Assignment-I

1. State and prove Weierstrass's theorem for the sequence of functions.
2. Expand $f(z) = e^z + e^{1/z^2}$ in a Laurent series for $|z| > 0$.
3. Expand
$$f(z) = \frac{1}{(z+1)(z^2+2)}$$
in a Laurent series valid for
(a) $1 < |z| < \sqrt{2}$, (b) $|z| > \sqrt{2}$.
4. Given $u(x, y) = x^2 - y^2$, find its harmonic conjugate $v(x, y)$ and the corresponding analytic function $f(z)$.
5. Prove that the sum of two harmonic functions is also harmonic.
6. State and prove Mean-value theorem for harmonic functions.
7. State and prove the Maximum Principle for harmonic functions.
8. State and prove Poisson Integral Formula for Harmonic Functions.
9. Given that $u(x, y)$ is harmonic, find a corresponding analytic function $f(z)$ such that $u(x, y)$ is the real part of $f(z)$.
10. State and prove Schwarz's theorem regarding harmonic functions.