

Financial Engineering

Assignment-4

- 1 Suppose there are three financial market scenarios $Q = \{w_1, w_2, w_3\}$ with different probabilities of occurrence. Consider the following table showing the returns on two different stocks in these three scenarios

Scenario	Probability	Return K1 %	Return K2 %
w_1	0.2	-10	-30
w_2	0.5	0	20
w_3	0.3	20	15

- (a) What are the expected returns on the stocks?
 (b) Suppose 60% of the available fund is invested in stock 1 and the remaining is invested in stock 2, then what is the expected return of the portfolio?
 (c) Compute the weights if the expected return on a portfolio is 20%.

- 2 Consider the following data for two different stocks

Scenario	Probability	Return K ₁ %	Return K ₂ %
w_1	0.4	-10	20
w_2	0.2	0	20
w_3	0.4	20	10

Suppose a portfolio comprises of 40% of total investment in stock 1 and 60% in stock 2. Compare the risk of the portfolio with the risks of its individual components. What will be the risk situation if a portfolio is designed with investment of 80% in stock 1 and the remaining in stock 2.

- 3 Prove that if short sales are not allowed then the risk of the portfolio can not exceed the greater of the risks of the individual components of the portfolio.
- 4 Suppose the portfolios are constructed using three securities a_1, a_2, a_3 with expected returns, $\mu_1 = 20\%$, $\mu_2 = 13\%$, $\mu_3 = 4\%$, standard deviations of returns, $\sigma_1 = 25\%$, $\sigma_2 = 28\%$, $\sigma_3 = 20\%$, and the correlation between returns, $\rho_{12} = 0.3$, $\rho_{13} = 0.15$ and $\rho_{23} = 0.4$. Among all the attainable portfolios, find the one with minimum variance. What are the weights of the three securities in this portfolio? Also compute the expected return and standard deviation of this portfolio.
- 5 Consider the following data for three different stocks.

Prob	return K1	return K2	return K3
0.1	0.3	0.08	-0.1
0.5	0.13	0.11	0.34
0.2	0.15	0.4	0.11
0.2	0.25	0.12	0.15

Among all the attainable portfolios, find the one with minimum variance. What are the weights of the three securities in this portfolio? Also compute the expected return and standard deviation of this portfolio.

- 6 Consider the following data

	μ	σ
Asset 1	10%	5%
Asset 2	8%	2%

For each correlation coefficient $\rho = -1, -0.5, 0, 0.5, 1$, what is the combination of the two assets that yields the minimum standard deviation and what is the minimum value of the standard deviation?

- 7 Consider three risky assets with the variance-covariance matrix and expected returns (all data in %) as follows.

Variance-covariance matrix (C)			Return (M)
10	4	0	5
4	12	6	6
0	6	10	1

Find two efficient portfolios. Also construct the portfolio giving the return of 2.8% with minimum risk. Will this portfolio be also efficient?

- 8 Suppose an investor is interested in constructing a portfolio with one risk-free asset a_1 , and three risky assets a_2 , a_3 and a_4 . Let the expected returns of a_1 , a_2 , a_3 and a_4 be 6%, 10%, 12% and 18% respectively. Let the variance-covariance matrix C of the three risky assets be $C = \begin{pmatrix} 4 & 20 & 40 \\ 20 & 10 & 70 \\ 40 & 70 & 14 \end{pmatrix}$. Determine all efficient portfolios for the investor.

- 9 Assume that the following assets are correctly priced according to the security market line. Derive the security market line. $\mu_1 = 6\%$, $\beta_1 = 0.5$, $\mu_2 = 12\%$, $\beta_2 = 1.5$. What is the expected return on an asset with $\beta = 2$?

- 10 If the following two assets are correctly priced according to the security market line, what is the return of the market portfolio? What is the risk-free return?

$$\mu_1 = 9.5\%, \quad \beta_1 = 0.8, \quad \mu_2 = 13.5\%, \quad \beta_2 = 1.3$$