No. :

Date :

Solⁿ 1. $B(0) = Rs\,100,\ B(1) = Rs\,110$

$S(0) = Rs\,80,\ V(0) = Rs\,100000$

$$S(1) = \begin{cases} Rs\,100, & \text{with prob } p = 0.8 \\ Rs\,60, & \text{with prob } p = 0.2 \end{cases}$$

$V(0) = 100000 = x\,S(0) + y\,B(0)$

$x = 100000 \times \frac{3}{5} \times \frac{1}{80} = 750$ stocks

$y = 100000 \times \frac{2}{5} \times \frac{1}{100} = 400$ bonds

portfolio (750, 400)

$V(1) = x\,S(1) + y\,B(1)$

$$= \begin{cases} 750 \times 100 + 400 \times 110 = 119000, & \text{with prob } p = 0.8 \\ 750 \times 60 + 400 \times 110 = 89000, & \text{with prob } p = 0.2 \end{cases}$$

$$K_V = \frac{V(1) - V(0)}{V(0)} = \begin{cases} 0.19, & \text{with prob } p = 0.8 \\ -0.11, & \text{with prob } p = 0.2 \end{cases}$$

$E(K_V) = 0.19 \times 0.8 - 0.11 \times 0.2 = 0.13$

$Var(K_V) = (0.19 - 0.13)^2 0.8 + (-0.11 - 0.13)^2 0.2$

$= 0.0144$

Risk, $\sigma(K_V) = 0.12$

Solⁿ 2. $B(0) = Rs\,90,\ B(1) = Rs\,100$

$S(0) = Rs\,25,\ V(0) = ?,\ V(1) = ?$

$$S(1) = \begin{cases} Rs\,30, & \text{with prob } p \\ Rs\,20, & \text{with prob } 1-p \end{cases}$$

portfolio (10, 15)

$V(0) = x\,S(0) + y\,B(0) = Rs\,1600$

$$V(1) = \begin{cases} Rs\,1800, & \text{with prob } p \\ Rs\,1700, & \text{with prob } 1-p \end{cases}$$

Punk

No. :

Date :

$$K_v = \frac{V(1)-V(0)}{V(0)} = \begin{cases} 0.125, & \text{with prob } p \\ 0.0625, & \text{with prob } 1-p \end{cases}$$

$$E(K_v) = 0.125p + 0.0625(1-p)$$
$$= 0.0625(1+p)$$

Sol^{n}3.

$B(0) = Rs\ 100,\ B(1) = Rs\ 110$

$S(0) = Rs\ 80,\ V(0) = 10{,}000\ Rs$

$$S(1) = \begin{cases} Rs\ 100, & \text{with prob } p = 0.8 \\ Rs\ 60, & \text{with prob } p = 0.2 \end{cases}$$

$$x = 10000 \times \frac{1}{2} \times \frac{1}{80} = 62.5 \text{ stocks}$$

$$y = 10000 \times \frac{1}{2} \times \frac{1}{100} = 50 \text{ bonds}$$

portfolio $(62.5, 50)$

$$V(1) = x\,S(1) + y\,B(1)$$
$$= \begin{cases} 62.5\times100 + 50\times110 = 11750, & \text{with prob } p = 0.8 \\ 62.5\times60 + 50\times110 = 9250, & \text{with prob } p = 0.2 \end{cases}$$

$$K_v = \frac{V(1)-V(0)}{V(0)} = \begin{cases} 0.175, & \text{with prob } p = 0.8 \\ -0.075, & \text{with prob } p = 0.2 \end{cases}$$

$$E(K_v) = 0.175\times0.8 - 0.075\times0.2 = 0.125$$

$$Var(K_v) = (0.175-0.125)^2\,0.8 + (-0.075-0.125)^2\,0.2$$
$$= 0.01$$

Risk, $\sigma(K_v) = 0.1$

Sol^{n}4.

$B(0) = Rs\ 90,\ B(1) = Rs\ 100,\ S(0) = Rs\ 25$

$$S(1) = \begin{cases} Rs\ 30, & \text{with prob } p \\ Rs\ 20, & \text{with prob } 1-p \end{cases}$$

$$V(1) = \begin{cases} 1160, & \text{if stock goes up} \\ 1040, & \text{if stock goes down} \end{cases}$$

Let no. of stocks be x & no. of bonds be y

ie. portfolio (x, y)

No. :

Date :

$V(1) = x\,S(1) + y\,B(1)$

$30x + 100y = 1160$ ——— (i)

$20x + 100y = 1040$ ——— (ii)

solving (i) & (ii) we get,

$x = 12$ & $y = 8$

ie. portfolio (12, 8)

$V(0) = x\,S(0) + y\,B(0)$

$= 300 + 720 = 1020$ Rs.

Solⁿ 5. $B(0) = \text{Rs } 100$, $B(1) = \text{Rs } 110$

$S(0) = \text{Rs } 80$, $K = \text{Rs } 100$ (strike)

$$S(1) = \begin{cases} \text{Rs } 100, & \text{with prob } p = 0.8 \\ \text{Rs } 60, & \text{with prob } p = 0.2 \end{cases}$$

(i) $$C(1) = \begin{cases} S(1) - K = 0, & \text{with prob } p = 0.8 \\ 0, & \text{with prob } p = 0.2 \end{cases}$$

using replicating portfolio procedure,

$100x + 110y = 0$

$60x + 110y = 0$

above 2 eqⁿ gives $x = y = 0$ ie.

$C(0) = 0$ Rs.

$$P(1) = \begin{cases} 0, & \text{with prob } p = 0.8 \\ K - S(1) = 40, & \text{with prob } p = 0.2 \end{cases}$$

using replicating portfolio procedure,

$100x + 110y = 0$

$60x + 110y = 40$

solving above 2 eqⁿ we get,

$x = -1$ & $y = 10/11$

ie. we get,

$P(0) = x\,S(0) + y\,B(0) = -80 + \frac{1000}{11} = \frac{120}{11} \approx \text{Rs. } 10.91$

(ii) Since wealth distribution equally in given stock, given call & given put

No. of shares $= \frac{300}{80} = \frac{15}{4} = 3.75$

for call option, money will not be invested as it is not beneficial to use it.

No. of put options $= \frac{300}{10.91} = 27.5$

Hence, final wealth is given by,

$$V(1) = \begin{cases} 3.75 \times 100 + 300 + 0 = 675 \text{ Rs}, & \text{with prob } p = 0.8 \\ 3.75 \times 60 + 300 + 27.5 \times 40 = 1625 \text{ Rs}, & \text{with prob } p = 0.2 \end{cases}$$

Sol^n 6. $B(0) = \text{Rs } 100$, $B(1) = \text{Rs } 110$

$S(0) = \text{Rs } 100$, $V(0) = \text{Rs } 1000$, $K = \text{Rs } 100$

$$S(1) = \begin{cases} \text{Rs } 120, & \text{with prob } p \\ \text{Rs } 80, & \text{with prob } 1-p \end{cases}$$

Let options be call options. Then we have,

$$C(1) = \begin{cases} 20, & \text{with prob } p \\ 0, & \text{with prob } 1-p \end{cases}$$

using replicating portfolio procedure,

$120x + 110y = 20$ ——— (i)

$80x + 110y = 0$ ——— (ii)

solving (i) & (ii) we get,

$x = \frac{1}{2}$ & $y = \frac{-4}{11}$

ie. we get,

$C(0) = 100 \times \frac{1}{2} - 100 \times \frac{4}{11} = \text{Rs } \frac{150}{11}$

we have to split in fifty-fifty for stocks & options,

no. of stocks $= \frac{500}{100} = 5$ stocks

no. of options $= 500 \times \frac{11}{150} = \frac{110}{3}$ options

putting values of no. of stocks & no. of options we get,

No. :

Date :

final wealth of investor is given as,

$$V(1) = \begin{cases} 5\times 120 + 36.67\times 20 = 1333.33 \text{ Rs}, & \text{with prob } p \\ 5\times 80 + 36.67\times 0 = 400 \text{ Rs}, & \text{with prob } 1-p \end{cases}$$

Solⁿ 7. No Arbitrage Principle

There is no admissible portfolio with initial value $V(0)=0$ such that $V(1)>0$ with a non zero probability

Lets suppose $V(0)=0$

Rs 10000 is borrowed from bank

(i) buy $\frac{10000}{80} = 125$ pounds from dealer B

(ii) invest in bank for an year & get $125(1+0.06) = 132.5$ pounds

(iii) sell pounds to dealer A & get $Rs(132.5\times 79) = Rs\, 10467.5$

(iv) return the borrowed amount with interest to bank ie.

$Rs\, 10000(1+0.04) = Rs\, 10400$

(v) profit $= Rs\, 10467.5 - Rs\, 10400 = Rs\, 67.5 > 0$

Hence, an arbitrage opportunity exists.

Solⁿ 8. $B(0) = Rs\, 100$, $B(1) = Rs\, 110$

$S(0) = Rs\, 50$

Let the forward price be F

(i) Short forward contract,

if we sell at a fixed price F

(a) borrow Rs 50

(b) buy asset for $S(0) = Rs\, 50$

portfolio $(1, -\frac{1}{2}, -1)$

Now we will sell the asset at F & return the amount 55 Rs to borrower

profit $= Rs\,(F-55)$

Now for no arbitrage condition,

$F - 55 \le 0$

$F \le 55$

No. :

Date :

(ii) Long forward contract,,

if we buy at F at time $t=1$ then,

(a) Sell short the asset at Rs 50

(b) investing risk free.

we get 55 Rs from investment we get the asset at F, we will return the asset to the owner.

profit = Rs (55 - F)

Now, for no arbitrage condition,

$55 - F \leq 0$

$55 \leq F$

from inequalities we obtain from cases (i) & (ii), we get,

$F \leq 55$ & $55 \leq F$

which implies $F = 55$ Rs

Solⁿ 9. The investor makes a gain if the price of stock is above Rs 26 at the time of exercise (if we ignore the time value of money)

Solⁿ 10. $S(0) =$ Rs 5000 per 100 gm = Rs 50000 per kg

Storage cost, $c =$ Rs 0.5 per gm per year = Rs 125 per kg per quater

$r = \frac{9\%}{4} = \frac{0.09}{4} = 0.0225$

$R = 1 + r = 1 + 0.0225 = 1.0225$

$T = 6$ months = 2 quaters

So, forward price is given as

$$F(0,2) = S(0)R^2 + \sum_{i=1}^{2} cR^i$$

$$F(0,2) = 50{,}000(1.0225)^2 + 125[1.0225 + (1.0225)^2]$$

$$= \text{Rs. } 52533.81328$$