FINANCIAL ENGINEERING

ASSIGNMENT - 3

1. Consider the binomial model for trading in stock, t = 1, 2 where at each time the stock can go up by the factor u or down by the factor d. The sample space $Q = \{(u,u),(u,d),(d,u),(d,d)\}$. Create one non-trivial $\sigma - field$ and the largest $\sigma - field$ on Q.

2. Let X and Y be i.i. d. random variables each having uniform distribution on the interval$(-\pi, \pi)$. Let $Z(t) = cos(tX + Y),\ t \geq 0.$ Is $\{Z(t), t \geq 0\}$ wide sense stationary process?

3. Let $\{W(t), t \geq 0\}$ be a Brownian motion. Prove that $\{tW(1/t), t \geq 0\}$ where $tW(1/t)$ is taken to be zero when $t = 0$, is a Brownian motion.

4. Consider two i.i.d random variables X and Y each having uniform distribution between the intervals 0 and 1. Define $Z = X + Y$. Prove that $E(X/Z) = Z/2$

5. Consider $\Omega = \{a, b, c, d\}$. Construct 4 distinct $\sigma - field$ $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4$ such that $\mathcal{F}_1 \subset \mathcal{F}_2 \subset \mathcal{F}_3 \subset \mathcal{F}_4$.

6. Let $X_1, X_{2,\ldots,}$ be $i.i.d$ random variables each having normal distribution with zero mean and unit variance. Show that the sequence $Y_n = \exp\{(\sum_{i=1}^{n} X_n) - \frac{n}{2}\}$ forms a martingale.

7. Let $(N(t), t \geq 0\}$ be a Poisson process with parameter 1. Which of the following are martingales.
   (i) $\{N(t) - t, t \geq 0\}$.
   (ii) $\{N^2(t) - t, t \geq 0\}$.
   (iii) $\{\{N(t) - t\}^2 - t, t \geq 0\}$.

8. Let $\{W(t), t \geq 0\}$be a Wiener process. ls $\exp\{\sigma W(t) - \frac{\sigma^2}{2} t\}$ a martingale where σ is a positive constant?

9. Find the stochastic differential of $W^2(t)$.

10. Consider a stock whose value $S(t)$ follows sde $dS = r.Sdt + \sigma.SdW$ and has a current price S(0). What is the probability that a call option is in the money based on a strike price $K = 1.25\ S(0)$ at time of expiration T? Given that $T = 0.5, r = 0.04$ and $\sigma = 0.10$.

11. Let Z be a normally distributed random variable, with mean 0 and variance 1, $Z \sim N\ (0, 1)$. Then consider the continuous time stochastic process $X = \sqrt{t}Z$, Show that the distribution of X is normal with mean 0 and variance t. Is $X(t)$ a Brownian motion?

12. What is the distribution of $W(s) + W(t)$, for $0 \leq s \leq t$?

13. A stock price is currently Rs.50. Assume that the expected return from the stock is 18% per annum and its volatility is 30% per annum. What is the probability distribution for the stock price in two years? Calculate the mean and standard deviation of the distribution. Determine the 95% confidence interval.