Bayes Theorem :-

$$P(\omega_j/x) = \frac{P(x/\omega_j)\,P(\omega_j)}{P(x)},$$ where $P(x) = \sum_{k=1}^{N} P(x/\omega_k).P(\omega_k)$

where ω_j is the j^{th} class and x is the feature vector.

$P(\omega_j) \rightarrow$ A priori probability of class ω_j

$P(\omega_j/x) \rightarrow$ Posterior prob. of class ω_j given the observation x.

$P(x/\omega_j) \rightarrow$ Likelihood (Class conditional prob. of x given Class ω_j)

$P(x) \rightarrow$ A normalization constant that does not affect the decision.

Discrete Random variable

PMF (Prob. Mass function) $\rightarrow$ $P_x(X) = P(X = x)$

CDF (Cumulative distribution funⁿ) $\rightarrow$ $F_X(x) = P(X \le x)$

Continous Random variable

$$P(a < X < b) = F(a < x < b) = \int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(X)$ is Cumulative distribution function

& $f(x)$ " prob. density function.

(i) $f(x) = \frac{d}{dx} F(x)$ (ii) $f(x) \geq 0$

(iii) $F(x) = \int_{-\infty}^{\infty} f(x)\,dx = 1$

Expectation or mean of Prob. distribution

$\mu = E[x] = \sum x f[x]$ For discrete variables | $\mu = E[x] = \int x f(x)\,dx$ Continous variables.

Variance   For discrete variables

$$Var[x] = \sigma^2 = E[(x-\mu)^2]$$
$$= E[x^2 + \mu^2 - 2x\mu]$$
$$= E[x^2] + E[\mu^2] - E[2x\mu]$$
$$= E[x^2] + \mu^2 - 2\mu E[x]$$
$$= E[x^2] + \mu^2 - 2\mu . \mu$$
$$\boxed{Var[x] = E[x^2] - \mu^2}$$

For continous variables

$$Var[x] = \sigma^2(x) = \int_a^b x^2 f(x)\,dx - \mu^2$$

Standard deviation

$$\sigma = sqrt(variance)$$
$$= \sqrt{E[x^2] - \mu^2}$$

Covariance

$$Cov(x, y) = E(xy) - E(x)E(y)$$

or

$$Cov(x, y) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n}$$

Correlation :- Karl Pearson's Product moment method

$$-1 \le \rho \le 1$$

$-1 \le \rho \le 0 \rightarrow$ -ve Correlation.

$0 < \rho \le 1 \rightarrow$ +ve Correlation.

$$\boxed{\rho = \frac{Cov(x, y)}{\sigma_x \sigma_y}}$$

$\rightarrow$ use when directly Cov. & Std. deviation are given

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}, \quad \sigma_y = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n}}$$

or

$$\sigma_x = \sqrt{E(x^2) - E(x)^2}, \quad \sigma_y = \sqrt{E(y^2) - E(y^2)}$$