# Chapter 4

# Syntax Analysis

This chapter is devoted to parsing methods that are typically used in compilers We first present the basic concepts then techniques suitable for hand implementation and finally algorithms that have been used in automated tools Since programs may contain syntactic errors we discuss extensions of the parsing methods for recovery from common errors

By design every programming language has precise rules that prescribe the syntactic structure of well formed programs In C for example a program is made up of functions a function out of declarations and statements a statement out of expressions and so on The syntax of programming language constructs can be specified by context free grammars or BNF Backus Naur Form notation introduced in Section 2 2 Grammars offer significant benefits for both language designers and compiler writers

A grammar gives a precise yet easy to understand syntactic specification of a programming language

From certain classes of grammars we can construct automatically an efficient parser that determines the syntactic structure of a source program As a side benefit the parser construction process can reveal syntactic ambiguities and trouble spots that might have slipped through the initial design phase of a language

The structure imparted to a language by a properly designed grammar is useful for translating source programs into correct object code and for detecting errors

A grammar allows a language to be evolved or developed iteratively by adding new constructs to perform new tasks These new constructs can be integrated more easily into an implementation that follows the grammatical structure of the language

From Chapter 4 of *Compilers: Principles, Techniques, and Tools*, Second Edition. Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. Copyright © 2007 by Pearson Education, Inc. All rights reserved.

# 4 1 Introduction

In this section we examine the way the parser fits into a typical compiler We then look at typical grammars for arithmetic expressions Grammars for expressions suffice for illustrating the essence of parsing since parsing techniques for expressions carry over to most programming constructs This section ends with a discussion of error handling since the parser must respond gracefully to finding that its input cannot be generated by its grammar

## 4 1 1 The Role of the Parser

In our compiler model the parser obtains a string of tokens from the lexical analyzer as shown in Fig 4 1 and verifies that the string of token names can be generated by the grammar for the source language We expect the parser to report any syntax errors in an intelligible fashion and to recover from commonly occurring errors to continue processing the remainder of the program Conceptually for well formed programs the parser constructs a parse tree and passes it to the rest of the compiler for further processing In fact the parse tree need not be constructed explicitly since checking and translation actions can be interspersed with parsing as we shall see Thus the parser and the rest of the front end could well be implemented by a single module

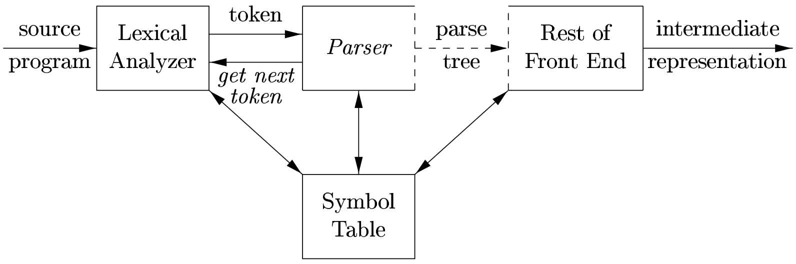


Figure 4 1 Position of parser in compiler model

There are three general types of parsers for grammars universal top down and bottom up Universal parsing methods such as the Cocke Younger Kasami algorithm and Earley s algorithm can parse any grammar see the bibliographic notes These general methods are however too inefficient to use in production compilers

The methods commonly used in compilers can be classified as being either top down or bottom up As implied by their names top down methods build parse trees from the top root to the bottom leaves while bottom up methods start from the leaves and work their way up to the root In either case the input to the parser is scanned from left to right one symbol at a time

The most efficient top down and bottom up methods work only for subclasses of grammars but several of these classes particularly LL and LR grammars are expressive enough to describe most of the syntactic constructs in modern programming languages Parsers implemented by hand often use LL grammars for example the predictive parsing approach of Section 2 4 2 works for LL grammars Parsers for the larger class of LR grammars are usually constructed using automated tools

In this chapter we assume that the output of the parser is some representation of the parse tree for the stream of tokens that comes from the lexical analyzer In practice there are a number of tasks that might be conducted during parsing such as collecting information about various tokens into the symbol table performing type checking and other kinds of semantic analysis and generating intermediate code We have lumped all of these activities into the rest of the front end box in Fig 4 1 These activities will be covered in detail in subsequent chapters

### 4 1 2 Representative Grammars

Some of the grammars that will be examined in this chapter are presented here for ease of reference Constructs that begin with keywords like **while** or **int** are relatively easy to parse because the keyword guides the choice of the grammar production that must be applied to match the input We therefore concentrate on expressions which present more of challenge because of the associativity and precedence of operators

Associativity and precedence are captured in the following grammar which is similar to ones used in Chapter 2 for describing expressions terms and factors $E$ represents expressions consisting of terms separated by signs $T$ represents terms consisting of factors separated by signs and $F$ represents factors that can be either parenthesized expressions or identifiers

$$\begin{array}{lll} E & E \quad T \mid T & \\ T & T \quad F \mid F & 4\ 1 \\ F & \quad E \quad \mid \mathbf{id} & \end{array}$$

Expression grammar 4 1 belongs to the class of LR grammars that are suitable for bottom up parsing This grammar can be adapted to handle additional operators and additional levels of precedence However it cannot be used for top down parsing because it is left recursive

The following non left recursive variant of the expression grammar 4 1 will be used for top down parsing

$$\begin{array}{lll} E & T\,E' & \\ E' & \quad T\,E' \mid & \\ T & F\,T' & 4\ 2 \\ T' & \quad F\,T' \mid & \\ F & \quad E \quad \mid \mathbf{id} & \end{array}$$

The following grammar treats and alike so it is useful for illustrating techniques for handling ambiguities during parsing

$$E \qquad E \quad E \mid E \quad E \mid \quad E \quad \mid \textbf{id} \qquad 4\;3$$

Here $E$ represents expressions of all types Grammar 4 3 permits more than one parse tree for expressions like $a$ $b$ $c$

### 4 1 3 Syntax Error Handling

The remainder of this section considers the nature of syntactic errors and general strategies for error recovery Two of these strategies called panic mode and phrase level recovery are discussed in more detail in connection with specific parsing methods

If a compiler had to process only correct programs its design and implementation would be simplified greatly However a compiler is expected to assist the programmer in locating and tracking down errors that inevitably creep into programs despite the programmer s best efforts Strikingly few languages have been designed with error handling in mind even though errors are so commonplace Our civilization would be radically different if spoken languages had the same requirements for syntactic accuracy as computer languages Most programming language specifications do not describe how a compiler should respond to errors error handling is left to the compiler designer Planning the error handling right from the start can both simplify the structure of a compiler and improve its handling of errors

Common programming errors can occur at many different levels

*Lexical* errors include misspellings of identifiers keywords or operators e g the use of an identifier `elipseSize` instead of `ellipseSize` and missing quotes around text intended as a string

*Syntactic* errors include misplaced semicolons or extra or missing braces that is or As another example in C or Java the appearance of a `case` statement without an enclosing `switch` is a syntactic error however this situation is usually allowed by the parser and caught later in the processing as the compiler attempts to generate code

*Semantic* errors include type mismatches between operators and operands e g the return of a value in a Java method with result type `void`

*Logical* errors can be anything from incorrect reasoning on the part of the programmer to the use in a C program of the assignment operator instead of the comparison operator The program containing may be well formed however it may not reflect the programmer s intent

The precision of parsing methods allows syntactic errors to be detected very efficiently Several parsing methods such as the LL and LR methods detect an error as soon as possible that is when the stream of tokens from the lexical analyzer cannot be parsed further according to the grammar for the language More precisely they have the *viable prefix property* meaning that they detect that an error has occurred as soon as they see a prefix of the input that cannot be completed to form a string in the language

Another reason for emphasizing error recovery during parsing is that many errors appear syntactic whatever their cause and are exposed when parsing cannot continue A few semantic errors such as type mismatches can also be detected efficiently however accurate detection of semantic and logical errors at compile time is in general a difficult task

The error handler in a parser has goals that are simple to state but challenging to realize

- Report the presence of errors clearly and accurately
- Recover from each error quickly enough to detect subsequent errors
- Add minimal overhead to the processing of correct programs

Fortunately common errors are simple ones and a relatively straightforward error handling mechanism often suffices

How should an error handler report the presence of an error At the very least it must report the place in the source program where an error is detected because there is a good chance that the actual error occurred within the previous few tokens A common strategy is to print the offending line with a pointer to the position at which an error is detected

### 4 1 4 Error Recovery Strategies

Once an error is detected how should the parser recover Although no strategy has proven itself universally acceptable a few methods have broad applicability The simplest approach is for the parser to quit with an informative error message when it detects the first error Additional errors are often uncovered if the parser can restore itself to a state where processing of the input can continue with reasonable hopes that the further processing will provide meaningful diagnostic information If errors pile up it is better for the compiler to give up after exceeding some error limit than to produce an annoying avalanche of spurious errors

The balance of this section is devoted to the following recovery strategies panic mode phrase level error productions and global correction

#### Panic Mode Recovery

With this method on discovering an error the parser discards input symbols one at a time until one of a designated set of *synchronizing tokens* is found The synchronizing tokens are usually delimiters such as semicolon or whose role in the source program is clear and unambiguous The compiler designer must select the synchronizing tokens appropriate for the source language While panic mode correction often skips a considerable amount of input without checking it for additional errors it has the advantage of simplicity and unlike some methods to be considered later is guaranteed not to go into an infinite loop

### Phrase Level Recovery

On discovering an error a parser may perform local correction on the remaining input that is it may replace a prefix of the remaining input by some string that allows the parser to continue A typical local correction is to replace a comma by a semicolon delete an extraneous semicolon or insert a missing semicolon The choice of the local correction is left to the compiler designer Of course we must be careful to choose replacements that do not lead to infinite loops as would be the case for example if we always inserted something on the input ahead of the current input symbol

Phrase level replacement has been used in several error repairing compilers as it can correct any input string Its major drawback is the difficulty it has in coping with situations in which the actual error has occurred before the point of detection

### Error Productions

By anticipating common errors that might be encountered we can augment the grammar for the language at hand with productions that generate the erroneous constructs A parser constructed from a grammar augmented by these error productions detects the anticipated errors when an error production is used during parsing The parser can then generate appropriate error diagnostics about the erroneous construct that has been recognized in the input

### Global Correction

Ideally we would like a compiler to make as few changes as possible in processing an incorrect input string There are algorithms for choosing a minimal sequence of changes to obtain a globally least cost correction Given an incorrect input string $x$ and grammar $G$ these algorithms will find a parse tree for a related string $y$ such that the number of insertions deletions and changes of tokens required to transform $x$ into $y$ is as small as possible Unfortunately these methods are in general too costly to implement in terms of time and space so these techniques are currently only of theoretical interest

Do note that a closest correct program may not be what the programmer had in mind Nevertheless the notion of least cost correction provides a yardstick for evaluating error recovery techniques and has been used for finding optimal replacement strings for phrase level recovery

# 4 2 Context Free Grammars

Grammars were introduced in Section 2 2 to systematically describe the syntax of programming language constructs like expressions and statements Using a syntactic variable *stmt* to denote statements and variable *expr* to denote expressions the production

$$\textit{stmt} \quad \textbf{if} \quad \textit{expr} \quad \textit{stmt}\ \textbf{else}\ \textit{stmt} \qquad 4\ 4$$

specifies the structure of this form of conditional statement Other productions then define precisely what an *expr* is and what else a *stmt* can be

This section reviews the definition of a context free grammar and introduces terminology for talking about parsing In particular the notion of derivations is very helpful for discussing the order in which productions are applied during parsing

## 4 2 1 The Formal Definition of a Context Free Grammar

From Section 2 2 a context free grammar grammar for short consists of terminals nonterminals a start symbol and productions

1 *Terminals* are the basic symbols from which strings are formed The term token name is a synonym for terminal and frequently we will use the word token for terminal when it is clear that we are talking about just the token name We assume that the terminals are the first components of the tokens output by the lexical analyzer In 4 4 the terminals are the keywords **if** and **else** and the symbols and

2 *Nonterminals* are syntactic variables that denote sets of strings In 4 4 *stmt* and *expr* are nonterminals The sets of strings denoted by nonterminals help define the language generated by the grammar Nonterminals impose a hierarchical structure on the language that is key to syntax analysis and translation

3 In a grammar one nonterminal is distinguished as the *start symbol* and the set of strings it denotes is the language generated by the grammar Conventionally the productions for the start symbol are listed first

4 The productions of a grammar specify the manner in which the terminals and nonterminals can be combined to form strings Each *production* consists of

   a A nonterminal called the *head* or *left side* of the production this production defines some of the strings denoted by the head

   b The symbol Sometimes has been used in place of the arrow

   c A *body* or *right side* consisting of zero or more terminals and nonterminals The components of the body describe one way in which strings of the nonterminal at the head can be constructed

**Example 4 5** The grammar in Fig 4 2 defines simple arithmetic expressions In this grammar the terminal symbols are

**id**

The nonterminal symbols are *expression term* and *factor* and *expression* is the start symbol □

$$\begin{array}{rl}
\mathit{expression} & \mathit{expression} \quad \mathit{term} \\
\mathit{expression} & \mathit{expression} \quad \mathit{term} \\
\mathit{expression} & \mathit{term} \\
\mathit{term} & \mathit{term} \quad \mathit{factor} \\
\mathit{term} & \mathit{term} \quad \mathit{factor} \\
\mathit{term} & \mathit{factor} \\
\mathit{factor} & \quad \mathit{expression} \\
\mathit{factor} & \mathbf{id}
\end{array}$$

Figure 4 2 Grammar for simple arithmetic expressions

### 4 2 2 Notational Conventions

To avoid always having to state that these are the terminals these are the nonterminals and so on the following notational conventions for grammars will be used throughout the remainder of this book

1 These symbols are terminals

a Lowercase letters early in the alphabet such as *a b c*

b Operator symbols such as and so on

c Punctuation symbols such as parentheses comma and so on

d The digits 0 1 9

e Boldface strings such as **id** or **if** each of which represents a single terminal symbol

2 These symbols are nonterminals

a Uppercase letters early in the alphabet such as *A B C*

b The letter *S* which when it appears is usually the start symbol

c Lowercase italic names such as *expr* or *stmt*

d When discussing programming constructs uppercase letters may be used to represent nonterminals for the constructs For example nonterminals for expressions terms and factors are often represented by *E T* and *F* respectively

3 Uppercase letters late in the alphabet such as $X$ $Y$ $Z$ represent *grammar symbols* that is either nonterminals or terminals

4 Lowercase letters late in the alphabet chiefly $u$ $v$ $z$ represent possibly empty strings of terminals

5 Lowercase Greek letters for example represent possibly empty strings of grammar symbols Thus a generic production can be written as $A$ where $A$ is the head and the body

6 A set of productions $A \quad {}_1$ $A \quad {}_2$ $A \quad {}_k$ with a common head $A$ call them $A$ *productions* may be written $A \quad {}_1 \mid {}_2 \mid \quad \mid {}_k$ Call ${}_1 \quad {}_2 \quad {}_k$ the *alternatives* for $A$

7 Unless stated otherwise the head of the first production is the start symbol

**Example 4 6** Using these conventions the grammar of Example 4 5 can be rewritten concisely as

$$\begin{array}{lll} E & & E \quad T \mid E \quad T \mid T \\ T & & T \quad F \mid T \quad F \mid F \\ F & & \quad E \quad \mid \mathbf{id} \end{array}$$

The notational conventions tell us that $E$ $T$ and $F$ are nonterminals with $E$ the start symbol The remaining symbols are terminals □

### 4 2 3 Derivations

The construction of a parse tree can be made precise by taking a derivational view in which productions are treated as rewriting rules Beginning with the start symbol each rewriting step replaces a nonterminal by the body of one of its productions This derivational view corresponds to the top down construction of a parse tree but the precision afforded by derivations will be especially helpful when bottom up parsing is discussed As we shall see bottom up parsing is related to a class of derivations known as rightmost derivations in which the rightmost nonterminal is rewritten at each step

For example consider the following grammar with a single nonterminal $E$ which adds a production $E \quad E$ to the grammar 4 3

$$E \quad E \quad E \mid E \quad E \mid \quad E \mid \quad E \quad \mid \mathbf{id} \qquad 4\ 7$$

The production $E \quad E$ signifies that if $E$ denotes an expression then $E$ must also denote an expression The replacement of a single $E$ by $E$ will be described by writing

$$E \quad E$$

which is read $E$ derives $E$ The production $E$ $E$ can be applied to replace any instance of $E$ in any string of grammar symbols by $E$ e g $E$ $E$ $E$ $E$ or $E$ $E$ $E$ $E$ We can take a single $E$ and repeatedly apply productions in any order to get a sequence of replacements For example

$$E \quad E \quad E \quad \mathbf{id}$$

We call such a sequence of replacements a *derivation* of **id** from $E$ This derivation provides a proof that the string **id** is one particular instance of an expression

For a general definition of derivation consider a nonterminal $A$ in the middle of a sequence of grammar symbols as in $A$ where and are arbitrary strings of grammar symbols Suppose $A$ is a production Then we write $A$ The symbol means derives in one step When a sequence of derivation steps $_1$ $_2$ $_n$ rewrites $_1$ to $_n$ we say $_1$ *derives* $_n$ Often we wish to say derives in zero or more steps For this purpose we can use the symbol Thus

1 for any string and

2 If and then

Likewise means derives in one or more steps

If $S$ where $S$ is the start symbol of a grammar $G$ we say that is a *sentential form* of $G$ Note that a sentential form may contain both terminals and nonterminals and may be empty A *sentence* of $G$ is a sentential form with no nonterminals The *language generated by* a grammar is its set of sentences Thus a string of terminals $w$ is in $L$ $G$ the language generated by $G$ if and only if $w$ is a sentence of $G$ or $S$ $w$ A language that can be generated by a grammar is said to be a *context free language* If two grammars generate the same language the grammars are said to be *equivalent*

The string **id** **id** is a sentence of grammar 4 7 because there is a derivation

$$E \quad E \quad E \quad E \; E \quad \mathbf{id} \; E \quad \mathbf{id} \; \mathbf{id} \qquad 4\;8$$

The strings $E$ $E$ $E$ **id** **id** are all sentential forms of this grammar We write $E$ **id** **id** to indicate that **id** **id** can be derived from $E$

At each step in a derivation there are two choices to be made We need to choose which nonterminal to replace and having made this choice we must pick a production with that nonterminal as head For example the following alternative derivation of **id** **id** differs from derivation 4 8 in the last two steps

$$E \quad E \quad E \quad E \; E \quad E \; \mathbf{id} \quad \mathbf{id} \; \mathbf{id} \qquad 4\;9$$

Each nonterminal is replaced by the same body in the two derivations but the order of replacements is different

To understand how parsers work we shall consider derivations in which the nonterminal to be replaced at each step is chosen as follows

1 In *leftmost* derivations the leftmost nonterminal in each sentential is always chosen If is a step in which the leftmost nonterminal in is replaced we write $\underset{lm}{}$

2 In *rightmost* derivations the rightmost nonterminal is always chosen we write $\underset{rm}{}$ in this case

Derivation 4 8 is leftmost so it can be rewritten as

$$E \underset{lm}{} \quad E \underset{lm}{} \quad E \underset{lm}{} \quad E \quad E \underset{lm}{} \quad \textbf{id} \quad E \underset{lm}{} \quad \textbf{id} \quad \textbf{id}$$

Note that 4 9 is a rightmost derivation

Using our notational conventions every leftmost step can be written as $wA \underset{lm}{} w$ where $w$ consists of terminals only $A$ is the production applied and is a string of grammar symbols To emphasize that derives by a leftmost derivation we write $\underset{lm}{}$ If $S \underset{lm}{}$ then we say that is a *left sentential form* of the grammar at hand

Analogous definitions hold for rightmost derivations Rightmost derivations are sometimes called *canonical* derivations

### 4 2 4 Parse Trees and Derivations

A parse tree is a graphical representation of a derivation that filters out the order in which productions are applied to replace nonterminals Each interior node of a parse tree represents the application of a production The interior node is labeled with the nonterminal $A$ in the head of the production the children of the node are labeled from left to right by the symbols in the body of the production by which this $A$ was replaced during the derivation

For example the parse tree for **id** **id** in Fig 4 3 results from the derivation 4 8 as well as derivation 4 9

The leaves of a parse tree are labeled by nonterminals or terminals and read from left to right constitute a sentential form called the *yield* or *frontier* of the tree

To see the relationship between derivations and parse trees consider any derivation $_1$ $_2$ $_n$ where $_1$ is a single nonterminal $A$ For each sentential form $_i$ in the derivation we can construct a parse tree whose yield is $_i$ The process is an induction on $i$

**BASIS** The tree for $_1$ $A$ is a single node labeled $A$

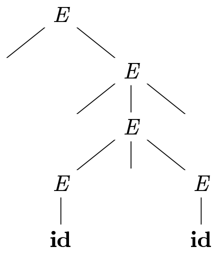


Figure 4 3 Parse tree for id id

**INDUCTION** Suppose we already have constructed a parse tree with yield $_{i\ 1}$ $X_1X_2$ $X_k$ note that according to our notational conventions each grammar symbol $X_i$ is either a nonterminal or a terminal Suppose $_i$ is derived from $_{i\ 1}$ by replacing $X_j$ a nonterminal by $Y_1Y_2$ $Y_m$ That is at the $i$th step of the derivation production $X_j$ is applied to $_{i\ 1}$ to derive $_i$ $X_1X_2$ $X_{j\ 1}$ $X_{j\ 1}$ $X_k$

To model this step of the derivation nd the $j$th non leaf from the left in the current parse tree This leaf is labeled $X_j$ Give this leaf $m$ children labeled $Y_1$ $Y_2$ $Y_m$ from the left As a special case if $m$ 0 then and we give the $j$th leaf one child labeled

**Example 4 10** The sequence of parse trees constructed from the derivation 4 8 is shown in Fig 4 4 In the rst step of the derivation $E$ $E$ To model this step add two children labeled and $E$ to the root $E$ of the initial tree The result is the second tree

In the second step of the derivation $E$ $E$ Consequently add three children labeled $E$ and to the leaf labeled $E$ of the second tree to obtain the third tree with yield $E$ Continuing in this fashion we obtain the complete parse tree as the sixth tree □

Since a parse tree ignores variations in the order in which symbols in senten tial forms are replaced there is a many to one relationship between derivations and parse trees For example both derivations 4 8 and 4 9 are associated with the same nal parse tree of Fig 4 4

In what follows we shall frequently parse by producing a leftmost or a rightmost derivation since there is a one to one relationship between parse trees and either leftmost or rightmost derivations Both leftmost and rightmost derivations pick a particular order for replacing symbols in sentential forms so they too lter out variations in the order It is not hard to show that every parse tree has associated with it a unique leftmost and a unique rightmost derivation

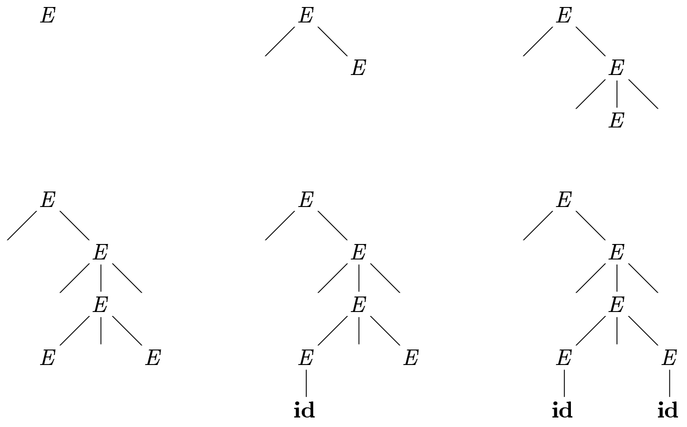


Figure 4 4 Sequence of parse trees for derivation 4 8

### 4 2 5 Ambiguity

From Section 2 2 4 a grammar that produces more than one parse tree for some sentence is said to be *ambiguous* Put another way an ambiguous grammar is one that produces more than one leftmost derivation or more than one rightmost derivation for the same sentence

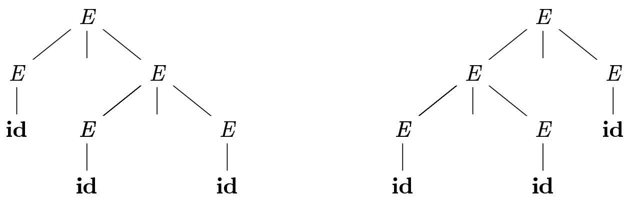
**Example 4 11** The arithmetic expression grammar 4 3 permits two distinct leftmost derivations for the sentence **id id id**

$$\begin{array}{llcll}
E & E \quad E & \qquad & E & E \quad E \\
 & \mathbf{id} \quad E & & & E \quad E \quad E \\
 & \mathbf{id} \quad E \quad E & & & \mathbf{id} \quad E \quad E \\
 & \mathbf{id} \quad \mathbf{id} \quad E & & & \mathbf{id} \quad \mathbf{id} \quad E \\
 & \mathbf{id} \quad \mathbf{id} \quad \mathbf{id} & & & \mathbf{id} \quad \mathbf{id} \quad \mathbf{id}
\end{array}$$

The corresponding parse trees appear in Fig 4 5

Note that the parse tree of Fig 4 5 a reflects the commonly assumed precedence of and while the tree of Fig 4 5 b does not That is it is customary to treat operator as having higher precedence than corresponding to the fact that we would normally evaluate an expression like $a \quad b \quad c$ as $a \quad b \quad c$ rather than as $a \quad b \quad c$ □

For most parsers it is desirable that the grammar be made unambiguous for if it is not we cannot uniquely determine which parse tree to select for a sentence In other cases it is convenient to use carefully chosen ambiguous grammars together with *disambiguating rules* that throw away undesirable parse trees leaving only one tree for each sentence



a

b

Figure 4 5 Two parse trees for **id id id**

## 4 2 6 Verifying the Language Generated by a Grammar

Although compiler designers rarely do so for a complete programming language grammar it is useful to be able to reason that a given set of productions generates a particular language Troublesome constructs can be studied by writing a concise abstract grammar and studying the language that it generates We shall construct such a grammar for conditional statements below

A proof that a grammar $G$ generates a language $L$ has two parts show that every string generated by $G$ is in $L$ and conversely that every string in $L$ can indeed be generated by $G$

**Example 4 12** Consider the following grammar

$$S \qquad S \;\; S \mid \qquad\qquad 4\ 13$$

It may not be initially apparent but this simple grammar generates all strings of balanced parentheses and only such strings To see why we shall show first that every sentence derivable from $S$ is balanced and then that every balanced string is derivable from $S$ To show that every sentence derivable from $S$ is balanced we use an inductive proof on the number of steps $n$ in a derivation

**BASIS** The basis is $n$ 1 The only string of terminals derivable from $S$ in one step is the empty string which surely is balanced

**INDUCTION** Now assume that all derivations of fewer than $n$ steps produce balanced sentences and consider a leftmost derivation of exactly $n$ steps Such a derivation must be of the form

$$S \underset{lm}{} \quad S\;S \underset{lm}{} \quad x\;S \underset{lm}{} \quad x\;y$$

The derivations of $x$ and $y$ from $S$ take fewer than $n$ steps so by the inductive hypothesis $x$ and $y$ are balanced Therefore the string $x$ $y$ must be balanced That is it has an equal number of left and right parentheses and every prefix has at least as many left parentheses as right

Having thus shown that any string derivable from $S$ is balanced we must next show that every balanced string is derivable from $S$ To do so use induction on the length of a string

**BASIS** If the string is of length 0 it must be which is balanced

**INDUCTION** First observe that every balanced string has even length Assume that every balanced string of length less than $2n$ is derivable from $S$ and consider a balanced string $w$ of length $2n$ $n$ 1 Surely $w$ begins with a left parenthesis Let $x$ be the shortest nonempty prefix of $w$ having an equal number of left and right parentheses Then $w$ can be written as $w$ $x$ $y$ where both $x$ and $y$ are balanced Since $x$ and $y$ are of length less than $2n$ they are derivable from $S$ by the inductive hypothesis Thus we can find a derivation of the form

$$S \qquad S\,S \qquad x\,S \qquad x\,y$$

proving that $w$ $x$ $y$ is also derivable from $S$ □

### 4 2 7 Context Free Grammars Versus Regular Expressions

Before leaving this section on grammars and their properties we establish that grammars are a more powerful notation than regular expressions Every construct that can be described by a regular expression can be described by a grammar but not vice versa Alternatively every regular language is a context free language but not vice versa

For example the regular expression **a|b abb** and the grammar

$$\begin{array}{ll} A_0 & aA_0 \mid bA_0 \mid aA_1 \\ A_1 & bA_2 \\ A_2 & bA_3 \\ A_3 & \end{array}$$

describe the same language the set of strings of $a$ s and $b$ s ending in $abb$

We can construct mechanically a grammar to recognize the same language as a nondeterministic finite automaton NFA The grammar above was constructed from the NFA in Fig 3 24 using the following construction

1 For each state $i$ of the NFA create a nonterminal $A_i$

2 If state $i$ has a transition to state $j$ on input $a$ add the production $A_i$ $aA_j$ If state $i$ goes to state $j$ on input add the production $A_i$ $A_j$

3 If $i$ is an accepting state add $A_i$

4 If $i$ is the start state make $A_i$ be the start symbol of the grammar

On the other hand the language $L \quad \{a^nb^n \mid n \quad 1\}$ with an equal number of $a$ s and $b$ s is a prototypical example of a language that can be described by a grammar but not by a regular expression To see why suppose $L$ were the language defined by some regular expression We could construct a DFA $D$ with a finite number of states say $k$ to accept $L$ Since $D$ has only $k$ states for an input beginning with more than $k$ $a$ s $D$ must enter some state twice say $s_i$ as in Fig 4 6 Suppose that the path from $s_i$ back to itself is labeled with a sequence $a^{j \quad i}$ Since $a^ib^i$ is in the language there must be a path labeled $b^i$ from $s_i$ to an accepting state $f$ But then there is also a path from the initial state $s_0$ through $s_i$ to $f$ labeled $a^jb^i$ as shown in Fig 4 6 Thus $D$ also accepts $a^jb^i$ which is not in the language contradicting the assumption that $L$ is the language accepted by $D$

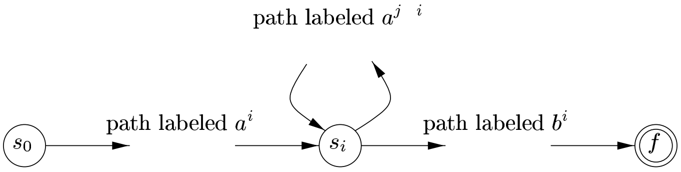


Figure 4 6 DFA $D$ accepting both $a^ib^i$ and $a^jb^i$

Colloquially we say that finite automata cannot count meaning that a finite automaton cannot accept a language like $\{a^nb^n \mid n \quad 1\}$ that would require it to keep count of the number of $a$ s before it sees the $b$ s Likewise a grammar can count two items but not three as we shall see when we consider non context free language constructs in Section 4 3 5

### 4 2 8 Exercises for Section 4 2

**Exercise 4 2 1** Consider the context free grammar

$$S \qquad S\,S \quad \mid S\,S \quad \mid a$$

and the string $aa \quad a$

a Give a leftmost derivation for the string

b Give a rightmost derivation for the string

c Give a parse tree for the string

d Is the grammar ambiguous or unambiguous Justify your answer

e Describe the language generated by this grammar

**Exercise 4 2 2** Repeat Exercise 4 2 1 for each of the following grammars and strings a $S$ $0\ S\ 1 \mid 0\ 1$ with string 000111

b $S$ $S\ S \mid\ \ S\ S \mid a$ with string $aaa$

c $S$ $S\ \ S\ \ S \mid$ with string

d $S$ $S\ \ S \mid S\ S \mid\ \ S\ \ \mid S\ \ \mid a$ with string $a\ \ a\ \ a$

e $S$ $L\ \ \mid a$ and $L$ $L\ \ S \mid S$ with string $a\ a\ \ a\ \ a$

f $S$ $a\ S\ b\ S \mid b\ S\ a\ S \mid$ with string $aabbab$

g The following grammar for boolean expressions

$$\begin{array}{ll} \textit{bexpr} & \textit{bexpr}\ \textbf{or}\ \textit{bterm} \mid \textit{bterm} \\ \textit{bterm} & \textit{bterm}\ \textbf{and}\ \textit{bfactor} \mid \textit{bfactor} \\ \textit{bfactor} & \textbf{not}\ \textit{bfactor} \mid\ \ \textit{bexpr}\ \ \mid \textbf{true} \mid \textbf{false} \end{array}$$

**Exercise 4 2 3** Design grammars for the following languages

a The set of all strings of 0s and 1s such that every 0 is immediately followed by at least one 1

b The set of all strings of 0s and 1s that are *palindromes* that is the string reads the same backward as forward

c The set of all strings of 0s and 1s with an equal number of 0s and 1s

d The set of all strings of 0s and 1s with an unequal number of 0s and 1s

e The set of all strings of 0s and 1s in which 011 does not appear as a substring

f The set of all strings of 0s and 1s of the form $xy$ where $x\ /\ y$ and $x$ and $y$ are of the same length

**Exercise 4 2 4** There is an extended grammar notation in common use In this notation square and curly braces in production bodies are metasymbols like or | with the following meanings

*i* Square braces around a grammar symbol or symbols denotes that these constructs are optional Thus production $A$ $X\ \ Y\ \ Z$ has the same effect as the two productions $A$ $X\ Y\ Z$ and $A$ $X\ Z$

*ii* Curly braces around a grammar symbol or symbols says that these symbols may be repeated any number of times including zero times Thus $A$ $X\ \{Y\ Z\}$ has the same effect as the infinite sequence of productions $A$ $X$ $A$ $X\ Y\ Z$ $A$ $X\ Y\ Z\ Y\ Z$ and so on

Show that these two extensions do not add power to grammars that is any language that can be generated by a grammar with these extensions can be generated by a grammar without the extensions

**Exercise 4 2 5** Use the braces described in Exercise 4 2 4 to simplify the following grammar for statement blocks and conditional statements

*stmt* **if** *expr* **then** *stmt* **else** *stmt*<br>
| **if** *stmt* **then** *stmt*<br>
| **begin** *stmtList* **end**<br>
*stmtList* *stmt* *stmtList* | *stmt*

**Exercise 4 2 6** Extend the idea of Exercise 4 2 4 to allow any regular expression of grammar symbols in the body of a production Show that this extension does not allow grammars to define any new languages

**Exercise 4 2 7** A grammar symbol $X$ terminal or nonterminal is *useless* if there is no derivation of the form $S$ $wXy$ $wxy$ That is $X$ can never appear in the derivation of any sentence

a Give an algorithm to eliminate from a grammar all productions containing useless symbols

b Apply your algorithm to the grammar

$S$ 0 | $A$<br>
$A$ $AB$<br>
$B$ 1

**Exercise 4 2 8** The grammar in Fig 4 7 generates declarations for a single numerical identifier these declarations involve four different independent properties of numbers

*stmt* **declare id** *optionList*<br>
*optionList* *optionList option* |<br>
*option* *mode* | *scale* | *precision* | *base*<br>
*mode* **real** | **complex**<br>
*scale* **fixed** | **floating**<br>
*precision* **single** | **double**<br>
*base* **binary** | **decimal**

Figure 4 7 A grammar for multi attribute declarations

a Generalize the grammar of Fig 4 7 by allowing $n$ options $A_i$ for some fixed $n$ and for $i$ 1 2 $n$ where $A_i$ can be either $a_i$ or $b_i$ Your grammar should use only $O$ $n$ grammar symbols and have a total length of productions that is $O$ $n$

b The grammar of Fig 4 7 and its generalization in part a allow declarations that are contradictory and or redundant such as

```
declare foo real fixed real floating
```

We could insist that the syntax of the language forbid such declarations that is every declaration generated by the grammar has exactly one value for each of the $n$ options If we do then for any fixed $n$ there is only a finite number of legal declarations The language of legal declarations thus has a grammar and also a regular expression as any finite language does The obvious grammar in which the start symbol has a production for every legal declaration has $n$ productions and a total production length of $O\ n \quad n$ You must do better a total production length that is $O\ n2^n$

c Show that any grammar for part b must have a total production length of at least $2^n$

d What does part c say about the feasibility of enforcing nonredundancy and noncontradiction among options in declarations via the syntax of the programming language

## 4 3 Writing a Grammar

Grammars are capable of describing most but not all of the syntax of programming languages For instance the requirement that identifiers be declared before they are used cannot be described by a context free grammar Therefore the sequences of tokens accepted by a parser form a superset of the programming language subsequent phases of the compiler must analyze the output of the parser to ensure compliance with rules that are not checked by the parser

This section begins with a discussion of how to divide work between a lexical analyzer and a parser We then consider several transformations that could be applied to get a grammar more suitable for parsing One technique can eliminate ambiguity in the grammar and other techniques left recursion elimination and left factoring are useful for rewriting grammars so they become suitable for top down parsing We conclude this section by considering some programming language constructs that cannot be described by any grammar

### 4 3 1 Lexical Versus Syntactic Analysis

As we observed in Section 4 2 7 everything that can be described by a regular expression can also be described by a grammar We may therefore reasonably ask Why use regular expressions to define the lexical syntax of a language There are several reasons

1 Separating the syntactic structure of a language into lexical and non lexical parts provides a convenient way of modularizing the front end of a compiler into two manageable sized components

2 The lexical rules of a language are frequently quite simple and to describe them we do not need a notation as powerful as grammars

3 Regular expressions generally provide a more concise and easier to under stand notation for tokens than grammars

4 More efficient lexical analyzers can be constructed automatically from regular expressions than from arbitrary grammars

There are no firm guidelines as to what to put into the lexical rules as op posed to the syntactic rules Regular expressions are most useful for describing the structure of constructs such as identifiers constants keywords and white space Grammars on the other hand are most useful for describing nested structures such as balanced parentheses matching begin end s corresponding if then else s and so on These nested structures cannot be described by regular expressions

### 4 3 2 Eliminating Ambiguity

Sometimes an ambiguous grammar can be rewritten to eliminate the ambiguity As an example we shall eliminate the ambiguity from the following dangling else grammar

$$\begin{array}{lcl} \mathit{stmt} & & \textbf{if } \mathit{expr} \textbf{ then } \mathit{stmt} \\ & | & \textbf{if } \mathit{expr} \textbf{ then } \mathit{stmt} \textbf{ else } \mathit{stmt} \\ & | & \textbf{other} \end{array} \qquad 4\ 14$$

Here **other** stands for any other statement According to this grammar the compound conditional statement

$$\textbf{if } E_1 \textbf{ then } S_1 \textbf{ else if } E_2 \textbf{ then } S_2 \textbf{ else } S_3$$

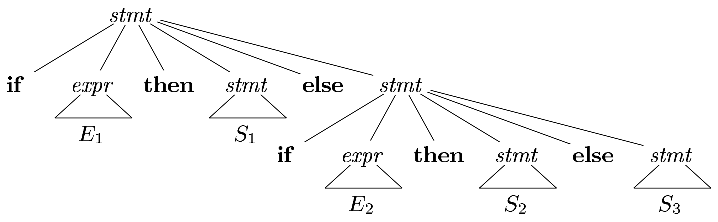


Figure 4 8 Parse tree for a conditional statement has the parse tree shown in Fig 4 8 [1] Grammar 4 14 is ambiguous since the string

$$\textbf{if } E_1 \textbf{ then if } E_2 \textbf{ then } S_1 \textbf{ else } S_2 \qquad 4\ 15$$

has the two parse trees shown in Fig 4 9

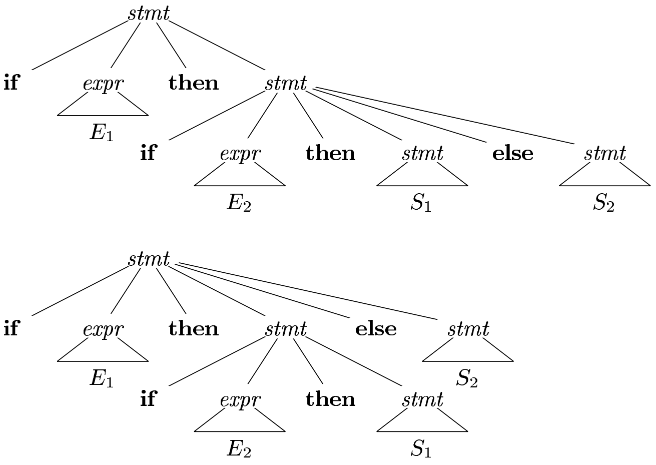


Figure 4 9 Two parse trees for an ambiguous sentence

In all programming languages with conditional statements of this form the first parse tree is preferred The general rule is Match each **else** with the closest unmatched **then** [2] This disambiguating rule can theoretically be incorporated directly into a grammar but in practice it is rarely built into the productions

**Example 4 16** We can rewrite the dangling else grammar 4 14 as the following unambiguous grammar The idea is that a statement appearing between a **then** and an **else** must be matched that is the interior statement must not end with an unmatched or open **then** A matched statement is either an **if then else** statement containing no open statements or it is any other kind of unconditional statement Thus we may use the grammar in Fig 4 10 This grammar generates the same strings as the dangling else grammar 4 14 but it allows only one parsing for string 4 15 namely the one that associates each **else** with the closest previous unmatched **then** □

[1] The subscripts on $E$ and $S$ are just to distinguish different occurrences of the same nonterminal and do not imply distinct nonterminals

[2] We should note that C and its derivatives are included in this class Even though the C family of languages do not use the keyword **then** its role is played by the closing parenthesis for the condition that follows **if**

$$\begin{array}{rcl}
\textit{stmt} & & \textit{matched\_stmt} \\
 & \mid & \textit{open\_stmt} \\
\textit{matched\_stmt} & & \textbf{if } \textit{expr} \textbf{ then } \textit{matched\_stmt} \textbf{ else } \textit{matched\_stmt} \\
 & \mid & \textbf{other} \\
\textit{open\_stmt} & & \textbf{if } \textit{expr} \textbf{ then } \textit{stmt} \\
 & \mid & \textbf{if } \textit{expr} \textbf{ then } \textit{matched\_stmt} \textbf{ else } \textit{open\_stmt}
\end{array}$$

Figure 4 10 Unambiguous grammar for if then else statements

### 4 3 3 Elimination of Left Recursion

A grammar is *left recursive* if it has a nonterminal $A$ such that there is a derivation $A \quad A$ for some string Top down parsing methods cannot handle left recursive grammars so a transformation is needed to eliminate left recursion In Section 2 4 5 we discussed *immediate left recursion* where there is a production of the form $A \quad A$ Here we study the general case In Section 2 4 5 we showed how the left recursive pair of productions $A \quad A \mid$ could be replaced by the non left recursive productions

$$\begin{array}{ll}
A & A' \\
A' & A' \mid
\end{array}$$

without changing the strings derivable from $A$ This rule by itself suffices for many grammars

**Example 4 17** The non left recursive expression grammar 4 2 repeated here

$$\begin{array}{ll}
E & T\,E' \\
E' & \quad T\,E' \mid \\
T & F\,T' \\
T' & \quad F\,T' \mid \\
F & \quad E \quad \mid \textbf{id}
\end{array}$$

is obtained by eliminating immediate left recursion from the expression grammar 4 1 The left recursive pair of productions $E \quad E \quad T \mid T$ are replaced by $E \quad T\,E'$ and $E' \quad T\,E' \mid$ The new productions for $T$ and $T'$ are obtained similarly by eliminating immediate left recursion □

Immediate left recursion can be eliminated by the following technique which works for any number of $A$ productions First group the productions as

$$A \quad A_{\ 1} \mid A_{\ 2} \mid \quad \mid A_{\ m} \mid {}_{1} \mid {}_{2} \mid \quad \mid {}_{n}$$

where no ${}_i$ begins with an $A$ Then replace the $A$ productions by

$$\begin{array}{ll} A & {}_1A' \mid {}_2A' \mid \quad \mid {}_nA' \\ A' & {}_1A' \mid {}_2A' \mid \quad \mid {}_mA' \mid \end{array}$$

The nonterminal $A$ generates the same strings as before but is no longer left recursive This procedure eliminates all left recursion from the $A$ and $A'$ productions provided no ${}_i$ is but it does not eliminate left recursion involving derivations of two or more steps For example consider the grammar

$$\begin{array}{ll} S & A\,a \mid b \\ A & A\,c \mid S\,d \mid \end{array} \tag{4 18}$$

The nonterminal $S$ is left recursive because $S$ $Aa$ $Sda$ but it is not immediately left recursive

Algorithm 4 19 below systematically eliminates left recursion from a grammar It is guaranteed to work if the grammar has no cycles derivations of the form $A$ $A$ or productions productions of the form $A$ Cycles can be eliminated systematically from a grammar as can productions see Exercises 4 4 6 and 4 4 7

**Algorithm 4 19** Eliminating left recursion

INPUT Grammar $G$ with no cycles or productions

OUTPUT An equivalent grammar with no left recursion

METHOD Apply the algorithm in Fig 4 11 to $G$ Note that the resulting non left recursive grammar may have productions □

```
1   arrange the nonterminals in some order $A_1$ $A_2$       $A_n$
2   for  each $i$ from 1 to $n$  {
3          for  each $j$ from 1 to $i$   1  {
4                 replace each production of the form $A_i$   $A_j$  by the
                     productions $A_i$     ${}_1$ | ${}_2$ |     | ${}_k$  where
                     $A_j$     ${}_1$ | ${}_2$ |     | ${}_k$ are all current $A_j$ productions
5          }
6          eliminate the immediate left recursion among the $A_i$ productions
7   }
```

Figure 4 11 Algorithm to eliminate left recursion from a grammar

The procedure in Fig 4 11 works as follows In the first iteration for $i$ 1 the outer for loop of lines 2 through 7 eliminates any immediate left recursion among $A_1$ productions Any remaining $A_1$ productions of the form $A_1$ $A_l$ must therefore have $l$ 1 After the $i$ 1st iteration of the outer for loop all nonterminals $A_k$ where $k$ $i$ are cleaned that is any production $A_k$ $A_l$ must have $l$ $k$ As a result on the $i$th iteration the inner loop of lines 3 through 5 progressively raises the lower limit in any production $A_i$ $A_m$ until we have $m$ $i$ Then eliminating immediate left recursion for the $A_i$ productions at line 6 forces $m$ to be greater than $i$

**Example 4 20** Let us apply Algorithm 4 19 to the grammar 4 18 Technically the algorithm is not guaranteed to work because of the production but in this case the production $A$ turns out to be harmless

We order the nonterminals $S$ $A$ There is no immediate left recursion among the $S$ productions so nothing happens during the outer loop for $i$ 1 For $i$ 2 we substitute for $S$ in $A$ $S\ d$ to obtain the following $A$ productions

$$A \quad A\,c \mid A\,a\,d \mid b\,d \mid$$

Eliminating the immediate left recursion among these $A$ productions yields the following grammar

$$\begin{array}{ll} S & A\,a \mid b \\ A & b\,d\,A' \mid A' \\ A' & c\,A' \mid a\,d\,A' \mid \end{array}$$

□

### 4 3 4 Left Factoring

Left factoring is a grammar transformation that is useful for producing a grammar suitable for predictive or top down parsing When the choice between two alternative $A$ productions is not clear we may be able to rewrite the productions to defer the decision until enough of the input has been seen that we can make the right choice

For example if we have the two productions

$$\begin{array}{lll} \textit{stmt} & & \textbf{if } \textit{expr} \textbf{ then } \textit{stmt} \textbf{ else } \textit{stmt} \\ & \mid & \textbf{if } \textit{expr} \textbf{ then } \textit{stmt} \end{array}$$

on seeing the input **if** we cannot immediately tell which production to choose to expand *stmt* In general if $A$ $_1 \mid$ $_2$ are two $A$ productions and the input begins with a nonempty string derived from we do not know whether to expand $A$ to $_1$ or $_2$ However we may defer the decision by expanding $A$ to $A'$ Then after seeing the input derived from we expand $A'$ to $_1$ or to $_2$ That is left factored the original productions become

$$\begin{array}{ll} A & A' \\ A' & _1 \mid \ _2 \end{array}$$

**Algorithm 4 21** Left factoring a grammar

INPUT Grammar $G$

OUTPUT An equivalent left factored grammar

**METHOD** For each nonterminal $A$ find the longest prefix common to two or more of its alternatives If $\quad / \quad$ i e there is a nontrivial common prefix replace all of the $A$ productions $A \quad {}_1 \mid {}_2 \mid \quad \mid {}_n \mid$ where represents all alternatives that do not begin with by

$$\begin{array}{ll} A & A' \mid \\ A' & {}_1 \mid {}_2 \mid \quad \mid {}_n \end{array}$$

Here $A'$ is a new nonterminal Repeatedly apply this transformation until no two alternatives for a nonterminal have a common prefix □

**Example 4 22** The following grammar abstracts the dangling else problem

$$\begin{array}{ll} S & i\,E\,t\,S \mid i\,E\,t\,S\,e\,S \mid a \\ E & b \end{array} \qquad 4\ 23$$

Here $i$ $t$ and $e$ stand for **if** **then** and **else** $E$ and $S$ stand for conditional expression and statement Left factored this grammar becomes

$$\begin{array}{ll} S & i\,E\,t\,S\,S' \mid a \\ S' & e\,S \mid \\ E & b \end{array} \qquad 4\ 24$$

Thus we may expand $S$ to $iEtSS'$ on input $i$ and wait until $iEtS$ has been seen to decide whether to expand $S'$ to $eS$ or to Of course these grammars are both ambiguous and on input $e$ it will not be clear which alternative for $S'$ should be chosen Example 4 33 discusses a way out of this dilemma □

### 4 3 5 Non Context Free Language Constructs

A few syntactic constructs found in typical programming languages cannot be specified using grammars alone Here we consider two of these constructs using simple abstract languages to illustrate the difficulties

**Example 4 25** The language in this example abstracts the problem of checking that identifiers are declared before they are used in a program The language consists of strings of the form $wcw$ where the first $w$ represents the declaration of an identifier $w$ $c$ represents an intervening program fragment and the second $w$ represents the use of the identifier

The abstract language is $L_1 \quad \{wcw \mid w$ is in $\mathbf{a|b}$ $\}$ $L_1$ consists of all words composed of a repeated string of $a$ s and $b$ s separated by $c$ such as *aabcaab* While it is beyond the scope of this book to prove it the non context freedom of $L_1$ directly implies the non context freedom of programming languages like C and Java which require declaration of identifiers before their use and which allow identifiers of arbitrary length

For this reason a grammar for C or Java does not distinguish among identifiers that are different character strings Instead all identifiers are represented by a token such as **id** in the grammar In a compiler for such a language the semantic analysis phase checks that identifiers are declared before they are used □

**Example 4 26** The non context free language in this example abstracts the problem of checking that the number of formal parameters in the declaration of a function agrees with the number of actual parameters in a use of the function The language consists of strings of the form $a^n b^m c^n d^m$ Recall $a^n$ means $a$ written $n$ times Here $a^n$ and $b^m$ could represent the formal parameter lists of two functions declared to have $n$ and $m$ arguments respectively while $c^n$ and $d^m$ represent the actual parameter lists in calls to these two functions

The abstract language is $L_2 \quad \{a^n b^m c^n d^m \mid n \quad 1 \text{ and } m \quad 1\}$ That is $L_2$ consists of strings in the language generated by the regular expression **a b c d** such that the number of $a$ s and $c$ s are equal and the number of $b$ s and $d$ s are equal This language is not context free

Again the typical syntax of function declarations and uses does not concern itself with counting the number of parameters For example a function call in C like language might be specified by

$$\begin{array}{rll} stmt & & \textbf{id} \quad expr\_list \\ expr\_list & & expr\_list \quad expr \\ & \mid & expr \end{array}$$

with suitable productions for *expr* Checking that the number of parameters in a call is correct is usually done during the semantic analysis phase □

### 4 3 6 Exercises for Section 4 3

**Exercise 4 3 1** The following is a grammar for regular expressions over symbols $a$ and $b$ only using in place of | for union to avoid conflict with the use of vertical bar as a metasymbol in grammars

$$\begin{array}{ll} rexpr & rexpr \quad rterm \mid rterm \\ rterm & rterm\ rfactor \mid rfactor \\ rfactor & rfactor \quad \mid rprimary \\ rprimary & \textbf{a} \mid \textbf{b} \end{array}$$

a Left factor this grammar

b Does left factoring make the grammar suitable for top down parsing

c In addition to left factoring eliminate left recursion from the original grammar

d Is the resulting grammar suitable for top down parsing

**Exercise 4 3 2** Repeat Exercise 4 3 1 on the following grammars a The grammar of Exercise 4 2 1

b The grammar of Exercise 4 2 2 a

c The grammar of Exercise 4 2 2 c

d The grammar of Exercise 4 2 2 e

e The grammar of Exercise 4 2 2 g

**Exercise 4 3 3** The following grammar is proposed to remove the dangling else ambiguity discussed in Section 4 3 2

$$\begin{array}{lll} stmt & & \textbf{if } expr \textbf{ then } stmt \\ & | & matchedStmt \\ matchedStmt & & \textbf{if } expr \textbf{ then } matchedStmt \textbf{ else } stmt \\ & | & \textbf{other} \end{array}$$

Show that this grammar is still ambiguous

## 4 4 Top Down Parsing

Top down parsing can be viewed as the problem of constructing a parse tree for the input string starting from the root and creating the nodes of the parse tree in preorder depth first as discussed in Section 2 3 4 Equivalently top down parsing can be viewed as finding a leftmost derivation for an input string

**Example 4 27** The sequence of parse trees in Fig 4 12 for the input **id** **id** **id** is a top down parse according to grammar 4 2 repeated here

$$\begin{array}{lll} E & & T\ E' \\ E' & & \ T\ E' \mid \\ T & & F\ T' \\ T' & & \ F\ T' \ \mid \\ F & & \ E \ \mid \textbf{id} \end{array} \qquad 4\ 28$$

This sequence of trees corresponds to a leftmost derivation of the input □

At each step of a top down parse the key problem is that of determining the production to be applied for a nonterminal say $A$ Once an $A$ production is chosen the rest of the parsing process consists of matching the terminal symbols in the production body with the input string

The section begins with a general form of top down parsing called recursive descent parsing which may require backtracking to find the correct $A$ production to be applied Section 2 4 2 introduced predictive parsing a special case of recursive descent parsing where no backtracking is required Predictive parsing chooses the correct $A$ production by looking ahead at the input a fixed number of symbols typically we may look only at one that is the next input symbol

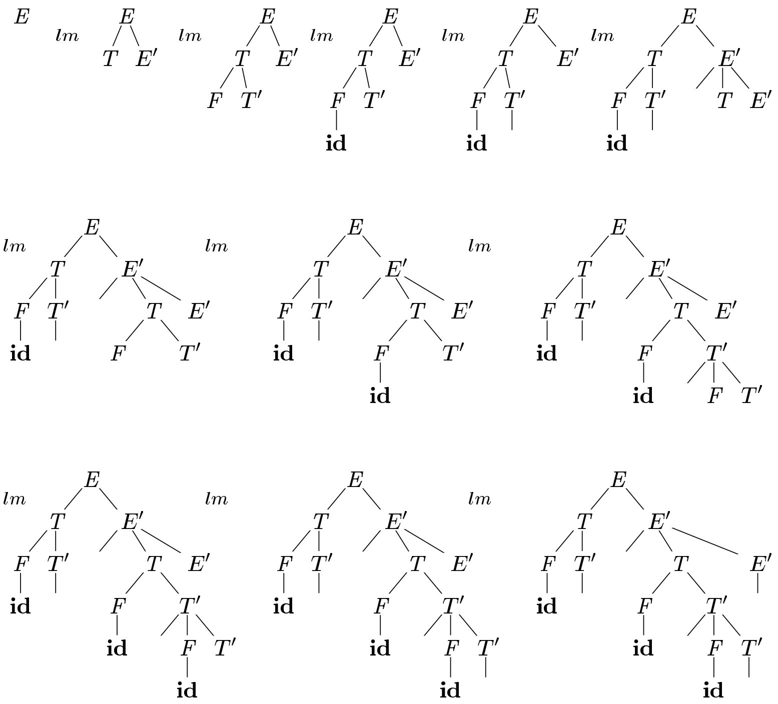


Figure 4 12 Top down parse for **id** **id** **id**

For example consider the top down parse in Fig 4 12 which constructs a tree with two nodes labeled $E'$ At the first $E'$ node in preorder the production $E'$ $TE'$ is chosen at the second $E'$ node the production $E'$ is chosen A predictive parser can choose between $E'$ productions by looking at the next input symbol

The class of grammars for which we can construct predictive parsers looking $k$ symbols ahead in the input is sometimes called the $LL$ $k$ class We discuss the LL 1 class in Section 4 4 3 but introduce certain computations called FIRST and FOLLOW in a preliminary Section 4 4 2 From the FIRST and FOLLOW sets for a grammar we shall construct predictive parsing tables which make explicit the choice of production during top down parsing These sets are also useful during bottom up parsing as we shall see

In Section 4 4 4 we give a nonrecursive parsing algorithm that maintains a stack explicitly rather than implicitly via recursive calls Finally in Section 4 4 5 we discuss error recovery during top down parsing

### 4 4 1 Recursive Descent Parsing

```
    void A   {
1       Choose an A production  A      $X_1X_2$      $X_k$
2       for   i    1 to k   {
3           if   $X_i$ is a nonterminal
4               call procedure $X_i$
5           else if   $X_i$ equals the current input symbol a
6               advance the input to the next symbol
7           else     an error has occurred
        }
    }
```

Figure 4 13 A typical procedure for a nonterminal in a top down parser

A recursive descent parsing program consists of a set of procedures one for each nonterminal Execution begins with the procedure for the start symbol which halts and announces success if its procedure body scans the entire input string Pseudocode for a typical nonterminal appears in Fig 4 13 Note that this pseudocode is nondeterministic since it begins by choosing the $A$ production to apply in a manner that is not specified

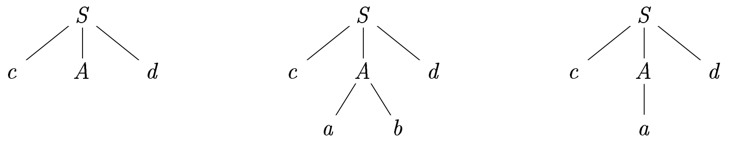
General recursive descent may require backtracking that is it may require repeated scans over the input However backtracking is rarely needed to parse programming language constructs so backtracking parsers are not seen frequently Even for situations like natural language parsing backtracking is not very efficient and tabular methods such as the dynamic programming algorithm of Exercise 4 4 9 or the method of Earley see the bibliographic notes are preferred

To allow backtracking the code of Fig 4 13 needs to be modified First we cannot choose a unique $A$ production at line 1 so we must try each of several productions in some order Then failure at line 7 is not ultimate failure but suggests only that we need to return to line 1 and try another $A$ production Only if there are no more $A$ productions to try do we declare that an input error has been found In order to try another $A$ production we need to be able to reset the input pointer to where it was when we first reached line 1 Thus a local variable is needed to store this input pointer for future use

**Example 4 29** Consider the grammar

$$\begin{array}{ll} S & c\ A\ d \\ A & a\ b\ \mid\ a \end{array}$$

To construct a parse tree top down for the input string $w$ *cad* begin with a tree consisting of a single node labeled $S$ and the input pointer pointing to $c$ the first symbol of $w$ $S$ has only one production so we use it to expand $S$ and obtain the tree of Fig 4 14 a The leftmost leaf labeled $c$ matches the first symbol of input $w$ so we advance the input pointer to $a$ the second symbol of $w$ and consider the next leaf labeled $A$



a b c

Figure 4 14 Steps in a top down parse

Now we expand $A$ using the first alternative $A$ $a$ $b$ to obtain the tree of Fig 4 14 b We have a match for the second input symbol $a$ so we advance the input pointer to $d$ the third input symbol and compare $d$ against the next leaf labeled $b$ Since $b$ does not match $d$ we report failure and go back to $A$ to see whether there is another alternative for $A$ that has not been tried but that might produce a match

In going back to $A$ we must reset the input pointer to position 2 the position it had when we first came to $A$ which means that the procedure for $A$ must store the input pointer in a local variable

The second alternative for $A$ produces the tree of Fig 4 14 c The leaf $a$ matches the second symbol of $w$ and the leaf $d$ matches the third symbol Since we have produced a parse tree for $w$ we halt and announce successful completion of parsing □

A left recursive grammar can cause a recursive descent parser even one with backtracking to go into an infinite loop That is when we try to expand a nonterminal $A$ we may eventually find ourselves again trying to expand $A$ without having consumed any input

## 4 4 2 FIRST and FOLLOW

The construction of both top down and bottom up parsers is aided by two functions FIRST and FOLLOW associated with a grammar $G$ During top down parsing FIRST and FOLLOW allow us to choose which production to apply based on the next input symbol During panic mode error recovery sets of tokens produced by FOLLOW can be used as synchronizing tokens

Define *FIRST* where is any string of grammar symbols to be the set of terminals that begin strings derived from If then is also in FIRST For example in Fig 4 15 $A$ $c$ so $c$ is in FIRST $A$

For a preview of how FIRST can be used during predictive parsing consider two $A$ productions $A$ | where FIRST and FIRST are disjoint sets We can then choose between these $A$ productions by looking at the next input

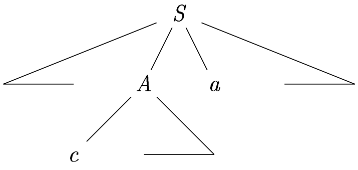


Figure 4 15 Terminal $c$ is in FIRST $A$ and $a$ is in FOLLOW $A$

symbol $a$ since $a$ can be in at most one of FIRST and FIRST not both For instance if $a$ is in FIRST choose the production $A$ This idea will be explored when LL 1 grammars are defined in Section 4 4 3

Define *FOLLOW* $A$ for nonterminal $A$ to be the set of terminals $a$ that can appear immediately to the right of $A$ in some sentential form that is the set of terminals $a$ such that there exists a derivation of the form $S$ $Aa$ for some and as in Fig 4 15 Note that there may have been symbols between $A$ and $a$ at some time during the derivation but if so they derived and disappeared In addition if $A$ can be the rightmost symbol in some sentential form then is in FOLLOW $A$ recall that is a special endmarker symbol that is assumed not to be a symbol of any grammar

To compute FIRST $X$ for all grammar symbols $X$ apply the following rules until no more terminals or can be added to any FIRST set

1 If $X$ is a terminal then FIRST $X$ $\{X\}$

2 If $X$ is a nonterminal and $X$ $Y_1Y_2$ $Y_k$ is a production for some $k$ 1 then place $a$ in FIRST $X$ if for some $i$ $a$ is in FIRST $Y_i$ and is in all of FIRST $Y_1$ FIRST $Y_{i\ 1}$ that is $Y_1$ $Y_{i\ 1}$ If is in FIRST $Y_j$ for all $j$ 1 2 $k$ then add to FIRST $X$ For example everything in FIRST $Y_1$ is surely in FIRST $X$ If $Y_1$ does not derive then we add nothing more to FIRST $X$ but if $Y_1$ then we add FIRST $Y_2$ and so on

3 If $X$ is a production then add to FIRST $X$

Now we can compute FIRST for any string $X_1X_2$ $X_n$ as follows Add to FIRST $X_1X_2$ $X_n$ all non symbols of FIRST $X_1$ Also add the non symbols of FIRST $X_2$ if is in FIRST $X_1$ the non symbols of FIRST $X_3$ if is in FIRST $X_1$ and FIRST $X_2$ and so on Finally add to FIRST $X_1X_2$ $X_n$ if for all $i$ is in FIRST $X_i$

To compute FOLLOW $A$ for all nonterminals $A$ apply the following rules until nothing can be added to any FOLLOW set

1 Place in FOLLOW $S$ where $S$ is the start symbol and is the input right endmarker

2 If there is a production $A$ $B$ then everything in FIRST except is in FOLLOW $B$

3 If there is a production $A$ $B$ or a production $A$ $B$ where FIRST contains then everything in FOLLOW $A$ is in FOLLOW $B$

**Example 4 30** Consider again the non left recursive grammar 4 28 Then

1 FIRST $F$ FIRST $T$ FIRST $E$ $\{$ **id**$\}$ To see why note that the two productions for $F$ have bodies that start with these two terminal symbols **id** and the left parenthesis $T$ has only one production and its body starts with $F$ Since $F$ does not derive FIRST $T$ must be the same as FIRST $F$ The same argument covers FIRST $E$

2 FIRST $E'$ $\{$ $\}$ The reason is that one of the two productions for $E'$ has a body that begins with terminal and the other s body is When ever a nonterminal derives we place in FIRST for that nonterminal

3 FIRST $T'$ $\{$ $\}$ The reasoning is analogous to that for FIRST $E'$

4 FOLLOW $E$ FOLLOW $E'$ $\{$ $\}$ Since $E$ is the start symbol FOLLOW $E$ must contain The production body $E$ explains why the right parenthesis is in FOLLOW $E$ For $E'$ note that this nonterminal appears only at the ends of bodies of $E$ productions Thus FOLLOW $E'$ must be the same as FOLLOW $E$

5 FOLLOW $T$ FOLLOW $T'$ $\{$ $\}$ Notice that $T$ appears in bodies only followed by $E'$ Thus everything except that is in FIRST $E'$ must be in FOLLOW $T$ that explains the symbol However since FIRST $E'$ contains i e $E'$ and $E'$ is the entire string following $T$ in the bodies of the $E$ productions everything in FOLLOW $E$ must also be in FOLLOW $T$ That explains the symbols and the right parenthesis As for $T'$ since it appears only at the ends of the $T$ productions it must be that FOLLOW $T'$ FOLLOW $T$

6 FOLLOW $F$ $\{$ $\}$ The reasoning is analogous to that for $T$ in point 5

□

### 4 4 3 LL 1 Grammars

Predictive parsers that is recursive descent parsers needing no backtracking can be constructed for a class of grammars called LL 1 The rst L in LL 1 stands for scanning the input from left to right the second L for producing a leftmost derivation and the 1 for using one input symbol of lookahead at each step to make parsing action decisions

## Transition Diagrams for Predictive Parsers

Transition diagrams are useful for visualizing predictive parsers For example the transition diagrams for nonterminals $E$ and $E'$ of grammar 4 28 appear in Fig 4 16 a To construct the transition diagram from a grammar first eliminate left recursion and then left factor the grammar Then for each nonterminal $A$

1 Create an initial and final return state

2 For each production $A$ $X_1X_2$ $X_k$ create a path from the initial to the final state with edges labeled $X_1$ $X_2$ $X_k$ If $A$ the path is an edge labeled

Transition diagrams for predictive parsers differ from those for lexical analyzers Parsers have one diagram for each nonterminal The labels of edges can be tokens or nonterminals A transition on a token terminal means that we take that transition if that token is the next input symbol A transition on a nonterminal $A$ is a call of the procedure for $A$

With an LL 1 grammar the ambiguity of whether or not to take an edge can be resolved by making transitions the default choice

Transition diagrams can be simplified provided the sequence of grammar symbols along paths is preserved We may also substitute the diagram for a nonterminal $A$ in place of an edge labeled $A$ The diagrams in Fig 4 16 a and b are equivalent if we trace paths from $E$ to an accepting state and substitute for $E'$ then in both sets of diagrams the grammar symbols along the paths make up strings of the form $T$ $T$ $T$ The diagram in b can be obtained from a by transformations akin to those in Section 2 5 4 where we used tail recursion removal and substitution of procedure bodies to optimize the procedure for a nonterminal

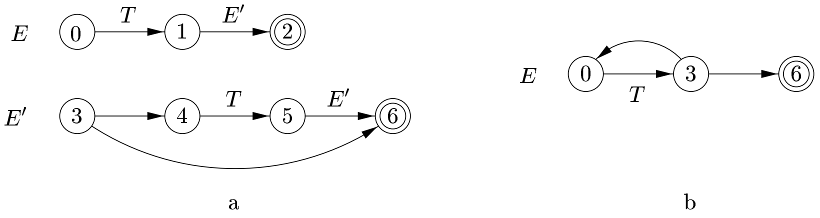
The class of LL 1 grammars is rich enough to cover most programming constructs although care is needed in writing a suitable grammar for the source language For example no left recursive or ambiguous grammar can be LL 1

A grammar $G$ is LL 1 if and only if whenever $A$ | are two distinct productions of $G$ the following conditions hold

1 For no terminal $a$ do both and derive strings beginning with $a$

2 At most one of and can derive the empty string

3 If then does not derive any string beginning with a terminal in FOLLOW $A$ Likewise if then does not derive any string beginning with a terminal in FOLLOW $A$



a

b

Figure 4 16 Transition diagrams for nonterminals $E$ and $E'$ of grammar 4 28

The first two conditions are equivalent to the statement that FIRST and FIRST are disjoint sets The third condition is equivalent to stating that if is in FIRST then FIRST and FOLLOW $A$ are disjoint sets and likewise if is in FIRST

Predictive parsers can be constructed for LL 1 grammars since the proper production to apply for a nonterminal can be selected by looking only at the current input symbol Flow of control constructs with their distinguishing key words generally satisfy the LL 1 constraints For instance if we have the productions

*stmt* **if** *expr* *stmt* **else** *stmt*
| **while** *expr* *stmt*
| *stmt_list*

then the keywords **if** **while** and the symbol tell us which alternative is the only one that could possibly succeed if we are to find a statement

The next algorithm collects the information from FIRST and FOLLOW sets into a predictive parsing table $M$ $A$ $a$ a two dimensional array where $A$ is a nonterminal and $a$ is a terminal or the symbol the input endmarker The algorithm is based on the following idea the production $A$ is chosen if the next input symbol $a$ is in FIRST The only complication occurs when or more generally In this case we should again choose $A$ if the current input symbol is in FOLLOW $A$ or if the on the input has been reached and is in FOLLOW $A$

**Algorithm 4 31** Construction of a predictive parsing table

**INPUT** Grammar $G$

**OUTPUT** Parsing table $M$

**METHOD** For each production $A$ of the grammar do the following

1 For each terminal $a$ in FIRST add $A$ to $M$ $A$ $a$

2 If is in FIRST then for each terminal $b$ in FOLLOW $A$ add $A$ to $M$ $A$ $b$ If is in FIRST and is in FOLLOW $A$ add $A$ to $M$ $A$ as well

If after performing the above there is no production at all in $M\ A\ a$ then set $M\ A\ a$ to **error** which we normally represent by an empty entry in the table □

**Example 4 32** For the expression grammar 4 28 Algorithm 4 31 produces the parsing table in Fig 4 17 Blanks are error entries nonblanks indicate a production with which to expand a nonterminal

| NON TERMINAL | INPUT SYMBOL | | | | | |
|---|---|---|---|---|---|---|
| | **id** | | | | | |
| $E$ | $E\quad TE'$ | | | $E\quad TE'$ | | |
| $E'$ | | $E'\quad TE'$ | | | $E'$ | $E'$ |
| $T$ | $T\quad FT'$ | | | $T\quad FT'$ | | |
| $T'$ | | $T'$ | $T'\quad FT'$ | | $T'$ | $T'$ |
| $F$ | $F$ **id** | | | $F\quad E$ | | |

Figure 4 17 Parsing table $M$ for Example 4 32

Consider production $E\quad TE'$ Since

$$\text{FIRST } TE' \qquad \text{FIRST } T \qquad \{\ \ \textbf{id}\}$$

this production is added to $M\ E$ and $M\ E$ **id** Production $E'\qquad TE'$ is added to $M\ E'$ since FIRST $TE'$ $\{\ \}$ Since FOLLOW $E'$ $\{\ \ \}$ production $E'$ is added to $M\ E'$ and $M\ E'$ □

Algorithm 4 31 can be applied to any grammar $G$ to produce a parsing table $M$ For every LL 1 grammar each parsing table entry uniquely identifies a production or signals an error For some grammars however $M$ may have some entries that are multiply defined For example if $G$ is left recursive or ambiguous then $M$ will have at least one multiply defined entry Although left recursion elimination and left factoring are easy to do there are some grammars for which no amount of alteration will produce an LL 1 grammar

The language in the following example has no LL 1 grammar at all

**Example 4 33** The following grammar which abstracts the dangling else problem is repeated here from Example 4 22

$$\begin{array}{ll} S & iEtSS' \mid a \\ S' & eS \mid \\ E & b \end{array}$$

The parsing table for this grammar appears in Fig 4 18 The entry for $M\ S'\ e$ contains both $S'\quad eS$ and $S'$

The grammar is ambiguous and the ambiguity is manifested by a choice in what production to use when an $e$ **else** is seen We can resolve this ambiguity

| NON TERMINAL | INPUT SYMBOL | | | | | |
|---|---|---|---|---|---|---|
| | $a$ | $b$ | $e$ | $i$ | $t$ | |
| $S$ | $S \quad a$ | | | $S \quad iEtSS'$ | | |
| $S'$ | | | $S'$<br>$S' \quad eS$ | | | $S'$ |
| $E$ | | $E \quad b$ | | | | |

Figure 4 18 Parsing table $M$ for Example 4 33

by choosing $S' \quad eS$ This choice corresponds to associating an **else** with the closest previous **then** Note that the choice $S'$ would prevent $e$ from ever being put on the stack or removed from the input and is surely wrong □

## 4 4 4 Nonrecursive Predictive Parsing

A nonrecursive predictive parser can be built by maintaining a stack explicitly rather than implicitly via recursive calls The parser mimics a leftmost deriva tion If $w$ is the input that has been matched so far then the stack holds a sequence of grammar symbols such that

$$S \quad \underset{lm}{\ } \quad w$$

The table driven parser in Fig 4 19 has an input bu er a stack containing a sequence of grammar symbols a parsing table constructed by Algorithm 4 31 and an output stream The input bu er contains the string to be parsed followed by the endmarker We reuse the symbol to mark the bottom of the stack which initially contains the start symbol of the grammar on top of

The parser is controlled by a program that considers $X$ the symbol on top of the stack and $a$ the current input symbol If $X$ is a nonterminal the parser chooses an $X$ production by consulting entry $M \ X \ a$ of the parsing table $M$ Additional code could be executed here for example code to construct a node in a parse tree Otherwise it checks for a match between the terminal $X$ and current input symbol $a$

The behavior of the parser can be described in terms of its *configurations* which give the stack contents and the remaining input The next algorithm describes how configurations are manipulated

**Algorithm 4 34** Table driven predictive parsing

INPUT A string $w$ and a parsing table $M$ for grammar $G$

OUTPUT If $w$ is in $L \ G$ a leftmost derivation of $w$ otherwise an error indication

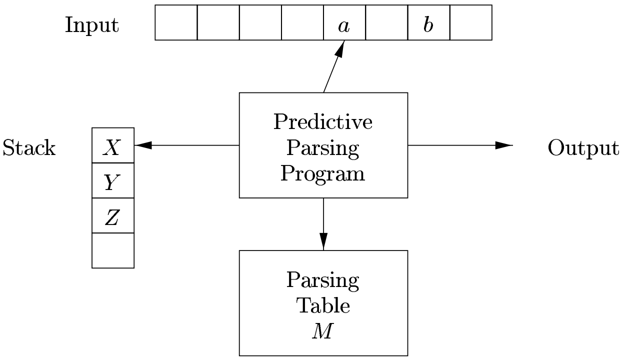


Figure 4 19 Model of a table driven predictive parser

**METHOD** Initially the parser is in a configuration with $w$ in the input buffer and the start symbol $S$ of $G$ on top of the stack above The program in Fig 4 20 uses the predictive parsing table $M$ to produce a predictive parse for the input □

```
let $a$ be the first symbol of $w$
let $X$ be the top stack symbol
while  $X$ /    {    stack is not empty
        if  $X$   $a$  pop the stack and let $a$ be the next symbol of $w$
        else if  $X$ is a terminal  error
        else if  $M X a$ is an error entry  error
        else if  $M X a$   $X$   $Y_1Y_2$   $Y_k$  {
                output the production $X$   $Y_1Y_2$   $Y_k$
                pop the stack
                push $Y_k\ Y_{k\ 1}$      $Y_1$ onto the stack  with $Y_1$ on top
        }
        let $X$ be the top stack symbol
}
```

Figure 4 20 Predictive parsing algorithm

**Example 4 35** Consider grammar 4 28 we have already seen its the parsing table in Fig 4 17 On input **id** **id** **id** the nonrecursive predictive parser of Algorithm 4 34 makes the sequence of moves in Fig 4 21 These moves correspond to a leftmost derivation see Fig 4 12 for the full derivation

$$E \underset{lm}{\ } TE' \underset{lm}{\ } FT'E' \underset{lm}{\ } \mathbf{id}\,T'E' \underset{lm}{\ } \mathbf{id}\,E' \underset{lm}{\ } \mathbf{id}\quad TE' \underset{lm}{\ }$$

| MATCHED | STACK | INPUT | ACTION |
|---|---|---|---|
| | $E$ | **id id id** | |
| | $TE'$ | **id id id** | output $E$ $TE'$ |
| | $FT'E'$ | **id id id** | output $T$ $FT'$ |
| | **id** $T'E'$ | **id id id** | output $F$ **id** |
| **id** | $T'E'$ | **id id** | match **id** |
| **id** | $E'$ | **id id** | output $T'$ |
| **id** | $TE'$ | **id id** | output $E'$ $TE'$ |
| **id** | $TE'$ | **id id** | match |
| **id** | $FT'E'$ | **id id** | output $T$ $FT'$ |
| **id** | **id** $T'E'$ | **id id** | output $F$ **id** |
| **id id** | $T'E'$ | **id** | match **id** |
| **id id** | $FT'E'$ | **id** | output $T'$ $FT'$ |
| **id id** | $FT'E'$ | **id** | match |
| **id id** | **id** $T'E'$ | **id** | output $F$ **id** |
| **id id id** | $T'E'$ | | match **id** |
| **id id id** | $E'$ | | output $T'$ |
| **id id id** | | | output $E'$ |

Figure 4 21 Moves made by a predictive parser on input **id id id**

Note that the sentential forms in this derivation correspond to the input that has already been matched in column MATCHED followed by the stack contents The matched input is shown only to highlight the correspondence For the same reason the top of the stack is to the left when we consider bottom up parsing it will be more natural to show the top of the stack to the right The input pointer points to the leftmost symbol of the string in the INPUT column □

### 4 4 5 Error Recovery in Predictive Parsing

This discussion of error recovery refers to the stack of a table driven predictive parser since it makes explicit the terminals and nonterminals that the parser hopes to match with the remainder of the input the techniques can also be used with recursive descent parsing

An error is detected during predictive parsing when the terminal on top of the stack does not match the next input symbol or when nonterminal $A$ is on top of the stack $a$ is the next input symbol and $M$ $A$ $a$ is **error** i e the parsing table entry is empty

#### Panic Mode

Panic mode error recovery is based on the idea of skipping over symbols on the input until a token in a selected set of synchronizing tokens appears Its effectiveness depends on the choice of synchronizing set The sets should be chosen so that the parser recovers quickly from errors that are likely to occur in practice Some heuristics are as follows

1 As a starting point place all symbols in FOLLOW $A$ into the synchronizing set for nonterminal $A$ If we skip tokens until an element of FOLLOW $A$ is seen and pop $A$ from the stack it is likely that parsing can continue

2 It is not enough to use FOLLOW $A$ as the synchronizing set for $A$ For example if semicolons terminate statements as in C then keywords that begin statements may not appear in the FOLLOW set of the nonterminal representing expressions A missing semicolon after an assignment may therefore result in the keyword beginning the next statement being skipped Often there is a hierarchical structure on constructs in a language for example expressions appear within statements which appear within blocks and so on We can add to the synchronizing set of a lower level construct the symbols that begin higher level constructs For example we might add keywords that begin statements to the synchronizing sets for the nonterminals generating expressions

3 If we add symbols in FIRST $A$ to the synchronizing set for nonterminal $A$ then it may be possible to resume parsing according to $A$ if a symbol in FIRST $A$ appears in the input

4 If a nonterminal can generate the empty string then the production deriving can be used as a default Doing so may postpone some error detection but cannot cause an error to be missed This approach reduces the number of nonterminals that have to be considered during error recovery

5 If a terminal on top of the stack cannot be matched a simple idea is to pop the terminal issue a message saying that the terminal was inserted and continue parsing In effect this approach takes the synchronizing set of a token to consist of all other tokens

**Example 4 36** Using FIRST and FOLLOW symbols as synchronizing tokens works reasonably well when expressions are parsed according to the usual grammar 4 28 The parsing table for this grammar in Fig 4 17 is repeated in Fig 4 22 with synch indicating synchronizing tokens obtained from the FOLLOW set of the nonterminal in question The FOLLOW sets for the nonterminals are obtained from Example 4 30

The table in Fig 4 22 is to be used as follows If the parser looks up entry $M A a$ and finds that it is blank then the input symbol $a$ is skipped If the entry is synch then the nonterminal on top of the stack is popped in an attempt to resume parsing If a token on top of the stack does not match the input symbol then we pop the token from the stack as mentioned above

| NON TERMINAL | INPUT SYMBOL | | | | | |
|---|---|---|---|---|---|---|
| | **id** | | | | | |
| $E$ | $E$ $TE'$ | | | $E$ $TE'$ | synch | synch |
| $E'$ | | $E$ $TE'$ | | | $E$ | $E$ |
| $T$ | $T$ $FT'$ | synch | | $T$ $FT'$ | synch | synch |
| $T'$ | | $T'$ | $T'$ $FT'$ | | $T'$ | $T'$ |
| $F$ | $F$ **id** | synch | synch | $F$ $E$ | synch | synch |

Figure 4 22 Synchronizing tokens added to the parsing table of Fig 4 17

On the erroneous input **id** **id** the parser and error recovery mechanism of Fig 4 22 behave as in Fig 4 23 □

| STACK | INPUT | REMARK |
|---|---|---|
| $E$ | **id** **id** | error skip |
| $E$ | **id** **id** | **id** is in FIRST $E$ |
| $TE'$ | **id** **id** | |
| $FT'E'$ | **id** **id** | |
| **id** $T'E'$ | **id** **id** | |
| $T'E'$ | **id** | |
| $FT'E'$ | **id** | |
| $FT'E'$ | **id** | error $M$ $F$ synch |
| $T'E'$ | **id** | $F$ has been popped |
| $E'$ | **id** | |
| $TE'$ | **id** | |
| $TE'$ | **id** | |
| $FT'E'$ | **id** | |
| **id** $T'E'$ | **id** | |
| $T'E'$ | | |
| $E'$ | | |

Figure 4 23 Parsing and error recovery moves made by a predictive parser

The above discussion of panic mode recovery does not address the important issue of error messages The compiler designer must supply informative error messages that not only describe the error they must draw attention to where the error was discovered

**Phrase level Recovery**

Phrase level error recovery is implemented by filling in the blank entries in the predictive parsing table with pointers to error routines These routines may change insert or delete symbols on the input and issue appropriate error messages They may also pop from the stack Alteration of stack symbols or the pushing of new symbols onto the stack is questionable for several reasons First the steps carried out by the parser might then not correspond to the derivation of any word in the language at all Second we must ensure that there is no possibility of an infinite loop Checking that any recovery action eventually results in an input symbol being consumed or the stack being shortened if the end of the input has been reached is a good way to protect against such loops

## 4 4 6 Exercises for Section 4 4

**Exercise 4 4 1** For each of the following grammars devise predictive parsers and show the parsing tables You may left factor and or eliminate left recursion from your grammars first

a The grammar of Exercise 4 2 2 a

b The grammar of Exercise 4 2 2 b

c The grammar of Exercise 4 2 2 c

d The grammar of Exercise 4 2 2 d

e The grammar of Exercise 4 2 2 e

f The grammar of Exercise 4 2 2 g

**Exercise 4 4 2** Is it possible by modifying the grammar in any way to construct a predictive parser for the language of Exercise 4 2 1 postfix expressions with operand $a$

**Exercise 4 4 3** Compute FIRST and FOLLOW for the grammar of Exercise 4 2 1

**Exercise 4 4 4** Compute FIRST and FOLLOW for each of the grammars of Exercise 4 2 2

**Exercise 4 4 5** The grammar $S \quad a\ S\ a \mid a\ a$ generates all even length strings of $a$ s We can devise a recursive descent parser with backtrack for this grammar If we choose to expand by production $S \quad a\ a$ first then we shall only recognize the string $aa$ Thus any reasonable recursive descent parser will try $S \quad a\ S\ a$ first

a Show that this recursive descent parser recognizes inputs $aa$ $aaaa$ and $aaaaaaaa$ but not $aaaaaa$

b What language does this recursive descent parser recognize

The following exercises are useful steps in the construction of a Chomsky Normal Form grammar from arbitrary grammars as defined in Exercise 4 4 8

**Exercise 4 4 6** A grammar is *free* if no production body is called an *production*

a Give an algorithm to convert any grammar into an free grammar that generates the same language with the possible exception of the empty string no free grammar can generate *Hint* First find all the nonterminals that are *nullable* meaning that they generate perhaps by a long derivation

b Apply your algorithm to the grammar $S$ $aSbS \mid bSaS \mid$

**Exercise 4 4 7** A *single production* is a production whose body is a single nonterminal i e a production of the form $A$ $B$

a Give an algorithm to convert any grammar into an free grammar with no single productions that generates the same language with the possible exception of the empty string *Hint* First eliminate productions and then find for which pairs of nonterminals $A$ and $B$ does $A$ $B$ by a sequence of single productions

b Apply your algorithm to the grammar 4 1 in Section 4 1 2

c Show that as a consequence of part a we can convert a grammar into an equivalent grammar that has no *cycles* derivations of one or more steps in which $A$ $A$ for some nonterminal $A$

**Exercise 4 4 8** A grammar is said to be in *Chomsky Normal Form* CNF if every production is either of the form $A$ $BC$ or of the form $A$ $a$ where $A$ $B$ and $C$ are nonterminals and $a$ is a terminal Show how to convert any grammar into a CNF grammar for the same language with the possible exception of the empty string no CNF grammar can generate

**Exercise 4 4 9** Every language that has a context free grammar can be recognized in at most $O$ $n^3$ time for strings of length $n$ A simple way to do so called the *Cocke Younger Kasami* or CYK algorithm is based on dynamic programming That is given a string $a_1a_2$ $a_n$ we construct an $n$ by $n$ table $T$ such that $T_{ij}$ is the set of nonterminals that generate the substring $a_ia_{i\ \ 1}$ $a_j$ If the underlying grammar is in CNF see Exercise 4 4 8 then one table entry can be filled in in $O$ $n$ time provided we fill the entries in the proper order lowest value of $j$ $i$ first Write an algorithm that correctly fills in the entries of the table and show that your algorithm takes $O$ $n^3$ time Having filled in the table how do you determine whether $a_1a_2$ $a_n$ is in the language

**Exercise 4 4 10** Show how having filled in the table as in Exercise 4 4 9 we can in $O\ n$ time recover a parse tree for $a_1 a_2 \quad a_n$ *Hint* modify the table so it records for each nonterminal $A$ in each table entry $T_{ij}$ some pair of nonterminals in other table entries that justified putting $A$ in $T_{ij}$

**Exercise 4 4 11** Modify your algorithm of Exercise 4 4 9 so that it will find for any string the smallest number of insert delete and mutate errors each error a single character needed to turn the string into a string in the language of the underlying grammar

| | | |
|---|---|---|
| *stmt* | | **if** *e* **then** *stmt stmtTail* |
| | \| | **while** *e* **do** *stmt* |
| | \| | **begin** *list* **end** |
| | \| | *s* |
| *stmtTail* | | **else** *stmt* |
| | \| | |
| *list* | | *stmt listTail* |
| *listTail* | | *list* |
| | \| | |

Figure 4 24 A grammar for certain kinds of statements

**Exercise 4 4 12** In Fig 4 24 is a grammar for certain statements You may take $e$ and $s$ to be terminals standing for conditional expressions and other statements respectively If we resolve the conflict regarding expansion of the optional else nonterminal *stmtTail* by preferring to consume an **else** from the input whenever we see one we can build a predictive parser for this grammar Using the idea of synchronizing symbols described in Section 4 4 5

a Build an error correcting predictive parsing table for the grammar

b Show the behavior of your parser on the following inputs

*i* **if** *e* **then** *s* **if** *e* **then** *s* **end**
*ii* **while** *e* **do begin** *s* **if** *e* **then** *s* **end**

## 4 5 Bottom Up Parsing

A bottom up parse corresponds to the construction of a parse tree for an input string beginning at the leaves the bottom and working up towards the root the top It is convenient to describe parsing as the process of building parse trees although a front end may in fact carry out a translation directly without building an explicit tree The sequence of tree snapshots in Fig 4 25 illustrates