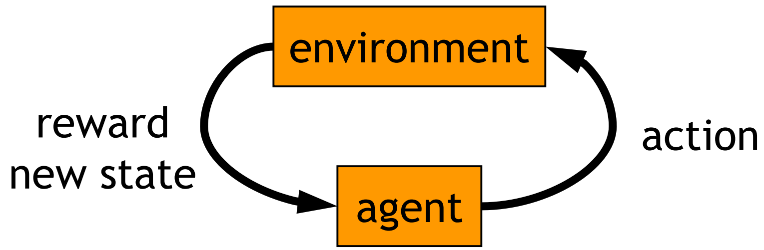
Reinforcement Learning

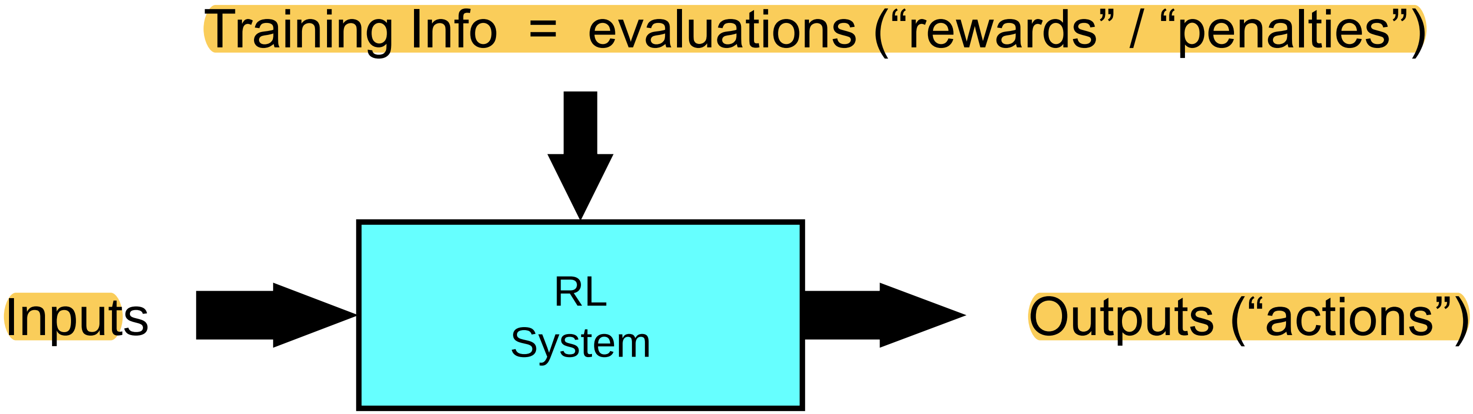
Learning Algorithms

- Supervised learning
  - classification, regression
- Unsupervised learning
  - clustering



- Reinforcement learning
  - more general than supervised/unsupervised learning
  - learn from interaction w/ environment to achieve a goal

Reinforcement Learning

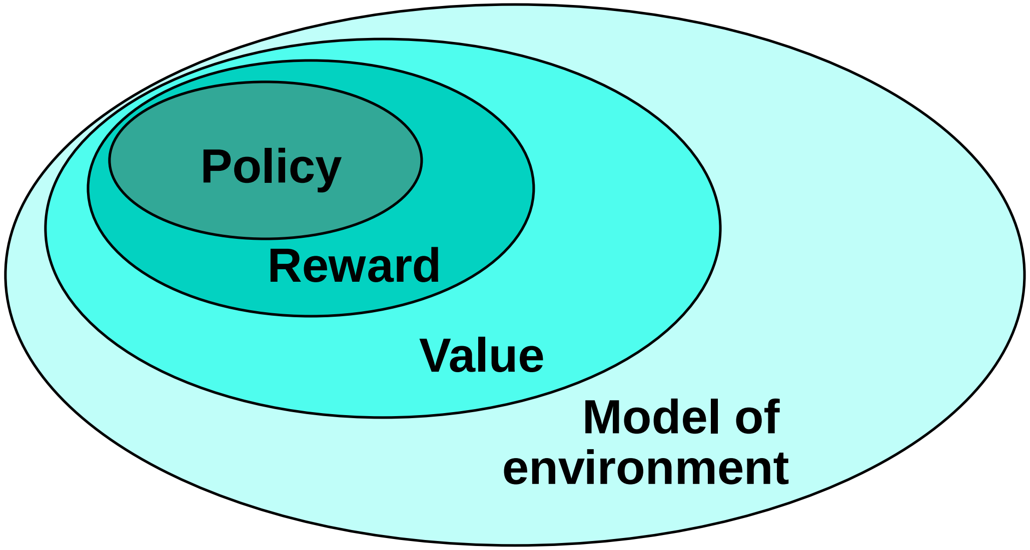


Objective: get as much reward as possible

Key Features of RL

- Learner is not told which actions to take
- Trial-and-Error search
- Possibility of delayed reward (sacrifice short-term gains for greater long-term gains).
- The need to ***explore*** and ***exploit***
- Considers the whole problem of a goal-directed agent interacting with an uncertain environment.

Element of RL



- **Policy**: what to do
- **Reward**: what is good
- **Value**: what is good because it *predicts* reward
- **Model**: what follows what

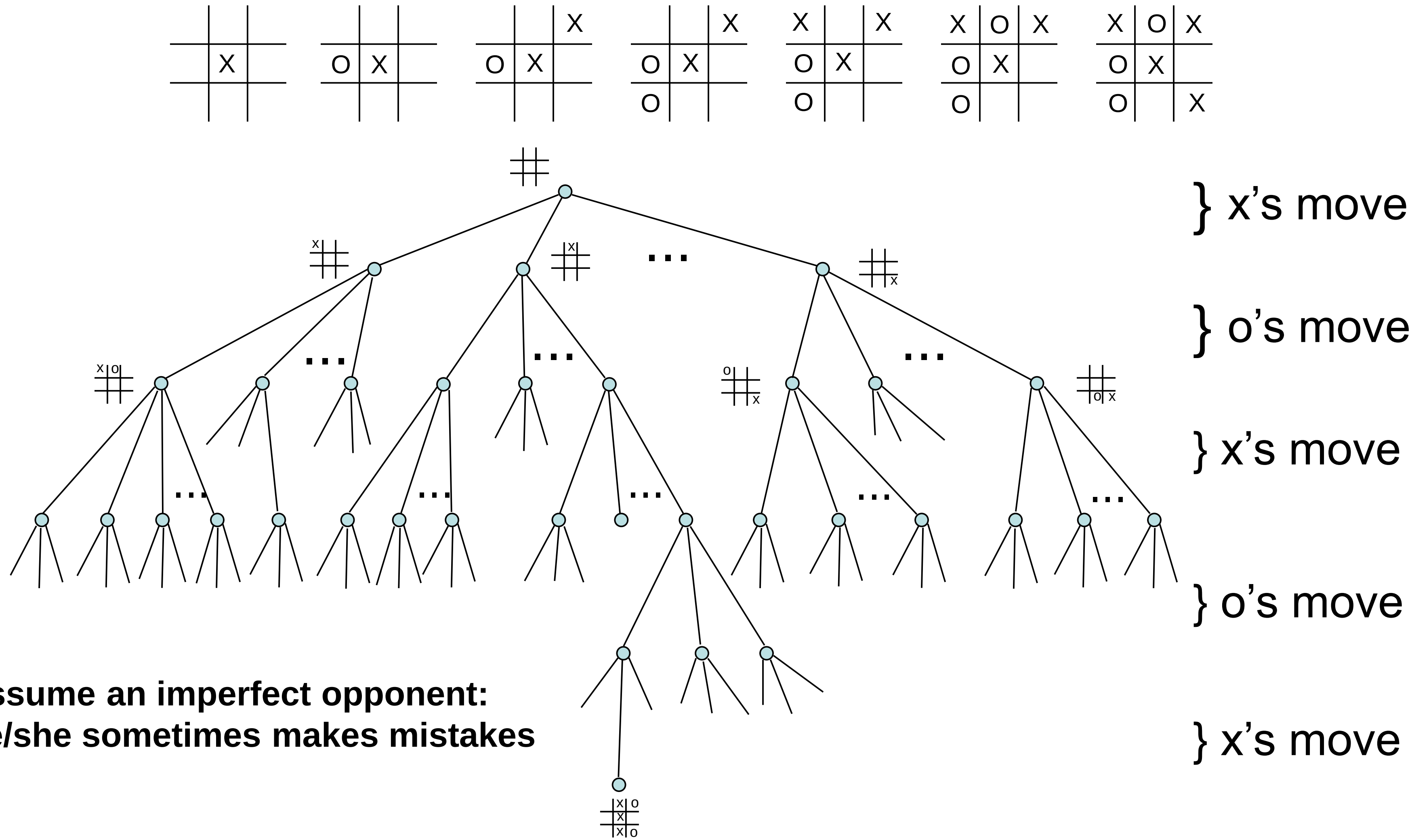
Outlines

- Examples

- Defining an RL problem
  - Markov Decision Processes

- Solving an RL problem
  - Dynamic Programming
  - Monte Carlo methods
  - Temporal-Difference learning

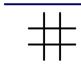
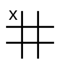
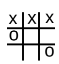
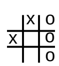
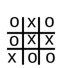
Example: Tic-Tac-Toe



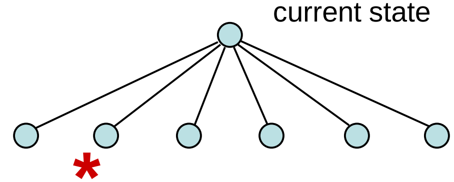
Assume an imperfect opponent: he/she sometimes makes mistakes

An RL Approach to Tic-Tac-Toe

1. Make a table with one entry per state:

State	$V(s)$ – estimated probability of winning	
	.5	?
	.5	?
⋮	⋮	
	1	win
⋮	⋮	
	0	loss
⋮	⋮	
	0	draw

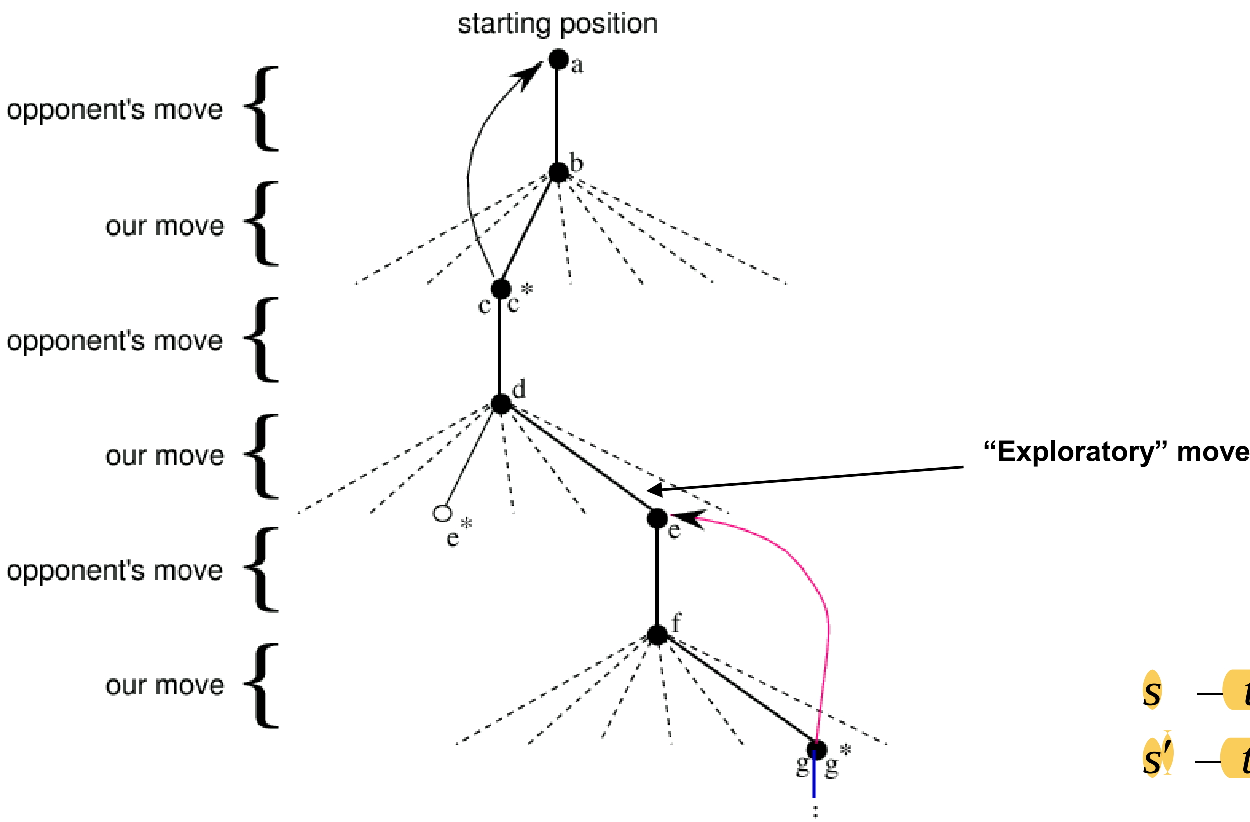
2. Now play lots of games. To pick our moves, look ahead one step:



Just pick the next state with the highest estimated prob. of winning — the largest $V(s)$; a ***greedy*** move.

But 10% of the time pick a move at random; an ***exploratory move***.

RL Learning Rule for Tic-Tac-Toe



s – the state *before* our greedy move

s' – the state *after* our greedy move

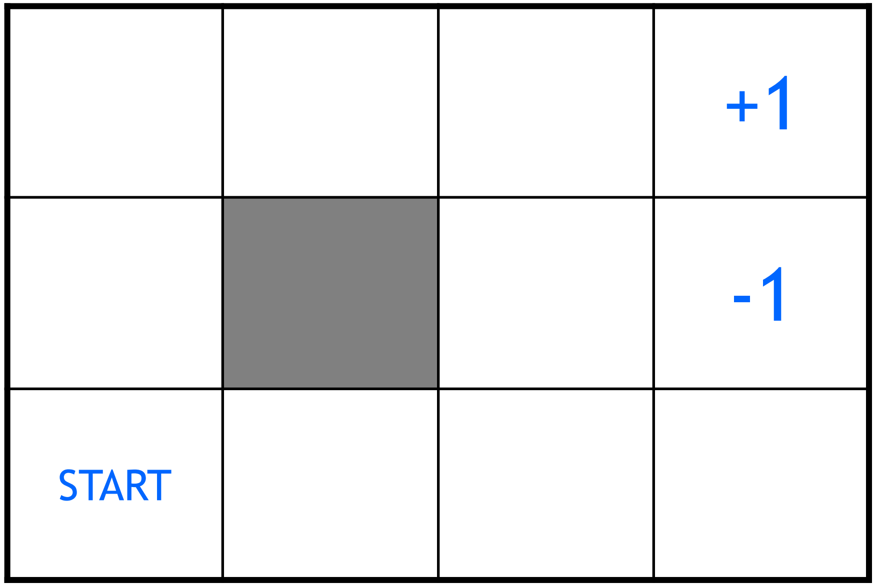
We increment each $V(s)$ toward $V(s')$ – ***a backup***:

$$V(s) \leftarrow V(s) + \alpha\left[V(s') - V(s)\right]$$

a small positive fraction, e.g., $\alpha = .1$

the ***step - size parameter***

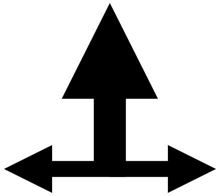
Robot in a Room



actions: UP, DOWN, LEFT, RIGHT

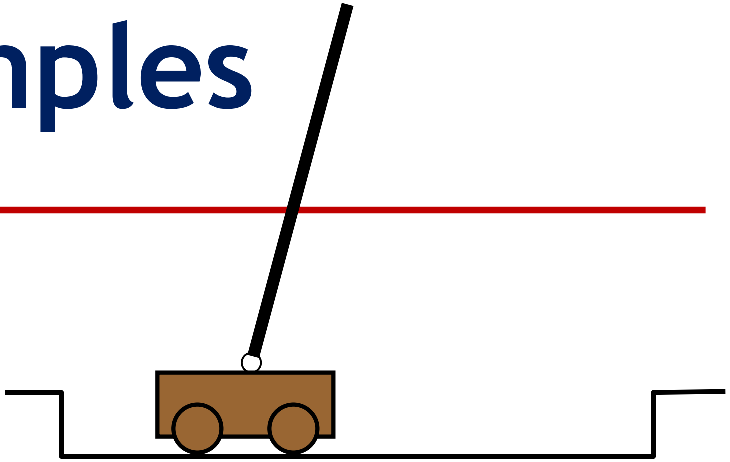
UP

80% move UP
10% move LEFT
10% move RIGHT



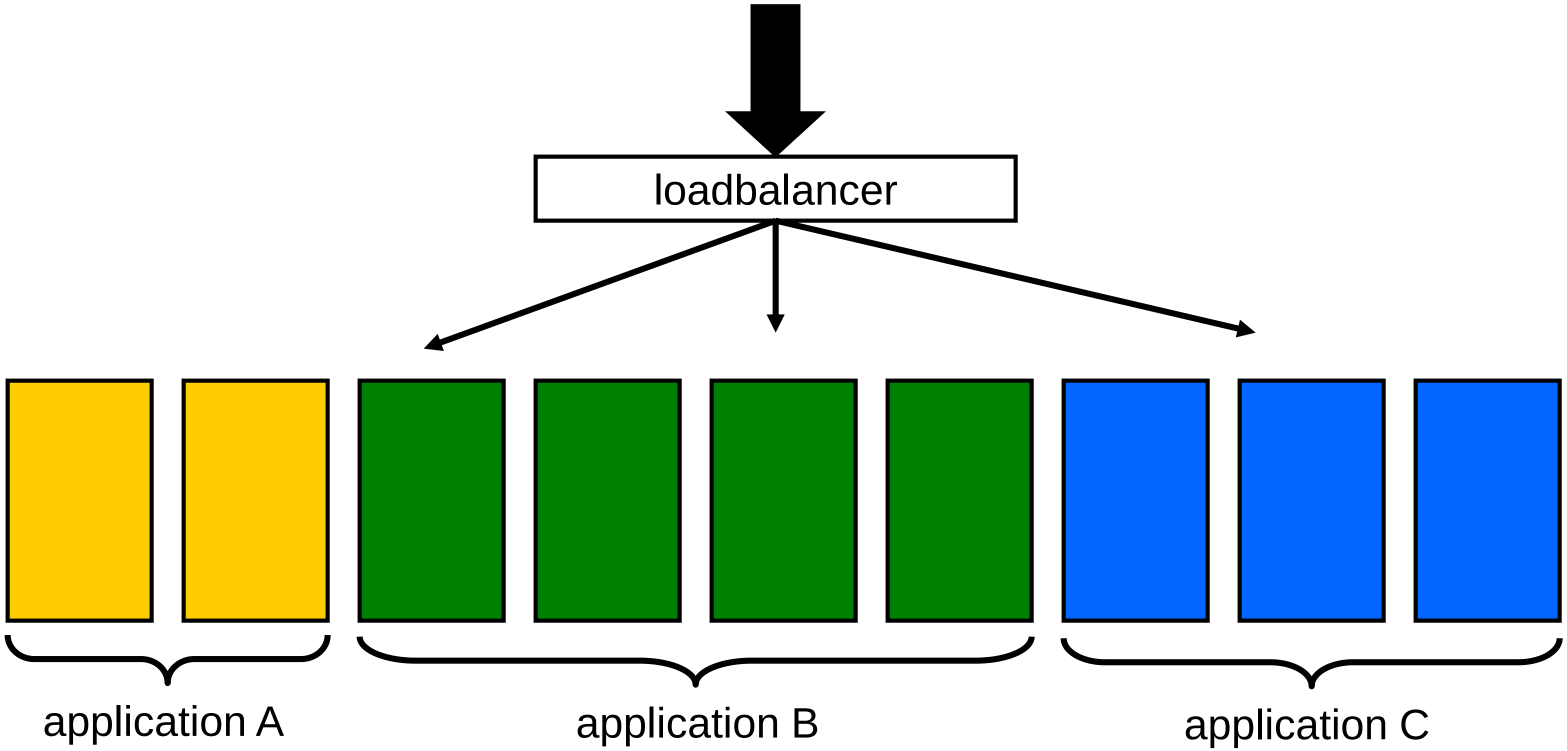
- reward +1 at [4,3], -1 at [4,2]
- reward -0.04 for each step
- what's the strategy to achieve max reward?
- what if the actions were deterministic?

Other examples



- Pole-balancing
- TD-Gammon [Gerry Tesauro]
- Helicopter [Andrew Ng]
- No teacher who would say “good” or “bad”
  - is reward “10” good or bad?
  - rewards could be delayed
- Similar to control theory
  - more general, fewer constraints
- Explore the environment and learn from experience
  - not just blind search, try to be smart about it

Resource allocation in datacenters

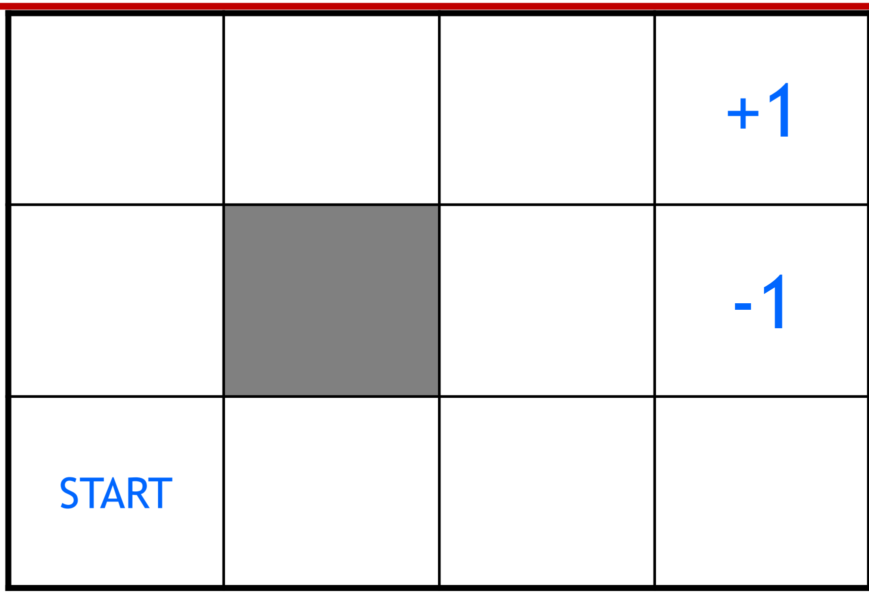


- A Hybrid Reinforcement Learning Approach to Autonomic Resource Allocation
  - Tesauro, Jong, Das, Bennani (IBM)
  - ICAC 2006

Outline

- Examples
- Defining an RL problem
  - Markov Decision Processes
- Solving an RL problem
  - Dynamic Programming
  - Monte Carlo methods
  - Temporal-Difference learning

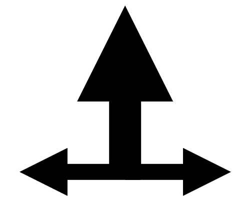
Robot in a room



actions: UP, DOWN, LEFT, RIGHT

UP

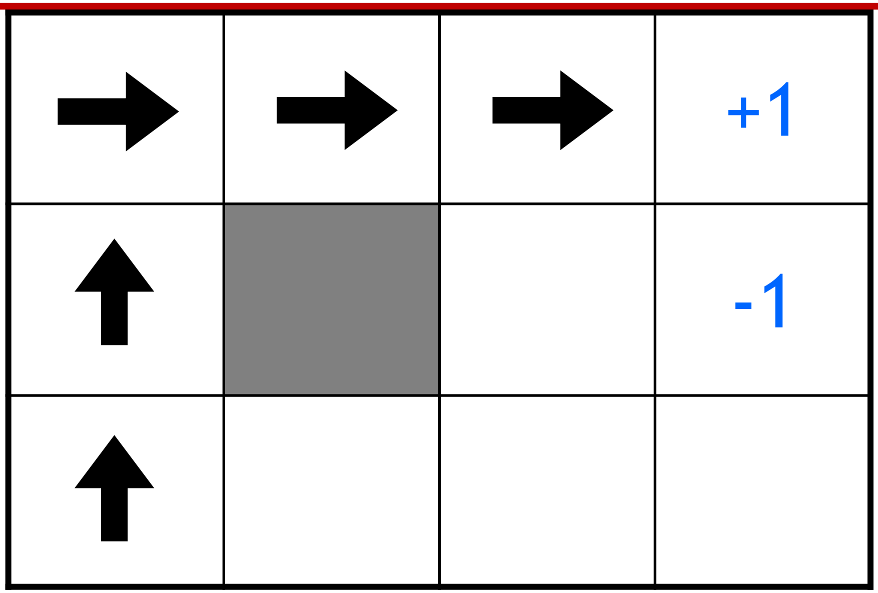
80% move UP
10% move LEFT
10% move RIGHT



reward +1 at [4,3], -1 at [4,2]
reward -0.04 for each step

- states
- actions
- rewards
- what is the solution?

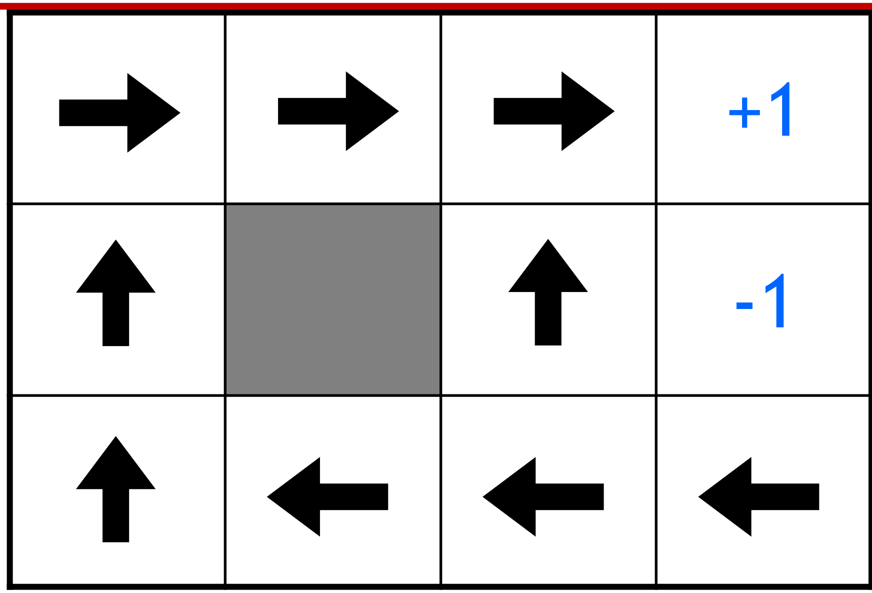
Is this a solution?



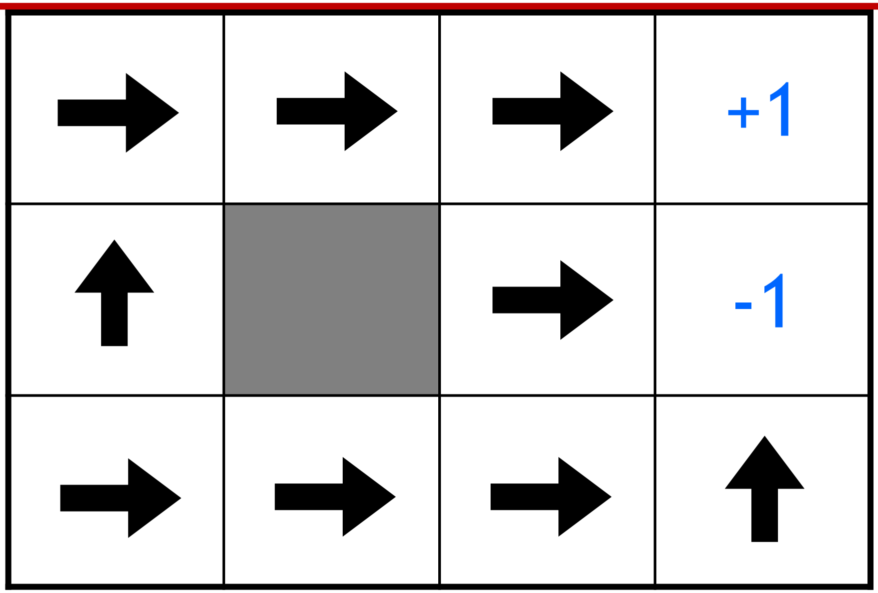
- only if actions deterministic
  - not in this case (actions are stochastic)

- solution/policy
  - mapping from each state to an action

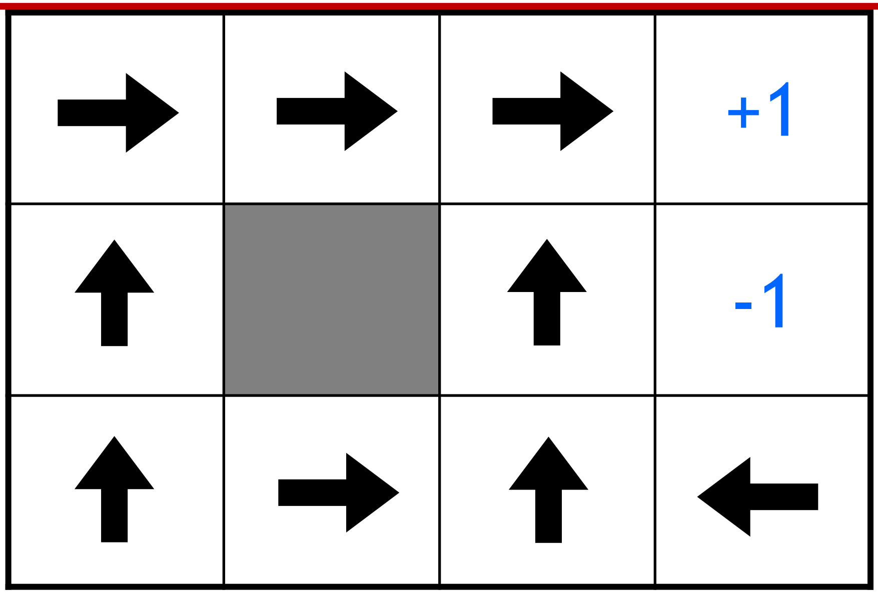
Optimal policy



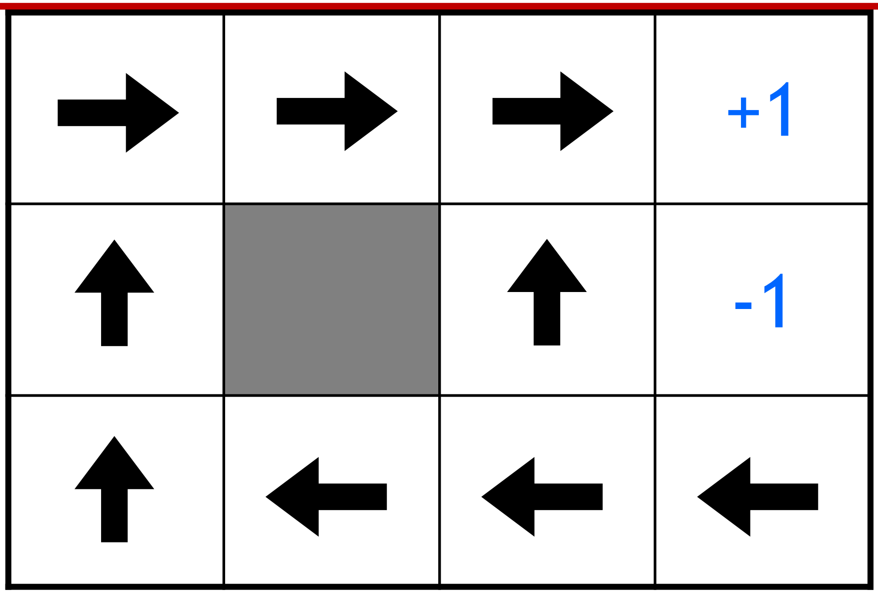
Reward for each step: -2



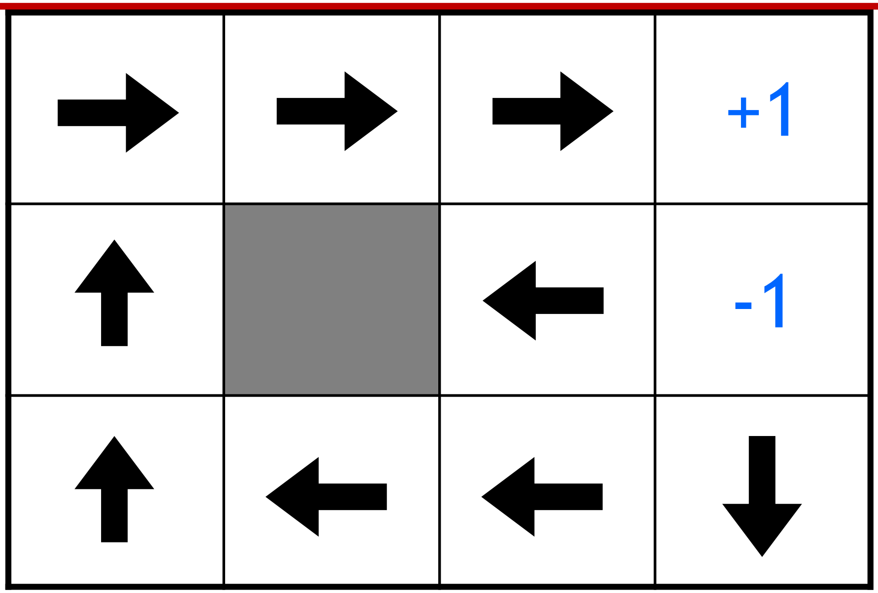
Reward for each step: -0.1



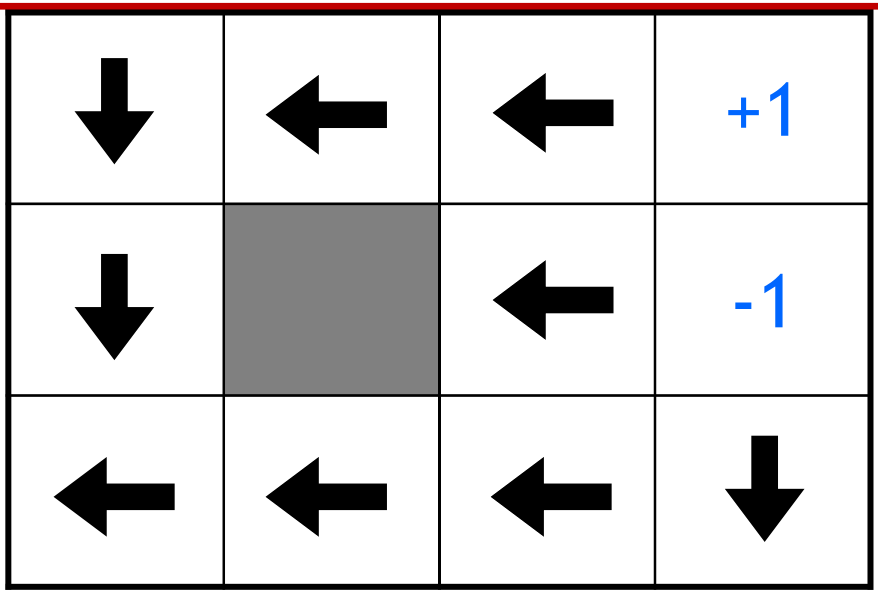
Reward for each step: -0.04



Reward for each step: -0.01

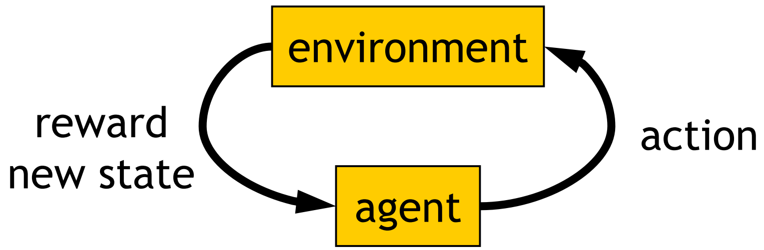


Reward for each step: +0.01



Markov Decision Process (MDP)

- set of states S, set of actions A, initial state S_0
- transition model P(s,a,s')
  - P([1,1], up, [1,2]) = 0.8
- reward function r(s)
  - r([4,3]) = +1
- goal: maximize cumulative reward in the long run



- policy: mapping from S to A
  - $\pi(s)$ or $\pi(s,a)$ (deterministic vs. stochastic)
- reinforcement learning
  - transitions and rewards usually not available
  - how to change the policy based on experience
  - how to explore the environment

Computing return from rewards

- episodic (vs. continuing) tasks
  - "game over" after N steps
  - optimal policy depends on N; harder to analyze

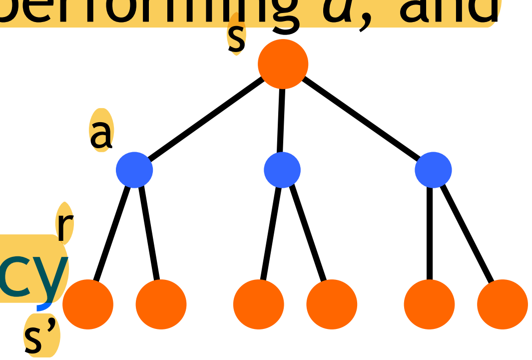
- additive rewards
  - $V(s_0, s_1, \ldots) = r(s_0) + r(s_1) + r(s_2) + \ldots$
  - infinite value for continuing tasks

- discounted rewards
  - $V(s_0, s_1, \ldots) = r(s_0) + \gamma^* r(s_1) + \gamma^{2*} r(s_2) + \ldots$
  - value bounded if rewards bounded

Value functions

- state value function: $V^\pi(s)$
  - expected return when starting in *s* and following π

- state-action value function: $Q^\pi(s,a)$
  - expected return when starting in *s*, performing *a*, and following π



- useful for finding the optimal policy
  - can estimate from experience
  - $V^\pi(s) = \sum_a \pi(s,a) \sum_{s'} P^a_{ss'} \left[r^a_{ss'} + \gamma V^\pi(s') \right] = \sum_a \pi(s,a) Q^\pi(s,a)$

- Bellman equation

Optimal value functions

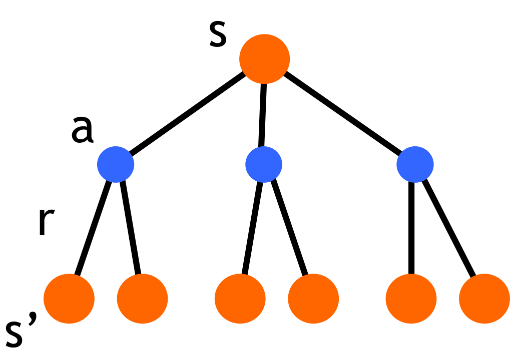
- there's a set of *optimal* policies
  - V^π defines partial ordering on policies
  - they share the same optimal value function

$$V^*(s) = \max_\pi V^\pi(s)$$

- Bellman optimality equation

$$V^*(s) = \max_a \sum_{s'} P^a_{ss'} \left[r^a_{ss'} + \gamma V^*(s') \right]$$

  - system of n non-linear equations
  - solve for V*(s)
  - easy to extract the optimal policy



- having Q*(s,a) makes it even simpler

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

Outline

- Examples
- Defining an RL problem
  - Markov Decision Processes
- Solving an RL problem
  - Dynamic Programming
  - Monte Carlo methods
  - Temporal-Difference learning

Dynamic programming

- main idea
  - use value functions to structure the search for good policies
  - need a perfect model of the environment
- two main components
  - policy evaluation: compute V^π from π
  - policy improvement: improve π based on V^π

  - start with an arbitrary policy
  - repeat evaluation/improvement until convergence

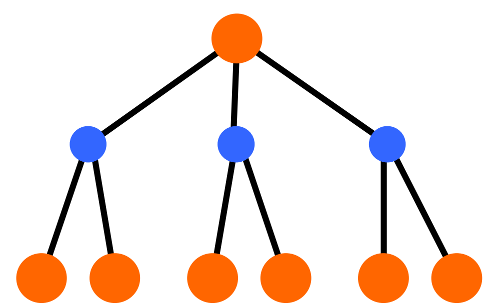
Policy evaluation/improvement

- policy evaluation: π -> V^π
  - Bellman eqn's define a system of n eqn's
  - could solve, but will use iterative version

$$V_{k+1}(s) = \sum_a \pi(s,a) \sum_{k'} P^a_{ss'} \left[r^a_{ss'} + \gamma V_k(s') \right]$$

  - start with an arbitrary value function V_0, iterate until V_k converges

$$\pi'(s) = \arg\max_a Q^\pi(s,a)$$
$$= \arg\max_a \sum_{s'} P^a_{ss'} \left[r^a_{ss'} + \gamma V^\pi(s') \right]$$



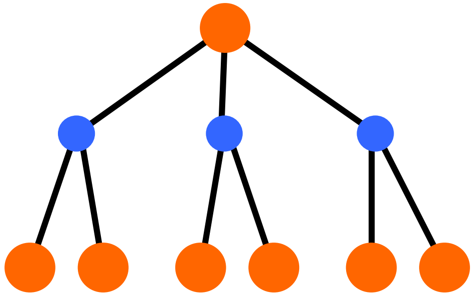
- policy improvement: V^π -> π'
  - π' either strictly better than π, or π' is optimal (if π = π')

Policy/Value iteration

- Policy iteration

$$\pi_0 \rightarrow^{E} V^{\pi_0} \rightarrow^{I} \pi_1 \rightarrow^{E} V^{\pi_1} \rightarrow^{I} \ldots \rightarrow^{I} \pi^* \rightarrow^{E} V^*$$

  - two nested iterations; too slow
  - don't need to converge to V^{π_k}
    - just move towards it



- Va $V_{k+1}(s) = \max_a \sum_{s'} P^a_{ss'} \left[r^a_{ss'} + \gamma V_k(s') \right]$

  - use Bellman optimality equation as an update
  - converges to V*

Using DP

- need complete model of the environment and rewards
  - robot in a room
    - state space, action space, transition model

- can we use DP to solve
  - robot in a room?
  - back gammon?
  - helicopter?

Outline

- examples
- defining an RL problem
  - Markov Decision Processes
- solving an RL problem
  - Dynamic Programming
  - Monte Carlo methods
  - Temporal-Difference learning
- miscellaneous
  - state representation
  - function approximation
  - rewards

Monte Carlo methods

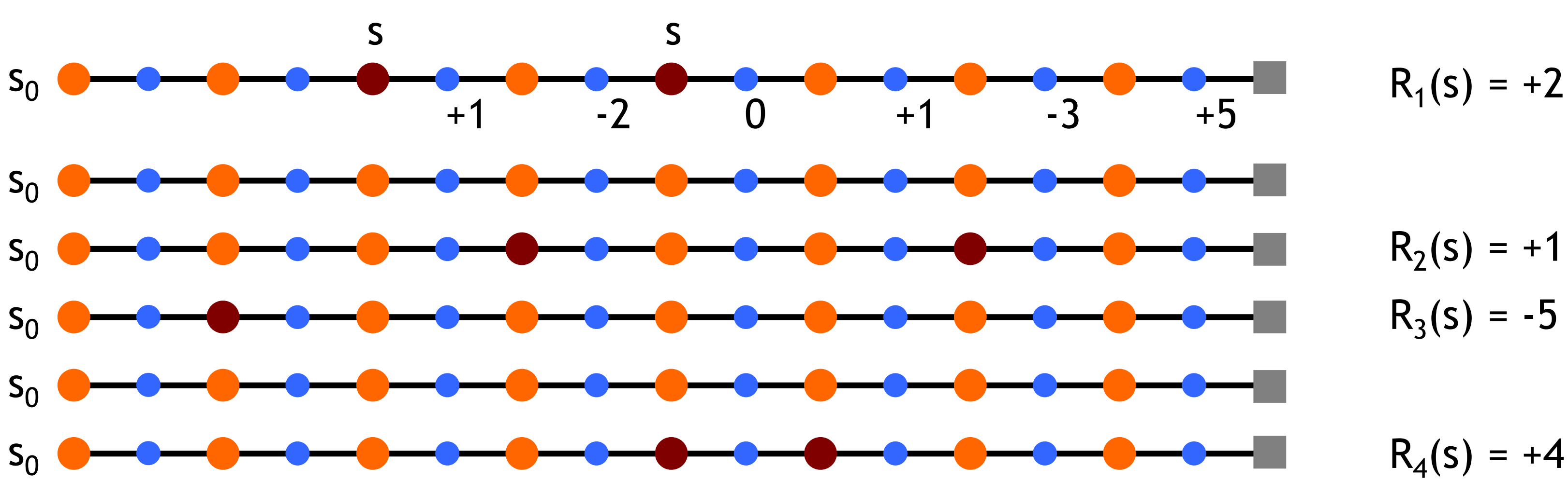
- don’t need full knowledge of environment
  - just experience, or
  - simulated experience

- but similar to DP
  - policy evaluation, policy improvement

- averaging sample returns
  - defined only for episodic tasks

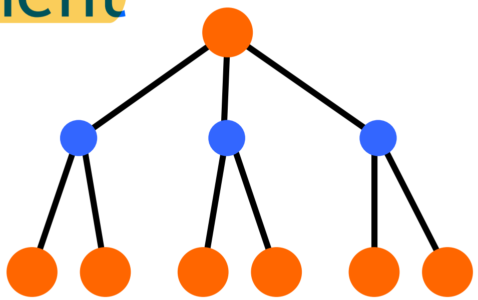
Monte Carlo policy evaluation

- want to estimate $V^{\pi}(s)$
  = expected return starting from s and following π
  - estimate as average of observed returns in state s
- first-visit MC
  - average returns following the first visit to state s



$V^{\pi}(s) \approx (2 + 1 - 5 + 4)/4 = 0.5$

Monte Carlo control



- V^{π} not enough for policy improvement
  - need exact model of environment
- estimate $Q^{\pi}(s,a)$

$$\pi'(s) = \arg\max_a Q^{\pi}(s,a)$$

$$\pi_0 \xrightarrow{E} Q^{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} Q^{\pi_1} \xrightarrow{I} \dots \xrightarrow{I} \pi^* \xrightarrow{E} Q^*$$

- MC control
  - update after each episode
- non-stationary environment

$$V(s) \leftarrow V(s) + \alpha\,[R - V(s)]$$

- a problem
  - greedy policy won't explore all actions

Maintaining exploration

- deterministic/greedy policy won’t explore all actions
  - don’t know anything about the environment at the beginning
  - need to try all actions to find the optimal one
- maintain exploration
  - use *soft* policies instead: $\pi(s,a)>0$ (for all s,a)
- ε-greedy policy
  - with probability 1-ε perform the optimal/greedy action
  - with probability ε perform a random action
  - will keep exploring the environment
  - slowly move it towards greedy policy: ε -> 0

Simulated experience

- 5-card draw poker
  - s_0: A♣, A♦, 6♠, A♥, 2♠
  - a_0: discard 6♠, 2♠
  - s_1: A♣, A♦, A♥, A♠, 9♠ + dealer takes 4 cards
  - return: +1 (probably)
- DP
  - list all states, actions, compute P(s,a,s')
    - P([A♣,A♦,6♠,A♥,2♠], [6♠,2♠], [A♠,9♠,4]) = 0.00192
- MC
  - all you need are sample episodes
  - let MC play against a random policy, or itself, or another algorithm

Summary of Monte Carlo

- don’t need model of environment
  - averaging of sample returns
  - only for episodic tasks
- learn from sample episodes or simulated experience
- can concentrate on “important” states
  - don’t need a full sweep
- need to maintain exploration
  - use soft policies

Outline

- examples
- defining an RL problem
  - Markov Decision Processes
- solving an RL problem
  - Dynamic Programming
  - Monte Carlo methods
  - Temporal-Difference learning
- miscellaneous
  - state representation
  - function approximation
  - rewards

Temporal Difference Learning

- combines ideas from MC and DP
  - like MC: learn directly from experience (don't need a model)
  - like DP: learn from values of successors
  - works for continuous tasks, usually faster than MC

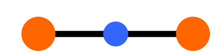
- con

$$V(s_t) \leftarrow V(s_t) + \alpha \left[R_t - V(s_t)\right]$$

target

  - have to wait until the end of episode to update

$$V(s_t) \leftarrow V(s_t) + \alpha \left[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)\right]$$



- simplest TD
  - update after every step, based on the successor

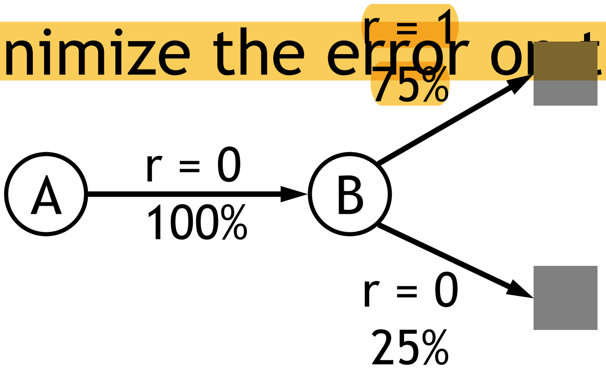
MC vs. TD

- observed the following 8 episodes:

  A – 0, B – 0 B – 1 B – 1 B - 1
  B – 1 B – 1 B – 1 B – 0

- MC and TD agree on V(B) = 3/4

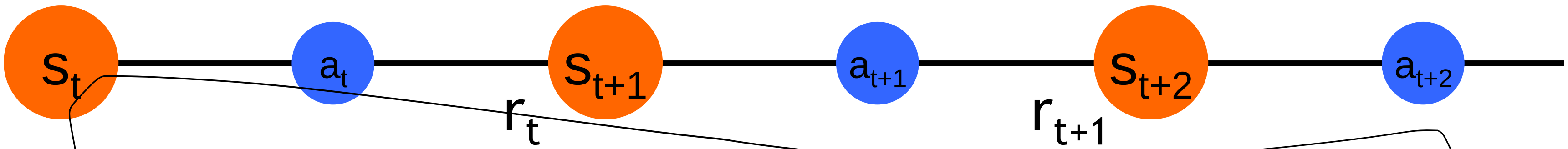
- MC: V(A) = 0
  - converges to values that minimize the error on training data



- TD: V(A) = 3/4
  - converges to ML estimate

Sarsa

- again, need Q(s,a), not just V(s)



$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_t + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t) \right]$$

- control
  - start with a random policy
  - update Q and π after each step
  - again, need ε-soft policies

Q-learning

- before: on-policy algorithms
  - start with a random policy, iteratively improve
  - converge to optimal

- Q-learning: off-policy

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$

  - Q directly approximates Q* (Bellman optimality eqn)
  - independent of the policy being followed
  - only requirement: keep updating each (s,a) pair

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[r_t + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t) \right]$$

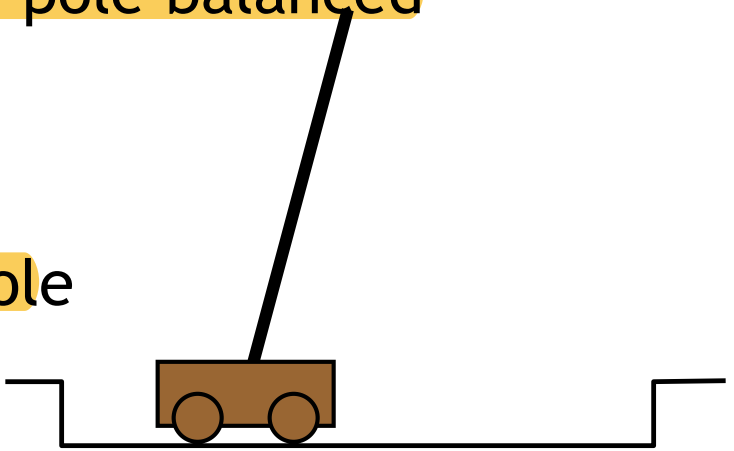
- Sarsa

Outline

- examples
- defining an RL problem
  - Markov Decision Processes
- solving an RL problem
  - Dynamic Programming
  - Monte Carlo methods
  - Temporal-Difference learning
- miscellaneous
  - state representation
  - function approximation
  - rewards

State representation

- pole-balancing
  - move car left/right to keep the pole balanced
- state representation
  - position and velocity of car
  - angle and angular velocity of pole
- what about *Markov property*?
  - would need more info
  - noise in sensors, temperature, bending of pole
- solution
  - coarse discretization of 4 state variables
    - left, center, right
  - totally non-Markov, but still works



Function approximation

- represent V_t as a parameterized function
  - linear regression, decision tree, neural net, …
  - linear regression: $V_t(s) = \vec{\theta}_t^T \vec{\phi}_s = \sum_{i=1}^{n} \theta_t(i)\phi_s(i)$

- update parameters instead of entries in a table
  - better generalization
    - fewer parameters and updates affect "similar" states as well

$$V(s_t) \leftarrow V(s_t) + \alpha \left[r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \right]$$

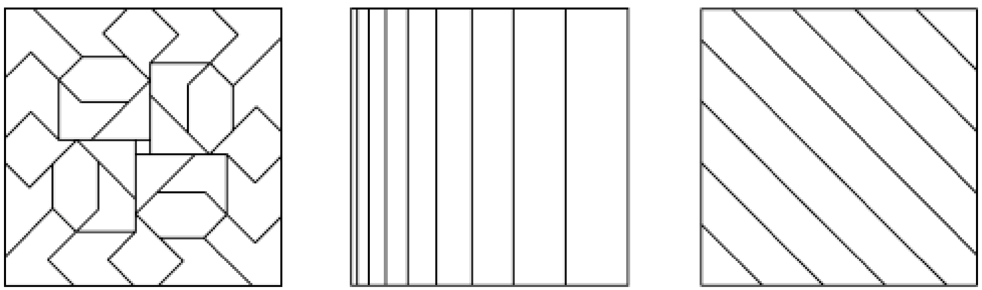
- TD update

$$V(\underbrace{s_t}_{\mathbf{x}}) \mapsto \underbrace{r_{t+1} + \gamma V(s_{t+1})}_{\mathbf{y}}$$

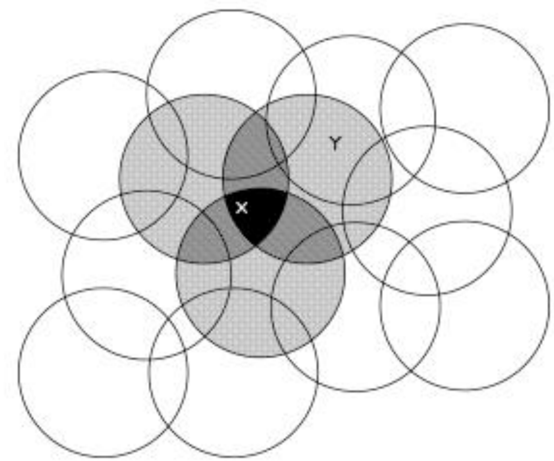
  - treat as one data point for regression
  - want method that can learn on-line (update after each step)

Features

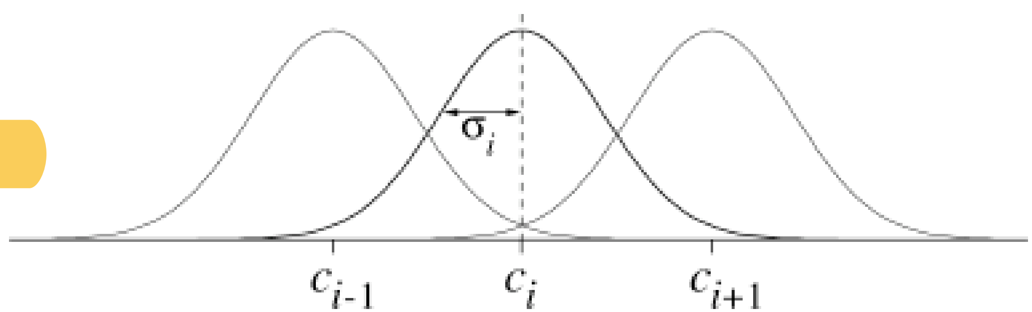
- tile coding, coarse coding
  - binary features



a) Irregular b) Log stripes c) Diagonal stripes

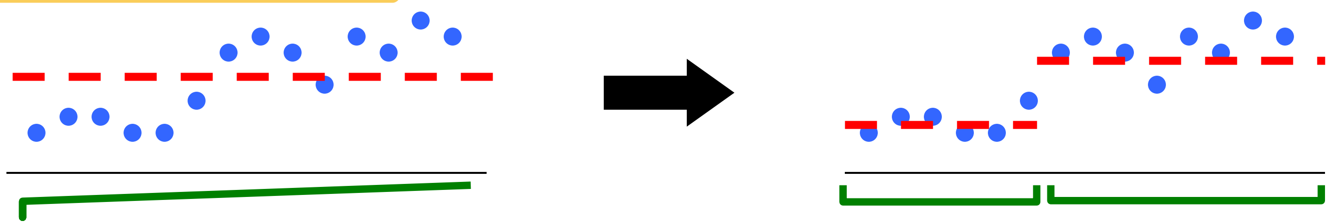


- radial basis functio
  - typically a Gaussiar
  - between 0 and 1

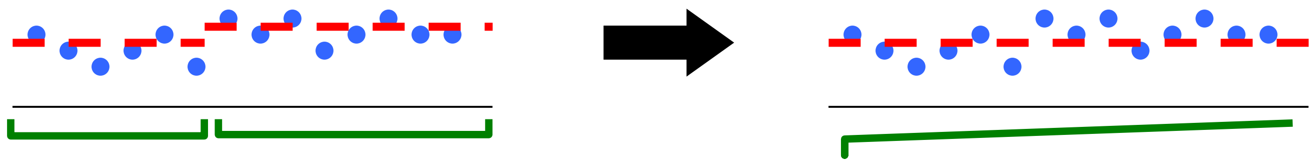


Splitting and aggregation

- want to discretize the state space
  - learn the best discretization during training
- splitting of state space
  - start with a single state
  - split a state when different *parts of that state* have different values



- state aggregation
  - start with many states
  - merge states with similar values



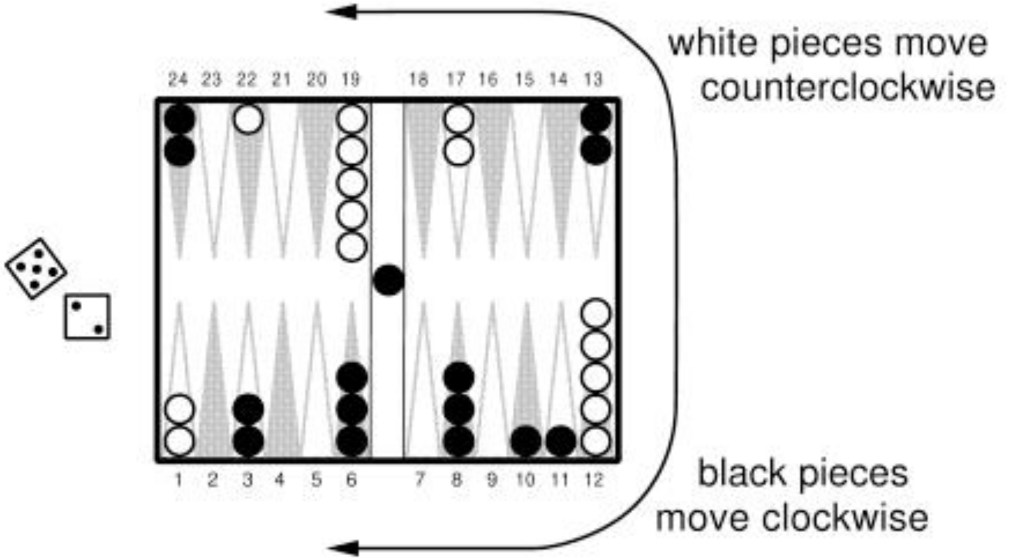
Designing rewards

- robot in a maze
  - episodic task, not discounted, +1 when out, 0 for each step
- chess
  - GOOD: +1 for winning, -1 losing
  - BAD: +0.25 for taking opponent's pieces
    - high reward even when lose
- rewards
  - rewards indicate what we want to accomplish
  - NOT how we want to accomplish it
- shaping

  - positive reward often very "far away"
  - rewards for achieving subgoals (domain knowledge)
  - also: adjust initial policy or initial value function

Case study: Back gammon

- rules
  - 30 pieces, 24 locations
  - roll 2, 5: move 2, 5
  - hitting, blocking
  - branching factor: 400



- implementation
  - use TD(λ) and neural nets
  - 4 binary features for each pos
  - no BG expert knowledge

- results
  - TD-Gammon 0.0: trained against itself (300,000 games)
    - as good as best previous BG computer program (also by Tesauro)
    - lot of expert input, hand-crafted features
  - TD-Gammon 1.0: add special features
  - TD-Gammon 2 and 3 (2-ply and 3-ply search)
    - 1.5M games, beat human champion

Summary

- Reinforcement learning
  - use when need to make decisions in uncertain environment
- solution methods
  - dynamic programming
    - need complete model
  - Monte Carlo
  - time-difference learning (Sarsa, Q-learning)
- most work
  - algorithms simple
  - need to design features, state representation, rewards