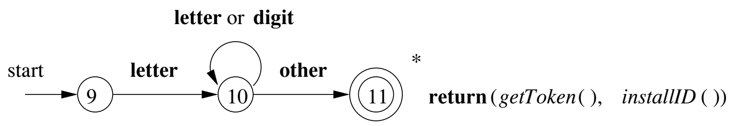


Figure 3 14 A transition diagram for **id** s and keywords

There are two ways that we can handle reserved words that look like iden ti ers

1 Install the reserved words in the symbol table initially A eld of the symbol table entry indicates that these strings are never ordinary identi ers and tells which token they represent We have supposed that this method is in use in Fig 3 14 When we nd an identi er a call to *installID* places it in the symbol table if it is not already there and returns a pointer to the symbol table entry for the lexeme found Of course any identi er not in the symbol table during lexical analysis cannot be a reserved word so its token is **id** The function *getToken* examines the symbol table entry for the lexeme found and returns whatever token name the symbol table says this lexeme represents either **id** or one of the keyword tokens that was initially installed in the table

2 Create separate transition diagrams for each keyword an example for the keyword `then` is shown in Fig 3 15 Note that such a transition diagram consists of states representing the situation after each successive letter of the keyword is seen followed by a test for a nonletter or digit i e any character that cannot be the continuation of an identi er It is necessary to check that the identi er has ended or else we would return token **then** in situations where the correct token was **id** with a lexeme like `thenextvalue` that has `then` as a proper pre x If we adopt this approach then we must prioritize the tokens so that the reserved word

tokens are recognized in preference to **id** when the lexeme matches both patterns We *do not* use this approach in our example which is why the states in Fig 3 15 are unnumbered

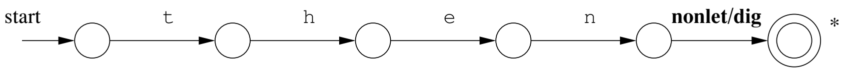


Figure 3 15 Hypothetical transition diagram for the keyword `then`

3 4 3 Completion of the Running Example

The transition diagram for **id** s that we saw in Fig 3 14 has a simple structure Starting in state 9 it checks that the lexeme begins with a letter and goes to state 10 if so We stay in state 10 as long as the input contains letters and digits When we rst encounter anything but a letter or digit we go to state 11 and accept the lexeme found Since the last character is not part of the identi er we must retract the input one position and as discussed in Section 3 4 2 we enter what we have found in the symbol table and determine whether we have a keyword or a true identi er

The transition diagram for token **number** is shown in Fig 3 16 and is so far the most complex diagram we have seen Beginning in state 12 if we see a digit we go to state 13 In that state we can read any number of additional digits However if we see anything but a digit dot or `E` we have seen a number in the form of an integer `123` is an example That case is handled by entering state 20 where we return token **number** and a pointer to a table of constants where the found lexeme is entered These mechanics are not shown on the diagram but are analogous to the way we handled identi ers

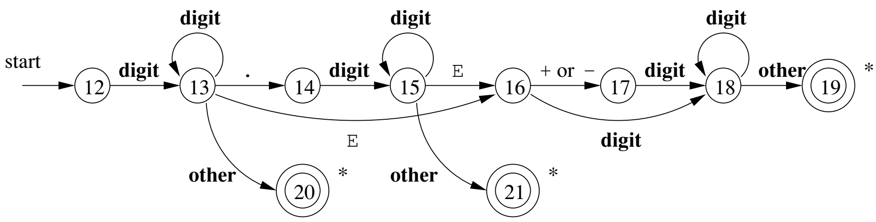


Figure 3 16 A transition diagram for unsigned numbers

If we instead see a dot in state 13 then we have an optional fraction State 14 is entered and we look for one or more additional digits state 15 is used for that purpose If we see an `E` then we have an optional exponent whose recognition is the job of states 16 through 19 Should we in state 15 instead see anything but `E` or a digit then we have come to the end of the fraction there is no exponent and we return the lexeme found via state 21

The nal transition diagram shown in Fig 3 17 is for whitespace In that diagram we look for one or more whitespace characters represented by **delim** in that diagram typically these characters would be blank tab newline and perhaps other characters that are not considered by the language design to be part of any token

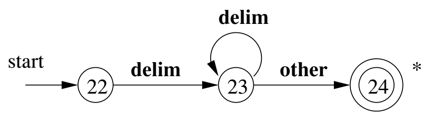


Figure 3 17 A transition diagram for whitespace

Note that in state 24 we have found a block of consecutive whitespace characters followed by a nonwhitespace character We retract the input to begin at the nonwhitespace but we do not return to the parser Rather we must restart the process of lexical analysis after the whitespace

3 4 4 Architecture of a Transition Diagram Based Lexical Analyzer

There are several ways that a collection of transition diagrams can be used to build a lexical analyzer Regardless of the overall strategy each state is represented by a piece of code We may imagine a variable `state` holding the number of the current state for a transition diagram A switch based on the value of `state` takes us to code for each of the possible states where we nd the action of that state Often the code for a state is itself a switch statement or multiway branch that determines the next state by reading and examining the next input character

Example 3 10 In Fig 3 18 we see a sketch of `getRelop` a C function whose job is to simulate the transition diagram of Fig 3 13 and return an object of type `TOKEN` that is a pair consisting of the token name which must be **relop** in this case and an attribute value the code for one of the six comparison operators in this case `getRelop` rst creates a new object `retToken` and initializes its rst component to `RELOP` the symbolic code for token **relop**

We see the typical behavior of a state in case 0 the case where the current state is 0 A function `nextChar` obtains the next character from the input and assigns it to local variable c We then check c for the three characters we expect to nd making the state transition dictated by the transition diagram of Fig 3 13 in each case For example if the next input character is we go to state 5

If the next input character is not one that can begin a comparison operator then a function `fail` is called What `fail` does depends on the global error recovery strategy of the lexical analyzer It should reset the `forward` pointer to `lexemeBegin` in order to allow another transition diagram to be applied to

```
TOKEN getRelop

    TOKEN retToken   new RELOP
    while 1       repeat character processing until a return
                  or failure occurs
        switch state
            case 0  c   nextChar
                    if   c          state   1
                    else if   c          state   5
                    else if   c          state   6
                    else fail       lexeme is not a relop
                    break
            case 1

            case 8  retract
                    retToken attribute   GT
                    return retToken
```

Figure 3 18 Sketch of implementation of **relop** transition diagram

the true beginning of the unprocessed input It might then change the value of `state` to be the start state for another transition diagram which will search for another token Alternatively if there is no other transition diagram that remains unused `fail` could initiate an error correction phase that will try to repair the input and nd a lexeme as discussed in Section 3 1 4

We also show the action for state 8 in Fig 3 18 Because state 8 bears a we must retract the input pointer one position i e put c back on the input stream That task is accomplished by the function `retract` Since state 8 represents the recognition of lexeme we set the second component of the returned object which we suppose is named `attribute` to `GT` the code for this operator □

To place the simulation of one transition diagram in perspective let us consider the ways code like Fig 3 18 could t into the entire lexical analyzer

1 We could arrange for the transition diagrams for each token to be tried se quentially Then the function `fail` of Example 3 10 resets the pointer `forward` and starts the next transition diagram each time it is called This method allows us to use transition diagrams for the individual key words like the one suggested in Fig 3 15 We have only to use these before we use the diagram for **id** in order for the keywords to be reserved words

2 We could run the various transition diagrams in parallel feeding the next input character to all of them and allowing each one to make what ever transitions it required If we use this strategy we must be careful to resolve the case where one diagram nds a lexeme that matches its pattern while one or more other diagrams are still able to process input The normal strategy is to take the longest pre x of the input that matches any pattern That rule allows us to prefer identi er `thenext` to keyword `then` or the operator to for example

3 The preferred approach and the one we shall take up in the following sections is to combine all the transition diagrams into one We allow the transition diagram to read input until there is no possible next state and then take the longest lexeme that matched any pattern as we discussed in item 2 above In our running example this combination is easy because no two tokens can start with the same character i e the rst character immediately tells us which token we are looking for Thus we could simply combine states 0 9 12 and 22 into one start state leaving other transitions intact However in general the problem of combining transition diagrams for several tokens is more complex as we shall see shortly

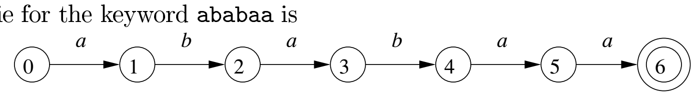
3 4 5 Exercises for Section 3 4

Exercise 3 4 1 Provide transition diagrams to recognize the same languages as each of the regular expressions in Exercise 3 3 2

Exercise 3 4 2 Provide transition diagrams to recognize the same languages as each of the regular expressions in Exercise 3 3 5

The following exercises up to Exercise 3 4 12 introduce the Aho Corasick algorithm for recognizing a collection of keywords in a text string in time pro portional to the length of the text and the sum of the length of the keywords This algorithm uses a special form of transition diagram called a *trie* A trie is a tree structured transition diagram with distinct labels on the edges leading from a node to its children Leaves of the trie represent recognized keywords

Knuth Morris and Pratt presented an algorithm for recognizing a single keyword $b_1 b_2$ b_n in a text string Here the trie is a transition diagram with n 1 states 0 through n State 0 is the initial state and state n represents acceptance that is discovery of the keyword From each state s from 0 through n 1 there is a transition to state s 1 labeled by symbol $b_{s\;1}$ For example the trie for the keyword `ababaa` is



In order to process text strings rapidly and search those strings for a key word it is useful to de ne for keyword $b_1 b_2$ b_n and position s in that keyword corresponding to state s of its trie a *failure function* f s computed as in

Fig 3 19 The objective is that b_1b_2 $b_{f \ s}$ is the longest proper pre x of b_1b_2 b_s that is also a su x of b_1b_2 b_s The reason $f \ s$ is important is that if we are trying to match a text string for b_1b_2 b_n and we have matched the rst s positions but we then fail i e the next position of the text string does not hold $b_{s \ 1}$ then $f \ s$ is the longest pre x of b_1b_2 b_n that could possibly match the text string up to the point we are at Of course the next character of the text string must be $b_{f \ s \ 1}$ or else we still have problems and must consider a yet shorter pre x which will be $b_{f \ f \ s}$

```
1   t  0
2   f 1   0
3   for s  1 s  n s      {
4       while t  0    b_s 1    b_t 1  t   f t
5       if b_s 1    b_t 1 {
6           t  t  1
7           f s  1   t
        }
8       else f s  1   0
    }
```

Figure 3 19 Algorithm to compute the failure function for keyword b_1b_2 b_n

As an example the failure function for the trie constructed above for `ababaa` is

s	1	2	3	4	5	6
$f \ s$	0	0	1	2	3	1

For instance states 3 and 1 represent pre xes `aba` and `a` respectively $f \ 3 \quad 1$ because `a` is the longest proper pre x of `aba` that is also a su x of `aba` Also $f \ 2 \quad 0$ because the longest proper pre x of `ab` that is also a su x is the empty string

Exercise 3 4 3 Construct the failure function for the strings

a `abababaab`

b `aaaaaa`

c `abbaabb`

Exercise 3 4 4 Prove by induction on s that the algorithm of Fig 3 19 correctly computes the failure function

Exercise 3 4 5 Show that the assignment $t \quad f \ t$ in line 4 of Fig 3 19 is executed at most n times Show that therefore the entire algorithm takes only $O \ n$ time on a keyword of length n

Having computed the failure function for a keyword b_1b_2 b_n we can scan a string a_1a_2 a_m in time O m to tell whether the keyword occurs in the string The algorithm shown in Fig 3 20 slides the keyword along the string trying to make progress by matching the next character of the keyword with the next character of the string If it cannot do so after matching s characters then it slides the keyword right s f s positions so only the rst f s characters of the keyword are considered matched with the string

```
1   s  0
2   for i  1 i  m i   {
3         while s  0   a_i   b_s 1  s  f s
4         if a_i   b_s 1  s  s  1
5         if s   n return  yes
    }
6   return  no
```

Figure 3 20 The KMP algorithm tests whether string a_1a_2 a_m contains a single keyword b_1b_2 b_n as a substring in O m n time

Exercise 3 4 6 Apply Algorithm KMP to test whether keyword `ababaa` is a substring of

a `abababaab`

b `abababbaa`

Exercise 3 4 7 Show that the algorithm of Fig 3 20 correctly tells whether the keyword is a substring of the given string *Hint* proceed by induction on i Show that for all i the value of s after line 4 is the length of the longest pre x of the keyword that is a su x of a_1a_2 a_i

Exercise 3 4 8 Show that the algorithm of Fig 3 20 runs in time O m n assuming that function f is already computed and its values stored in an array indexed by s

Exercise 3 4 9 The *Fibonacci strings* are de ned as follows

1 s_1 `b`

2 s_2 `a`

3 s_k s_k $_1s_k$ $_2$ for k 2

For example s_3 `ab` s_4 `aba` and s_5 `abaab`

a What is the length of s_n

b Construct the failure function for s_6

c Construct the failure function for s_7

d Show that the failure function for any s_n can be expressed by f 1 f 2 0 and for 2 j $|s_n|$ f j is j $|s_{k\ 1}|$ where k is the largest integer such that $|s_k|$ j 1

e In the KMP algorithm what is the largest number of consecutive applica tions of the failure function when we try to determine whether keyword s_k appears in text string $s_{k\ 1}$

Aho and Corasick generalized the KMP algorithm to recognize any of a set of keywords in a text string In this case the trie is a true tree with branching from the root There is one state for every string that is a pre x not necessarily proper of any keyword The parent of a state corresponding to string $b_1 b_2$ b_k is the state that corresponds to $b_1 b_2$ $b_{k\ 1}$ A state is accepting if it corresponds to a complete keyword For example Fig 3 21 shows the trie for the keywords `he` `she` `his` and `hers`

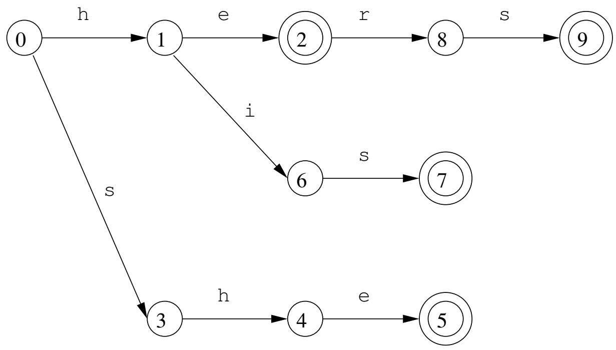


Figure 3 21 Trie for keywords `he` `she` `his` `hers`

The failure function for the general trie is de ned as follows Suppose s is the state that corresponds to string $b_1 b_2$ b_n Then f s is the state that corresponds to the longest proper su x of $b_1 b_2$ b_n that is also a pre x of *some* keyword For example the failure function for the trie of Fig 3 21 is

s	1	2	3	4	5	6	7	8	9
f s	0	0	0	1	2	0	3	0	3

Exercise 3 4 10 Modify the algorithm of Fig 3 19 to compute the failure function for general tries *Hint* The major di erence is that we cannot simply test for equality or inequality of $b_{s\ 1}$ and $b_{t\ 1}$ in lines 4 and 5 of Fig 3 19 Rather from any state there may be several transitions out on several charac ters as there are transitions on both `e` and `i` from state 1 in Fig 3 21 Any of those transitions could lead to a state that represents the longest su x that is also a pre x

Exercise 3 4 11 Construct the tries and compute the failure function for the following sets of keywords

a `aaa` `abaaa` and `ababaaa`

b `all` `fall` `fatal` `llama` and `lame`

c `pipe` `pet` `item` `temper` and `perpetual`

Exercise 3 4 12 Show that your algorithm from Exercise 3 4 10 still runs in time that is linear in the sum of the lengths of the keywords

3 5 The Lexical Analyzer Generator `Lex`

In this section we introduce a tool called `Lex` or in a more recent implementation `Flex` that allows one to specify a lexical analyzer by specifying regular expressions to describe patterns for tokens The input notation for the `Lex` tool is referred to as the *Lex language* and the tool itself is the *Lex compiler* Behind the scenes the Lex compiler transforms the input patterns into a transition diagram and generates code in a le called `lex yy c` that simulates this transition diagram The mechanics of how this translation from regular expressions to transition diagrams occurs is the subject of the next sections here we only learn the Lex language

3 5 1 Use of `Lex`

Figure 3 22 suggests how `Lex` is used An input le which we call `lex l` is written in the Lex language and describes the lexical analyzer to be generated The Lex compiler transforms `lex l` to a C program in a le that is always named `lex yy c` The latter le is compiled by the C compiler into a le called `a out` as always The C compiler output is a working lexical analyzer that can take a stream of input characters and produce a stream of tokens

The normal use of the compiled C program referred to as `a out` in Fig 3 22 is as a subroutine of the parser It is a C function that returns an integer which is a code for one of the possible token names The attribute value whether it be another numeric code a pointer to the symbol table or nothing is placed in a global variable `yylval` [2] which is shared between the lexical analyzer and parser thereby making it simple to return both the name and an attribute value of a token

[2]Incidentally the `yy` that appears in `yylval` and `lex yy c` refers to the `Yacc` parser generator which we shall describe in Section 4 9 and which is commonly used in conjunction with `Lex`

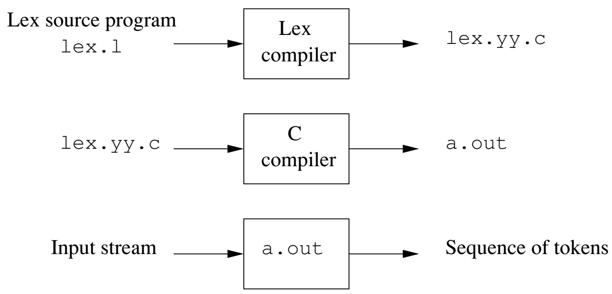


Figure 3 22 Creating a lexical analyzer with `Lex`

3 5 2 Structure of `Lex` Programs

A `Lex` program has the following form

declarations

translation rules

auxiliary functions

The declarations section includes declarations of variables *manifest constants* identi ers declared to stand for a constant e g the name of a token and regular de nitions in the style of Section 3 3 4

The translation rules each have the form

Pattern { Action }

Each pattern is a regular expression which may use the regular de nitions of the declaration section The actions are fragments of code typically written in C although many variants of `Lex` using other languages have been created

The third section holds whatever additional functions are used in the actions Alternatively these functions can be compiled separately and loaded with the lexical analyzer

The lexical analyzer created by `Lex` behaves in concert with the parser as follows When called by the parser the lexical analyzer begins reading its remaining input one character at a time until it nds the longest pre x of the input that matches one of the patterns P_i It then executes the associated action A_i Typically A_i will return to the parser but if it does not e g because P_i describes whitespace or comments then the lexical analyzer proceeds to nd additional lexemes until one of the corresponding actions causes a return to the parser The lexical analyzer returns a single value the token name to the parser but uses the shared integer variable `yylval` to pass additional information about the lexeme found if needed

Example 3 11 Figure 3 23 is a `Lex` program that recognizes the tokens of Fig 3 12 and returns the token found A few observations about this code will introduce us to many of the important features of `Lex`

In the declarations section we see a pair of special brackets and Anything within these brackets is copied directly to the le `lex yy c` and is not treated as a regular de nition It is common to place there the de nitions of the manifest constants using C `define` statements to associate unique integer codes with each of the manifest constants In our example we have listed in a comment the names of the manifest constants `LT IF` and so on but have not shown them de ned to be particular integers [3]

Also in the declarations section is a sequence of regular de nitions These use the extended notation for regular expressions described in Section 3 3 5 Regular de nitions that are used in later de nitions or in the patterns of the translation rules are surrounded by curly braces Thus for instance *delim* is de ned to be a shorthand for the character class consisting of the blank the tab and the newline the latter two are represented as in all UNIX commands by backslash followed by `t` or `n` respectively Then *ws* is de ned to be one or more delimiters by the regular expression `delim`

Notice that in the de nition of *id* and *number* parentheses are used as grouping metasymbols and do not stand for themselves In contrast `E` in the de nition of *number* stands for itself If we wish to use one of the `Lex` meta symbols such as any of the parentheses or to stand for themselves we may precede them with a backslash For instance we see in the de nition of *number* to represent the dot since that character is a metasymbol representing any character as usual in UNIX regular expressions

In the auxiliary function section we see two such functions `installID` and `installNum` Like the portion of the declaration section that appears between everything in the auxiliary section is copied directly to le `lex yy c` but may be used in the actions

Finally let us examine some of the patterns and rules in the middle section of Fig 3 23 First *ws* an identi er declared in the rst section has an associated empty action If we nd whitespace we do not return to the parser but look for another lexeme The second token has the simple regular expression pattern `if` Should we see the two letters `if` on the input and they are not followed by another letter or digit which would cause the lexical analyzer to nd a longer pre x of the input matching the pattern for **id** then the lexical analyzer consumes these two letters from the input and returns the token name `IF` that is the integer for which the manifest constant `IF` stands Keywords `then` and `else` are treated similarly

The fth token has the pattern de ned by *id* Note that although keywords like `if` match this pattern as well as an earlier pattern `Lex` chooses whichever

[3]If `Lex` is used along with `Yacc` then it would be normal to de ne the manifest constants in the `Yacc` program and use them without de nition in the `Lex` program Since `lex yy c` is compiled with the `Yacc` output the constants thus will be available to the actions in the `Lex` program

```
   definitions of manifest constants
 LT  LE  EQ  NE  GT  GE
 IF  THEN  ELSE  ID  NUMBER  RELOP


  regular definitions
delim        t n
ws          delim
letter      A Za z
digit       0 9
id          letter   letter   digit
number      digit      digit     E      digit



 ws            no action and no return
if          return IF
then        return THEN
else        return ELSE
 id         yylval    int  installID    return ID
 number     yylval    int  installNum    return NUMBER
            yylval   LT  return RELOP
            yylval   LE  return RELOP
            yylval   EQ  return RELOP
            yylval   NE  return RELOP
            yylval   GT  return RELOP
            yylval   GE  return RELOP



int installID       function to install the lexeme  whose
                    first character is pointed to by yytext
                    and whose length is yyleng  into the
                    symbol table and return a pointer
                    thereto


int installNum       similar to installID  but puts numer
                     ical constants into a separate table
```

Figure 3 23 Lex program for the tokens of Fig 3 12 pattern is listed rst in situations where the longest matching pre x matches two or more patterns The action taken when *id* is matched is threefold

1 Function `installID` is called to place the lexeme found in the symbol table

2 This function returns a pointer to the symbol table which is placed in global variable `yylval` where it can be used by the parser or a later component of the compiler Note that `installID` has available to it two variables that are set automatically by the lexical analyzer that `Lex` generates

   a `yytext` is a pointer to the beginning of the lexeme analogous to `lexemeBegin` in Fig 3 3

   b `yyleng` is the length of the lexeme found

3 The token name `ID` is returned to the parser

The action taken when a lexeme matching the pattern *number* is similar using the auxiliary function `installNum` □

3 5 3 Con ict Resolution in `Lex`

We have alluded to the two rules that `Lex` uses to decide on the proper lexeme to select when several pre xes of the input match one or more patterns

1 Always prefer a longer pre x to a shorter pre x

2 If the longest possible pre x matches two or more patterns prefer the pattern listed rst in the `Lex` program

Example 3 12 The rst rule tells us to continue reading letters and digits to nd the longest pre x of these characters to group as an identi er It also tells us to treat as a single lexeme rather than selecting as one lexeme and as the next lexeme The second rule makes keywords reserved if we list the keywords before **id** in the program For instance if **then** is determined to be the longest pre x of the input that matches any pattern and the pattern **then** precedes `id` as it does in Fig 3 23 then the token `THEN` is returned rather than `ID` □

3 5 4 The Lookahead Operator

`Lex` automatically reads one character ahead of the last character that forms the selected lexeme and then retracts the input so only the lexeme itself is consumed from the input However sometimes we want a certain pattern to be matched to the input only when it is followed by a certain other characters If so we may use the slash in a pattern to indicate the end of the part of the pattern that matches the lexeme What follows is additional pattern that must be matched before we can decide that the token in question was seen but what matches this second pattern is not part of the lexeme

Example 3 13 In Fortran and some other languages keywords are not re served That situation creates problems such as a statement

```
IF I J    3
```

where IF is the name of an array not a keyword This statement contrasts with statements of the form

```
IF   condition   THEN
```

where IF is a keyword Fortunately we can be sure that the keyword IF is always followed by a left parenthesis some text the condition that may contain parentheses a right parenthesis and a letter Thus we could write a Lex rule for the keyword IF like

```
IF              letter
```

This rule says that the pattern the lexeme matches is just the two letters IF The slash says that additional pattern follows but does not match the lexeme In this pattern the rst character is the left parentheses Since that character is a Lex metasymbol it must be preceded by a backslash to indicate that it has its literal meaning The dot and star match any string without a newline Note that the dot is a Lex metasymbol meaning any character except newline It is followed by a right parenthesis again with a backslash to give that character its literal meaning The additional pattern is followed by the symbol *letter* which is a regular de nition representing the character class of all letters

Note that in order for this pattern to be foolproof we must preprocess the input to delete whitespace We have in the pattern neither provision for whitespace nor can we deal with the possibility that the condition extends over lines since the dot will not match a newline character

For instance suppose this pattern is asked to match a pre x of input

```
IF A   B C   D THEN
```

the rst two characters match IF the next character matches the next nine characters match and the next two match and *letter* Note the fact that the rst right parenthesis after C is not followed by a letter is irrelevant we only need to nd some way of matching the input to the pattern We conclude that the letters IF constitute the lexeme and they are an instance of token **if** □

3 5 5 Exercises for Section 3 5

Exercise 3 5 1 Describe how to make the following modi cations to the Lex program of Fig 3 23

a Add the keyword while

b Change the comparison operators to be the C operators of that kind

c Allow the underscore _ as an additional letter

d Add a new pattern with token STRING The pattern consists of a double quote any string of characters and a nal double quote However if a double quote appears in the string it must be escaped by preceding it with a backslash and therefore a backslash in the string must be represented by two backslashes The lexical value which is the string without the surrounding double quotes and with backslashes used to es cape a character removed Strings are to be installed in a table of strings

Exercise 3 5 2 Write a Lex program that copies a le replacing each non empty sequence of white space by a single blank

Exercise 3 5 3 Write a Lex program that copies a C program replacing each instance of the keyword float by double

Exercise 3 5 4 Write a Lex program that converts a le to Pig latin Speci cally assume the le is a sequence of words groups of letters separated by whitespace Every time you encounter a word

1 If the rst letter is a consonant move it to the end of the word and then add ay

2 If the rst letter is a vowel just add ay to the end of the word

All nonletters are copied intact to the output

Exercise 3 5 5 In SQL keywords and identi ers are case insensitive Write a Lex program that recognizes the keywords SELECT FROM and WHERE in any combination of capital and lower case letters and token ID which for the purposes of this exercise you may take to be any sequence of letters and digits beginning with a letter You need not install identi ers in a symbol table but tell how the install function would di er from that described for case sensitive identi ers as in Fig 3 23