Fundamentals of Hypothesis Testing

Source: Levine et al

Basic Concepts (contd.)

- Law of Large numbers
- Central Limit Theorem
- Hypothesis Testing – Concepts and Steps
- Type I and Type II Errors
- Illustrative Examples

What is a Hypothesis?

- A hypothesis is a claim (assertion) about a population parameter:
  - population mean

Example: The mean monthly cell phone bill in this city is μ = Rs. 500

  - population proportion

Example: The proportion of adults in this city with cell phones is π = 0.68

The Null Hypothesis, H_0

- States the claim or assertion to be tested

Example: The mean diameter of a manufactured bolt is 30mm $(H_0 : \mu = 30)$

- Is always about a population parameter, not about a sample statistic

$$H_0 : \mu = 30$$

$$H_0 : \bar{X} = 30$$

The Null Hypothesis, H_0

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty

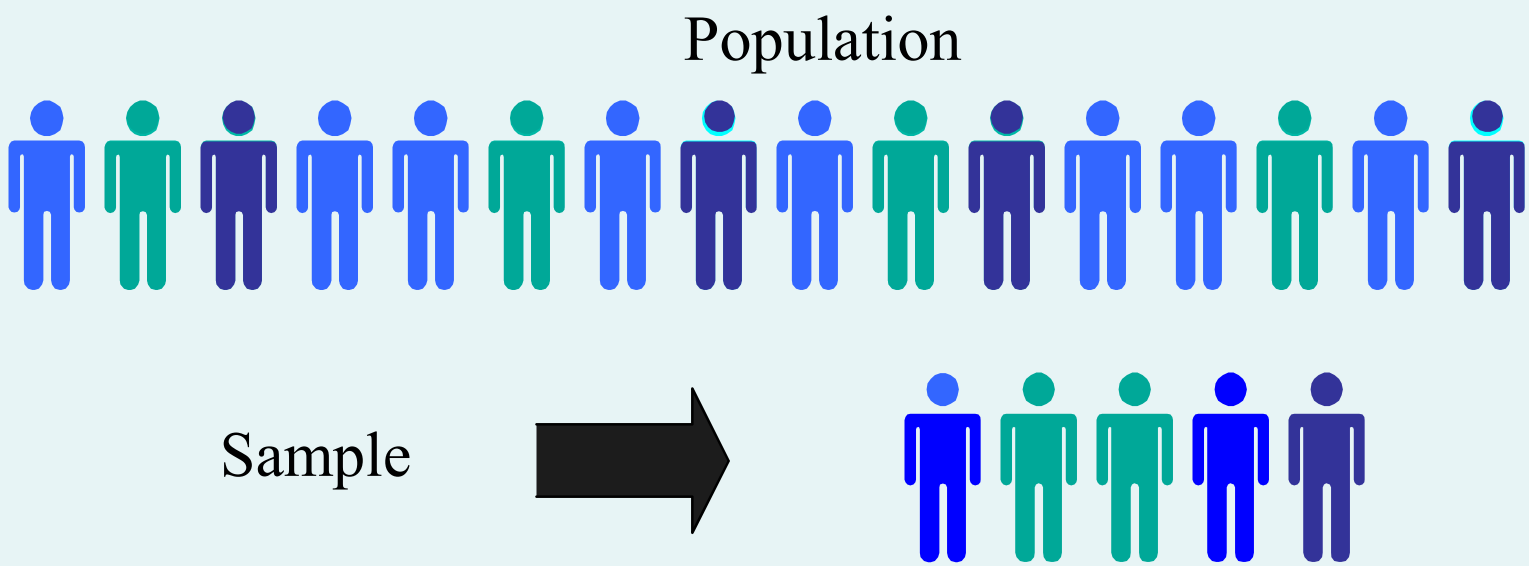
- Refers to the status quo or historical value
- Always contains "=", or "≤", or "≥" sign
- May or may not be rejected

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis
  - e.g., The average diameter of a manufactured bolt is not equal to 30mm ($H_1: \mu \neq 30$)
- Challenges the status quo
- May or may not be proven
- Is generally the hypothesis that the researcher is trying to prove

The Hypothesis Testing Process

- Claim: The population mean age is 50.
  - H_0: $\mu = 50$, H_1: $\mu \neq 50$
- Sample the population and find sample mean.



The Hypothesis Testing Process

(continued)

- Suppose the sample mean age was $\overline{X} = 20$.
- This is significantly lower than the claimed mean population age of 50.
- If the null hypothesis were true, the probability of getting such a different sample mean would be very small, so you reject the null hypothesis .
- In other words, getting a sample mean of 20 is so unlikely if the population mean was 50, you conclude that the population mean must not be 50.

The Hypothesis Testing Process

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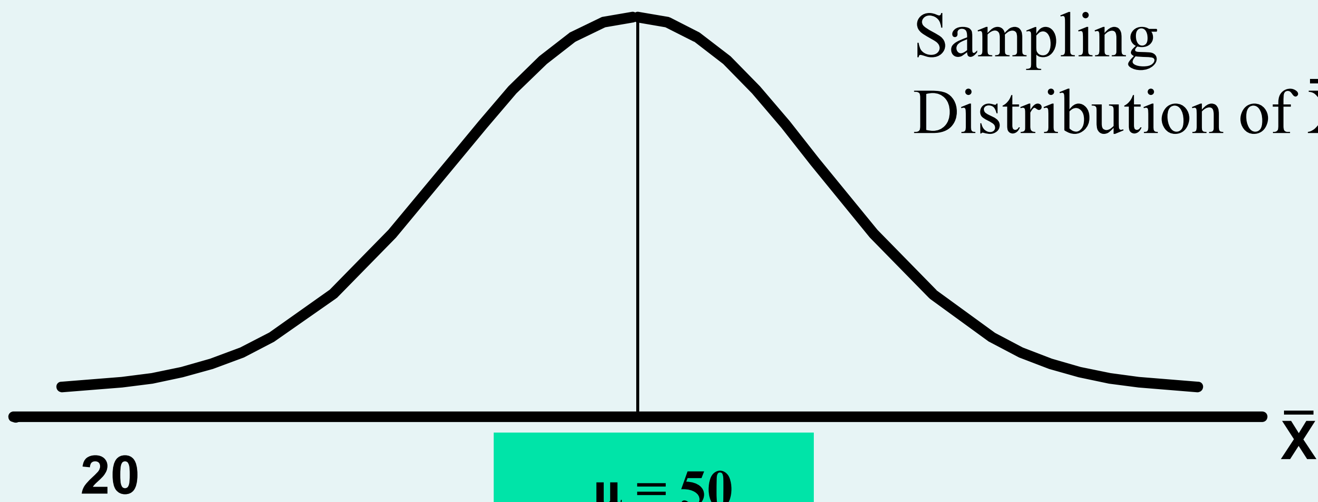


$\mu = 50$
If H_0 is true

If it is unlikely that you would get a sample mean of this value ...

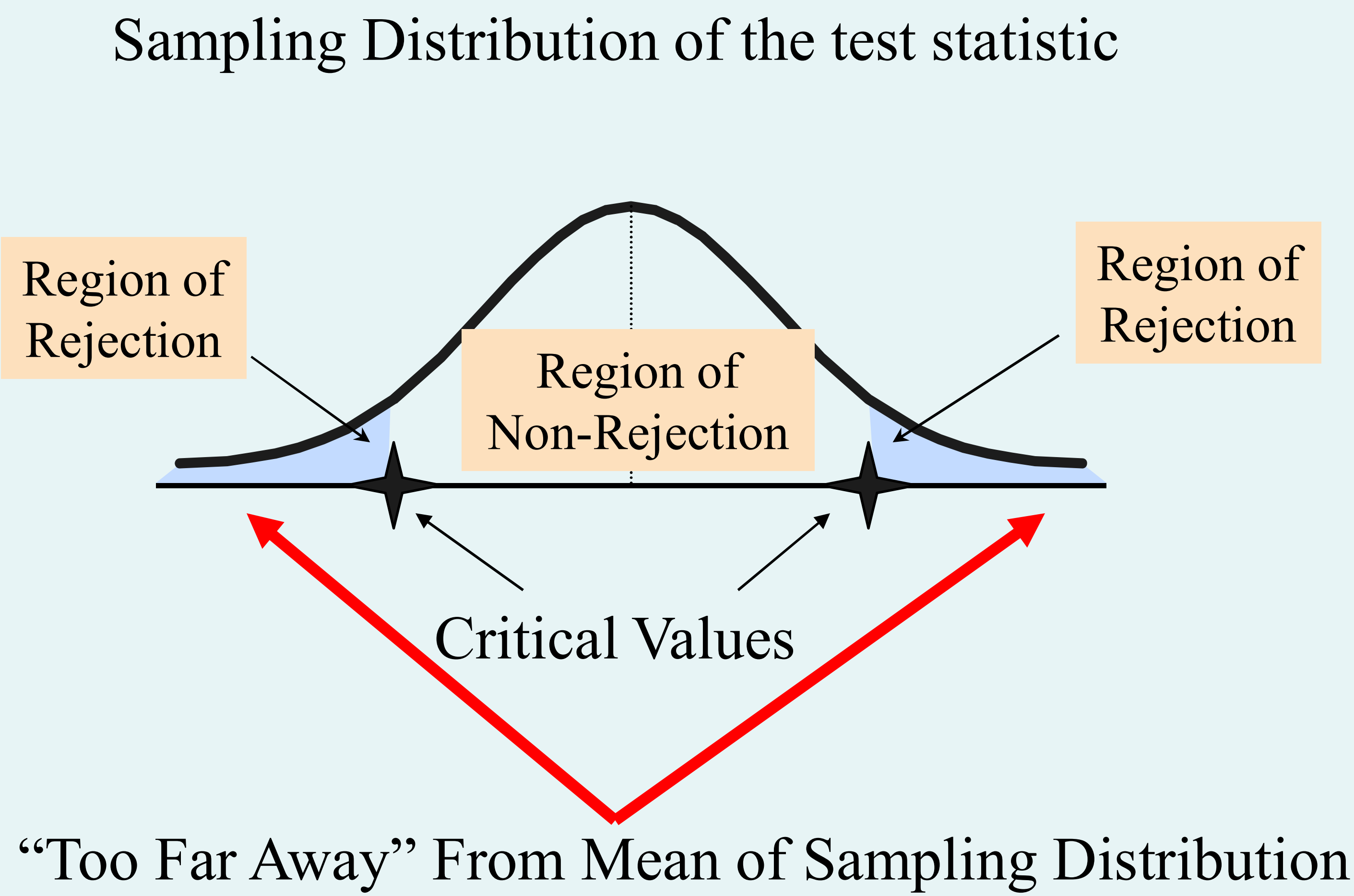
... When in fact this were the population mean…

... then you reject the null hypothesis that $\mu = 50$.

The Test Statistic and Critical Values

- If the sample mean is close to the stated population mean, the null hypothesis is not rejected.
- If the sample mean is far from the stated population mean, the null hypothesis is rejected.
- How far is "far enough" to reject H_0?
- The critical value of a test statistic creates a "line in the sand" for decision making -- it answers the question of how far is far enough.

The Test Statistic and Critical Values



6 Steps in Hypothesis Testing

1. State the null hypothesis, H_0 and the alternative hypothesis, H_1
2. Choose the level of significance, α, and the sample size, n
3. Determine the appropriate test statistic and sampling distribution
4. Determine the critical values that divide the rejection and non-rejection regions

6 Steps in Hypothesis Testing

(continued)

5. Collect data and compute the value of the test statistic under the assumption that H_0 is true.
6. Make the statistical decision and state the managerial conclusion. If the test statistic falls into the nonrejection region, do not reject the null hypothesis H_0. If the test statistic falls into the rejection region, reject the null hypothesis. Express the managerial conclusion in the context of the problem

Possible Errors in Hypothesis Test Decision Making

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error
  - The probability of a Type I Error is α
    - Called level of significance of the test
    - Set by researcher in advance
- **Type II Error**
  - Failure to reject a false null hypothesis
  - The probability of a Type II Error is $\boldsymbol{\beta}$

Possible Errors in Hypothesis Test Decision Making

(continued)

Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	No Error Probability $1 - \alpha$	Type II Error Probability β
Reject H_0	Type I Error Probability α	No Error Power $1 - \beta$

Possible Errors in Hypothesis Test Decision Making

(continued)

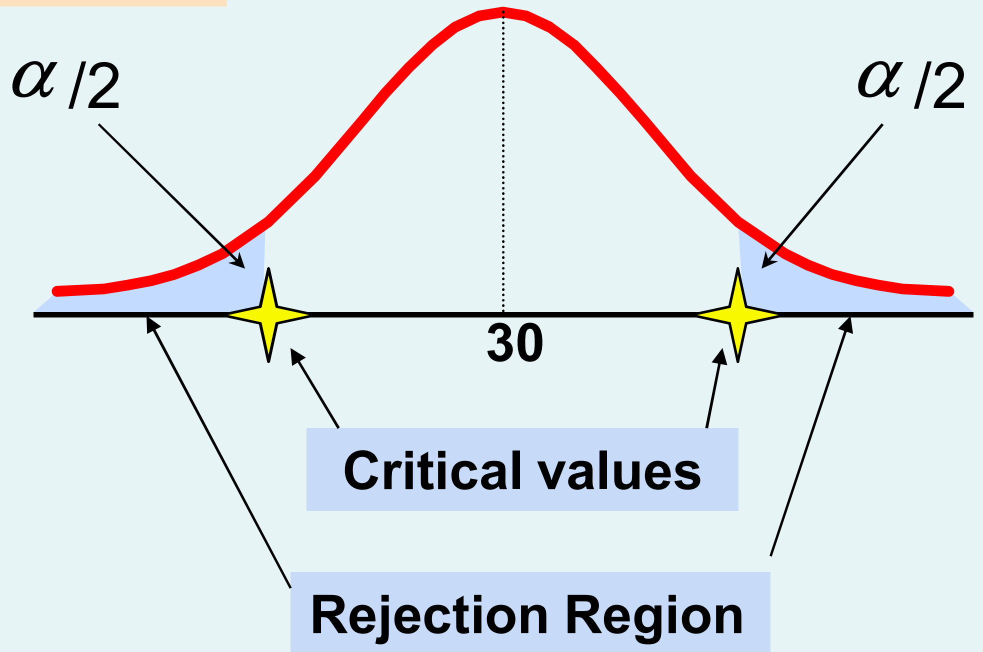
- The confidence coefficient (1-α) is the probability of not rejecting H_0 when it is true.
- The confidence level of a hypothesis test is (1-α)*100%.
- The power of a statistical test (1-β) is the probability of rejecting H_0 when it is false.

Level of Significance and the Rejection Region

H_0: $\mu = 30$

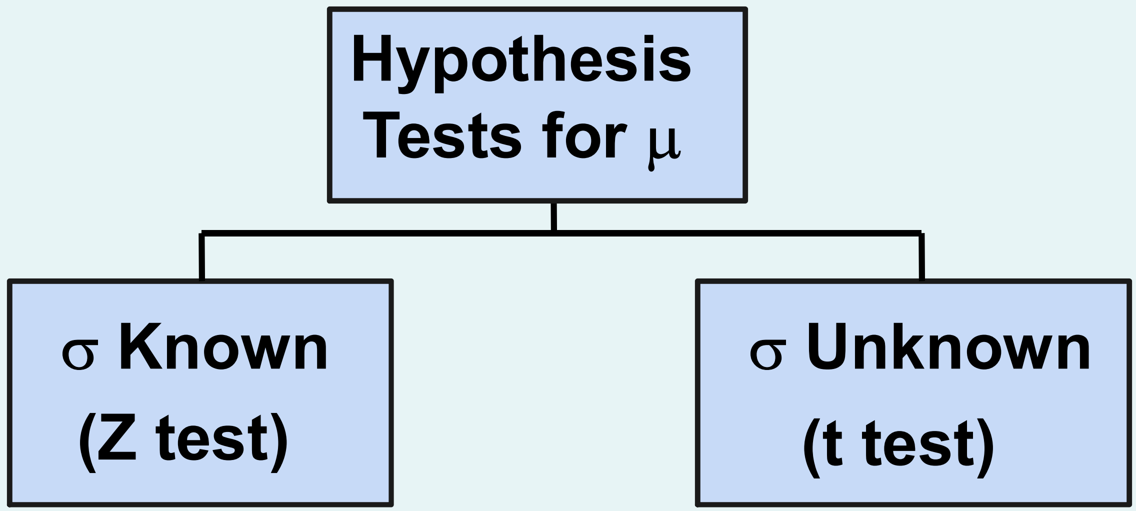
H_1: $\mu \neq 30$

Level of significance = α



This is a two-tail test because there is a rejection region in both tails

Hypothesis Tests for the Mean



Z Test of Hypothesis for the Mean (σ Known)

- Convert sample statistic ($\bar{X}$) to a Z_{STAT} test statistic

Hypothesis Tests for μ

- **σ Known (Z test)**
- **σ Unknown (t test)**

The test statistic is:

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Popular Tests

- Testing means of two Populations (Z test and t test)
- Paired t test
- Testing means of multiple populations (ANOVA test: uses F statistic)
- Testing significance of correlation coefficient
- Chi square test of independence of attributes
- ----
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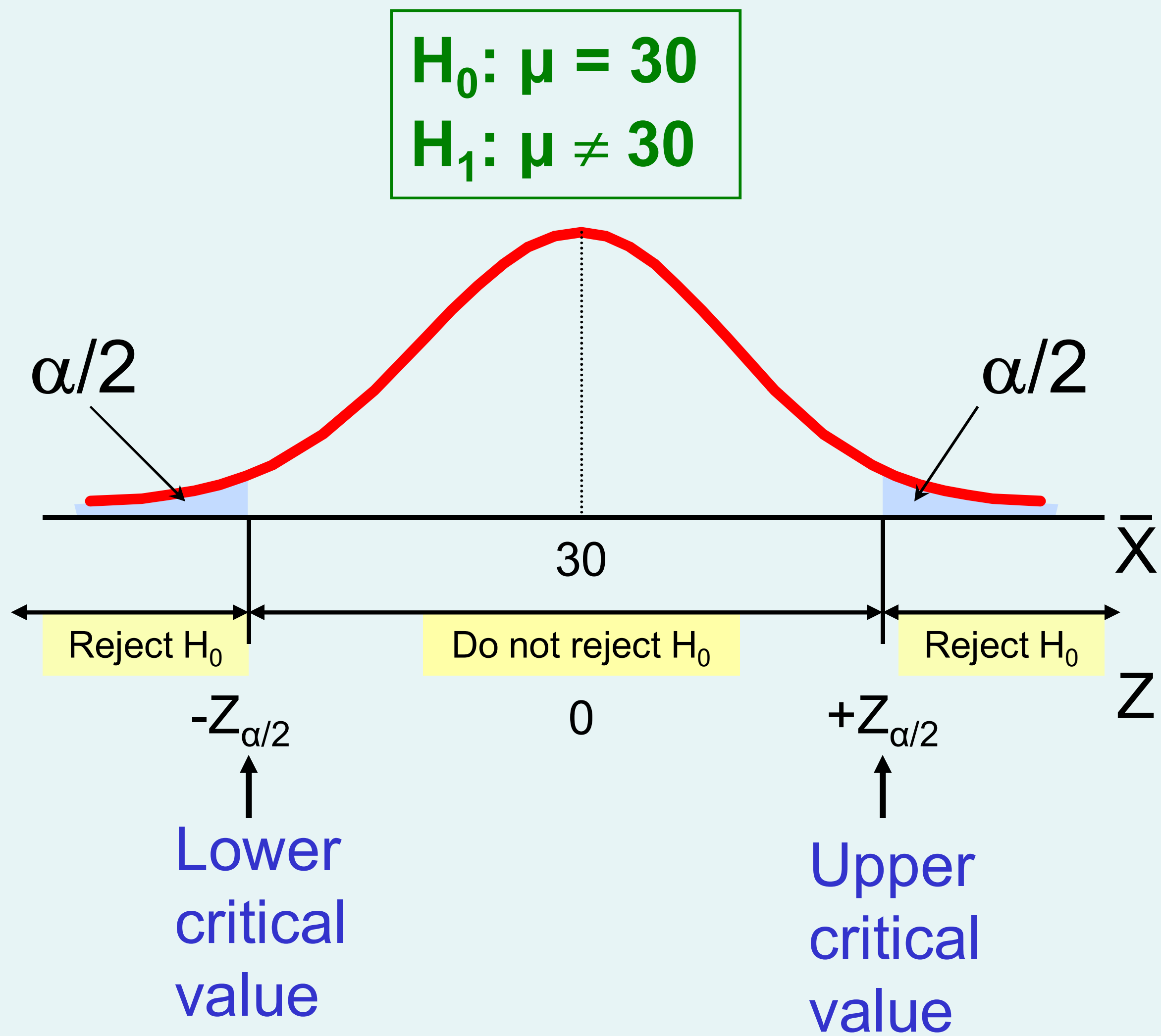
Critical Value Approach to Testing

- For a two-tail test for the mean, σ known:
- Convert sample statistic ($\bar{X}$) to test statistic (Z_{STAT})
- Determine the critical Z values for a specified level of significance α from a table or computer
- Decision Rule: If the test statistic falls in the rejection region, reject H_0 ; otherwise do not reject H_0

Two-Tail Tests

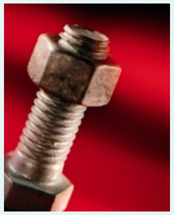
- There are two cutoff values (critical values), defining the regions of rejection



Hypothesis Testing Example

Test the claim that the true mean diameter of a manufactured bolt is 30mm. (Assume $\sigma = 0.8$)

1. State the appropriate null and alternative hypotheses
   - H_0: $\mu = 30$ H_1: $\mu \neq 30$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test



Hypothesis Testing Example

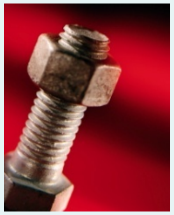
(continued)

3. Determine the appropriate technique
   - σ is assumed known so this is a Z test.
4. Determine the critical values
   - For $\alpha = 0.05$ the critical Z values are ±1.96
5. Collect the data and compute the test statistic
   - Suppose the sample results are

     $n = 100, \quad \bar{X} = 29.84 \quad (\sigma = 0.8$ is assumed known)

So the test statistic is:

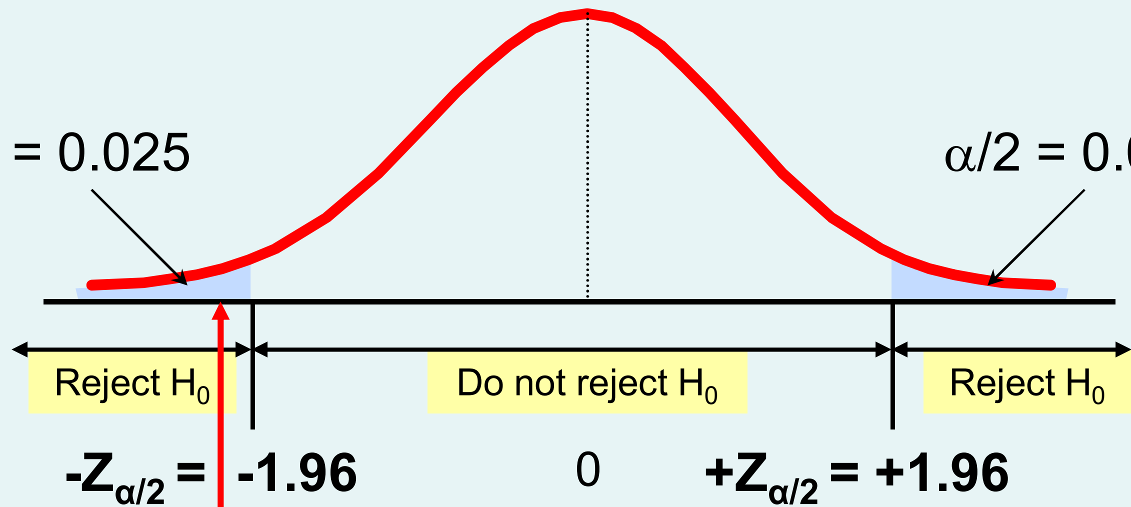
$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{29.84 - 30}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{0.08} = -2.0$$



Hypothesis Testing Example

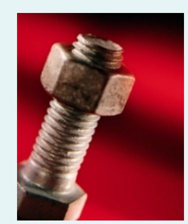
(continued)

- 6. Is the test statistic in the rejection region?



Reject H_0 if $Z_{STAT} < -1.96$ or $Z_{STAT} > 1.96$; otherwise do not reject H_0

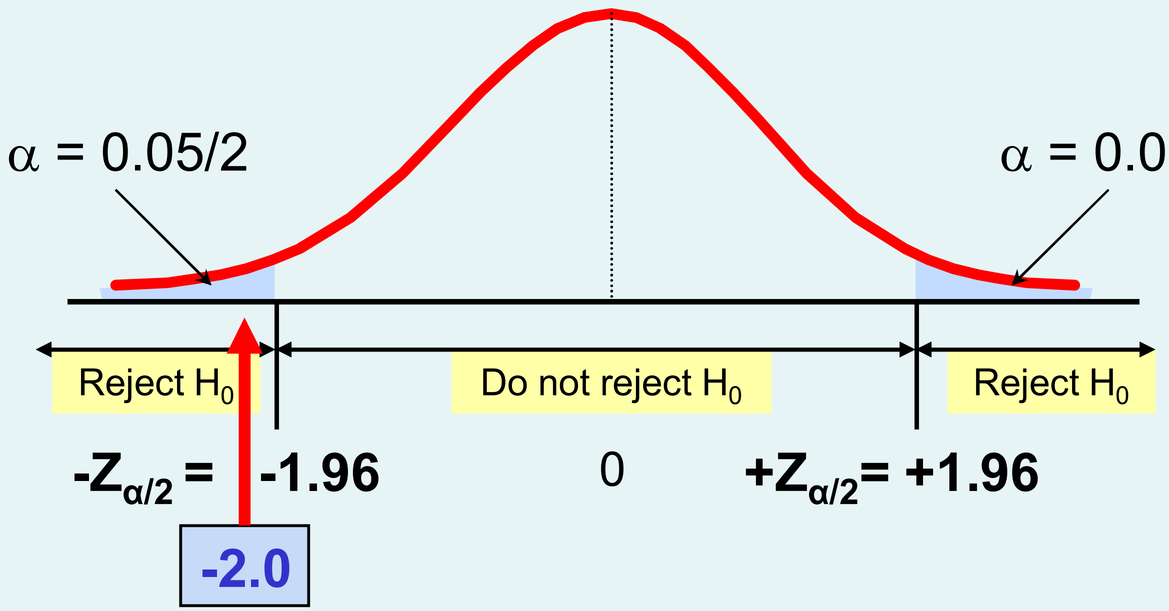
Here, $Z_{STAT} = -2.0 < -1.96$, so the test statistic is in the rejection region



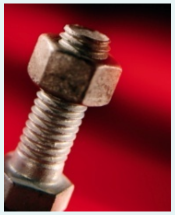
Hypothesis Testing Example

(continued)

6 (continued). Reach a decision and interpret the result



Since $Z_{STAT} = -2.0 < -1.96$, reject the null hypothesis and conclude there is sufficient evidence that the mean diameter of a manufactured bolt is not equal to 30



p-Value Approach to Testing

- p-value: Probability of obtaining a test statistic equal to or more extreme than the observed sample value given H_0 is true
  - The p-value is also called the observed level of significance
  - It is the smallest value of α for which H_0 can be rejected

p-Value Approach to Testing: Interpreting the p-value

- Compare the p-value with α
  - If p-value $< \alpha$, reject H_0
  - If p-value $\geq \alpha$, do not reject H_0
- Remember
  - If the p-value is low then H_0 must go

The 5 Step p-value approach to Hypothesis Testing

1. State the null hypothesis, H_0 and the alternative hypothesis, H_1
2. Choose the level of significance, α, and the sample size, n
3. Determine the appropriate test statistic and sampling distribution
4. Collect data and compute the value of the test statistic under the assumption that H_0 is true and the p-value
5. Make the statistical decision and state the managerial conclusion. If the p-value is $< \alpha$ then reject H_0, otherwise do not reject H_0. State the managerial conclusion in the context of the problem

p-value Hypothesis Testing Example

Test the claim that the true mean diameter of a manufactured bolt is 30mm. (Assume $\sigma = 0.8$)

1. State the appropriate null and alternative hypotheses
   - H_0: $\mu = 30$ H_1: $\mu \neq 30$ (This is a two-tail test)
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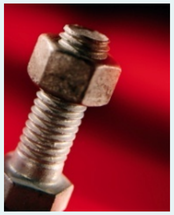


p-value Hypothesis Testing Example

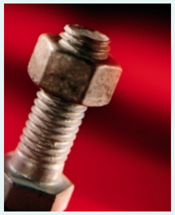
(continued)

3. Determine the appropriate technique
   - σ is assumed known so this is a Z test.
4. Collect the data, compute the test statistic and the p-value
   - Suppose the sample results are

     n = 100, $\bar{X} = 29.84$ (σ = 0.8 is assumed known)

So the test statistic is:

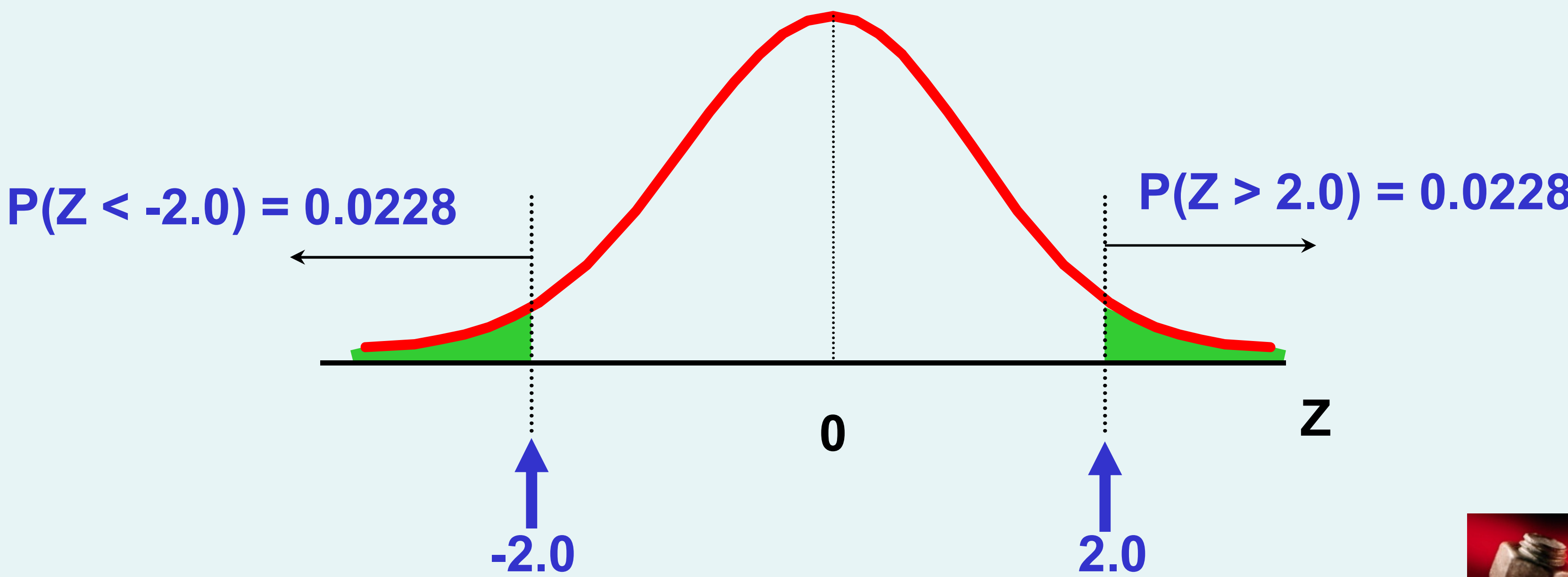
$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{29.84 - 30}{\frac{0.8}{\sqrt{100}}} = \frac{-0.16}{0.08} = -2.0$$



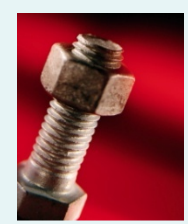
p-Value Hypothesis Testing Example: Calculating the p-value

4. (continued) Calculate the p-value.

- How likely is it to get a Z_{STAT} of -2 (or something further from the mean (0), in either direction) if H_0 is true?



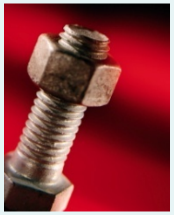
p-value = 0.0228 + 0.0228 = 0.0456



p-value Hypothesis Testing Example

(continued)

- 5. Is the p-value < α?
  - Since p-value = 0.0456 < α = 0.05 Reject H_0
- 5. (continued) State the managerial conclusion in the context of the situation.
  - There is sufficient evidence to conclude the average diameter of a manufactured bolt is not equal to 30mm.



Do You Ever Truly Know σ?

- Probably not!
- In virtually all real world business situations, σ is not known.
- If there is a situation where σ is known then μ is also known (since to calculate σ you need to know μ.)
- If you truly know μ there would be no need to gather a sample to estimate it.

Hypothesis Testing: σ Unknown

- If the population standard deviation is unknown, you instead use the sample standard deviation S.
- Because of this change, you use the t distribution instead of the Z distribution to test the null hypothesis about the mean.
- When using the t distribution you must assume the population you are sampling from follows a normal distribution.
- All other steps, concepts, and conclusions are the same.

t Test of Hypothesis for the Mean (σ Unknown)

- Convert sample statistic ($\bar{X}$) to a t_{STAT} test statistic

Hypothesis Tests for μ

σ Known (Z test)

σ Unknown (t test)

The test statistic is:

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$