Artificial Neural Network

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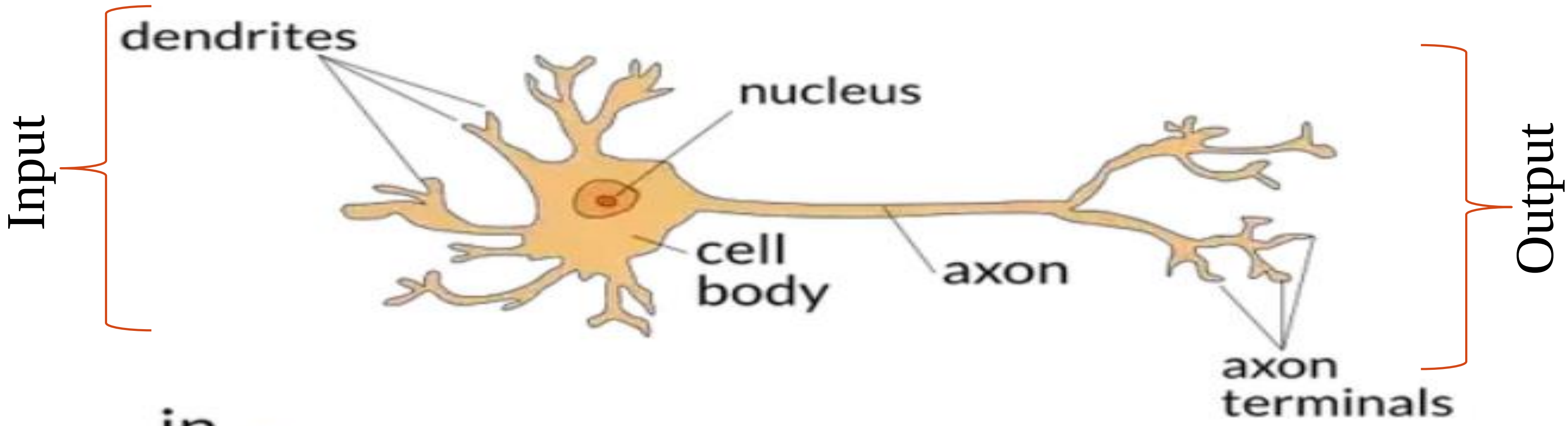
Introduction

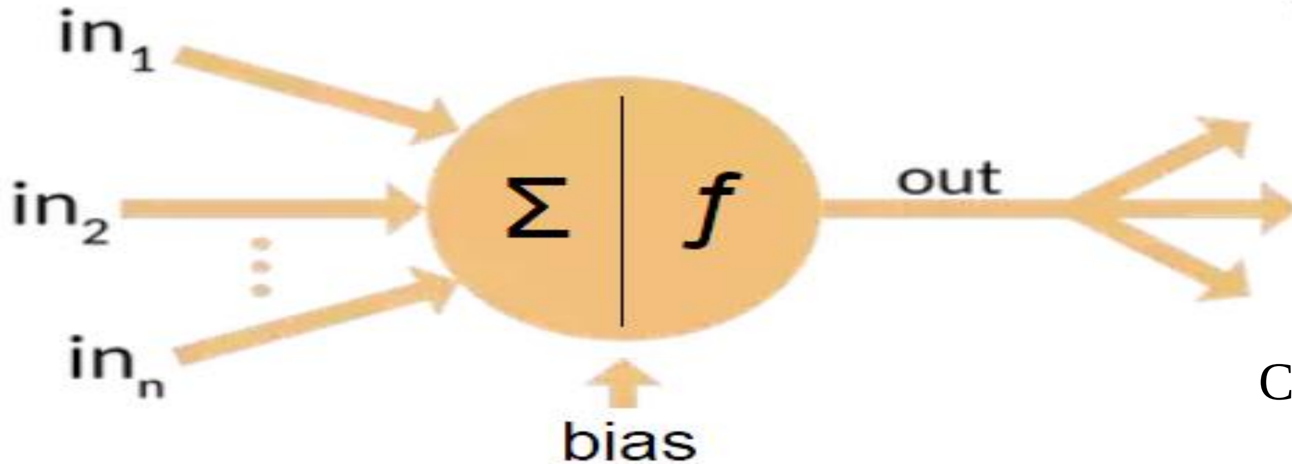
- Artificial neural networks (ANNs) provide a practical method for learning
  - ✓ real-valued functions
  - ✓ discrete-valued functions
  - ✓ vector-valued functions
- Robust to errors in training data
- Successfully applied to such problems as
  - ✓ interpreting visual scenes
  - ✓ speech recognition
  - ✓ learning robot control strategies

Introduction…

- **ANN learning well-suit to problems which the training data corresponds to noisy, complex data (inputs from cameras or microphones)**
- Can also be used for problems with symbolic representations
- **Most appropriate for problems where**
  - ✓ Instances have many attribute-value pairs
  - ✓ Target function output may be discrete-valued, real-valued, or a vector of several real- or discrete-valued attributes
  - ✓ Training examples may contain errors
  - ✓ Long training times are acceptable
  - ✓ Fast evaluation of the learned target function may be required
  - ✓ The ability for humans to understand the learned target function is not important.

Human Brain Processing





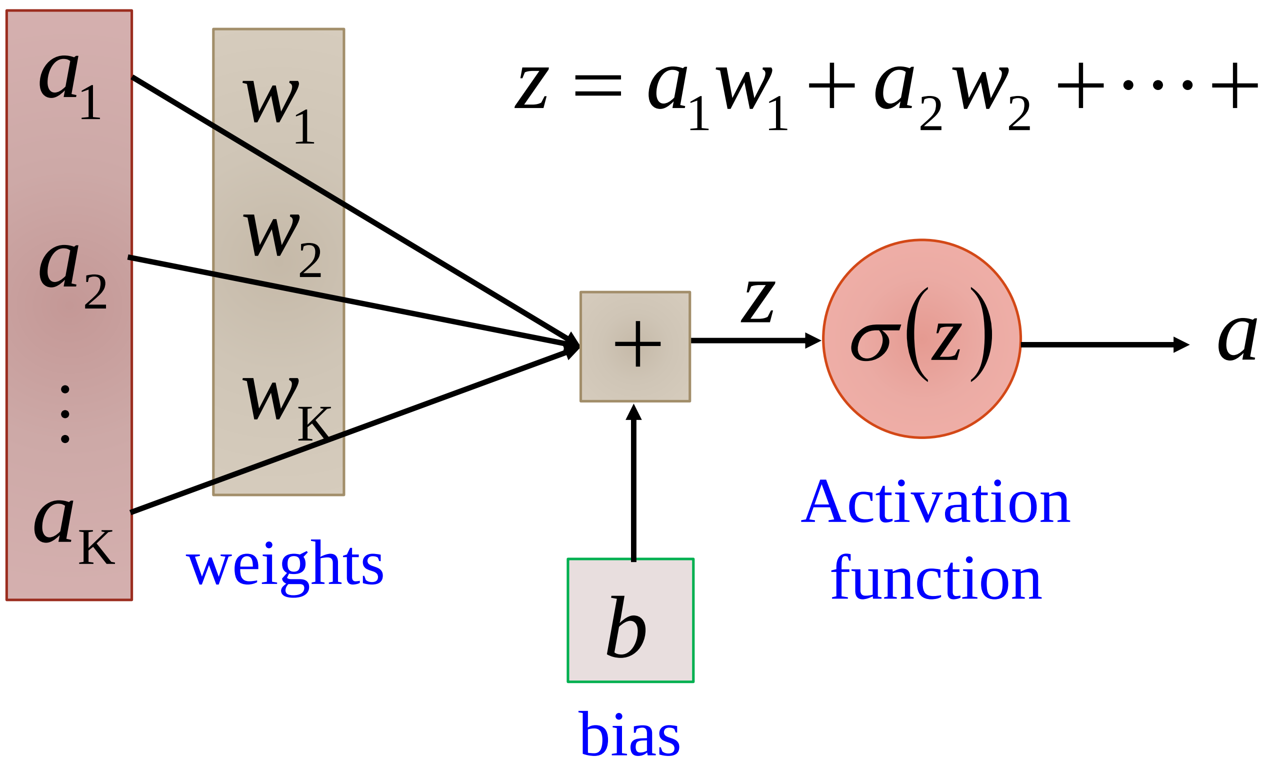
Dendrites: **Input**
Cell body: **Processor**
Synaptic: **Link**
Axon: **Output**

The human brain is made up of billions of simple processing units – neurons.

Neuron

$$f: R^K \rightarrow R$$

$$z = a_1 w_1 + a_2 w_2 + \cdots + a_K w_K + b$$



Neuron...

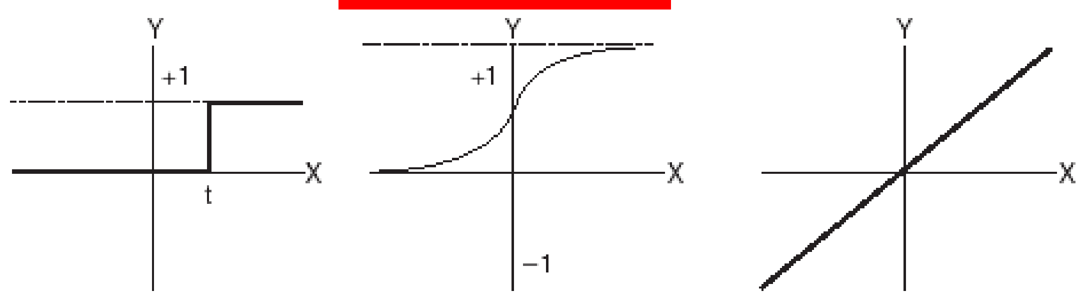
- Artificial neurons are based on biological neurons.
- Each neuron in the network receives one or more inputs.
- An activation function is applied to the inputs, which determines the output of the neuron – the activation level.

Activation Function works

$$X = \sum_{i=1}^{n} w_i x_i$$

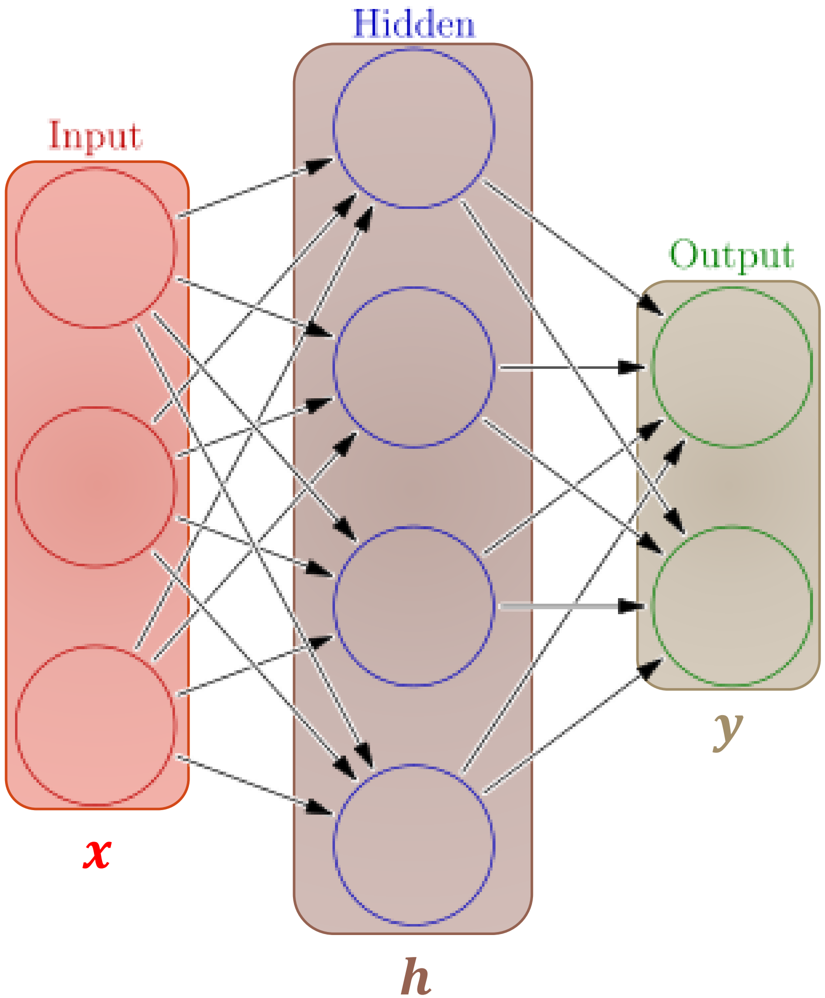
$$Y = \begin{cases} +1 & \text{for } X > t \\ 0 & \text{for } X \le t \end{cases}$$

Activation functions



(a) Step funtion (b) Sigmoid funtion (c) Linear funtion

Neural Network



Weights

$$h = \sigma(\mathbf{W}_1 x + b_1)$$

$$y = \sigma(W_2 h + b_2)$$

Activation functions

How do we train?

4 + 2 = 6 neurons (not counting inputs)

[3 x 4] + [4 x 2] = 20 weights

4 + 2 = 6 biases

26 learnable **parameters**

Training Perceptron

- Learning involves choosing values for the weights
- The perceptron is trained as follows:
  - ✓ First, inputs are given random weights (usually between –0.5 and 0.5).
  - ✓ An item of training data is presented. If the perceptron mis-classifies it, the weights are modified according to the following:
    - where t is the target output for the training example, o is the output generated by the perceptron and a is the learning rate, between 0 and 1 (usually small such as 0.1)
- Cycle through training examples until successfully classify all examples
  - ✓ Each cycle known as an **epoch**

Backpropagation

- Multilayer neural networks learn in the same way as perceptrons.
- However, there are many more weights, and it is important to assign credit (or blame) correctly when changing weights.
- *E* sums the errors over all of the network output units

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} \sum_{k \in outputs} (t_{kd} - o_{kd})^2$$

Backpropagation Algorithm

- Create a feed-forward network with n_{in} inputs, n_{hidden} hidden units, and n_{out} output units.
- Initialize all network weights to small random numbers.
- Until termination condition is met, Do
  - ✓ For each <*x*,*t*> in training examples, Do.
  - ✓ *Propagate the input forward through the network:*

1. Input the instance x to the network and compute the output o_u of every unit u in the network.
2. *Propagate the errors backward through the network:*
3. For each network output unit k, calculate its error term δ_k

$$\delta_k \leftarrow o_k(1-o_k)(t_k-o_k)$$

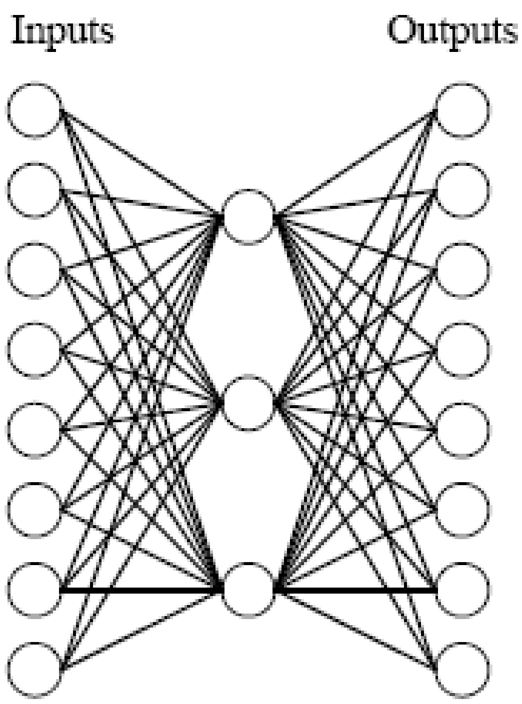
4. For each hidden unit h, calculate its error term δ_h

$$\delta_h \leftarrow o_h(1-o_h)\sum_{k\in outputs} w_{kh}\delta_k$$

5. Update each network weight w_{ji} where

$$w_{ji} \leftarrow w_{ji} + \Delta w_{ji} \qquad \Delta w_{ji} = \alpha\delta_j x_{ji}$$

Hidden Layer representation



Target Function:

Input		Output
10000000	$\rightarrow$	10000000
01000000	$\rightarrow$	01000000
00100000	$\rightarrow$	00100000
00010000	$\rightarrow$	00010000
00001000	$\rightarrow$	00001000
00000100	$\rightarrow$	00000100
00000010	$\rightarrow$	00000010
00000001	$\rightarrow$	00000001

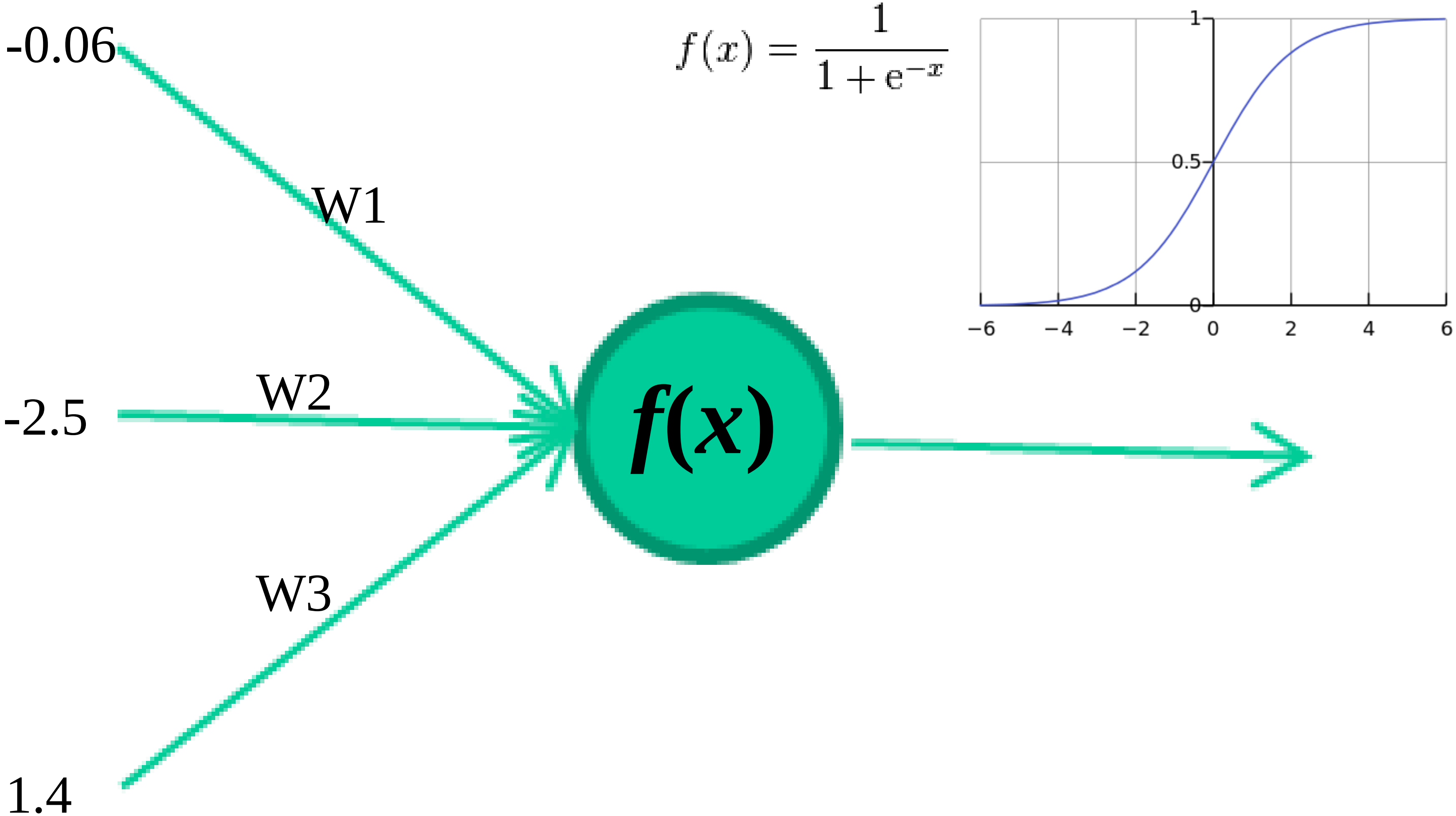
Can this be learned?

Yes

Input		Hidden Values		Output
10000000	→	.89 .04 .08	→	10000000
01000000	→	.15 .99 .99	→	01000000
00100000	→	.01 .97 .27	→	00100000
00010000	→	.99 .97 .71	→	00010000
00001000	→	.03 .05 .02	→	00001000
00000100	→	.01 .11 .88	→	00000100
00000010	→	.80 .01 .98	→	00000010
00000001	→	.60 .94 .01	→	00000001

Example 1 of NN

$$f(x) = \frac{1}{1 + e^{-x}}$$



Example 1 of NN…

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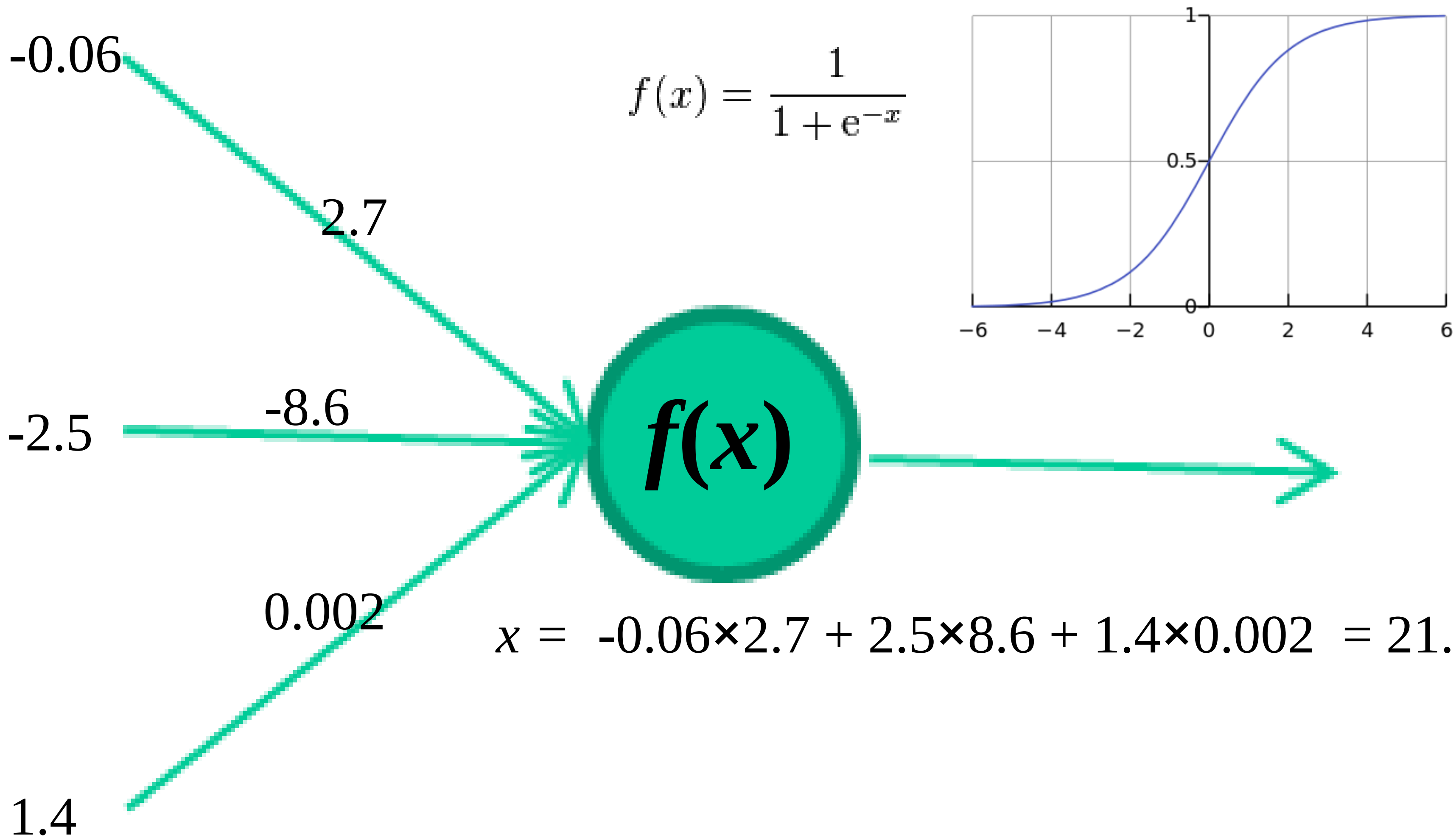
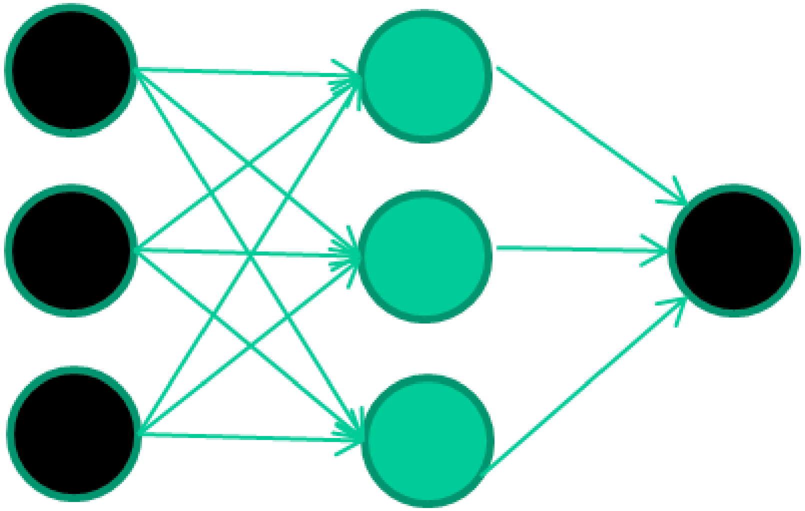


x = -0.06×2.7 + 2.5×8.6 + 1.4×0.002 = 21.34

Example 1 of NN…

A dataset

Fields			***class***
1.4	2.7	1.9	0
3.8	3.4	3.2	0
6.4	2.8	1.7	1
4.1	0.1	0.2	0
etc …			



Example 1 of NN…

Training the neural network

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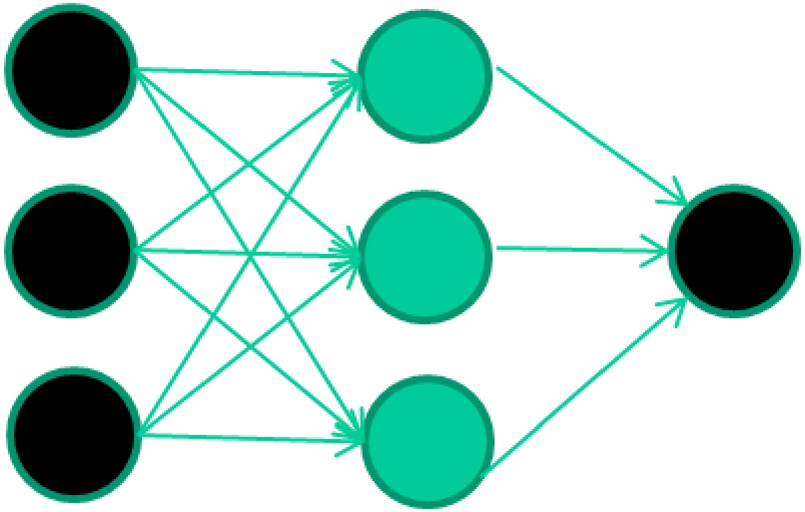

Example 1 of NN…

Training data

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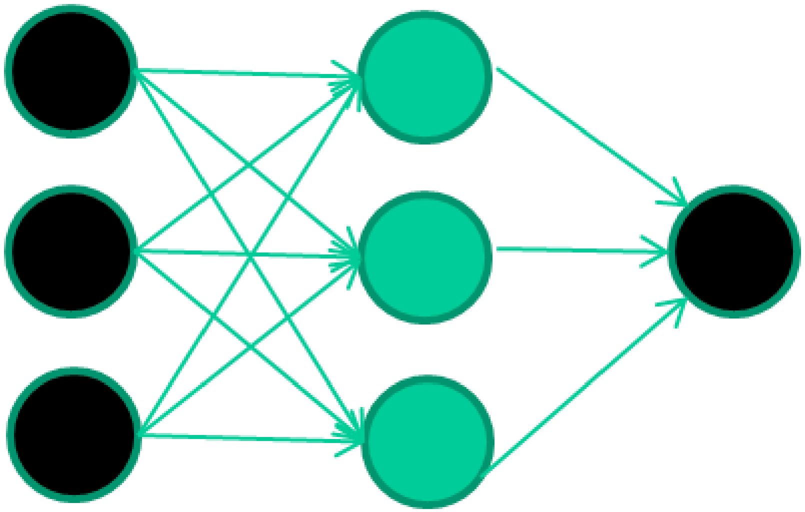
Initialise with random weights



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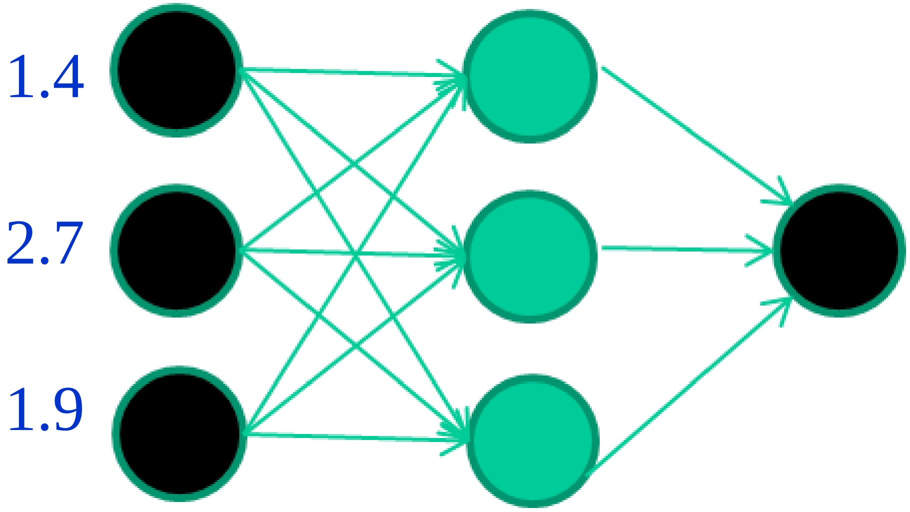
etc …			

Present a training pattern



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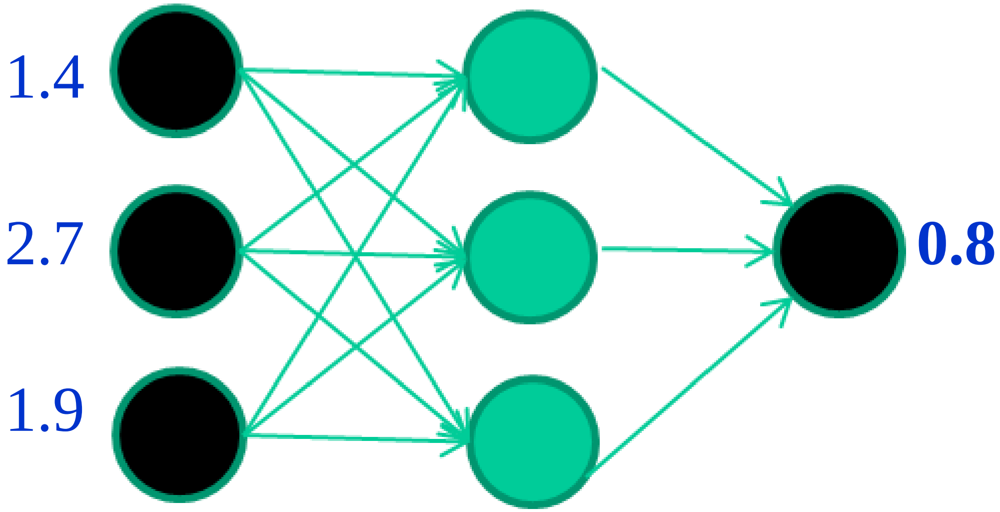
etc …

Feed it through to get output



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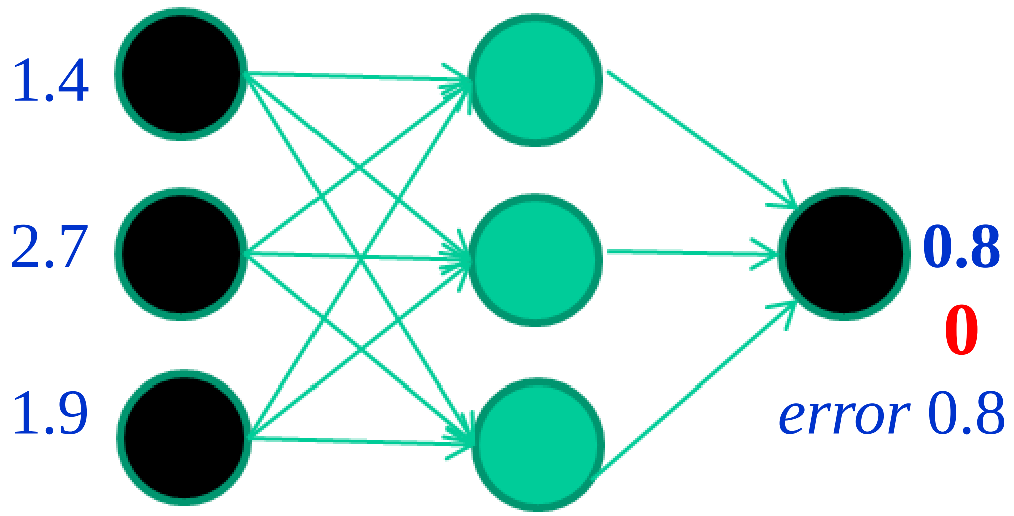
etc …

Compare with target output



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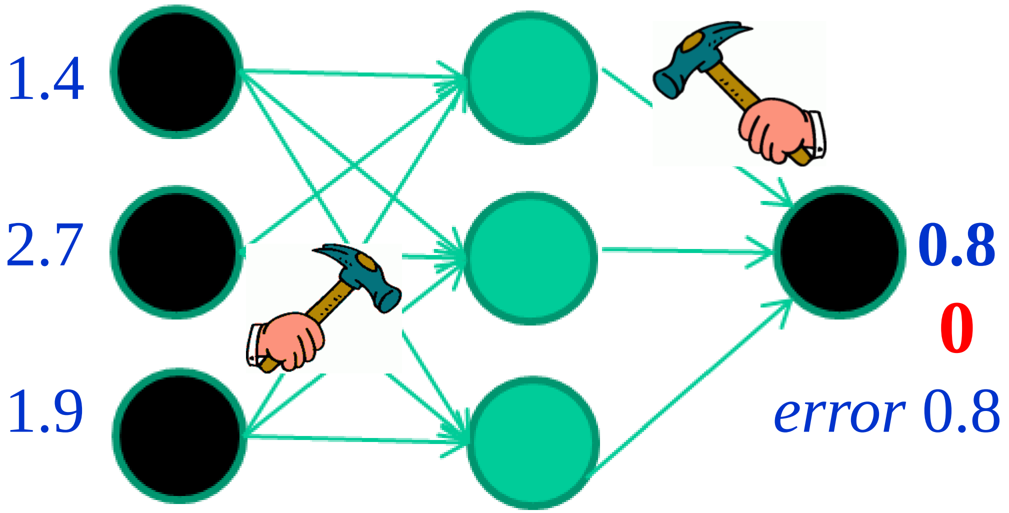
etc …			

Adjust weights based on error



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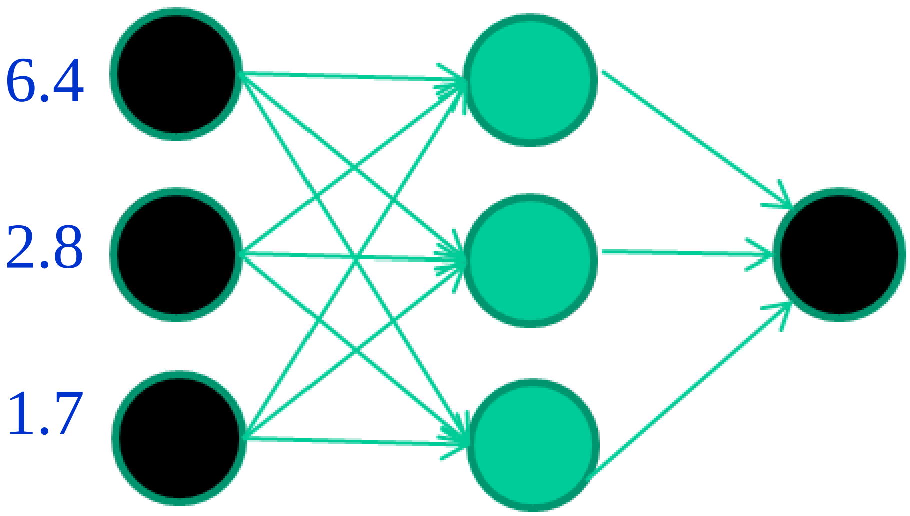

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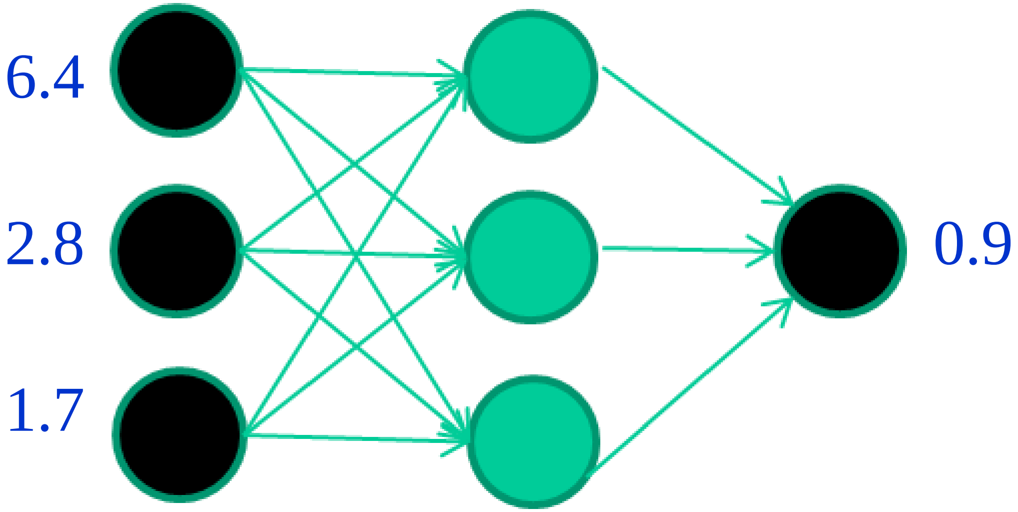

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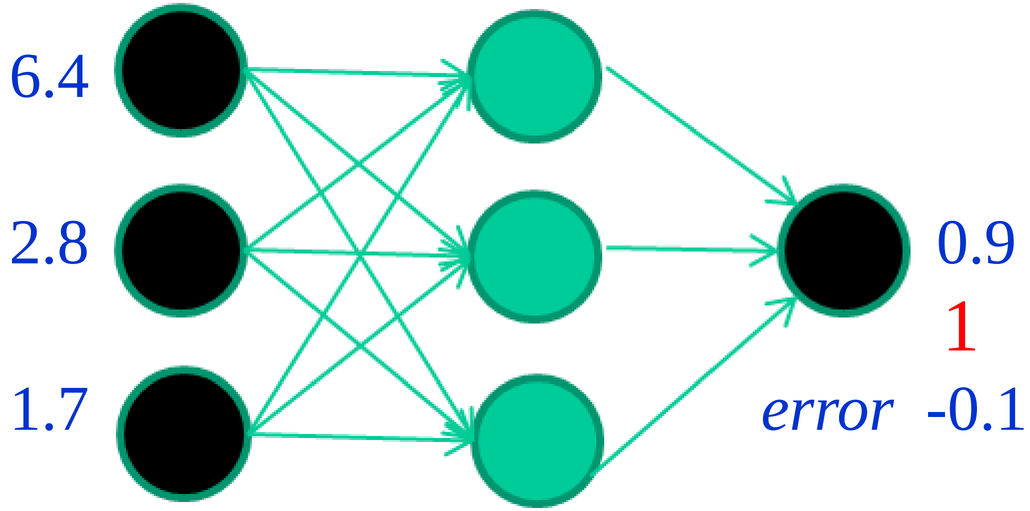
Compare with target output



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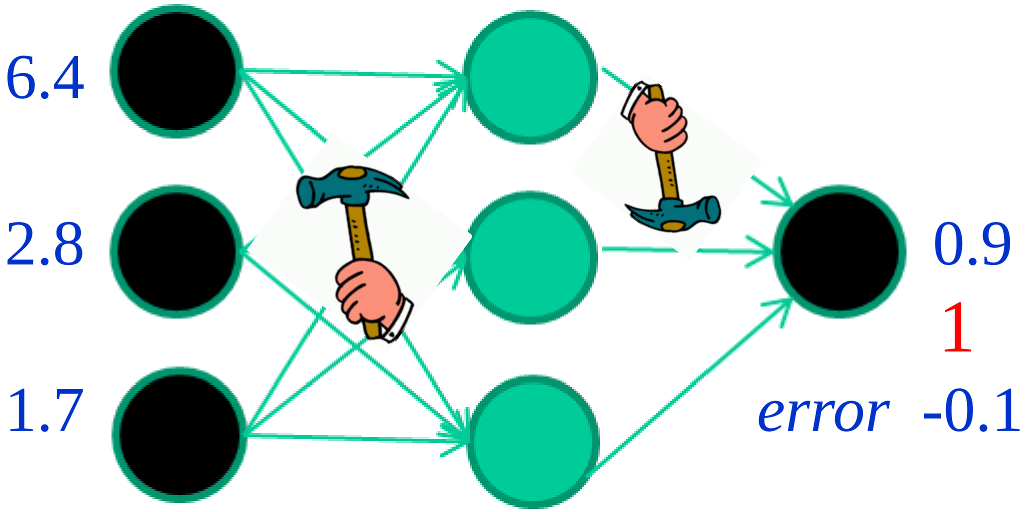

Example 1 of NN…

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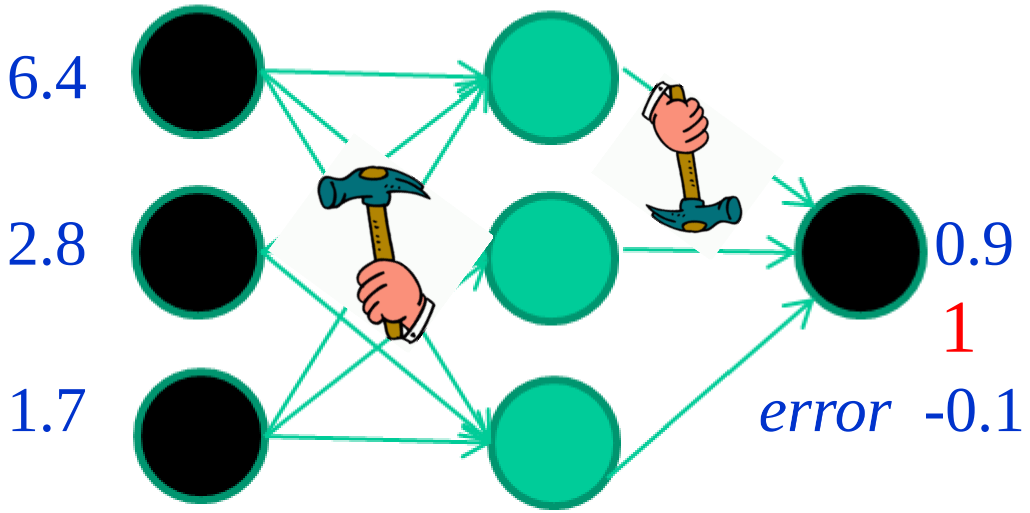
etc …			

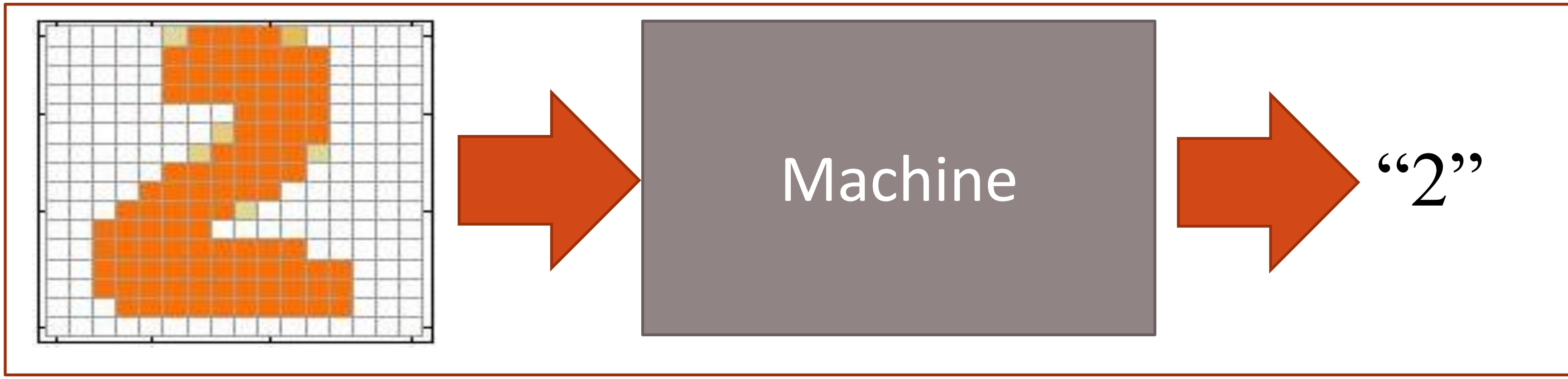
And so on ….

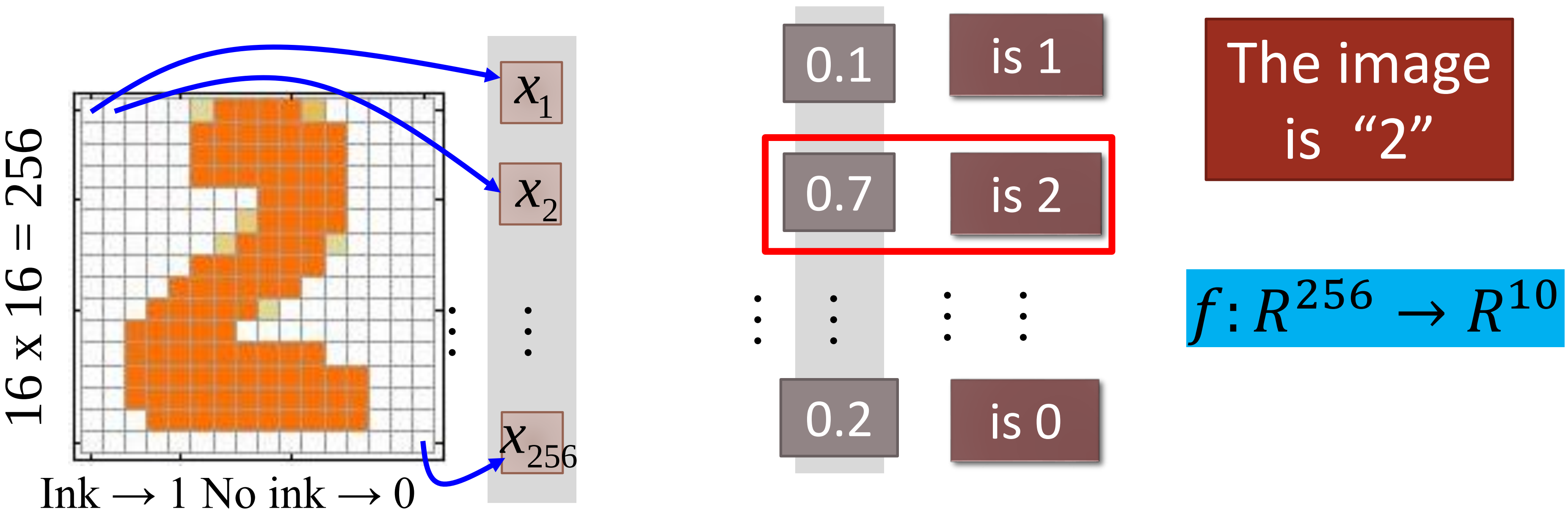


Repeat this thousands, maybe millions of times – each time taking a random training instance, and making slight weight adjustments

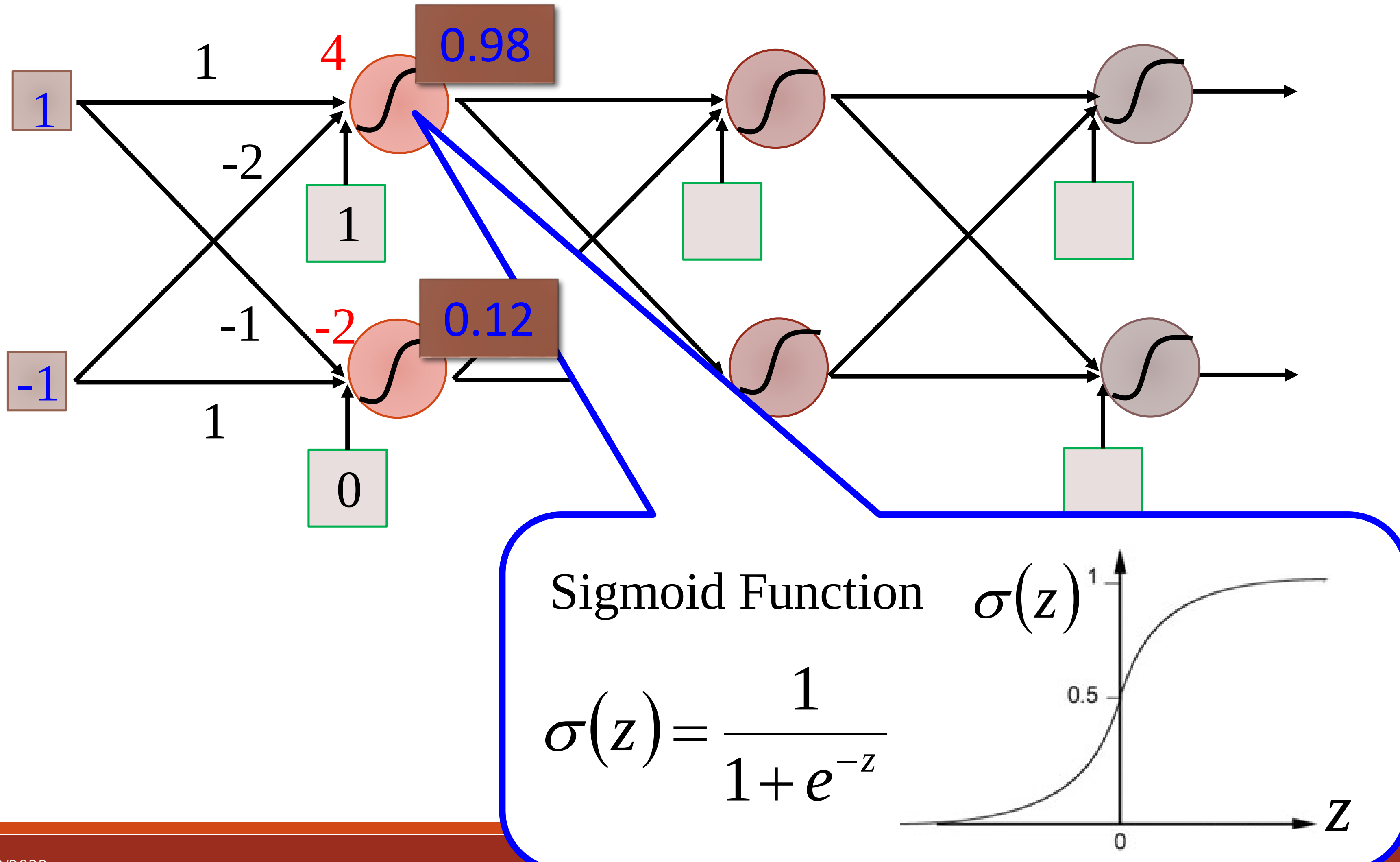
Algorithms for weight adjustment are designed to make changes that will reduce the error

Example of Digit Recognition



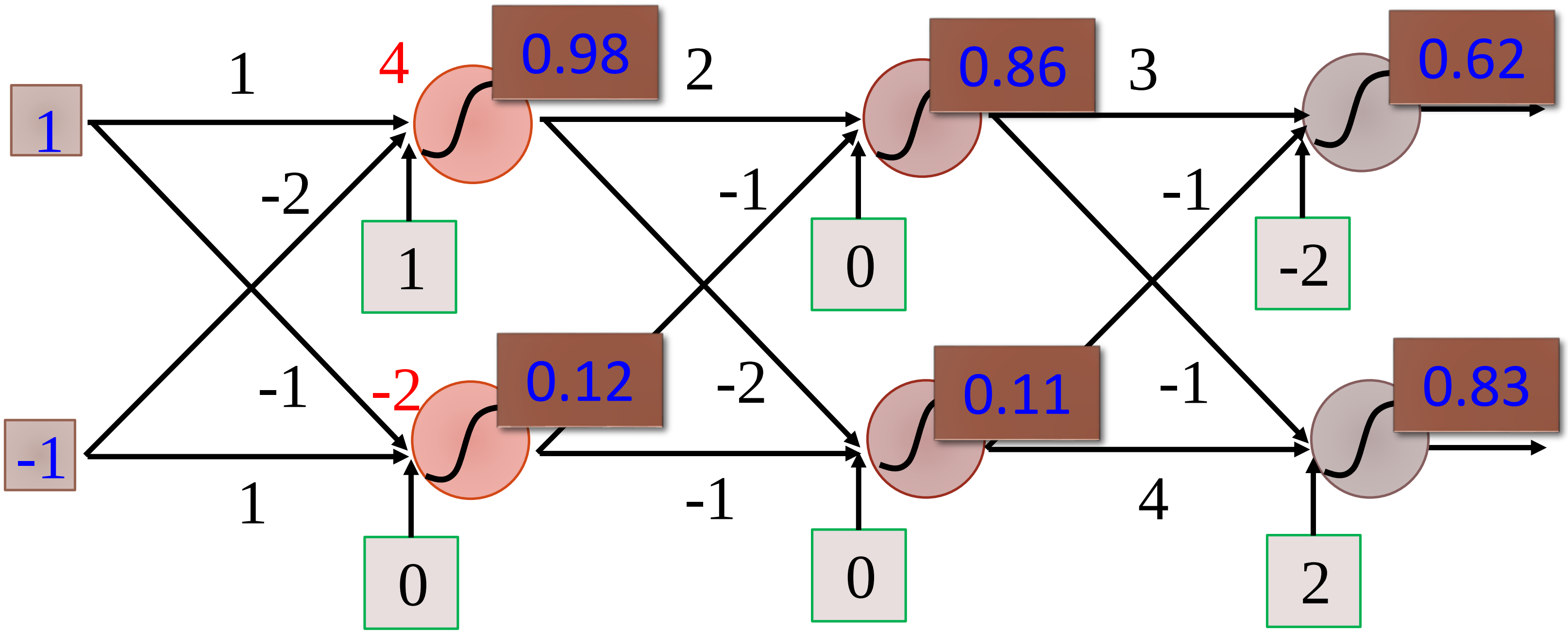


Example of Neural Network

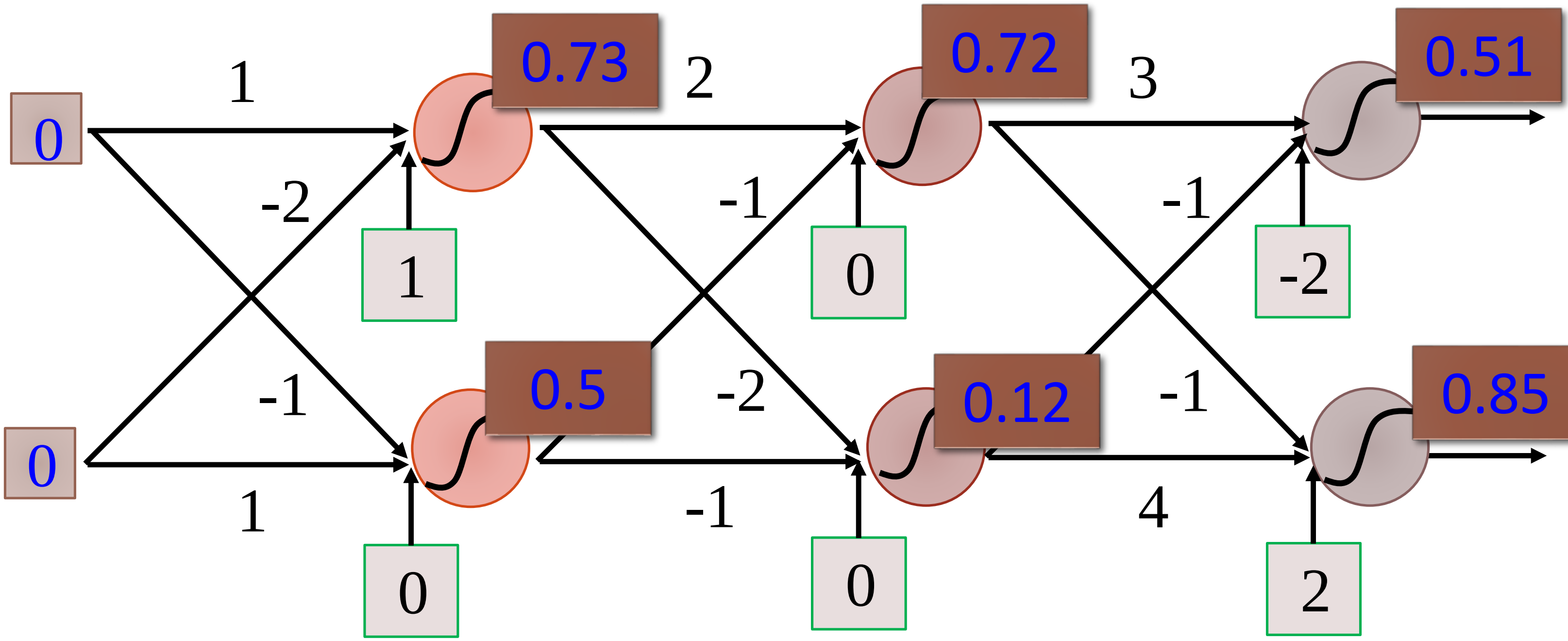


$$\sigma(z) = \frac{1}{1+e^{-z}}$$

Example of Neural Network



Example of Neural Network

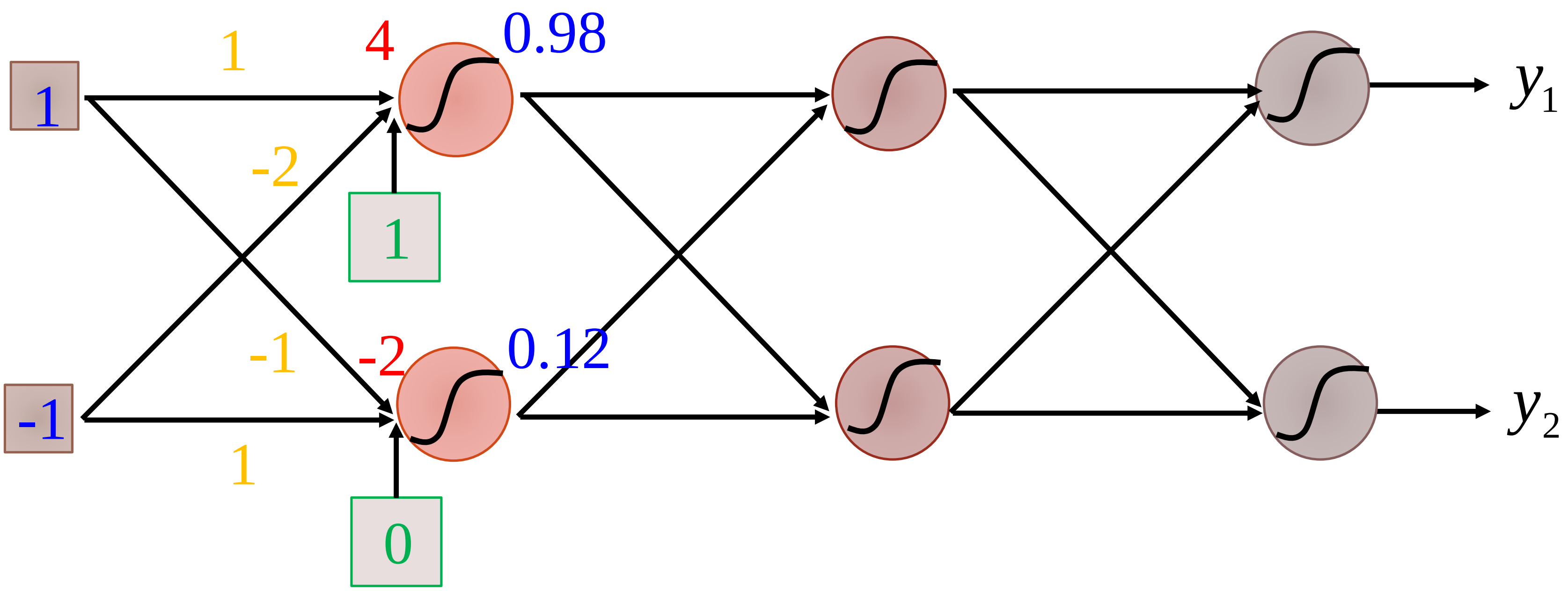


$f: R^2 \rightarrow R^2$

$$f\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 0.62 \\ 0.83 \end{bmatrix} \quad f\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0.51 \\ 0.85 \end{bmatrix}$$

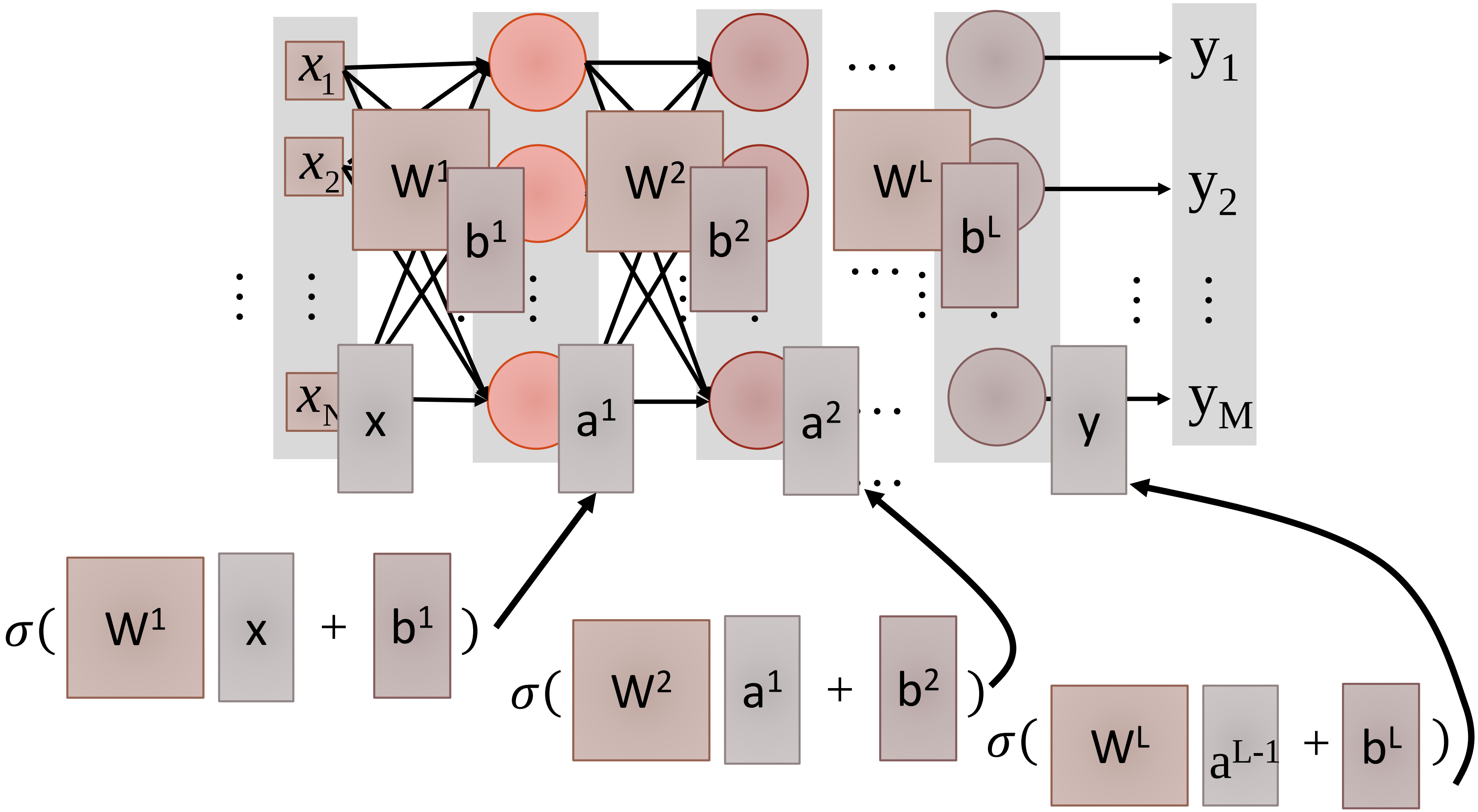
Different parameters define different function

Example of Neural Network

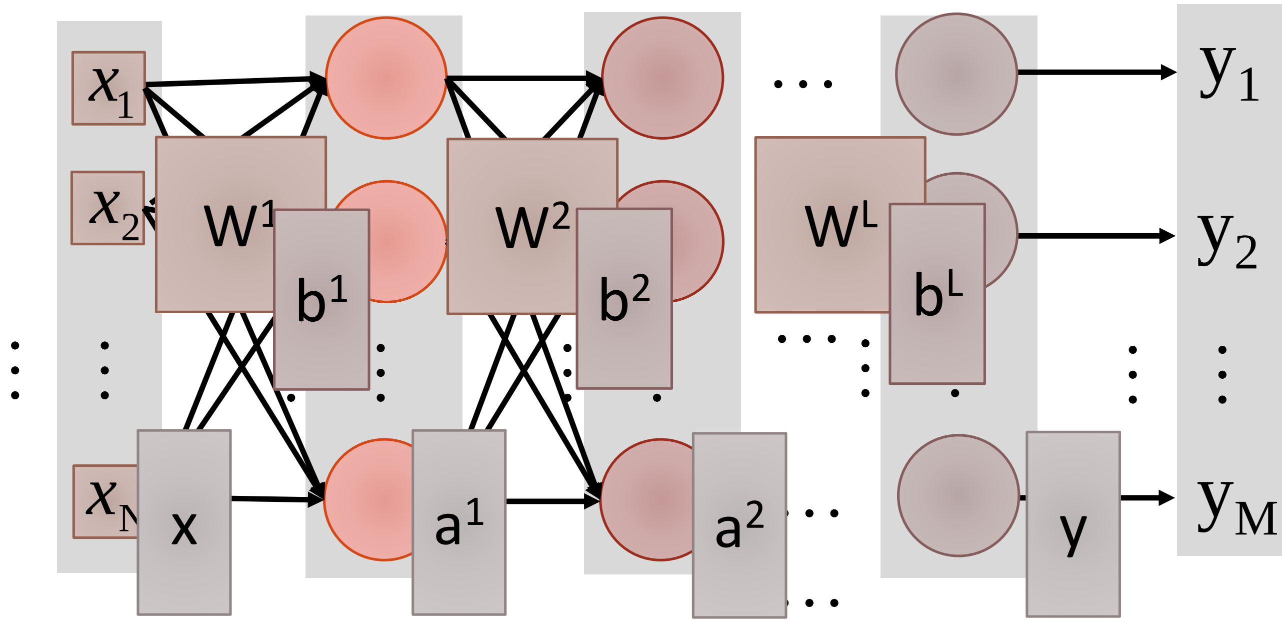


$$\sigma\Big(\underbrace{\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\begin{bmatrix} 4 \\ -2 \end{bmatrix}} \Big) = \begin{bmatrix} 0.98 \\ 0.12 \end{bmatrix}$$

Example of Neural Network



Neural Network



$$\mathbf{y} = f(\mathbf{x})$$

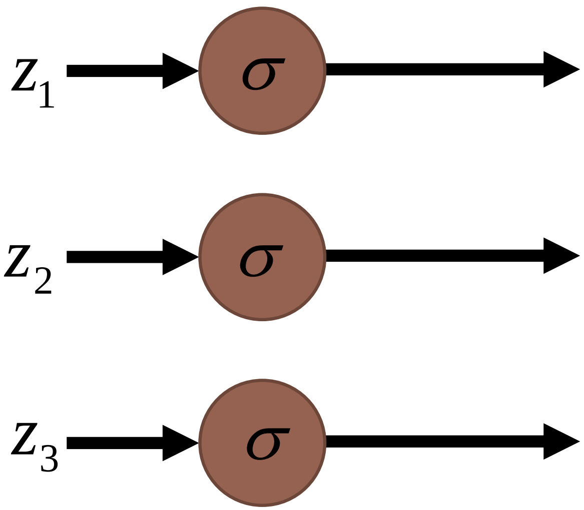
Using parallel computing techniques to speed up matrix operation

$$= \sigma(W^L \cdots \sigma(W^2\, \sigma(W^1\, \mathbf{x} + b^1) + b^2) \cdots + b^L)$$

Softmax

- Softmax layer as the output layer

<u>Ordinary Layer</u>



$$y_1 = \sigma(z_1)$$

$$y_2 = \sigma(z_2)$$

$$y_3 = \sigma(z_3)$$

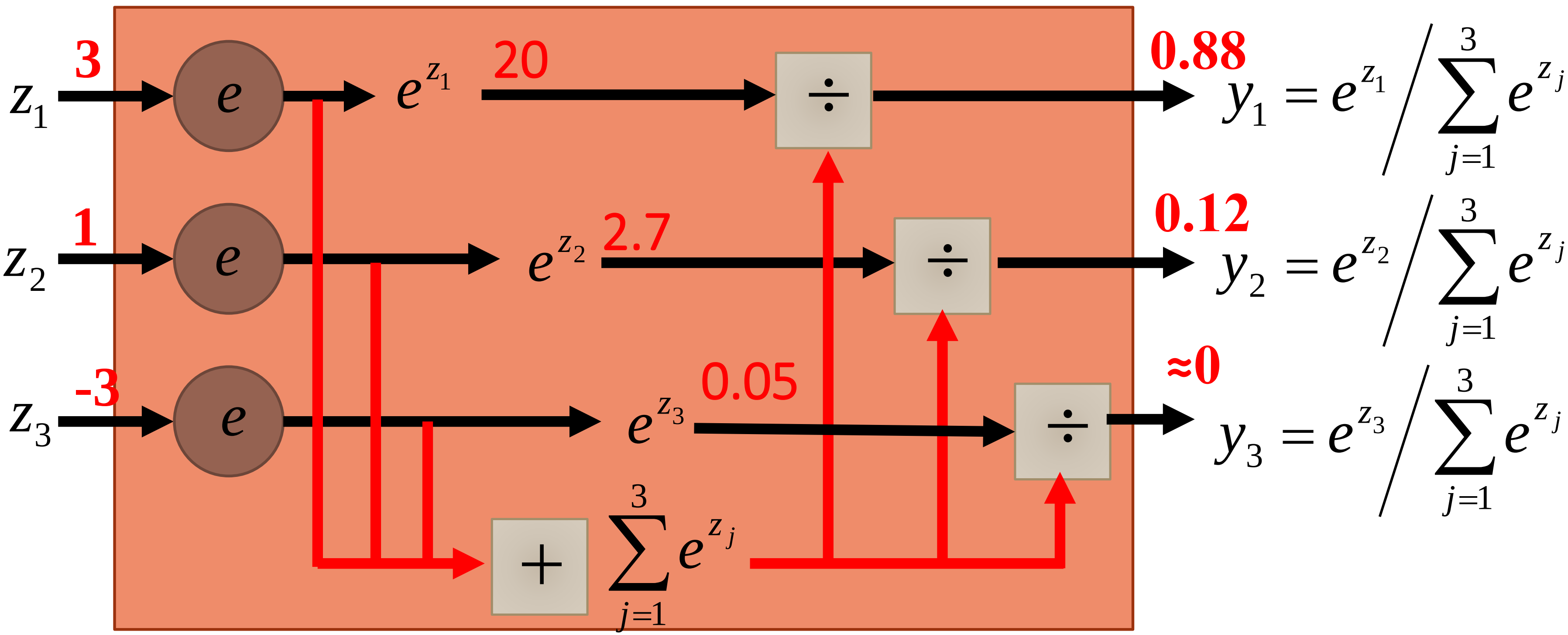
In general, the output of network can be any value.

May not be easy to interpret

Softmax

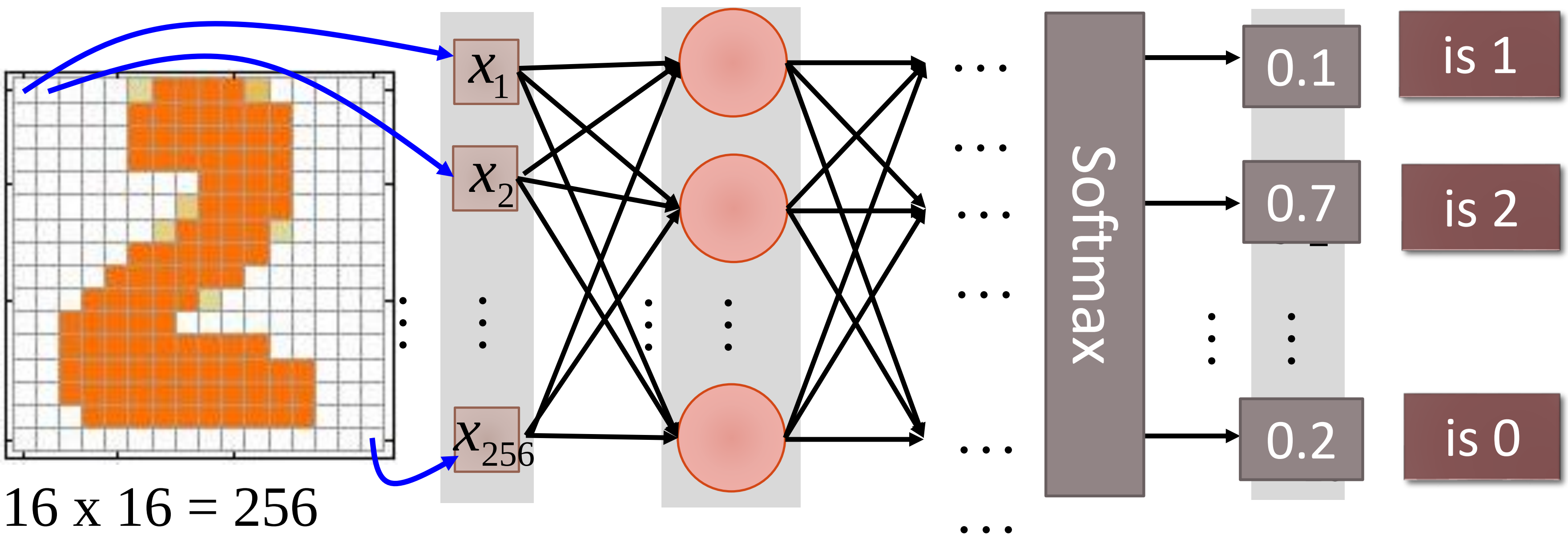
- Softmax layer as the output layer

Probability:
- $1 > y_i > 0$
- $\sum_i y_i = 1$



Network Parameters

$$\theta = \{W^1, b^1, W^2, b^2, \cdots W^L, b^L\}$$

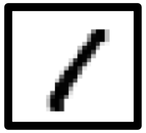


16 x 16 = 256

Ink → 1

No ink → 0

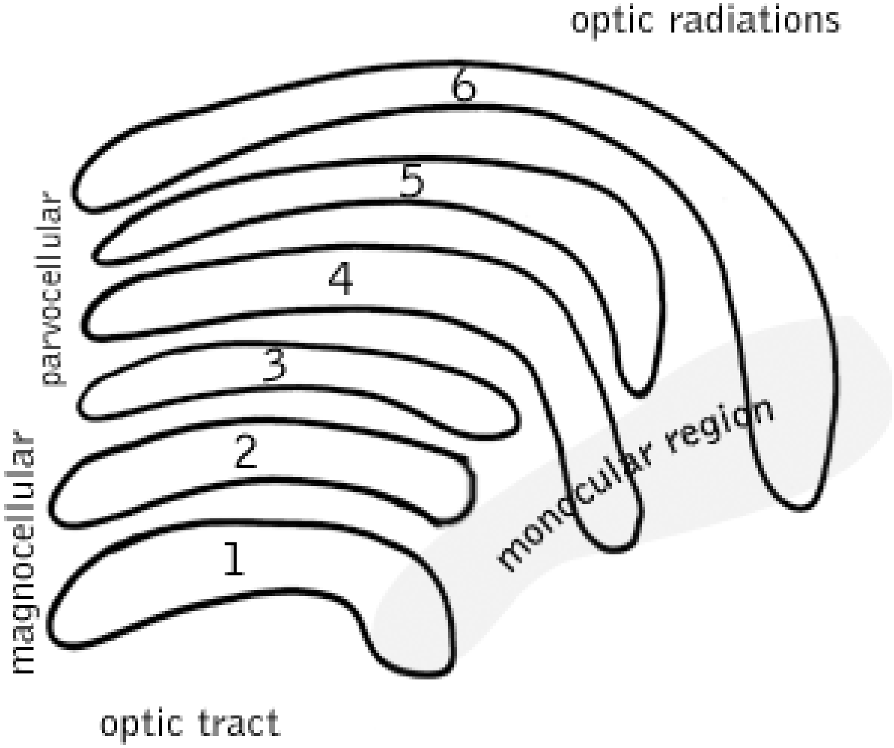
Set the network parameters such that ……

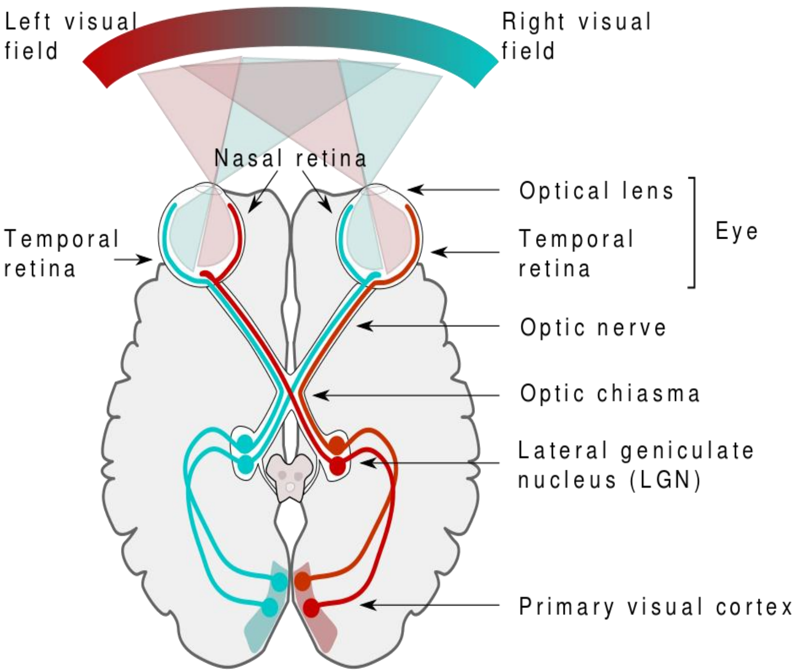
Input:  y_1 has the maximum value

Input:  y_2 has the maximum value

Visual Information Processing

- Visual information processed by our brain is multi-layered.



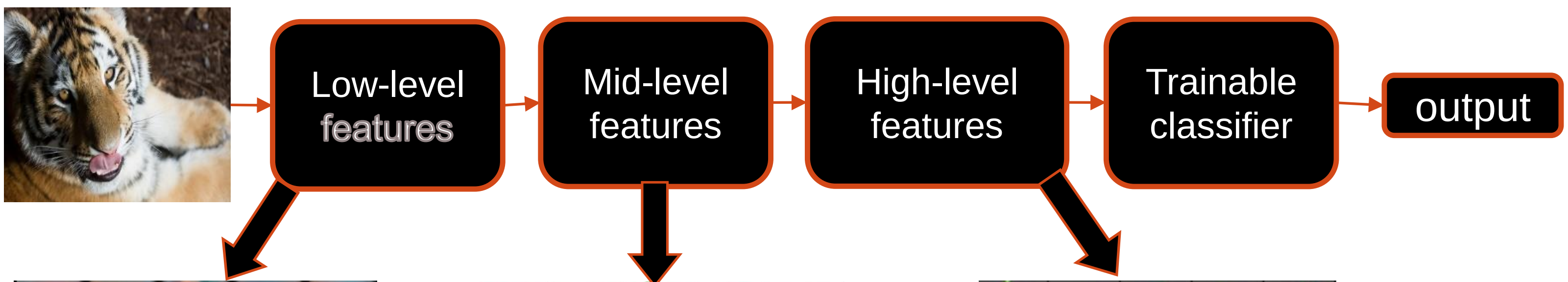


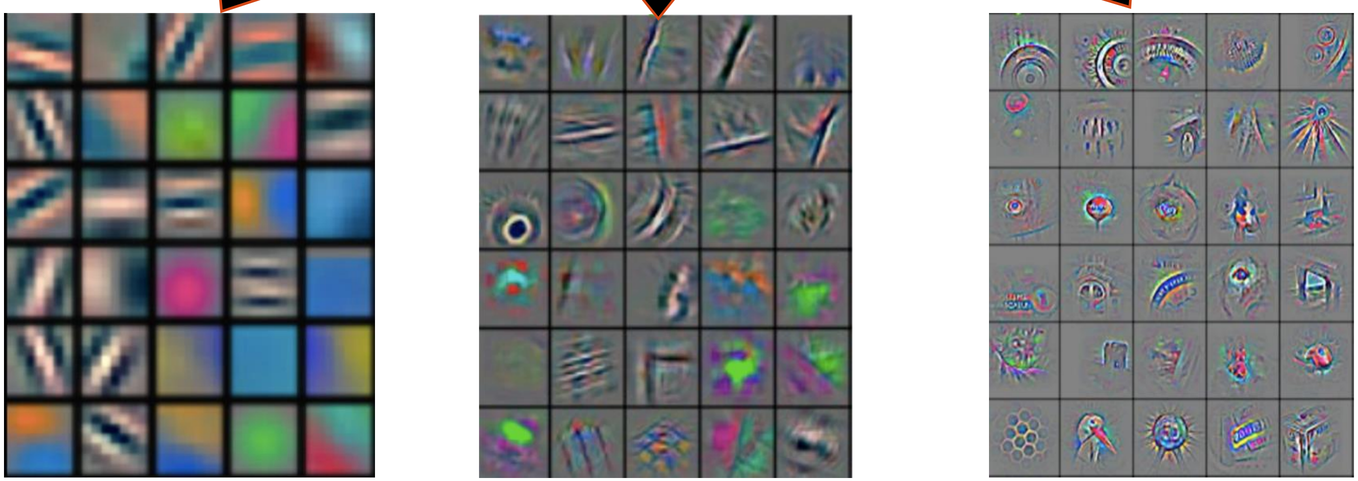
Enabling Factor of DL

- Training of deep networks was made computationally feasible by:
  - Faster CPU's
  - The move to parallel CPU architectures
  - **Advent of GPU computing**
- Neural networks are often represented as a matrix of weight vectors.
- GPU's are optimized for very fast matrix multiplication
- 2008 - Nvidia's CUDA library for GPU computing is released.

Hierarchical Learning

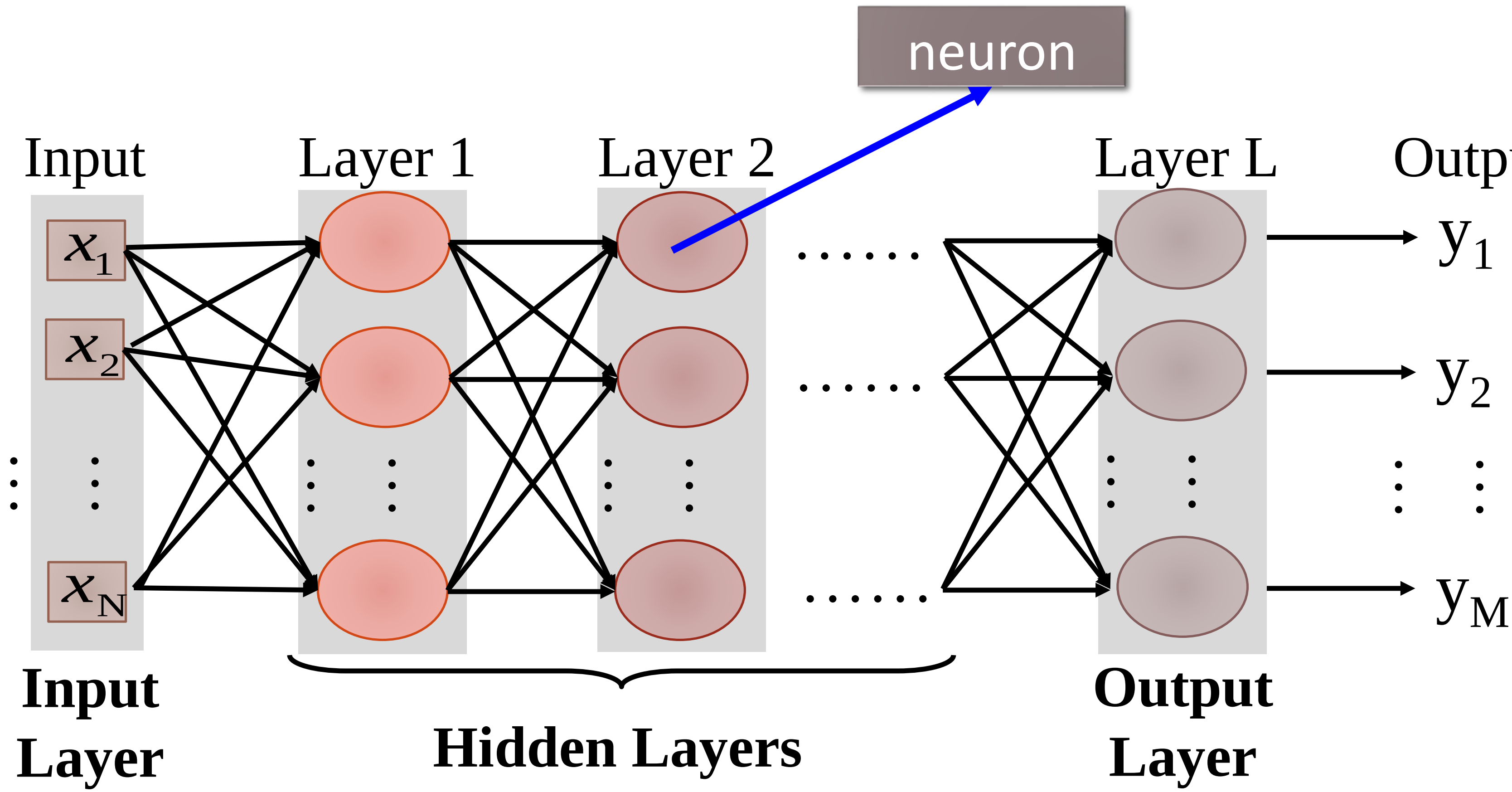
Inspired from visual information processing, a representation of Hierarchical Learning is developed, also know as **“Deep Learning”**





First in 1986 by **Rina Dechter**
Revolution **since 2012**

Deep Neural Network



Deep means many hidden layers

Why Deep Network?

Layer X Size	Word Error Rate (%)
1 X 2k	24.2
2 X 2k	20.4
3 X 2k	18.4
4 X 2k	17.8
5 X 2k	17.2
7 X 2k	17.1

Not surprised, more parameters, better performance

Seide, Frank, Gang Li, and Dong Yu. "Conversational Speech Transcription Using Context-Dependent Deep Neural Networks." *Interspeech*. 2011.

Why Deep Network?

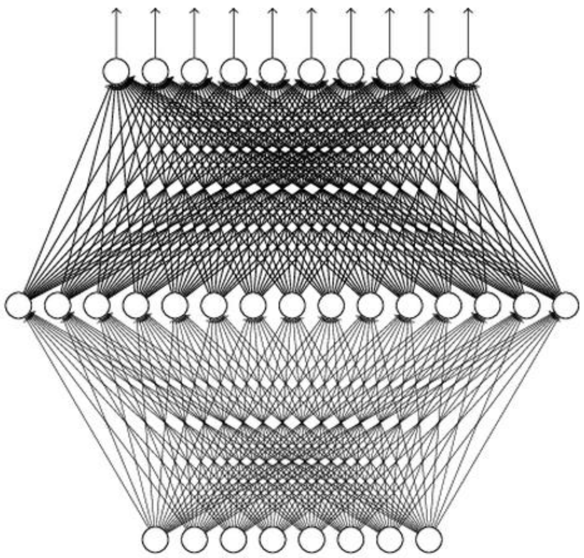
- Universal Theorem

  Any continuous function f

$$f : R^N \to R^M$$

  Can be realized by a network with one hidden layer

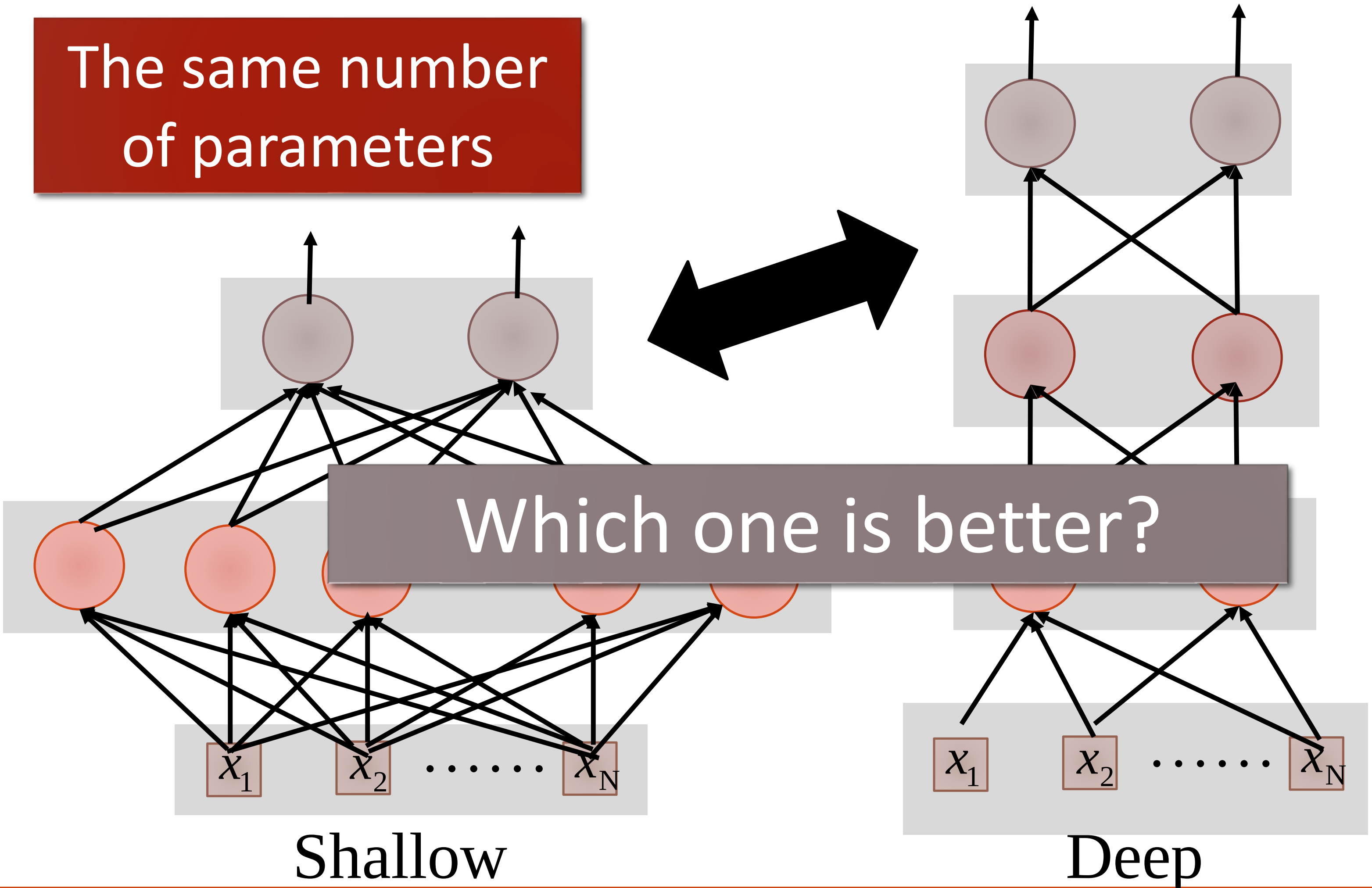
  (given **enough** hidden neurons)



Why "Deep" neural network not "Fat" neural network?

Fat + Short v.s. Thin + Tall

The same number of parameters



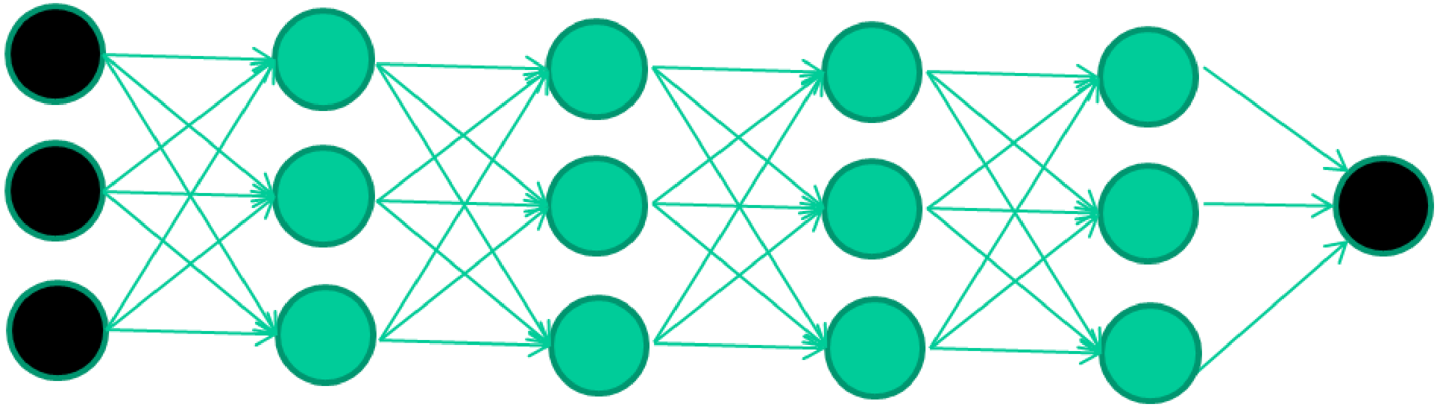
Which one is better?

Fat + Short v.s. Thin + Tall

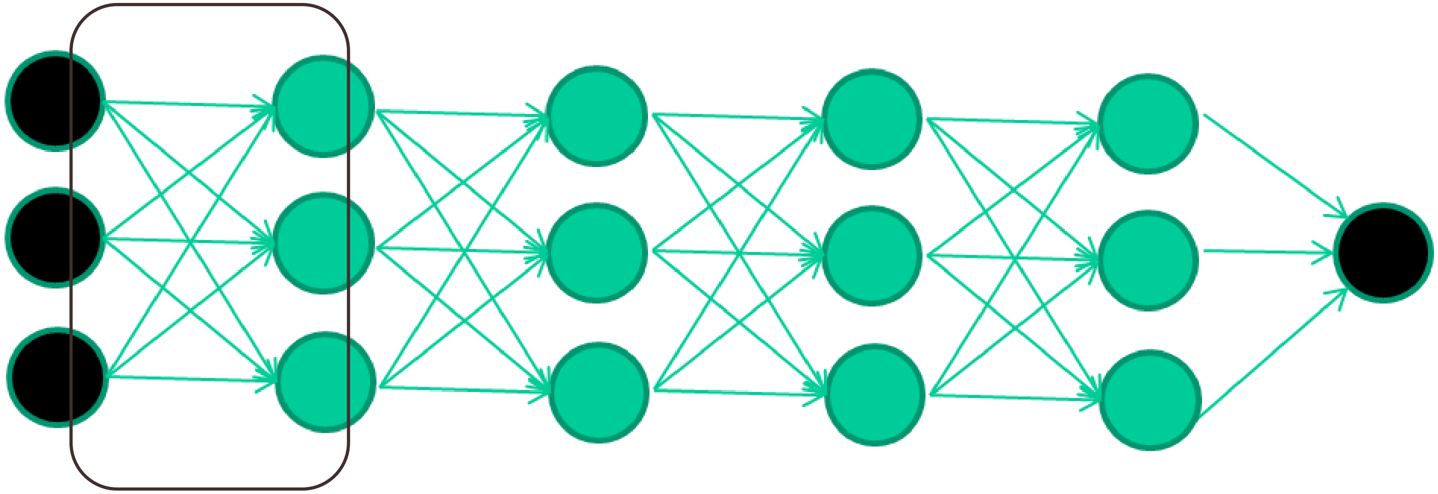
Layer X Size	Word Error Rate (%)	Layer X Size	Word Error Rate (%)
1 X 2k	24.2		
2 X 2k	20.4		
3 X 2k	18.4		
4 X 2k	17.8		
5 X 2k	17.2	1 X 3772	22.5
7 X 2k	17.1	1 X 4634	22.6
		1 X 16k	22.1

Seide, Frank, Gang Li, and Dong Yu. "Conversational Speech Transcription Using Context-Dependent Deep Neural Networks." *Interspeech*. 2011.

Training multi-layer NNs (DNN)

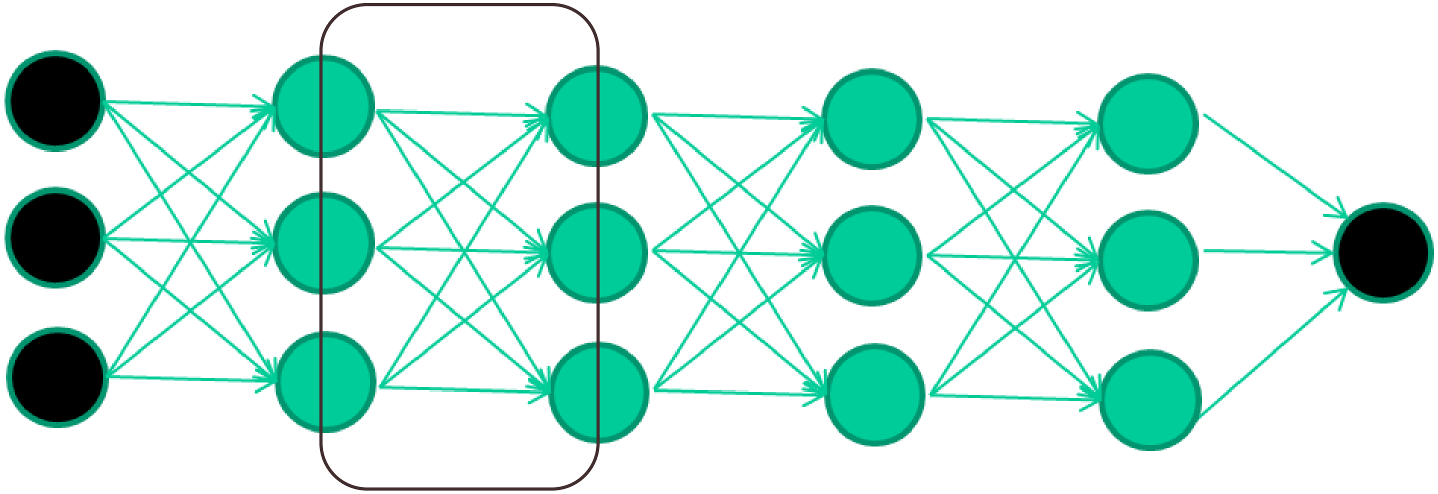


Training multi-layer NNs



Train **this** layer first

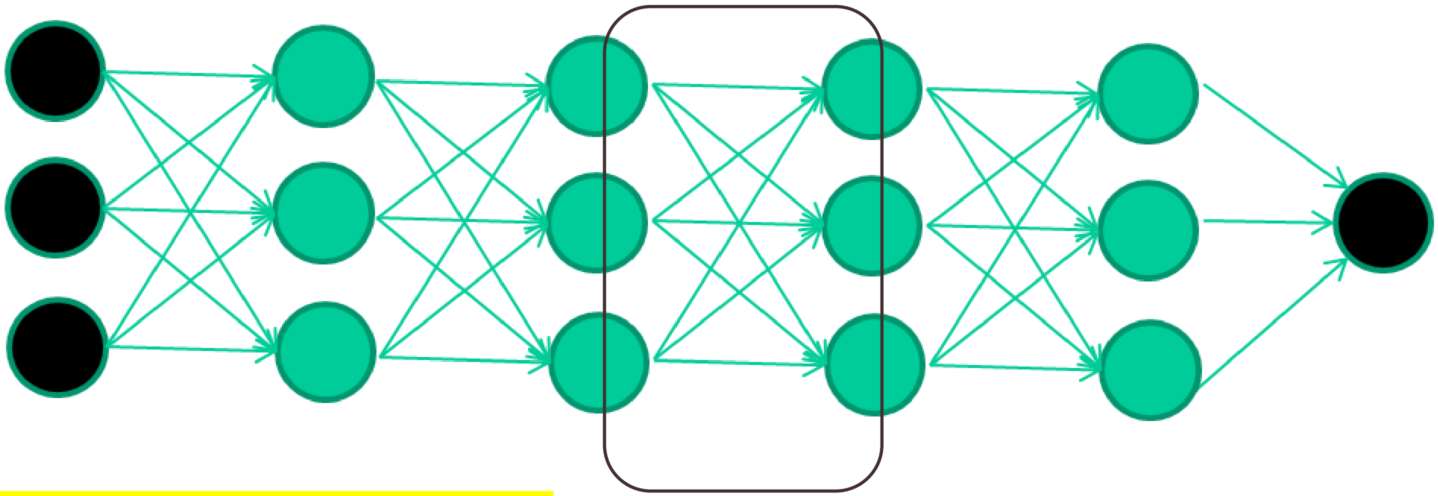
Training multi-layer NNs



Train **this** layer first

then **this** layer

Training multi-layer NNs

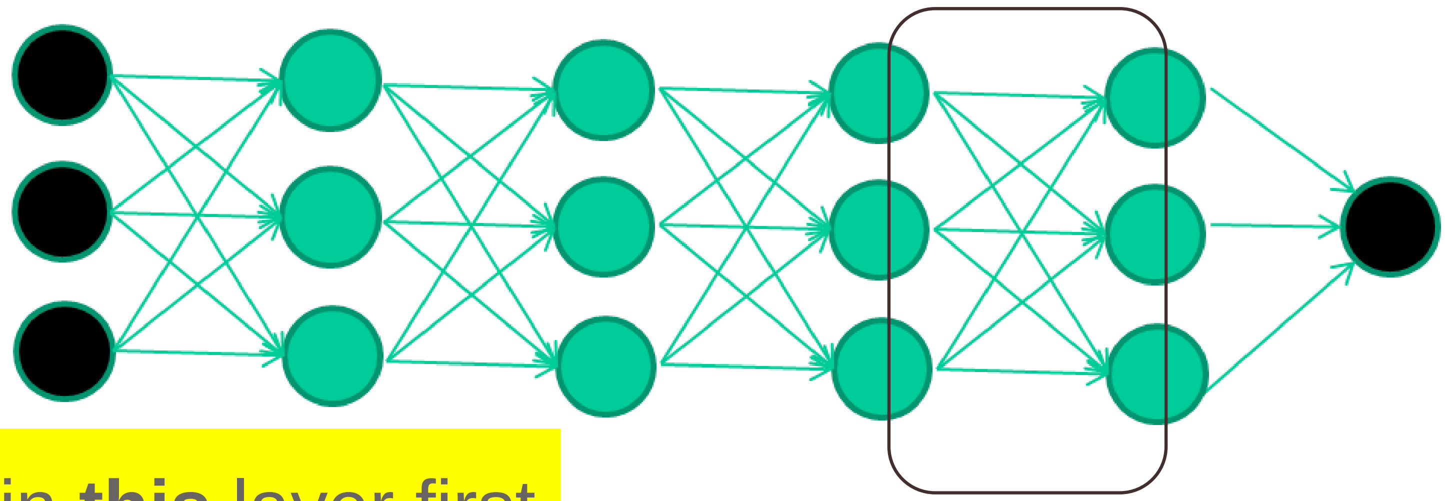


Train **this** layer first

then **this** layer

then **this** layer

Training multi-layer NNs



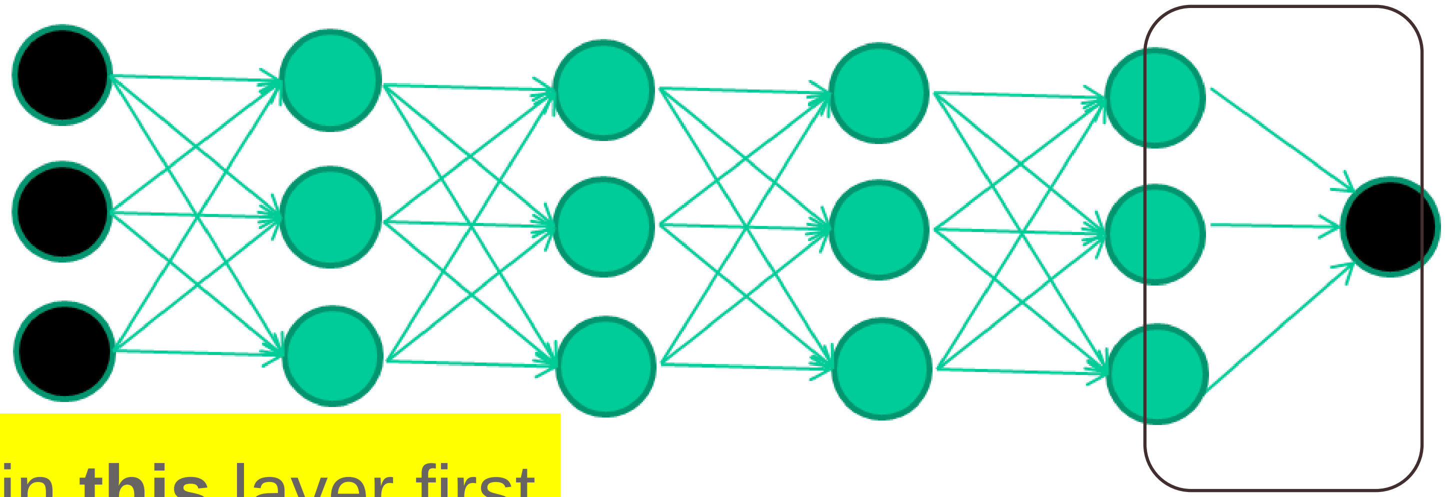
Train **this** layer first

then **this** layer

then **this** layer

then **this** layer

Training multi-layer NNs



Train **this** layer first

then **this** layer

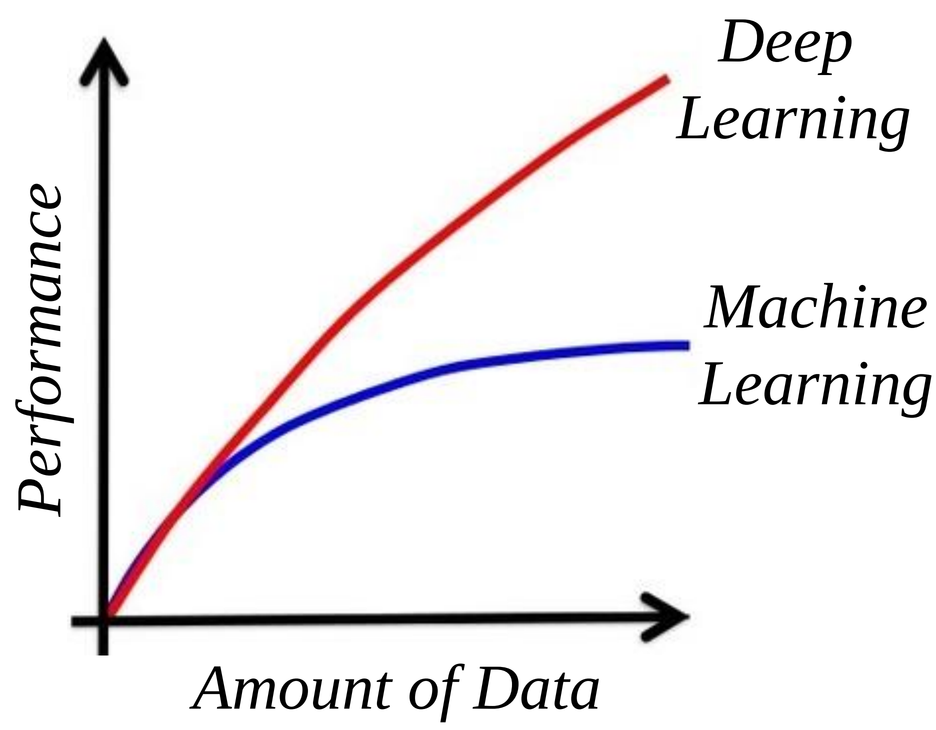
then **this** layer

then **this** layer

finally **this** layer

When to use Deep Learning?

- Data size is large
- High end infrastructure
- Lack of domain understanding
- Complex problem such as image classification, speech recognition etc.



Fuel of deep learning is the big data by Andrew Ng

Limitations of Deep Learning

- Very slow to train
- Models are very complex, with lot of parameters to optimize:
  - ✓Initialization of weights
  - ✓Layer-wise training algorithm
  - ✓Neural architecture
    - Number of layers
    - Size of layers
    - Type – regular, pooling, max pooling, soft max
  - ✓Fine-tuning of weights using back propagation

Thank you!

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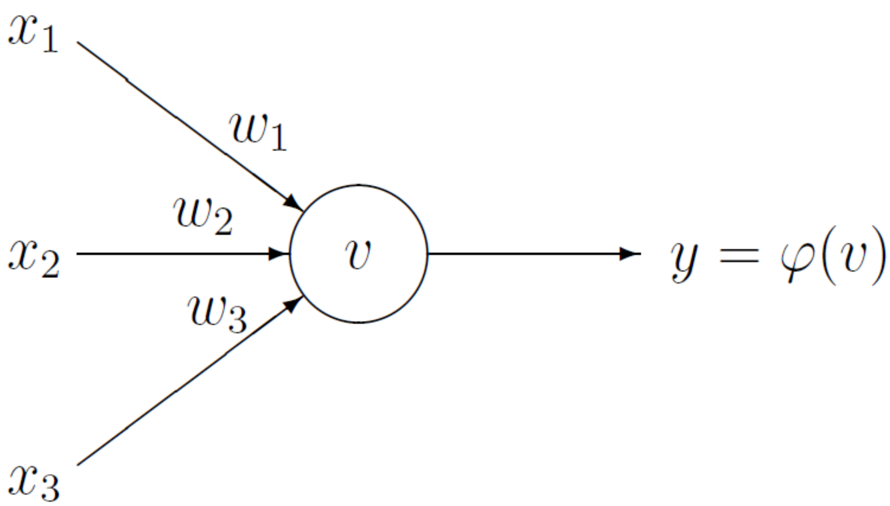
Problems on Neural Networks

Problem 1

- Consider a artificial Neurons, which has three inputs nodes x = (x1, x2, x3) that receive only binary signals (either 0 or 1). How many different input patterns this node can receive? What if the node had four inputs? Five? Can you give a formula that computes the number of binary input patterns for a given number of inputs?



Solutions

- There are three inputs, the number of combinations of 0 and 1 is 8.

x_1	0	1	0	1	0	1	0	1
x_2	0	0	1	1	0	0	1	1
x_3	0	0	0	0	1	1	1	1

- If there are four inputs nodes then number of combinations is 16.

x_1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
x_2	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
x_3	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
x_4	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

- If there are n-input nodes then the number combinations will be 2^n.

Problem 2

- Consider a artificial neurons have three inputs, the weights corresponding to the these inputs have (2, -4, 1), the activation function is unit step. Determine the output for following input values.

Pattern	P_1	P_2	P_3	P_4
x_1	1	0	1	1
x_2	0	1	0	1
x_3	0	1	1	1

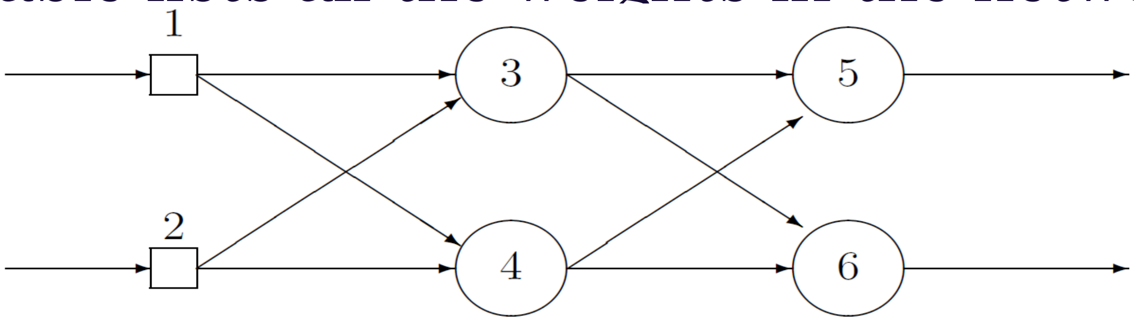
Solutions

- To find the output for each patterns
  - ✓ First calculate the weighted sum $\sum_i w_i x_i = w_1 \cdot x_1 + w_2 \cdot x_2 + w_3 \cdot x_3$
  - ✓ Apply the activation function i.e. unit step $\varphi(v) = \begin{cases} 1 & for\ v \geq 0 \\ 0 & otherwise \end{cases}$
  - ✓ The calculations for each input pattern are:

$$
\begin{aligned}
P_1 &: \quad v = 2 \cdot 1 - 4 \cdot 0 + 1 \cdot 0 = 2\,, \quad (2 > 0)\,, \quad y = \varphi(2) = 1 \\
P_2 &: \quad v = 2 \cdot 0 - 4 \cdot 1 + 1 \cdot 1 = -3\,, \quad (-3 < 0)\,, \quad y = \varphi(-3) = 0 \\
P_3 &: \quad v = 2 \cdot 1 - 4 \cdot 0 + 1 \cdot 1 = 3\,, \quad (3 > 0)\,, \quad y = \varphi(3) = 1 \\
P_4 &: \quad v = 2 \cdot 1 - 4 \cdot 1 + 1 \cdot 1 = -1\,, \quad (-1 < 0)\,, \quad y = \varphi(-1) = 0
\end{aligned}
$$

Problem 3

- Consider a feed forward neural network with one hidden layer. A weight on connection between nodes *i* and *j* is denoted by w_{ij} , such as w_{13} is the weight on the connection between nodes 1 and 3. The following table lists all the weights in the network.



$w_{13} = -2$ $w_{23} = 3$	$w_{35} = 1$ $w_{45} = -1$
$w_{14} = 4$ $w_{24} = -1$	$w_{36} = -1$ $w_{46} = 1$

- Node 3, 4, 5 and 6 uses unit step activation function. Compute the output of the n/w for following inputs.

Pattern:	P_1	P_2	P_3	P_4
Node 1:	0	1	0	1
Node 2:	0	0	1	1

Solutions

- In order to find the output of the network it is necessary to calculate weighted sums of hidden nodes 3 and 4:

$$v_3 = w_{13}x_1 + w_{23}x_2 \ , \quad v_4 = w_{14}x_1 + w_{24}x_2$$

- Then find the outputs from hidden nodes using activation function. $y_3 = \varphi(v_3) \ , \quad y_4 = \varphi(v_4)$
- Use the outputs of the hidden nodes y3 and y4 as the input values to the output layer (nodes 5 and 6), and find weighted sums of output nodes 5 and 6:

$$v_5 = w_{35}y_3 + w_{45}y_4 \ , \quad v_6 = w_{36}y_3 + w_{46}y_4$$

- Finally, compute the outputs from nodes 5 and 6 using

$$y_5 = \varphi(v_5) \ , \quad y_6 = \varphi(v_6)$$

Solutions

P_1: *Input pattern* $(0,0)$

$$\begin{aligned} v_3 &= -2\cdot 0 + 3\cdot 0 = 0, & y_3 &= \varphi(0) = 1 \\ v_4 &= 4\cdot 0 - 1\cdot 0 = 0, & y_4 &= \varphi(0) = 1 \\ v_5 &= 1\cdot 1 - 1\cdot 1 = 0, & y_5 &= \varphi(0) = 1 \\ v_6 &= -1\cdot 1 + 1\cdot 1 = 0, & y_6 &= \varphi(0) = 1 \end{aligned}$$

The output of the network is $(1,1)$.

P_2: *Input pattern* $(1,0)$

$$\begin{aligned} v_3 &= -2\cdot 1 + 3\cdot 0 = -2, & y_3 &= \varphi(-2) = 0 \\ v_4 &= 4\cdot 1 - 1\cdot 0 = 4, & y_4 &= \varphi(4) = 1 \\ v_5 &= 1\cdot 0 - 1\cdot 1 = -1, & y_5 &= \varphi(-1) = 0 \\ v_6 &= -1\cdot 0 + 1\cdot 1 = 1, & y_6 &= \varphi(1) = 1 \end{aligned}$$

The output of the network is $(0,1)$.

Solutions

P_3: *Input pattern* $(0, 1)$

$$
\begin{aligned}
v_3 &= -2 \cdot 0 + 3 \cdot 1 = 3, & y_3 &= \varphi(3) = 1 \\
v_4 &= 4 \cdot 0 - 1 \cdot 1 = -1, & y_4 &= \varphi(-1) = 0 \\
v_5 &= 1 \cdot 1 - 1 \cdot 0 = 1, & y_5 &= \varphi(1) = 1 \\
v_6 &= -1 \cdot 1 + 1 \cdot 0 = -1, & y_6 &= \varphi(-1) = 0
\end{aligned}
$$

The output of the network is $(1, 0)$.

P_4: *Input pattern* $(1, 1)$

$$
\begin{aligned}
v_3 &= -2 \cdot 1 + 3 \cdot 1 = 1, & y_3 &= \varphi(1) = 1 \\
v_4 &= 4 \cdot 1 - 1 \cdot 1 = 3, & y_4 &= \varphi(3) = 1 \\
v_5 &= 1 \cdot 1 - 1 \cdot 1 = 0, & y_5 &= \varphi(0) = 1 \\
v_6 &= -1 \cdot 1 + 1 \cdot 1 = 0, & y_6 &= \varphi(0) = 1
\end{aligned}
$$

The output of the network is $(1, 1)$.