Sol^n 1. (a) $E(R_{K1}) = 0.2(-10) + 0 \times 0.5 + 20 \times 0.3$

$= -2 + 0 + 6 = 4\%$

$E(R_{K2}) = 0.2(-30) + 20 \times 0.5 + 15 \times 0.3$

$= -6 + 10 + 4.5 = 8.5\%$

(b) 60% of available fund is invested in K1, then weight $K_1 = 0.6$ & $K_2 = 0.4$

$E(\text{Portfolio}) = W_{R_1} E(R_{K_1}) + W_{R_2} E(R_{K_2})$

$= 0.6 \times 4 + 0.4 \times 8.5$

$= 5.8\%$

(c) Let $W_{K_1} = x$, then $W_{K_2} = 1 - x$

$x(4) + (1-x)(8.5) = 20$

$4x + 8.5 - 8.5x = 20$

$-4.5x = 11.5$

$x = -2.56$

i.e. weight of stock 1 is -2.56 & hence, weight of stock 2 is 3.56

Sol^n 2. $E(R_{K_1}) = 0.4(-10) + 0.2(0) + 0.4(20)$

$= -4 + 8 = 4\%$

$E(R_{K_2}) = 0.4(20) + 0.2(20) + 0.4(10)$

$= 8 + 4 + 4 = 16\%$

$Var(R_{K_1}) = E(R_{K_1}^2) - (E(R_{K_1}))^2 = 0.0184$

$Var(R_{K_2}) = E(R_{K_2}^2) - (E(R_{K_2}))^2 = 0.0024$

$Cov(R_{K_1}, R_{K_2}) = E(R_{K_1} R_{K_2}) - E(R_{K_1}) E(R_{K_2}) = -0.0064$

$$\rho_{K_1 K_2} = \frac{Cov(R_{K_1}, R_{K_2})}{\sigma_{R_{K_1}} \sigma_{R_{K_2}}} = \frac{-0.0064}{0.1356 \times 0.049} = -0.96308$$

Portfolio comprises of 40% K_1 & 60% K_2

ie. we have $w_{K_1} = 0.4$ & $w_{K_2} = 0.6$

Risk (Portfolio), $\sigma_V^2 = w_{K_1}^2 \sigma_{K_1}^2 + w_{K_2}^2 \sigma_{K_2}^2 + 2 w_{K_1} w_{K_2} \sigma_{K_1} \sigma_{K_2} \rho_{K_1 K_2}$

$= (0.4)^2 (0.0184) + (0.6)^2 (0.0024) + 2(0.6 \times 0.4)(-0.96308)(0.0066)$

$= 0.000736$

$\sigma_V^2 < \sigma_{K_1}^2$ & $\sigma_V^2 < \sigma_{K_2}^2$ ie. portfolio risk is less than individual component risks

Portfolio comprises of 80% K_1 & 20% K_2

ie. we have $w_{K_1} = 0.8$ & $w_{K_2} = 0.2$

Risk (Portfolio), $\sigma_V^2 = w_{K_1}^2 \sigma_{K_1}^2 + w_{K_2}^2 \sigma_{K_2}^2 + 2 w_{K_1} w_{K_2} \sigma_{K_1} \sigma_{K_2} \rho_{K_1 K_2}$

$= (0.8)^2 (0.0184) + (0.2)^2 (0.0024) + 2(0.8 \times 0.2)(-0.96308)(0.0066)$

$= 0.00982$

$\sigma_V^2 < \sigma_{K_1}^2$ & $\sigma_V^2 > \sigma_{K_2}^2$ ie. portfolio risk lies b/w risk of individual component risks

Solⁿ 3. To prove, $\sigma_v^2 \leq \max(\sigma_1^2, \sigma_2^2)$

if short selling is not allowed

Let us assume that $\sigma_1^2 \leq \sigma_2^2$

If short sales are not allowed then $w_1, w_2 > 0$ &

$$w_1\sigma_1 + w_2\sigma_2 \leq (w_1 + w_2)\sigma_2 = \sigma_2 \quad \text{——— (i)}$$

$$(\text{As } w_1 + w_2 = 1)$$

Since we know that correlation coefficient satisfies $-1 \leq \rho \leq 1$. Then,

$$\sigma^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2$$

$$\leq (w_1\sigma_1 + w_2\sigma_2)^2 \leq \sigma_2^2 \quad (\text{from eq}^n \text{ (i)})$$

As we assumed $\sigma_1^2 \leq \sigma_2^2$,

So, $\sigma_v^2 \leq \max(\sigma_1^2, \sigma_2^2)$

Hence, proved.

$\underline{\underline{\text{Sol}^n 4.}}$ Securities a_1, a_2, a_3

expected returns $\mu_1 = 0.2, \mu_2 = 0.13, \mu_3 = 0.04$

std. deviations $\sigma_1 = 0.25, \sigma_2 = 0.28, \sigma_3 = 0.2$

correlations $\rho_{12} = 0.3, \rho_{13} = 0.15, \rho_{23} = 0.4$

from given data, Variance - Covariance matrix is,

$$C = \begin{bmatrix} 0.0625 & 0.0210 & 0.0075 \\ 0.0210 & 0.0784 & 0.0224 \\ 0.0075 & 0.0224 & 0.0400 \end{bmatrix}$$

& hence, $C^{-1} = \begin{bmatrix} 17.6031 & -4.4906 & -0.7859 \\ -4.4906 & 16.3302 & -8.3029 \\ -0.7859 & -8.3029 & 29.7970 \end{bmatrix}$

As we assumed $e = (1,1,1)^T$

So, weights, $w = \dfrac{C^{-1}e}{e^T C^{-1} e} = (0.3371, 0.0967, 0.5662)^T$

assume $m = (\mu_1, \mu_2, \mu_3)^T = (0.2, 0.13, 0.04)^T$

Expected return, $\mu = m^T w = 0.1026$

Variance, $\sigma^2 = w^T C w = 0.0273$

Standard deviation, $\sigma = 0.1654$

Sol^n 5. $\mu_1 = E(R_{K_1}) = 0.175$

$\mu_2 = E(R_{K_2}) = 0.167$

$\mu_3 = E(R_{K_3}) = 0.212$

Variance, $\sigma_1^2 = E(R_{K_1}^2) - (E(R_{K_1}))^2 = 0.0038$

Variance, $\sigma_2^2 = E(R_{K_2}^2) - (E(R_{K_2}))^2 = 0.0137$

Variance, $\sigma_3^2 = E(R_{K_3}^2) - (E(R_{K_3}))^2 = 0.0208$

$\sigma_1 = 0.0618, \sigma_2 = 0.117, \sigma_3 = 0.1441$

$Cov(R_{K_1}, R_{K_2}) = E(R_{K_1} R_{K_2}) - E(R_{K_1})E(R_{K_2}) = -0.0017$

$Cov(R_{K_2}, R_{K_3}) = E(R_{K_2} R_{K_3}) - E(R_{K_2})E(R_{K_3}) = -0.0051$

$Cov(R_{K_1}, R_{K_3}) = E(R_{K_1} R_{K_3}) - E(R_{K_1})E(R_{K_3}) = -0.0075$

$\rho_{12} = -0.23155, \rho_{23} = -0.30274, \rho_{31} = -0.80767$

from now obtained data, Variance - Covariance matrix,

$$C = \begin{bmatrix} 0.0038 & -0.0017 & -0.0075 \\ -0.0017 & 0.0137 & -0.0051 \\ -0.0075 & -0.0051 & 0.0208 \end{bmatrix} \;\&\; C^{-1} = \begin{bmatrix} 2663.4 & 738.08 & 1104.3 \\ 738.08 & 285.01 & 325.8 \\ 1104.3 & 325.8 & 510.88 \end{bmatrix}$$

As we assume, $e = (1,1,1)^T$ & $m = (0.175, 0.167, 0.212)^T$

Weights, $w = \dfrac{C^{-1}e}{e^T C^{-1} e} = (0.57799, 0.17303, 0.24899)^T$

Expected return, $\mu = m^T w = 0.18283$

Variance, $\sigma^2 = w^T C w = 0.00012828$

Standard deviation, $\sigma = 0.011326$

$\underline{\text{Sol}^n}$ 6. Given data: $\mu_1 = 0.1$, $\mu_2 = 0.08$

$\sigma_1 = 0.05$, $\sigma_2 = 0.02$

(i) for $\rho = -1$,,

$$\mu_{min} = \frac{\sigma_2 \mu_1 + \sigma_1 \mu_2}{\sigma_1 + \sigma_2} = 0.0857$$

$$s_{min} = \frac{0.02}{0.05 + 0.02} = 0.2857$$

ie. $w_1 = 0.2857$ & $w_2 = 1 - s_{min} = 0.7143$

& std deviation $= 0$

(ii) for $\rho = -0.5$,,

$$s_{min} = \frac{\sigma_1(\sigma_2 - \rho\sigma_1)}{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)} = 0.7692$$

ie. $w_2 = 0.7692$ & $w_1 = 1 - s_{min} = 0.2308$

$$\mu_{min} = w_1 \mu_1 + w_2 \mu_2 = 0.0846$$

$$\sigma^2_{min} = \frac{\sigma_1^2 \sigma_2^2 (1 - \rho^2)}{(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)} = 0.00019231$$

std. deviation, $\sigma = 0.013868$

(iii) for $\rho = 0$,,

$s_{min} = 0.8621$

ie. $w_2 = 0.8621$ & $w_1 = 0.1379$

$\mu_{min} = 0.0828$ & $\sigma_{min} = 0.1857$

(iv) for $\rho = 0.5$,,

$s_{min} = w_2 = 1.0526$ & $w_1 = -0.0526$

$\mu_{min} = 0.0789$ & $\sigma_{min} = 0.019868$

(v) for $\rho = 1$,,

$s_{min} = w_2 = 1.6667$ & $w_1 = -0.6667$

$\mu_{min} = 0.0667$ & $\sigma_{min} = 0$

Sol^n 7. Using given values & taking $\alpha=0, \beta=1$ we get,,

[redacted] $10v_1^{(1)} + 4v_2^{(1)} = 1$

$4v_1^{(1)} + 12v_2^{(1)} + 6v_3^{(1)} = 1$

$6v_2^{(1)} + 10v_3^{(1)} = 1$

Solving above set of eq^n we get,,

$V^{(1)} = (0.1, 0, 0.1)^T$

Now taking $\alpha=1, \beta=0$ we get,,

$10v_1^{(2)} + 4v_2^{(2)} = 5$

$4v_1^{(2)} + 12v_2^{(2)} + 6v_3^{(2)} = 6$

$6v_2^{(2)} + 10v_3^{(2)} = 1$

Solving above set of eq^n we get,,

$V^{(2)} = (0.3, 0.5, -0.2)^T$

Normalizing $V^{(1)}$ & $V^{(2)}$ we get required 2 portfolios,,

$W^{(1)} = (0.5, 0, 0.5)^T$ & $W^{(2)} = \left(\frac{1}{2}, \frac{5}{6}, \frac{-1}{3}\right)^T$

Expected return, $\bar{\mu}^{(1)} = m^T W^{(1)} = 3\%$

$\bar{\mu}^{(2)} = m^T W^{(2)} = 7.1667\%$

As for $\lambda = 1.048$ we get $\lambda\bar{\mu}^{(1)} + (1-\lambda)\bar{\mu}^{(2)} = 2.8$

Thus by 2 fund theorem, required portfolio is given by,,

$w = \lambda w^{(1)} + (1-\lambda) w^{(2)}$

$= (0.5, -0.04, 0.54)^T$

which is required portfolio weights.

This portfolio is not efficient.

Sol^n 8. The given information $m^T = (10, 12, 18)$, $\mu_{rf} = 6$

$$C = \begin{bmatrix} 4 & 20 & 40 \\ 20 & 10 & 70 \\ 40 & 70 & 14 \end{bmatrix} \text{ \& hence } C^{-1} = \begin{bmatrix} -0.0667 & 0.0353 & 0.0140 \\ 0.0353 & -0.0216 & 0.0073 \\ 0.0140 & 0.0073 & -0.0050 \end{bmatrix}$$

Therefore, weight vector of market portfolio is,,

$$w_M = \frac{C^{-1}(m - \mu_{rf} e)}{e^T C^{-1}(m - \mu_{rf} e)} = (0.4505, 0.3934, 0.1561)^T$$

Expected return, $\mu_M = m^T w_M = 12.0357\%$

std. deviation, $\sigma_M = (w_M^T C w_M)^{1/2} =$ 4.9004 [redacted]

$$\frac{\mu_M - \mu_{rf}}{\sigma_M} = \frac{12.0357 - 6}{4.9004} = 1.2317$$

So, from above values we derive equation of CML,,

$$\mu = \mu_{rf} + \left(\frac{\mu_M - \mu_{rf}}{\sigma_M}\right)\sigma$$

putting values of μ_{rf}, μ_M & σ_M we get,,

$$\mu = 6 + 1.2317\,\sigma$$

All the points on CML are efficient points for investors which lie on Markovitz bullet as well as

$\underline{\underline{\text{Sol}^n 9.}}$ Given data: $\mu_1 = 6\%$, $\beta_1 = 0.5$, $\mu_2 = 12\%$, $\beta_2 = 1.5$

Now for above data, we have

eq^n for 2 security market line for β_1 & β_2 as, points

$$0.06 - \mu_{rf} = 0.5(\mu_M - \mu_{rf})$$

$$0.12 - \mu_{rf} = 1.5(\mu_M - \mu_{rf})$$

Solving above set of equations we get,

$\mu_{rf} = 0.03$ & $\mu_M = 0.09$

$\therefore$ Equation for Security Market line is

$$\mu = 0.03 + 0.06\beta$$

Hence, when we have $\beta = 2$

$$\mu = 0.03 + 0.06(2) = 0.03 + 0.12 = 0.15$$

Therefore, expected return on asset is 15%

$\underline{\underline{\text{Sol}^n\ 10.}}$ Given data: $\mu_1 = 9.5\% = 0.095$ & $\beta_1 = 0.8$

$\mu_2 = 13.5\% = 0.135$ & $\beta_2 = 1.3$

Now for above data, we have

eq^n for 2 security market point for β_1 & β_2 as,

$$0.095 - \mu_{rf} = 0.8\,(\mu_M - \mu_{rf})$$

$$0.135 - \mu_{rf} = 1.3\,(\mu_M - \mu_{rf})$$

Solving above set of equations we get,

$\mu_{rf} = 0.031$ & $\mu_M = 0.111$

$\therefore$ Equation of Security Market line is

$$\mu = 0.031 + 0.08\,\beta$$

Risk free return = 3.1%

Return on Market portfolio = 11.1%