Total no. of pages :2
6th SEMESTER
END SEMESTER EXAMINATION

Roll No.____________
B.Tech (MC- Engg.)
May 2018

MC – 306 Financial Engineering

Time : 3 hrs

Max. Marks: 50

Note: *Q.No.1 is compulsory, answer any other three questions.* All questions carry equal mark. Statistical table is allowed. Assume missing data , if any.

1. (a) The current price of silver is $Rs.$ 5000 per 100gm. The storage cost is $Rs.$ 0.60 per gm per year payable quarterly in advance. Assuming that constant interest rate of 9% compounded quarterly, calculate the forward price of silver for 1kg for delivery in 6 months.

(b) Consider purchase of 100 units of 3-month Rs.25-strike European call option. It is given that the stock is currently selling for Rs.20; the continuous compounding risk free interest is 5%; the stocks volatility is 24% per annum. If the stock pays dividends continuously at the rate of 3% per annum, determine the price of block of 100 call options, assuming the Black-Scholes framework.

(c) Let $X(t) = \mu t + \sigma W(t), -\infty < \mu < \infty, 0 < \sigma < \infty$. Prove that $\{X(t), t \geq 0\}$ is a martingale for $\mu = 0$.

(d) Prove that if short sales are not allowed then the risk of the portfolio can not exceed the greater of the risks of the individual components of the portfolio.

2. (a) State and prove Put-Call parity formula.

(b) Consider a stock whose value S(t) follows sde $dS = r.Sdt + \sigma.SdW$ and has a current price S(0). What is the probability that a call option is in the money based on a strike price $K = 1.25\,S(0)$ at time of expiration T? Given that $T = 0.5, r = 0.04$ and $\sigma = 0.10$.

3. (a) Find the stochastic differential of $W^2(t)$.

(b) Use the first version of Ito-Doeblin formula to evaluate $\int_0^T 3W^3(t)dW(T)$

4. (a) Let $B(0) = Rs.\,100, B(1) = Rs.\,110$ and $S(0) = Rs.\,82$.

$$S(1) = \begin{cases} Rs.\,95, & with\ probability\ p = 0.80 \\ Rs.\,70, & with\ probability\ p = 0.20. \end{cases}$$

For K = Rs.90 and T = 1 year, determine P(0).

(b) Derive the expression for line which converts into Capital Market line. Explain the condition of CML with respect to efficient frontier.

5. (a) Find the expression for feasible region of 'n' asset portfolio in (σ, μ) – plane , and describe it.

(b) Suppose the portfolios are constructed using three securities a1, a2, a3 with expected returns, $\mu_1 = 20\%$, $\mu_2 = 13\%$, $\mu_3 = 4\%$ standard deviations of returns, $\sigma_1 = 25\%$, $\sigma_2 = 28\%$, $\sigma_3 = 20\%$, and the correlation between returns, $\rho_{12} = 0.3$, $\rho_{13} = 0.15$ and $\rho_{23} = 0.4$. What are the weights of the three securities in this portfolio with minimum risk, While desired expected return is 20%.

— 405 —

Total no. of pages :2

Roll No.______________

6th SEMESTER

B.Tech (MC- Engg.)

END SEMESTER EXAMINATION

May 2019

MC – 306 Financial Engineering

Time : 3 hrs

Max. Marks: 50

Note: *Q.No.1 is compulsory, answer any other three questions.* All questions carry equal mark. Statistical table is allowed. Assume missing data , if any.

1. (a) Define forward contract. Let $B(0) = Rs.100$, $B(1) = Rs.112$, $S(0) = RS.34$ and $T = 1$. Find the forward price F. Also find an arbitrage opportunity if F is taken to be Rs.38.60.

(b) The current stock price is Rs. 250. A six month call option on this stock with strike price Rs. 255 is priced using Black-Scholes formula. It is given that continuously compounded risk free rate is 4%, stock pays no dividend and the volatility of the stock is 20%. Determine the price of call and put options.

(c) Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. Prove that $\{N(t) - \lambda t;\ \lambda > 0\}$ is a martingale.

(d) A portfolio consisting of two assets a_1 and a_2 with weights w_1 and w_2, returns r_1 and r_2 and standard deviations σ_1 and σ_2 respectively. Also $\rho_{12} = 1$. Find the expression of weights and return for minimum risk of portfolio. Also find the value of minimum risk.

2. (a) Evaluate $\int_0^T w(t)dw(t)$ using quadratic variation.

(b) Consider a stock whose value S(t) follows sde $dS = r.Sdt + \sigma.SdW$ and has a current price Rs.40. What is the probability that a call option is exercised based on a strike price $K = Rs.52$ at time of expiration T? Given that $T = 0.5, r = 0.04$ and $\sigma = 0.20$.

3. (a) Let S(0) = \$50, r = 5%, u = 0.13 and d = −0.08. Find the price of a European call and put with strike price X = \$55 to be exercised after N = 3 time steps using CRR- formula.

(b) Find the SDE of $W^3(t)$ using Ito-Doeblin formula of version two.

4. (a) Define risk neutral probability, obtain its expression. Prove that under risk neutral probability after nth period $E\{S(n)\} = S(0)[1+r]^n$, where 'r' is risk free interest rate.

(b) Derive the expression for line which converts into Capital Market line.

5. (a) Prove that portfolio with minimum risk has weights given by

$$w = \frac{C^{-1}e}{e^T C^{-1} e},$$

where C is variance and covariance matrix, and $e^T = (1,1,\ldots 1) \in R^n$.

(b) Consider a portfolio of the assets a_1 and a_2 with no short sell and with the following statistical parameters $\mu_1 = 15\%$, $\mu_2 = 30\%$, $\sigma_1 = 20\%$, $\sigma_2 = 35\%$, $Cov(r_1, r_2) = -0.0035$, where r_1 & r_2 are return of the assets. Find the value of weights for minimum risk, expected return and minimum risk of the portfolio.

-154-

Total no. of pages :2
6th SEMESTER
SUP EXAMINATION

Roll No.____________
B.Tech (MC- Engg.)
Sep 2019

MC – 306 Financial Engineering

Time : 3 hrs

Max. Marks: 50

Note: *Q.No.1 is compulsory, answer any other three questions.* All questions carry equal mark. Statistical table is allowed. Assume missing data , if any.

1. (a) The current price of gold is Rs.25000 per 10 gm. The storage cost is Rs.200 per gm per year payable quarterly in advance. Assuming that constant interest rate of 9% compounded quarterly, calculate the forward price of gold for delivery in nine months.

   (b) The current stock price is Rs. 225. A six month call option on this stock with strike price Rs. 245 is priced using Black-Scholes formula. It is given that continuously compounded risk free rate is 6%, stock pays no dividend and the volatility of the stock is 18%. Determine the price of call and put options.

   (c) Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. Prove that $\{N(t); \lambda > 0\}$ is not a martingale.

   (d) Consider a portfolio of two assets. a1 & a2 with no short sell, with the following statistical parameters $\mu 1 = 5\%$, $\mu 2 = 10\%$, $\sigma 1 = 10\%$, $\sigma 2 = 40\%$, $\rho 12 = -0.05$. Find the value of minimum risk, the expected return and weight of the assets.

2. (a) Find the stochastic differential equation of $Cos(W(t))$ using Ito – Doeblin formula of version two.

   (b) Consider a stock whose value S(t) follows sde $dS = r.\,Sdt + \sigma.\,SdW$ and has a current price Rs.40. What is the probability that a call option is exercised based on a strike price $K = Rs.\,52$ at time of expiration T? Given that $T = 9$ months, $r = 0.04$ and $\sigma = 0.20$.

3. (a) Let $S(0) = \$50$, $r = 5\%$, $u = 0.3$ and $d = -0.1$. Find the price of a European call and put with strike price $X = \$60$ to be exercised after $N = 2$ time steps, using CRR- formula.

(b) Find the SDE of $W^2(t)$ using Ito-Doeblin formula of version two.

4. (a) State and prove the Put – Call parity formula for European call and put option, with current price of stock S(0) and exercise price X and exercise time T.

(b) Let $\{S_n\ ,\ n=0,1,2,\ldots\}$ be a symmetric random walk and F_n be a filtration. Show that $Y_n = (-1)^n \, Cos(\pi S_n)$ is a martingale with respect to F_n.

5. (a) For two asset portfolio prove that the variance of the portfolio can no exceed the greater of the variances σ_1^2 & σ_2^2 of the component assets, if there is no short sell.

(b) Using the following data:

Scenario	Probability	Return $K1$	Return $K2$
$\omega 1$ (recession)	0.4	-10%	20%
$\omega 2$ (stagnation)	0.2	0%	20%
$\omega 3$ (boom)	0.4	20%	10%

Find the weights in a portfolio with expected return $\mu_V = 26\%$ and compute the risk of this portfolio

... (n-1) edges.

(5)[CO2]

[a] Is the following statement true or false? If all edges in a flow network have distinct capacities, then there is a unique path for the possible maximum flow. Justify your answer with a (short) proof or give a counter example.

(3)[CO3,4]

1

Total no. of pages :2

Roll No.________________

6th SEMESTER

B.Tech (MC- Engg.)

END SEMESTER EXAMINATION

MAY 2023

MC – 306 Financial Engineering

Time : 3 hrs

Max. Marks: 50

Note: Q.No.1 is compulsory, answer any other three questions.
Statistical table is allowed. Assume missing data, if any.

1. (a) Let A(0)=100 , A(1)=110 , A(2)=121, and stock price can follow four possible scenario; 5 CO3

Scenario	S(0)	S(1)	S(2)
W1	50	60	72
W2	50	60	58
W3	50	45	55
W4	50	45	40

$S(i), i = 0,1,2$ is stock price at i^{th} time interval. Compute Risk Neutral Probability (RNP).

(b) Consider a portfolio of two assets a1 & a2 with no short sell, with the following statistical parameters $\mu_1 = 7.5\%, \mu_2 = 15\%$, $\sigma_1 = 12\%$, $\sigma_2 = 35\%$, $\rho_{12} = -0.22$. Find the value of minimum risk, the expected return and weight of the assets. 5 CO5

(c) The stock price is Rs.100. The continuously compounded risk free interest rate is 8% and the annual volatility is 20%. European Call options are written with a strike price of Rs.90 and time to expiration of 3 months. The stock will pay a dividend continuously at the rate of 2%. Use the Black – Scholes formula to find the price of one such call option. 5 CO3

(d) $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. Prove that $\{N(t) - \lambda t, t \geq 0\}$ is a martingale. 5 CO4

2. (a) A stock price following SDE $dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$ has an expected return of 16% and a volatility of 30%. The current price is Rs.38. What is the probability that a European call option on the stock with an exercise price of Rs.40 and a maturity date in 3 months will be exercised? 5 CO4

number of pendant vertices in a binary tree. (2)[CO1,4]

a tree with n vertices has $(n-1)$ edges. (5)[CO2]

...ing statement true or false? If all edges in a flow network have ...es, then there is a unique path for the possible maximum flow. ...answer with a (short) proof or give a counter example. (3)[CO3,4]

(b) Solve the SDE $dX(t) = X(t)\,dW(t)$ with $X(0) = 1$ and prove that its solution is $X(t) = \exp(W(t) - \frac{t}{2})$. — 5, CO4

3. (a) Evaluate $\int_0^T W^3(t)dW(t)$ using Ito – Doeblin formula of version two. — 5, CO4

(b) A stochastic process $\{S(t), t \geq 0\}$ is governed by
$dS(t) = aS(t)dt + bS(t)dW(t)$,
where a & b are constants. Find the SDE of $\sqrt{S(t)}$. — 5, CO4

4. (a) Consider a portfolio of two assets a_1 & a_2 with the following statistical parameters $\mu_1 = 10\%$, $\mu_2 = 20\%$, $\sigma_1 = 12\%$, $\sigma_2 = 25\%$, $\rho_{12} = -0.6$. Obtain the equation of Markovitz curve, and using that find value of minimum risk, the expected return. — 5, CO5

(b) Prove that if short sales are not allowed then the risk of the portfolio can not exceed the greater of the risks of the individual components of the portfolio. — 5, CO5

5. (a) A portfolio with three securities a_1, a_2, a_3 with expected returns, $\mu_1 = 20\%$, $\mu_2 = 13\%$, $\mu_3 = 4\%$, standard deviations of returns, $\sigma_1 = 25\%$, $\sigma_2 = 28\%$, $\sigma_3 = 20\%$, and the correlation between returns, $\rho_{12} = 0.3$, $\rho_{13}, = 0.15$ *and* $\rho_{23} = 0.4$. Compute the weights of individual assets in this portfolio for minimum variance. — 5, CO5

(b) Using the following data: — 5, CO5

Scenario	Probability	Return $K1$	Return $K2$
$\omega 1$ (recession)	0.2	−10%	30%
$\omega 2$ (stagnation)	0.5	24%	-10%
$\omega 3$ (boom)	0.3	20%	10%

Find the weights in a portfolio with expected return $\mu_P = 25\%$ and compute the risk of this portfolio

Total no. of pages :2

5^{th} SEMESTER

END SEMESTER EXAMINATION

Roll No.____________

B.Tech (MC- Engg.)

Jan 2021

MC – 303 Financial Engineering

Time : 3 hrs

Max. Marks: 70

Note: Q.No.1 is compulsory, answer any other three questions. Statistical table is allowed. Assume missing data , if any.

1. (a) The current price of gold is Rs.25000 per 10 gm. The storage cost is Rs.200 per gm per year payable quarterly in advance. Assuming that constant interest rate of 9% compounded quarterly, calculate the forward price of gold for delivery in nine months. 4

(b) A non-dividend paying stock is currently selling at Rs. 100 with annual volatility 20%. Assume that the continuously compounded risk-free interest rate is 5%. Using a two period CRR binomial option pricing model, find the price of one European *call option* on this stock with a strike price of Rs. 80 and time to expiration 4 years. 6

(c) Consider a portfolio of two assets a_1 & a_2 with no short sell, with the following statistical parameters $\mu_1 = 5\%$, $\mu_2 = 10\%$, $\sigma_1 = 10\%$, $\sigma_2 = 40\%$, $\rho_{12} = -0.05$. Find the value of minimum risk, the expected return and weight of the assets. 6

(d) The stock price is Rs.100. The annual continuously compounded risk free interest rate is 5% and the annual volatility relevant for the Black Scholes formula is 30%. Call options are written with a strike price of Rs.80 and time to expiration of 5 years. The stock will pay a dividend of Rs.20 In 2 years and another dividend of Rs. 30 in 3 years. Use the Black – Scholes formula to find the price of one such call option. 6

(e) Find the stochastic differential equation of $Cos(W(t))$ using Ito – Doeblin formula of version two. 6

2. (a) Define risk neutral probability, obtain its expression. Prove that under risk neutral probability after nth period $E(S(n)) = S(0)(1+r)^n$, where 'r' is risk free interest rate. 7

(b) Let $S(0) = \$50$, $r = 5\%$, $u = 0.3$ and $d = -0.1$. Find the price of a European call and put with strike price $X = 60$ dollars to be exercised after $N = 3$ time steps using CRR- formula. 7

3. (a) Let $\{S_n , n=0,1,2,....\}$ be a symmetric random walk and F_n be a filtration . Show that
$Y_n = (-1)^n Cos(\pi S_n)$ is a martingale with respect to F_n. 7

(b) Evaluate $\int_0^T W(t)\, dW(t)$ using quadratic variation .

4. (a) A stock being sold for Rs.45 and risk free interest rate is 6%and assume that a dividend of Rs.2 is paid after six months. Find the forward price of the contract on this stock with a delivery date as one year. Also find its value after nine months, the stock price at that time happen to be Rs.50.

(b) State and prove the Put – Call parity formula for European call and put option, with current price of stock S(0) and exercise price X and exercise time T. 7

5. (a) For two asset portfolio prove that the variance of the portfolio can not exceed the greater of the variances σ_1^2 & σ_2^2 of the component assets, if there is no short sell. 7

(b) Using the following data: 7

Scenario	Probability	Return $K1$	Return $K2$
$\omega 1$ (recession)	0.4	−10%	20%
$\omega 2$ (stagnation)	0.2	0%	20%
$\omega 3$ (boom)	0.4	20%	10%

Find the weights in a portfolio with expected return $\mu_V = 26\%$ and compute the risk of this portfolio

Total no. of pages :1
6th SEMESTER
SUPP EXAMINATION

Roll No.____________
B.Tech (MC- Engg.)
Aug 2018

MC – 306 Financial Engineering

Time : 3 hrs

Max. Marks: 50

Note: *Q.No.1 is compulsory, answer any other three questions.* All questions carry equal mark. Statistical table is allowed. Assume missing data , if any.

1. (a) Let $A(0) = 90$, $A(1) = 95$, $S(0) = 25$ dollars and let

$$S(1) = \begin{cases} 32 & \text{with probability } p, \\ 22 & \text{with probability } 1 - p, \end{cases}$$

where $0 < p < 1$. For a portfolio with $x = 10$ shares and $y = 15$ bonds calculate $V(0)$, $V(1)$ and K_V

(b) An investor paid \$92 for a bond with face value \$100 maturing in six months. When will the bond value reach \$98 if the interest rate remains constant?

(c) Find the stochastic differential of $\text{Cos}(W(t))$.

(d) The stock price is Rs.200. A 6-month European call option on the stock with strike price Rs.250 is priced using Black-Scholes formula. It is given that the continuously compounding risk free rate is 5%, stock pays no dividend. The volatility of the stock is 20%. Determine the price of call and put options.

2. (a) Let $S(0)$ = Rs.80, r = 10%, $u = 0.2$ and $d = -0.1$. Find the price of a European call and put with strike price X = Rs.100 to be exercised after $N = 2$ time steps using CRR- formula.

(b) Consider the following data
$S(0) = \text{Rs}. 50$, $K = \text{Rs}. 50$, $\sigma = 30\%$, $r = 8\%$. Assuming the Black Scholes frame work and that the stock pays no dividend, compute 3-months European call price and 3-months European put price using the Black-Scholes formula.

3. (a) Let $\{S_n, n = 0, 1, \ldots\}$ be a symmetric random walk and F_n be a filtration. Show that $Y_n = (-1)^n \text{Cos}(\pi S_n)$ is a martingale with respect to F_n.

(b) $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. Prove that , $\{N(t), t \geq 0\}$ is not a Martingale.

4. (a) If $S(0)$ is the price of asset at t=0, then prove that the forward price will be

$$F(0,T) = \frac{S(0)}{d(0,T)}$$

$d(0,T)$ is the discount factor between t=0 to t=T.

(b) Let $A(0) = 100$, $A(1) = 110$, $S(0)$ = Rs.100, strike price Rs.90 and

$$S(1) = \begin{cases} 115 & \text{with probability } 0.7 \\ 85 & \text{with probability } 0.3 \end{cases}$$

find put option price.

5. (a) Prove that portfolio with minimum risk has weights given by

$$w = \frac{C^{-1}e}{e^T C^{-1} e},$$

where C is variance and covariance matrix, and $e^T = (1,1,\ldots1) \in R^n$

(b) Using the following data:

Scenario	Probability	Return K1	Return K2
ω1 (recession)	0.3	−10%	20%
ω2 (stagnation)	0.3	0%	20%
ω3 (boom)	0.4	20%	10%

Find the weights in a portfolio with expected return μV =40% and compute the risk of this portfolio