Sol^n 1. Let $S(t)$ be the stock price at time t

All the options exercise at time T & strike price K

(i) 1 call & 1 put option

Case I: if $S(T) > K$

Total payoff = Call payoff + Put payoff

$= S(T) - K$

Case II: if $S(T) < K$

Total payoff = Call payoff + Put payoff

$= K - S(T)$

(ii) 2 calls & sold short 1 share of stock

Case I: if $S(T) < K$

Total payoff = 2 call payoff + short position payoff

$= -S(T)$

Case II: if $S(T) > K$

Total payoff = 2 call payoff + short position payoff

$= 2(S(T) - K) - S(T) = S(T) - 2K$

(iii) 1 share of stock & 1 call is sold

Case I: if $S(T) < K$

Total payoff = 1 short call payoff + long position payoff

$= S(T)$

Case II: if $S(T) > K$

Total payoff = 1 short call payoff + long position payoff

$= -(S(T) - K) + S(T) = K$

(iv) 1 call with strike price K_1 & sold 1 put with strike price K_2 (assume $K_1 < K_2$)

Case I: if $S(T) < K_1$

Total payoff = 1 short put payoff + 1 long call payoff

$= S(T) - K_1$

Case II: if $S(T) > K_2$

Total payoff = 1 short put payoff + 1 long call payoff

$= S(T) - K_2$

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Sol^{n}2. $S(0) = Rs\,50$, $r = 1\% = 0.01$, $R = 1 + r = 1.01$, $X = Rs\,48$, $T = 2$ month

$u = 1.1$ (grows by 10%)

$d = 0.9$ (falls by 10%)

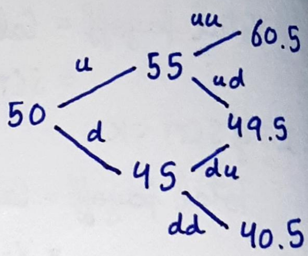
$S^u = S(0)\,u = 50 \times 1.1 = Rs\,55$

$S^d = S(0)\,d = 50 \times 0.9 = Rs\,45$

$S^{uu} = S(0)u^2 = 50 \times (1.1)^2 = Rs\,60.5$

$S^{ud} = S(0)\,ud = 50 \times 1.1 \times 0.9 = Rs\,49.5$

$S^{dd} = S(0)\,d^2 = 50 \times (0.9)^2 = Rs\,40.5$



$$p^* = \frac{R-d}{u-d} = \frac{1.01-0.9}{1.1-0.9} = \frac{11}{20}, \quad 1-p^* = \frac{9}{20}$$

$C^{uu} = [S^{uu} - X]^+ = \max(60.5 - 48; 0) = Rs\,12.5$

$C^{ud} = [S^{ud} - X]^+ = Rs\,1.5$

$C^{dd} = [S^{dd} - X]^+ = Rs\,0$

Using 2 step Binomial model,, (as $T = 2$)

$$C(0) = \frac{1}{R^2}\left[p^{*2} C^{uu} + 2p^*(1-p^*)C^{ud} + (1-p^*)^2 C^{dd}\right]$$

$$= \frac{1}{(1.01)^2}\left[\frac{121}{400} \times 60.5 + 99 \times \frac{99}{400}\right] = Rs\,4.4346$$

$C(0) = \underline{\underline{Rs\,4.43}}$

Sol^n 3. $S(0) = 60,\ K = 62,\ u = 1.1,\ d = 0.95,\ r = 0.03,\ T = 3$

$R = 1.03$

$S^u = S(0)\,u = 60 \times 1.1 = 66$

$S^d = S(0)\,d = 60 \times 0.95 = 57$

$S^{uu} = S(0)\,u^2 = 60 \times (1.1)^2 = 72.6$

$S^{ud} = S(0)\,ud = 60 \times 1.1 \times 0.95 = 62.7$

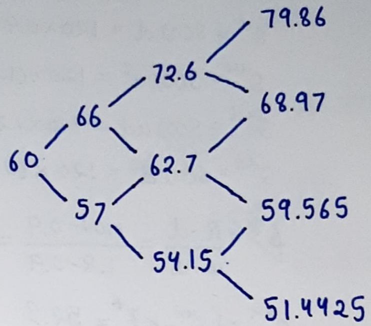
$S^{dd} = S(0)\,d^2 = 60 \times (0.95)^2 = 54.15$

$S^{uuu} = S(0)\,u^3 = 60 \times (1.1)^3 = 79.86$

$S^{uud} = S(0)\,u^2 d = 60 \times (1.1)^2 \times 0.95 = 68.97$

$S^{udd} = S(0)\,ud^2 = 60 \times$ [redacted] $1.1 \times (0.95)^2 = 59.565$

$S^{ddd} = S(0)\,d^3 = 60 \times (0.95)^3 = 51.4425$



$$p^* = \frac{R-d}{u-d} = \frac{1.03-0.95}{1.1-0.95} = \frac{8}{15}, \quad 1-p^* = \frac{7}{15}$$

$C^{uuu} = [S^{uuu} - K]^+ = 17.86$

$C^{uud} = [S^{uud} - K]^+ = 6.97$

$C^{udd} = [S^{udd} - K]^+ = 0$

$C^{ddd} = [S^{ddd} - K]^+ = 0$

Using 3 step Binomial model,, (as $T=3$)

$$C(0) = \frac{1}{R^3}\left[p^{*3} C^{uuu} + 3p^{*2}(1-p^*)C^{uud} + 3p^*(1-p^*)^2 C^{udd} + (1-p^*)^3 C^{ddd}\right]$$

$$= \frac{1}{(1.03)^3}\left[17.86 \times \left(\frac{8}{15}\right)^3 + 3 \times 6.97 \times \left(\frac{8}{15}\right)^2\left(\frac{7}{15}\right)\right]$$

$$C^E(0) = \underline{\underline{5.0196}}$$

Using Put-Call parity,,

$$P(0) = C(0) - S(0) + Ke^{-rT}$$

$$= 5.0196 - 60 + 62e^{-0.03 \times 3}$$

$$P^E(0) = \underline{\underline{1.6833}}$$

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Sol^n 4. $S(0) = 120$, $u = 1.2$, $d = 0.9$, $r = 1\% = 0.01$, $R = 1.01$, $K = 120$, $T = 2$

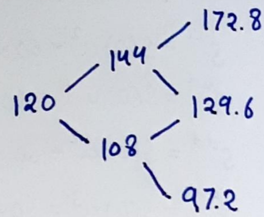
$S^u = S(0)\,u = 120 \times 1.2 = 144$

$S^d = S(0)\,d = 120 \times 0.9 = 108$

$S^{uu} = S(0)\,u^2 = 120 \times (1.2)^2 = 172.8$

$S^{ud} = S(0)\,ud = 120 \times 1.2 \times 0.9 = 129.6$

$S^{dd} = S(0)\,d^2 = 120 \times (0.9)^2 = 97.2$



$p^* = \frac{R-d}{u-d} = \frac{1.01-0.9}{1.2-0.9} = \frac{11}{30}$, $1-p^* = \frac{19}{30}$

$C^{uu} = [S^{uu} - K]^+ = 52.8$

$C^{ud} = [S^{ud} - K]^+ = 9.6$

$C^{dd} = [S^{dd} - K]^+ = 0$

$C^u = [S^u - K]^+ = 24$

$C^d = [S^d - K]^+ = 0$

Using 2 step Binomial model,, (as $T=2$)

$$C(0) = \frac{1}{R^2}\left[p^{*2} C^{uu} + 2p^*(1-p^*)C^{ud} + (1-p^*)^2 C^{dd}\right]$$

$$= \frac{1}{(1.01)^2}\left[52.8\left[\frac{11}{30}\right]^2 + 2 \times 9.6 \times \frac{11}{30} \times \frac{19}{30}\right]$$

$C(0) = \underline{11.3296}$

Let replicating strategy be $(x(1), y(1))$,,

$$x(1) = \frac{C^u - C^d}{S^u - S^d} = \frac{24-0}{144-108} = \frac{2}{3} = \underline{0.6667}$$

$$y(1) = -\frac{d\,C^u - u\,C^d}{(1+r)(u-d)} = \frac{-0.9 \times 24}{1.01 \times 0.3} = \frac{-7200}{101} = \underline{-71.287}$$

i.e. replicating position in stock which is delta, $x(1) = 2/3$

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$S(0) = \text{Rs } 100$, $\sigma = 18\% = 0.18$, $r = 4\% = 0.04$, $\Delta t = 2$ yr, $T = 3$ yr

$R = 1.0833$ $K = \text{Rs } 80$,, $R = e^{r\Delta t} = e^{0.08} = 1.0833$

$u = e^{\sigma\sqrt{\Delta t}} = e^{0.18\sqrt{2}} = 1.2899$

$d = \frac{1}{u} = 0.7753$

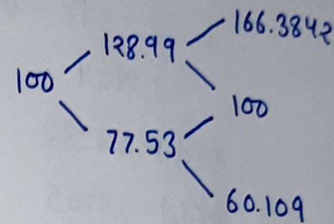
$S^{u} = S(0)\,u = 100 \times 1.2899 = \text{Rs } 128.99$

$S^{d} = S(0)\,d = 100 \times 0.7753 = \text{Rs } 77.53$

$S^{uu} = S(0)\,u^2 = 100 \times (1.2899)^2 = \text{Rs } 166.3842$

$S^{ud} = S(0)\,ud = 100 \times 1.2899 \times 0.7753 = \text{Rs } 100$

$S^{dd} = S(0)\,d^2 = 100 \times (0.7753)^2 = \text{Rs } 60.109$



$p^* = \frac{R-d}{u-d} = \frac{1.0833 - 0.7753}{1.2899 - 0.7753} = 0.5985$,, $1 - p^* = 0.4015$

$C^{uu} = [S^{uu} - K]^+ = 86.3842$ Rs

$C^{ud} = [S^{ud} - K]^+ = 20$ Rs

$C^{dd} = [S^{dd} - K]^+ = 0$ Rs

$C(0) = e^{-r\Delta t}\left[p^{*2} C^{uu} + 2p^*(1-p^*) C^{ud} + (1-p^*)^2 C^{dd}\right]$

$= e^{-0.08}\left[(0.5985)^2 \times 86.3842 + 2 \times 0.5985 \times 0.4015 \times 20\right]$

$C(0) = \text{Rs. } 37.5882$

Sol^n 6. $S(0) = \text{Rs } 51$, $K = \text{Rs } 50$, $\sigma = 0.3$, $r = 0.08$, $T = 3$ months $= 0.25$ yr.

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{51}{50}\right) + \left(0.08 + \frac{0.3^2}{2}\right)0.25}{0.3 \times \sqrt{0.25}}$$

$d_1 = 0.3404$

$d_2 = d_1 - \sigma\sqrt{T} = 0.3404 - 0.3\sqrt{0.25}$

$= 0.1904$

$\Phi(d_1) = 0.6332$

$\Phi(d_2) = 0.5755$

Using Black Scholes formula,,

$C(0) = S(0)\Phi(d_1) - Ke^{-rT}\Phi(d_2)$

$= 51 \times 0.6332 - 50e^{-0.08 \times 0.25} \times 0.5755$

$C(0) = \underline{\text{Rs } 4.0879}$

$\Phi(-d_1) = 0.3668$

$\Phi(-d_2) = 0.4245$

$P(0) = Ke^{-rT}\Phi(-d_2) - S(0)\Phi(-d_1)$

$= 50e^{-0.08 \times 0.25} \times 0.4245 - 51 \times 0.3668$

$P(0) = \text{Rs. } \underline{2.0979}$

Now using Put-Call parity,,

$P(0) = C(0) - S(0) + Ke^{-rT}$

$= 4.0879 - 51 + 50e^{-0.08 \times 0.25}$

$P(0) = \text{Rs. } \underline{2.0979}$

Both of the values are same.

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Soln 7. $S(0) = Rs\ 260$, $T = 6$ month $= 0.5$ yrs, $r = 0.04$, $\sigma = 0.25$, $K = Rs\ 256$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{260}{256}\right) + \left(0.04 + \frac{0.25^2}{2}\right)0.5}{0.25\sqrt{0.5}}$$

$d_1 = 0.2892$

$d_2 = d_1 - \sigma\sqrt{T} = 0.2892 - 0.25\sqrt{0.5}$

$d_2 = 0.1124$

$\phi(d_1) = 0.6138$

$\phi(d_2) = 0.5448$

Using Black Scholes formula,,

$C(0) = S(0)\phi(d_1) - Ke^{-rT}\phi(d_2)$

$= 260 \times 0.6138 - 256\, e^{-0.04 \times 0.5} \times 0.5448$

$C(0) = Rs\ 22.8804$

Soln 8. $S(0) = Rs\ 42$, $T = 6$ month $= 0.5$ yr, $r = 0.05$, $\sigma = 0.22$, $K = Rs\ 40$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{42}{40}\right) + \left(0.05 + \frac{0.22^2}{2}\right)0.5}{0.22\sqrt{0.5}}$$

$d_1 = 0.5521$

$d_2 = d_1 - \sigma\sqrt{T} = 0.5521 - 0.22\sqrt{0.5}$

$d_2 = 0.3965$

$\phi(-d_1) = 0.2904$

$\phi(-d_2) = 0.3459$

Using Black Scholes formula,,

$P(0) = Ke^{-rT}\phi(-d_2) - S(0)\phi(-d_1)$

$= 40\, e^{-0.05 \times 0.5} \times 0.3459 - 42 \times 0.2904$

$= Rs\ 1.2976$ for 1 stock put option

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2K2

Sol^n 9. $T = 3$ month $= 0.25$ yr, $K = 25$ Rs, $S(0) = $ Rs 20, $r = 0.05$, $\sigma = 0.24$, $r_{div} = 0.03$

$$S'(0) = S(0)\, e^{-r_{div} T} = 20\, e^{-0.03 \times 0.25}$$

$S'(0) = $ Rs 19.8505

$$d_1 = \frac{\ln\left(\frac{S'(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{19.8505}{25}\right) + \left(0.05 + \frac{0.24^2}{2}\right)0.25}{0.24\sqrt{0.25}}$$

$d_1 = -1.7579$

$d_2 = d_1 - \sigma\sqrt{T} = -1.7579 - 0.24\sqrt{0.25}$

$d_2 = -1.8779$

$\Phi(d_1) = 0.0394$

$\Phi(d_2) = 0.0302$

Using Black Scholes formula,

$$C(0) = S'(0)\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2)$$

$$= 19.8505 \times 0.0394 - 25e^{-0.05 \times 0.25} \times 0.0302$$

$C(0) = $ Rs $\underline{0.0365}$ for 1 call option

for 100 options ⇒ Rs. 3.65

Sol^n 10. $S(0) = $ Rs 100, $\sigma = 0.25$, $r = 0.05$, $T = 6$ month $= 0.5$ yr, $K = $ Rs 80

D shares of stock shorted

By assuming that portfolio value is non random,

$C - DS = 0$

$$\frac{\partial C}{\partial S} = D = \Phi(d_1)$$

$$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\ln\left(\frac{100}{80}\right) + \left(0.05 + \frac{0.25^2}{2}\right)0.5}{0.25\sqrt{0.5}}$$

$d_1 = 1.4921$

$\Phi(d_1) = 0.9322$

$D = \Phi(d_1) = \underline{0.9322}$

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